Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.3-Tangent/105-4.3.4.2-a+b-tan-^m-c+d-tan-^n-A+B-tan+C-tan^2-

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [171]. This is test number [105].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	98.83 (169)	1.17 (2)
Rubi	97.66 (167)	2.34 (4)
Maple	71.35 (122)	28.65 (49)
Fricas	66.08 (113)	33.92 (58)
Mupad	60.23 (103)	39.77 (68)
Giac	49.12 (84)	50.88 (87)
Maxima	49.12 (84)	50.88 (87)
Sympy	36.84 (63)	63.16 (108)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

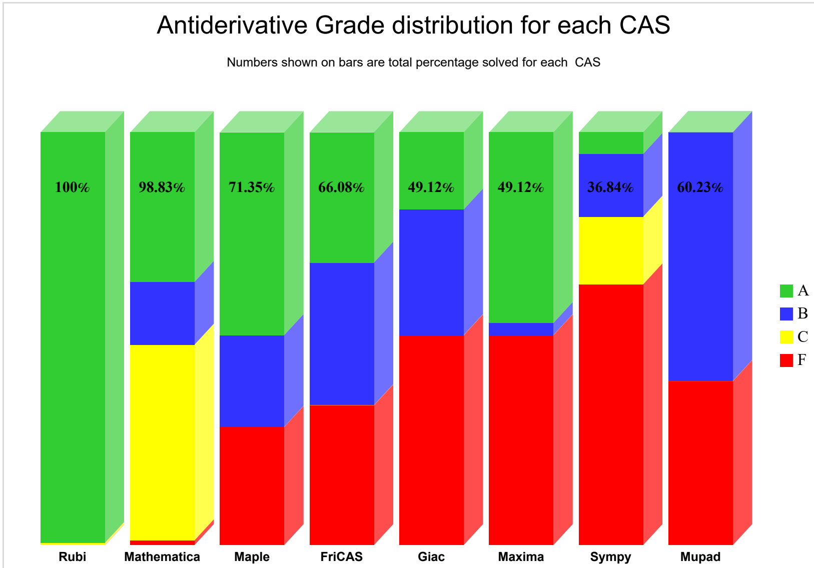
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

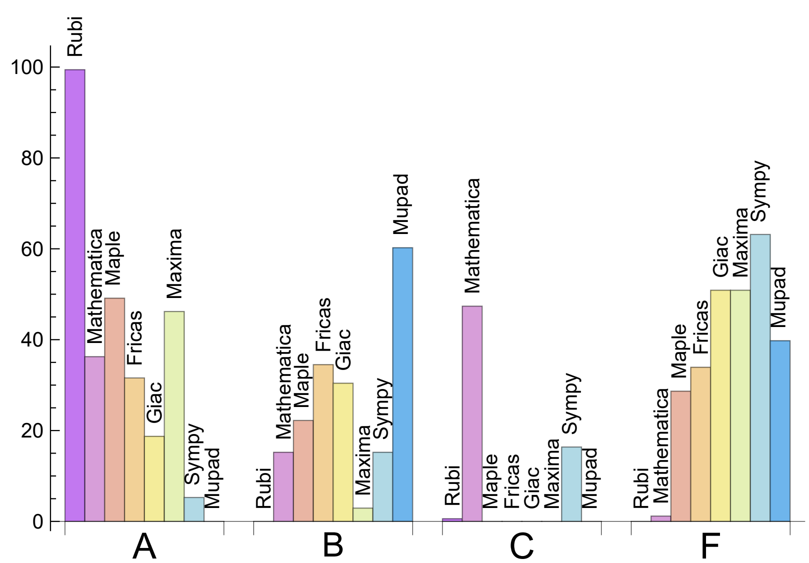
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.076	0.000	0.585	2.339
Maple	49.123	22.222	0.000	28.655
Maxima	46.199	2.924	0.000	50.877
Mathematica	36.257	15.205	47.368	1.170
Fricas	31.579	34.503	0.000	33.918
Giac	18.713	30.409	0.000	50.877
Sympy	5.263	15.205	16.374	63.158
Mupad	0.000	60.234	0.000	39.766

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	2	100.00	0.00	0.00
Rubi	4	100.00	0.00	0.00
Maple	49	26.53	73.47	0.00
Fricas	58	22.41	77.59	0.00
Mupad	68	0.00	100.00	0.00
Giac	87	14.94	85.06	0.00
Maxima	87	33.33	44.83	21.84
Sympy	108	75.00	5.56	19.44

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.36
Maple	0.40
Giac	1.97
Rubi	2.02
Sympy	3.63
Mathematica	4.88
Mupad	21.11
Fricas	26.31

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	335.57	1.06	302.00	1.04
Maxima	375.69	1.35	217.50	1.20
Mathematica	808.14	1.92	290.00	1.26
Giac	1752.95	5.51	492.00	2.30
Sympy	3297.92	13.67	711.00	2.70
Maple	3979.42	11.03	347.00	1.22
Mupad	13887.19	40.23	307.00	1.38
Fricas	15738.06	48.61	505.00	2.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

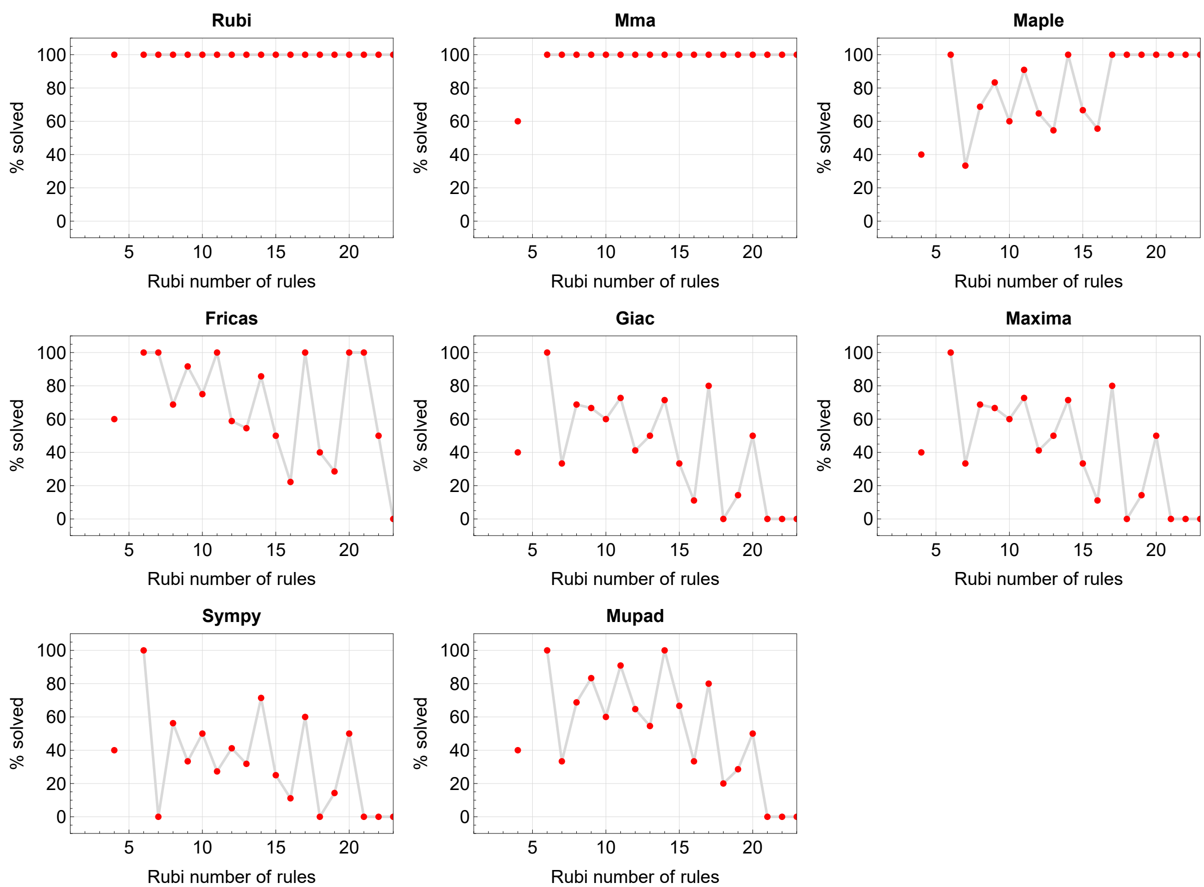


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

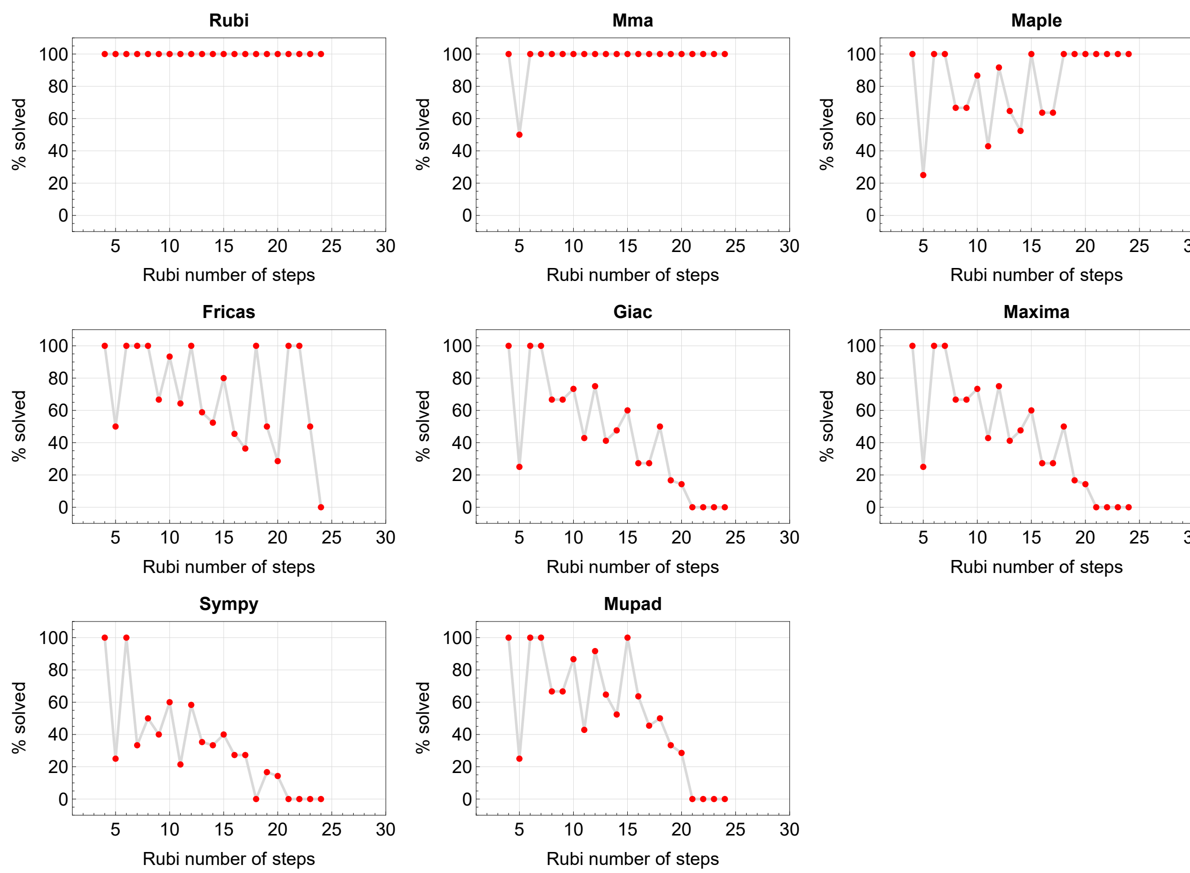


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

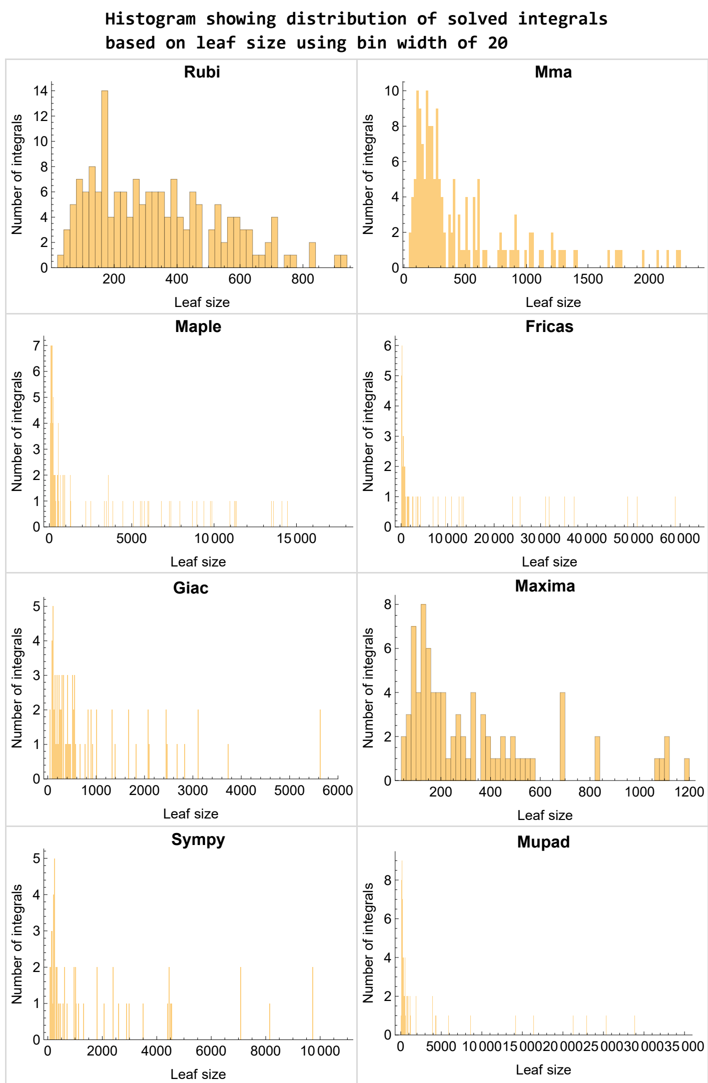


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

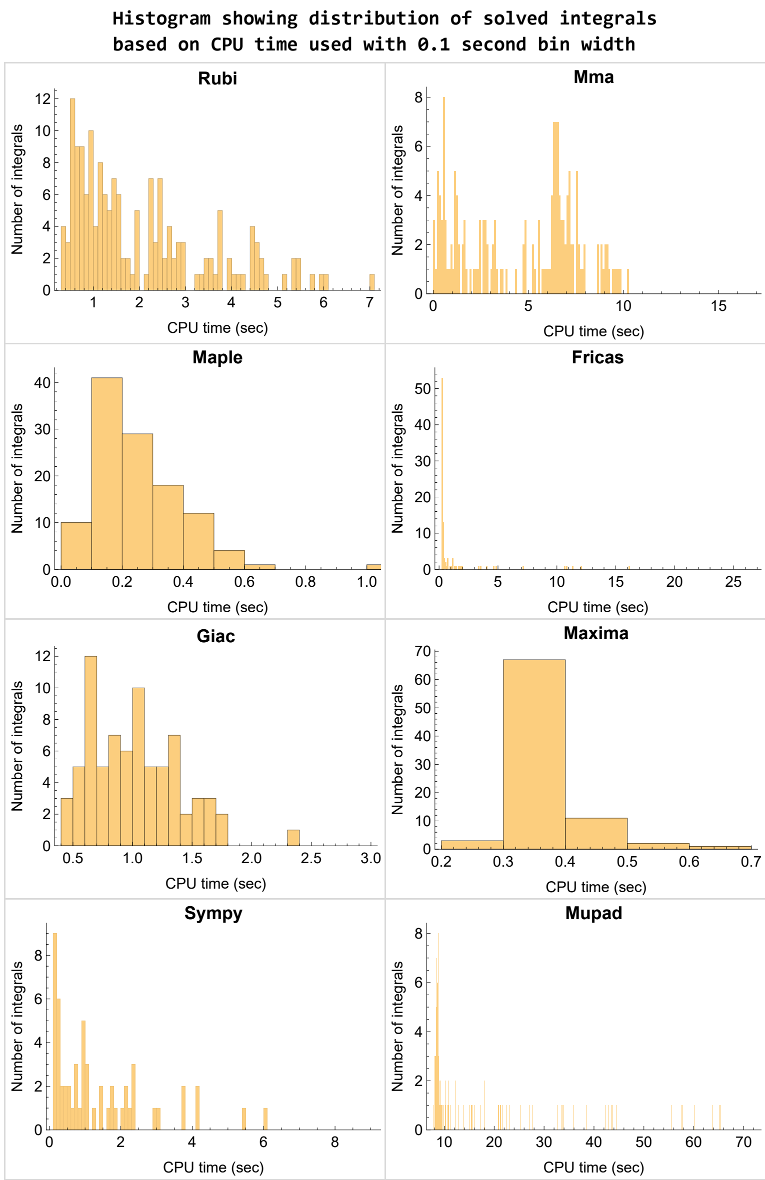


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

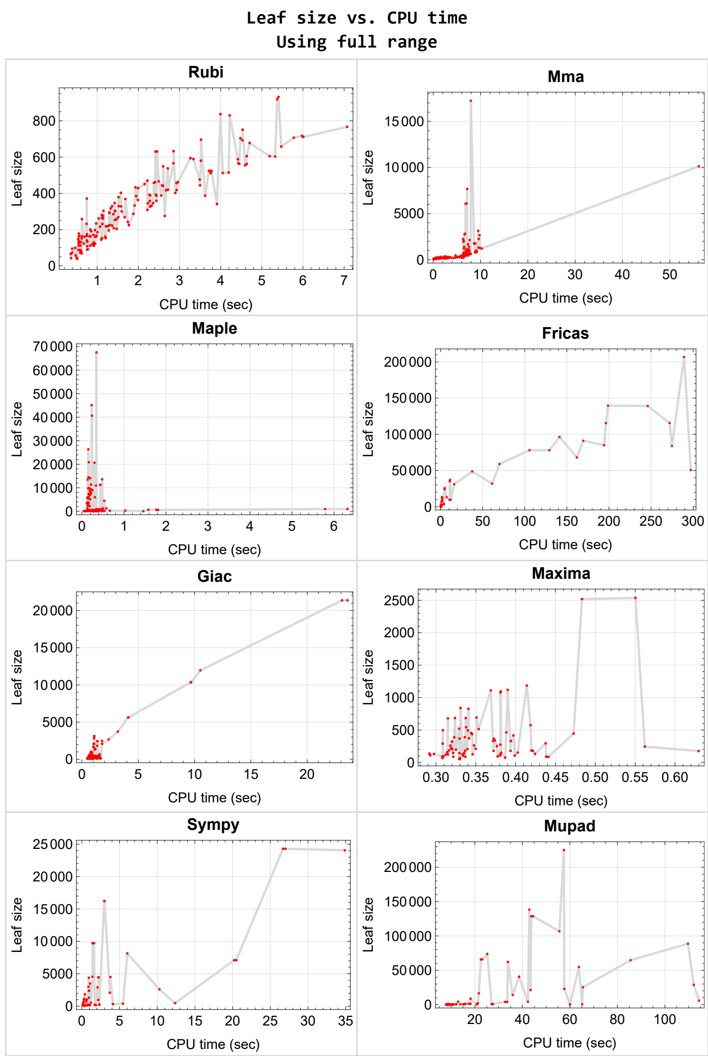


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127}

Mathematica {146}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

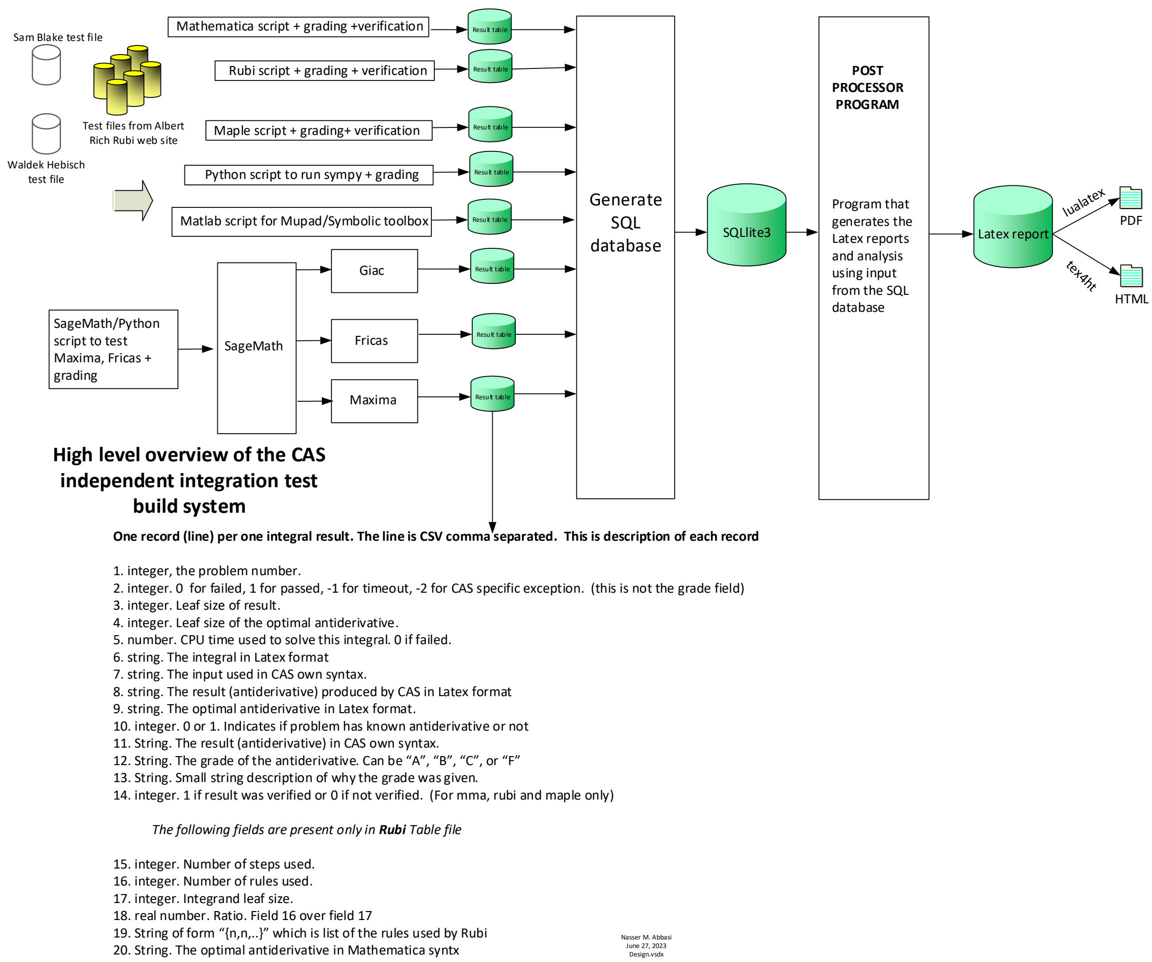
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170 }

B grade { }

C grade { 49 }

F normal fail { 108, 109, 146, 171 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 28, 45, 46, 47, 48, 53, 74, 76, 82, 88, 91, 92, 93, 94, 98, 99, 100, 101, 104, 105, 106, 107, 111, 112, 113, 114, 115, 120, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 161, 162, 163, 166, 167, 168, 169, 170 }

B grade { 75, 81, 83, 89, 90, 95, 96, 97, 102, 103, 108, 109, 110, 121, 126, 127, 138, 140, 143, 146, 153, 154, 159, 160, 165, 171 }

C grade { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 80, 84, 85, 86, 87, 116, 117, 118, 119, 122, 123, 124, 125, 139, 144, 145 }

F normal fail { 49, 164 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

B grade { 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 164, 165, 166, 167, 168, 169, 170, 171 }

F(-1) timedout fail { 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 64, 65, 66, 67, 70, 71, 72, 73, 74, 79, 80 }

B grade { 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 55, 56, 62, 63, 68, 69, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 104, 105, 106, 110, 111, 112, 113, 118, 119, 124, 125, 130, 131, 132, 137, 138, 148, 149, 150, 151, 155 }

C grade { }

F normal fail { 45, 46, 47, 48, 49, 164, 165, 166, 167, 168, 169, 170, 171 }

F(-1) timedout fail { 94, 95, 96, 101, 102, 103, 107, 108, 109, 114, 115, 116, 117, 120, 121, 122, 123, 126, 127, 128, 129, 133, 134, 135, 136, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 84, 85, 86, 87 }

B grade { 76, 82, 83, 88, 89 }

C grade { }

F normal fail { 45, 46, 49, 91, 92, 93, 100, 106, 112, 113, 128, 129, 130, 131, 135, 136, 137, 141, 148, 149, 150, 151, 156, 164, 166, 167, 168, 169, 170 }

F(-1) timedout fail { 47, 48, 90, 97, 98, 99, 104, 105, 110, 111, 116, 117, 118, 119, 122, 123, 124, 125, 132, 134, 138, 139, 142, 143, 144, 145, 147, 152, 153, 154, 155, 157, 158, 159, 160, 162, 163, 165, 171 }

F(-2) exception fail { 94, 95, 96, 101, 102, 103, 107, 108, 109, 114, 115, 120, 121, 126, 127, 133, 140, 146, 161 }

2.1.6 Giac

A grade { 3, 4, 11, 12, 13, 18, 19, 20, 21, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 44, 54, 55, 61, 67, 70, 71, 72, 73, 74, 79 }

B grade { 1, 2, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 22, 23, 24, 34, 35, 36, 40, 41, 42, 43, 50, 51, 52, 53, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 68, 69, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

C grade { }

F normal fail { 45, 46, 47, 126, 130, 164, 165, 166, 167, 168, 169, 170, 171 }

F(-1) timedout fail { 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 100, 101, 106, 110, 111, 112, 113, 114, 115, 117, 118, 119, 123, 124, 125 }

C grade { }

F normal fail { }

F(-1) timedout fail { 45, 46, 47, 48, 49, 90, 91, 96, 97, 98, 99, 102, 103, 104, 105, 107, 108, 109, 116, 120, 121, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 9, 10, 11, 18, 19, 20, 24 }

B grade { 3, 4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 21, 22, 23, 50, 51, 52, 53, 57, 58, 59, 60, 64, 65, 66 }

C grade { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 54, 55, 61, 62, 67, 68, 70, 71, 72, 73, 74, 77, 78, 79, 80 }

F normal fail { 45, 46, 47, 48, 49, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 171 }

F(-1) timedout fail { 83, 89, 107, 108, 109, 146 }

F(-2) exception fail { 38, 39, 40, 41, 42, 43, 44, 56, 63, 69, 75, 76, 81, 82, 84, 85, 86, 87, 88, 164, 170 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	88	86	85	139	937	84
N.S.	1	1.00	0.99	1.01	0.99	0.98	1.60	10.77	0.97
time (sec)	N/A	0.590	0.647	0.279	0.337	0.245	0.114	0.908	8.205

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	69	66	66	105	556	63
N.S.	1	1.00	1.02	1.05	1.00	1.00	1.59	8.42	0.95
time (sec)	N/A	0.359	0.329	0.046	0.308	0.244	0.103	0.652	7.970

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	59	46	50	50	82	50	58
N.S.	1	1.00	1.40	1.10	1.19	1.19	1.95	1.19	1.38
time (sec)	N/A	0.355	0.075	1.460	0.329	0.247	0.294	0.726	7.988

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	39	49	43	52	59	85	53	69
N.S.	1	1.05	1.32	1.16	1.41	1.59	2.30	1.43	1.86
time (sec)	N/A	0.510	0.064	0.286	0.330	0.257	0.412	0.902	8.309

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	45	88	53	68	73	116	119	87
N.S.	1	1.05	2.05	1.23	1.58	1.70	2.70	2.77	2.02
time (sec)	N/A	0.493	0.033	0.286	0.330	0.260	0.719	1.067	8.489

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	68	77	77	86	95	143	179	108
N.S.	1	1.03	1.17	1.17	1.30	1.44	2.17	2.71	1.64
time (sec)	N/A	0.618	0.507	0.339	0.322	0.242	1.072	1.308	7.977

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	89	101	95	104	121	173	237	127
N.S.	1	1.02	1.16	1.09	1.20	1.39	1.99	2.72	1.46
time (sec)	N/A	0.764	1.081	0.355	0.334	0.247	1.875	1.609	7.863

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	110	100	108	122	138	204	299	145
N.S.	1	1.02	0.93	1.00	1.13	1.28	1.89	2.77	1.34
time (sec)	N/A	0.909	1.255	0.348	0.315	0.254	2.321	1.337	8.637

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	160	221	148	147	146	250	2078	151
N.S.	1	1.08	1.49	1.00	0.99	0.99	1.69	14.04	1.02
time (sec)	N/A	0.934	6.256	0.096	0.321	0.273	0.155	1.794	8.465

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	172	120	120	119	194	1389	121
N.S.	1	1.00	1.54	1.07	1.07	1.06	1.73	12.40	1.08
time (sec)	N/A	0.538	1.961	0.063	0.312	0.255	0.130	1.192	8.432

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	96	87	91	91	151	95	91
N.S.	1	1.00	1.10	1.00	1.05	1.05	1.74	1.09	1.05
time (sec)	N/A	0.528	0.522	0.253	0.308	0.252	0.459	1.006	8.301

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	72	91	80	85	92	136	86	90
N.S.	1	1.03	1.30	1.14	1.21	1.31	1.94	1.23	1.29
time (sec)	N/A	0.577	0.311	0.224	0.441	0.282	0.750	1.336	8.589

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	100	84	93	112	158	118	100
N.S.	1	1.03	1.39	1.17	1.29	1.56	2.19	1.64	1.39
time (sec)	N/A	0.600	0.299	0.350	0.377	0.254	1.010	1.674	8.563

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	91	123	107	120	122	206	237	127
N.S.	1	1.03	1.40	1.22	1.36	1.39	2.34	2.69	1.44
time (sec)	N/A	0.763	0.384	0.385	0.380	0.255	1.760	0.868	8.697

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	121	152	136	149	157	252	334	156
N.S.	1	1.03	1.29	1.15	1.26	1.33	2.14	2.83	1.32
time (sec)	N/A	0.926	1.266	0.404	0.382	0.260	2.315	0.908	8.740

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	152	180	162	175	191	304	435	182
N.S.	1	1.01	1.19	1.07	1.16	1.26	2.01	2.88	1.21
time (sec)	N/A	1.116	3.147	0.464	0.630	0.252	4.123	0.978	8.709

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	209	180	179	178	313	2670	181
N.S.	1	1.00	1.27	1.09	1.08	1.08	1.90	16.18	1.10
time (sec)	N/A	0.744	1.765	0.109	0.422	0.265	0.166	2.372	8.544

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	130	139	143	142	248	158	142
N.S.	1	1.00	0.93	0.99	1.02	1.01	1.77	1.13	1.01
time (sec)	N/A	0.732	1.128	0.283	0.329	0.253	0.767	1.527	8.716

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	119	113	121	124	133	211	129	118
N.S.	1	1.02	0.97	1.03	1.06	1.14	1.80	1.10	1.01
time (sec)	N/A	0.864	0.504	0.265	0.333	0.274	0.976	1.621	9.325

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	121	113	118	125	145	214	152	114
N.S.	1	1.02	0.95	0.99	1.05	1.22	1.80	1.28	0.96
time (sec)	N/A	0.896	0.518	0.256	0.346	0.266	1.701	1.220	8.777

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	129	126	136	142	162	253	193	135
N.S.	1	1.02	0.99	1.07	1.12	1.28	1.99	1.52	1.06
time (sec)	N/A	0.910	0.487	0.243	0.335	0.276	2.326	1.285	8.766

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	161	164	172	180	181	323	390	169
N.S.	1	1.05	1.06	1.12	1.17	1.18	2.10	2.53	1.10
time (sec)	N/A	1.156	1.337	0.311	0.336	0.254	4.109	1.431	8.509

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	204	199	209	215	225	391	528	204
N.S.	1	1.07	1.04	1.09	1.13	1.18	2.05	2.76	1.07
time (sec)	N/A	1.438	0.807	0.314	0.320	0.259	5.461	1.477	8.472

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	242	237	243	250	266	462	670	238
N.S.	1	1.04	1.02	1.04	1.07	1.14	1.98	2.88	1.02
time (sec)	N/A	1.716	1.245	0.357	0.376	0.258	12.364	1.573	8.427

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	144	138	127	130	190	1306	135	144
N.S.	1	1.13	1.09	1.00	1.02	1.50	10.28	1.06	1.13
time (sec)	N/A	1.076	1.577	0.122	0.297	0.271	0.928	0.655	8.201

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	108	118	101	109	149	1020	110	117
N.S.	1	1.07	1.17	1.00	1.08	1.48	10.10	1.09	1.16
time (sec)	N/A	0.715	0.664	0.132	0.293	0.281	0.666	0.487	8.249

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	98	87	94	110	711	95	100
N.S.	1	1.12	1.15	1.02	1.11	1.29	8.36	1.12	1.18
time (sec)	N/A	0.383	0.205	0.078	0.309	0.264	0.563	0.499	8.910

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	66	88	76	541	94	93
N.S.	1	1.00	1.16	1.14	1.52	1.31	9.33	1.62	1.60
time (sec)	N/A	0.456	0.157	0.255	0.438	0.258	1.203	0.668	9.091

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	82	113	95	107	118	966	113	115
N.S.	1	1.02	1.41	1.19	1.34	1.48	12.08	1.41	1.44
time (sec)	N/A	0.592	0.369	0.282	0.381	0.271	2.169	0.802	9.197

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	111	138	122	131	177	2067	157	140
N.S.	1	1.08	1.34	1.18	1.27	1.72	20.07	1.52	1.36
time (sec)	N/A	0.812	0.981	0.316	0.424	0.267	3.717	1.075	9.882

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	153	163	152	158	234	2596	214	175
N.S.	1	1.12	1.19	1.11	1.15	1.71	18.95	1.56	1.28
time (sec)	N/A	1.202	1.517	0.350	0.314	0.280	10.291	1.355	10.245

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	225	193	172	220	434	4541	290	210
N.S.	1	1.08	0.93	0.83	1.06	2.09	21.83	1.39	1.01
time (sec)	N/A	1.292	6.086	0.191	0.319	0.310	1.408	0.688	9.236

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	171	146	155	197	311	3497	244	165
N.S.	1	1.09	0.93	0.99	1.25	1.98	22.27	1.55	1.05
time (sec)	N/A	0.881	2.900	0.128	0.325	0.289	1.084	0.561	8.649

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	126	140	145	185	221	2995	241	163
N.S.	1	1.10	1.22	1.26	1.61	1.92	26.04	2.10	1.42
time (sec)	N/A	0.545	2.304	0.091	0.316	0.256	0.869	0.535	8.715

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	122	190	141	177	222	2895	234	153
N.S.	1	1.10	1.71	1.27	1.59	2.00	26.08	2.11	1.38
time (sec)	N/A	0.659	2.448	0.292	0.372	0.281	2.073	0.869	8.615

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	161	159	163	208	323	4502	279	180
N.S.	1	1.18	1.16	1.19	1.52	2.36	32.86	2.04	1.31
time (sec)	N/A	0.978	2.607	0.405	0.350	0.314	3.788	1.175	10.179

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	222	193	196	262	465	8143	362	230
N.S.	1	1.16	1.01	1.02	1.36	2.42	42.41	1.89	1.20
time (sec)	N/A	1.391	3.807	0.536	0.348	0.317	6.028	1.233	11.128

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	363	275	263	389	890	0	505	335
N.S.	1	1.10	0.83	0.79	1.18	2.69	0.00	1.53	1.01
time (sec)	N/A	2.006	5.214	0.216	0.336	0.344	0.000	1.018	9.470

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	285	235	242	366	666	0	458	307
N.S.	1	1.14	0.94	0.97	1.46	2.66	0.00	1.83	1.23
time (sec)	N/A	1.421	6.388	0.152	0.373	0.308	0.000	0.834	8.518

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	216	288	223	333	478	0	410	280
N.S.	1	1.14	1.52	1.18	1.76	2.53	0.00	2.17	1.48
time (sec)	N/A	1.063	6.238	0.112	0.372	0.268	0.000	0.682	8.768

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	201	188	213	330	488	0	410	282
N.S.	1	1.12	1.05	1.19	1.84	2.73	0.00	2.29	1.58
time (sec)	N/A	0.816	4.350	0.140	0.394	0.285	0.000	0.689	8.648

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	197	243	208	321	482	0	409	279
N.S.	1	1.13	1.39	1.19	1.83	2.75	0.00	2.34	1.59
time (sec)	N/A	0.940	4.743	0.496	0.320	0.282	0.000	1.250	8.562

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	257	223	243	372	683	0	479	315
N.S.	1	1.20	1.04	1.13	1.73	3.18	0.00	2.23	1.47
time (sec)	N/A	1.474	3.311	0.665	0.341	0.329	0.000	1.241	10.872

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	333	288	289	454	917	0	560	380
N.S.	1	1.16	1.00	1.01	1.58	3.20	0.00	1.95	1.32
time (sec)	N/A	1.912	6.448	1.033	0.344	0.368	0.000	1.578	13.886

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	136	110	0	0	0	0	0	0
N.S.	1	1.03	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.628	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	148	115	0	0	0	0	0	0
N.S.	1	0.96	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	163	133	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	0.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	170	163	133	0	0	0	0	0	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.532	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	F	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	328	258	0	0	0	0	0	0	0
N.S.	1	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.625	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	369	300	347	416	415	1001	10353	477
N.S.	1	1.05	0.85	0.98	1.18	1.18	2.84	29.33	1.35
time (sec)	N/A	1.634	6.420	0.302	0.397	0.270	0.294	9.657	8.875

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	261	243	246	274	273	617	5631	300
N.S.	1	1.05	0.98	0.99	1.10	1.10	2.49	22.71	1.21
time (sec)	N/A	1.110	3.673	0.151	0.381	0.253	0.209	4.129	8.987

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	174	161	147	151	150	326	2475	153
N.S.	1	1.08	1.00	0.91	0.94	0.93	2.02	15.37	0.95
time (sec)	N/A	0.736	1.691	0.085	0.403	0.260	0.145	1.789	8.420

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	75	74	74	131	761	75
N.S.	1	1.00	1.04	1.03	1.01	1.01	1.79	10.42	1.03
time (sec)	N/A	0.381	0.521	0.053	0.387	0.239	0.116	0.778	8.703

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	162	148	173	183	226	2387	182	186
N.S.	1	1.04	0.95	1.11	1.17	1.45	15.30	1.17	1.19
time (sec)	N/A	0.806	1.195	0.148	0.420	0.366	0.964	0.550	9.559

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	281	216	321	338	556	9721	518	1875
N.S.	1	1.06	0.82	1.21	1.28	2.10	36.68	1.95	7.08
time (sec)	N/A	1.088	2.788	0.140	0.374	0.394	1.410	0.655	21.300

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	347	379	494	574	987	0	1006	502
N.S.	1	1.08	1.18	1.54	1.79	3.08	0.00	3.14	1.57
time (sec)	N/A	1.405	6.337	0.195	0.419	0.299	0.000	0.807	15.528

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	661	696	573	546	691	690	1819	21368	891
N.S.	1	1.05	0.87	0.83	1.05	1.04	2.75	32.33	1.35
time (sec)	N/A	3.407	6.735	0.447	0.351	0.276	0.393	23.581	8.861

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	470	383	392	463	462	1134	11957	561
N.S.	1	1.06	0.86	0.88	1.05	1.04	2.56	26.99	1.27
time (sec)	N/A	2.161	6.552	0.256	0.388	0.263	0.296	10.518	8.359

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	276	241	246	260	259	617	5631	300
N.S.	1	1.04	0.91	0.92	0.98	0.97	2.32	21.17	1.13
time (sec)	N/A	1.102	3.015	0.146	0.318	0.261	0.201	4.094	8.421

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	176	141	135	134	241	1825	141
N.S.	1	1.00	1.34	1.08	1.03	1.02	1.84	13.93	1.08
time (sec)	N/A	0.586	1.133	0.077	0.292	0.268	0.132	1.363	8.379

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	268	190	317	290	397	4444	331	325
N.S.	1	1.06	0.75	1.25	1.14	1.56	17.50	1.30	1.28
time (sec)	N/A	1.460	3.211	0.184	0.308	0.493	2.172	0.697	10.714

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	415	429	277	552	496	964	16225	893	3958
N.S.	1	1.03	0.67	1.33	1.20	2.32	39.10	2.15	9.54
time (sec)	N/A	1.929	6.407	0.309	0.309	0.589	2.972	0.837	32.728

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	631	1041	865	839	1699	0	1668	807
N.S.	1	1.06	1.74	1.45	1.41	2.85	0.00	2.79	1.35
time (sec)	N/A	2.404	7.045	0.439	0.331	0.742	0.000	1.015	27.621

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	603	633	419	546	680	679	1819	21368	891
N.S.	1	1.05	0.69	0.91	1.13	1.13	3.02	35.44	1.48
time (sec)	N/A	2.850	6.655	0.435	0.315	0.285	0.391	23.088	8.632

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	403	297	347	387	386	1001	10353	478
N.S.	1	1.04	0.76	0.89	0.99	0.99	2.57	26.61	1.23
time (sec)	N/A	1.545	6.368	0.237	0.323	0.279	0.275	9.706	8.378

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	212	210	202	201	410	3720	221
N.S.	1	1.00	1.11	1.10	1.06	1.05	2.15	19.48	1.16
time (sec)	N/A	0.834	2.612	0.106	0.320	0.259	0.170	3.192	8.170

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	387	255	501	436	623	7096	559	508
N.S.	1	1.07	0.70	1.38	1.20	1.72	19.55	1.54	1.40
time (sec)	N/A	2.327	4.894	0.253	0.345	0.794	20.196	0.968	12.193

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	574	589	1024	829	685	1512	24300	1329	701
N.S.	1	1.03	1.78	1.44	1.19	2.63	42.33	2.32	1.22
time (sec)	N/A	3.329	7.593	0.364	0.324	1.156	26.693	1.133	15.062

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	798	830	1409	1271	1119	2549	0	2441	1172
N.S.	1	1.04	1.77	1.59	1.40	3.19	0.00	3.06	1.47
time (sec)	N/A	4.153	7.565	0.496	0.390	1.444	0.000	1.358	17.390

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	359	258	500	445	627	7096	559	508
N.S.	1	1.07	0.77	1.48	1.32	1.86	21.06	1.66	1.51
time (sec)	N/A	2.420	4.863	0.245	0.473	0.773	20.489	0.964	12.131

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	253	190	317	294	390	4444	331	325
N.S.	1	1.07	0.81	1.34	1.25	1.65	18.83	1.40	1.38
time (sec)	N/A	1.425	3.228	0.208	0.438	0.468	2.234	0.672	10.243

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	163	148	173	178	212	2387	182	186
N.S.	1	1.04	0.95	1.11	1.14	1.36	15.30	1.17	1.19
time (sec)	N/A	0.793	1.176	0.147	0.394	0.325	0.924	0.528	9.169

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	117	100	106	118	966	106	109
N.S.	1	1.00	1.18	1.01	1.07	1.19	9.76	1.07	1.10
time (sec)	N/A	0.453	0.238	0.099	0.399	0.272	0.582	0.475	8.809

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	164	313	197	243	301	24052	268	196
N.S.	1	0.99	1.90	1.19	1.47	1.82	145.77	1.62	1.19
time (sec)	N/A	0.622	1.617	0.275	0.562	0.455	34.895	0.602	20.811

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	317	572	364	520	1345	0	832	393
N.S.	1	1.13	2.04	1.30	1.85	4.79	0.00	2.96	1.40
time (sec)	N/A	1.322	7.287	0.521	0.329	1.075	0.000	0.769	60.064

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	477	538	898	647	1096	3643	0	2080	65819
N.S.	1	1.13	1.88	1.36	2.30	7.64	0.00	4.36	137.99
time (sec)	N/A	2.722	8.915	1.580	0.381	3.390	0.000	1.095	22.517

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	579	594	1022	829	684	1477	24300	1327	701
N.S.	1	1.03	1.77	1.43	1.18	2.55	41.97	2.29	1.21
time (sec)	N/A	3.188	7.599	0.336	0.337	1.103	27.015	1.138	15.618

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	433	277	552	493	939	16225	893	3958
N.S.	1	1.04	0.66	1.32	1.18	2.25	38.91	2.14	9.49
time (sec)	N/A	1.938	5.568	0.270	0.340	0.561	3.011	0.781	33.596

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	304	221	321	319	505	9721	515	1875
N.S.	1	1.04	0.76	1.10	1.09	1.73	33.29	1.76	6.42
time (sec)	N/A	1.153	2.479	0.245	0.320	0.338	1.619	0.622	21.226

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	151	207	173	205	256	4396	291	184
N.S.	1	1.08	1.48	1.24	1.46	1.83	31.40	2.08	1.31
time (sec)	N/A	0.609	2.714	0.085	0.339	0.265	0.918	0.545	10.549

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	329	592	365	513	1275	0	832	430
N.S.	1	1.12	2.02	1.25	1.75	4.35	0.00	2.84	1.47
time (sec)	N/A	1.374	7.593	0.398	0.354	1.110	0.000	0.767	65.171

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	509	566	984	577	1185	4174	0	2823	73684
N.S.	1	1.11	1.93	1.13	2.33	8.20	0.00	5.55	144.76
time (sec)	N/A	2.839	9.124	1.811	0.414	3.540	0.000	1.074	25.217

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	841	919	1758	951	2519	9594	0	3115	128667
N.S.	1	1.09	2.09	1.13	3.00	11.41	0.00	3.70	152.99
time (sec)	N/A	5.324	8.883	6.321	0.483	10.632	0.000	1.102	44.526

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	804	837	454	1271	1110	2490	0	2441	1172
N.S.	1	1.04	0.56	1.58	1.38	3.10	0.00	3.04	1.46
time (sec)	N/A	3.999	7.366	0.581	0.369	1.326	0.000	1.388	18.168

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	597	631	1044	865	827	1618	0	1663	807
N.S.	1	1.06	1.75	1.45	1.39	2.71	0.00	2.79	1.35
time (sec)	N/A	2.400	6.980	0.400	0.341	0.649	0.000	1.047	27.059

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	379	378	493	543	897	0	1006	502
N.S.	1	1.08	1.07	1.40	1.54	2.55	0.00	2.86	1.43
time (sec)	N/A	1.485	6.404	0.171	0.338	0.291	0.000	0.809	15.424

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	234	261	262	367	566	0	531	327
N.S.	1	1.12	1.25	1.25	1.76	2.71	0.00	2.54	1.56
time (sec)	N/A	0.940	5.515	0.142	0.330	0.297	0.000	0.689	10.858

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	549	912	649	1078	3496	0	2078	65817
N.S.	1	1.13	1.87	1.33	2.21	7.18	0.00	4.27	135.15
time (sec)	N/A	2.586	9.018	1.771	0.381	4.009	0.000	1.036	23.069

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	861	932	1732	949	2537	9567	0	3115	128666
N.S.	1	1.08	2.01	1.10	2.95	11.11	0.00	3.62	149.44
time (sec)	N/A	5.329	8.626	5.789	0.550	12.033	0.000	1.100	43.729

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	476	1232	4473	0	35153	0	0	0
N.S.	1	1.03	2.66	9.64	0.00	75.76	0.00	0.00	0.00
time (sec)	N/A	3.444	6.545	0.531	0.000	10.873	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	332	314	3353	0	23984	0	0	0
N.S.	1	1.02	0.97	10.32	0.00	73.80	0.00	0.00	0.00
time (sec)	N/A	2.314	5.243	0.158	0.000	4.685	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	218	220	2218	0	12410	0	0	22955
N.S.	1	0.97	0.98	9.90	0.00	55.40	0.00	0.00	102.48
time (sec)	N/A	1.228	2.159	0.182	0.000	1.664	0.000	0.000	57.645

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	134	150	1312	0	2588	0	0	1199
N.S.	1	0.86	0.97	8.46	0.00	16.70	0.00	0.00	7.74
time (sec)	N/A	0.754	0.592	0.129	0.000	0.353	0.000	0.000	16.041

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	225	233	3576	0	0	0	0	62245
N.S.	1	0.96	1.00	15.28	0.00	0.00	0.00	0.00	266.00
time (sec)	N/A	1.763	0.761	0.153	0.000	0.000	0.000	0.000	33.934

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	327	764	5778	0	0	0	0	138318
N.S.	1	1.03	2.41	18.23	0.00	0.00	0.00	0.00	436.33
time (sec)	N/A	2.295	6.461	0.133	0.000	0.000	0.000	0.000	42.933

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	588	2819	9797	0	0	0	0	0
N.S.	1	1.08	5.19	18.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.276	6.721	0.150	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	566	1290	10952	0	84950	0	0	0
N.S.	1	1.03	2.35	19.91	0.00	154.45	0.00	0.00	0.00
time (sec)	N/A	4.502	6.574	0.335	0.000	194.267	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	402	510	7939	0	58971	0	0	0
N.S.	1	1.02	1.29	20.05	0.00	148.92	0.00	0.00	0.00
time (sec)	N/A	2.919	6.434	0.212	0.000	69.930	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	268	260	5107	0	31081	0	0	0
N.S.	1	0.98	0.95	18.71	0.00	113.85	0.00	0.00	0.00
time (sec)	N/A	1.666	4.837	0.194	0.000	16.158	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	166	202	2500	0	6846	0	0	4260
N.S.	1	0.89	1.08	13.37	0.00	36.61	0.00	0.00	22.78
time (sec)	N/A	0.991	1.319	0.138	0.000	0.950	0.000	0.000	42.332

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	275	266	6055	0	0	0	0	106783
N.S.	1	1.01	0.98	22.34	0.00	0.00	0.00	0.00	394.03
time (sec)	N/A	2.624	2.661	0.151	0.000	0.000	0.000	0.000	55.548

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	387	2738	9865	0	0	0	0	0
N.S.	1	1.04	7.36	26.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.694	6.611	0.167	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	563	7678	14441	0	0	0	0	0
N.S.	1	1.06	14.43	27.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.401	7.176	0.158	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	511	564	11280	0	91140	0	0	0
N.S.	1	1.02	1.12	22.43	0.00	181.19	0.00	0.00	0.00
time (sec)	N/A	3.783	6.575	0.436	0.000	169.464	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	345	324	7294	0	48734	0	0	0
N.S.	1	0.98	0.92	20.66	0.00	138.06	0.00	0.00	0.00
time (sec)	N/A	2.230	5.511	0.209	0.000	37.570	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	208	262	3562	0	10840	0	0	5863
N.S.	1	0.91	1.14	15.55	0.00	47.34	0.00	0.00	25.60
time (sec)	N/A	1.374	2.204	0.150	0.000	1.984	0.000	0.000	114.330

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	341	322	8698	0	0	0	0	0
N.S.	1	1.01	0.96	25.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.710	5.756	0.185	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	0	6112	14119	0	0	0	0	0
N.S.	1	0.00	12.92	29.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.015	0.196	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	0	17248	20663	0	0	0	0	0
N.S.	1	0.00	26.82	32.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	7.951	0.298	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	418	1200	5978	0	37247	0	0	28858
N.S.	1	1.03	2.95	14.69	0.00	91.52	0.00	0.00	70.90
time (sec)	N/A	2.869	6.508	0.308	0.000	11.353	0.000	0.000	112.142

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	414	5513	0	25627	0	0	21254
N.S.	1	1.00	1.44	19.21	0.00	89.29	0.00	0.00	74.06
time (sec)	N/A	1.820	6.361	0.155	0.000	4.807	0.000	0.000	43.418

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	185	192	3853	0	13473	0	0	16400
N.S.	1	0.95	0.99	19.86	0.00	69.45	0.00	0.00	84.54
time (sec)	N/A	0.961	1.630	0.142	0.000	1.749	0.000	0.000	21.650

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	112	129	3463	0	3194	0	0	4326
N.S.	1	0.84	0.97	26.04	0.00	24.02	0.00	0.00	32.53
time (sec)	N/A	0.590	0.238	0.115	0.000	0.385	0.000	0.000	12.950

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	195	194	13474	0	0	0	0	25341
N.S.	1	0.93	0.92	64.16	0.00	0.00	0.00	0.00	120.67
time (sec)	N/A	1.231	0.454	0.136	0.000	0.000	0.000	0.000	65.482

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	358	521	20870	0	0	0	0	225004
N.S.	1	1.09	1.59	63.82	0.00	0.00	0.00	0.00	688.09
time (sec)	N/A	2.252	6.259	0.165	0.000	0.000	0.000	0.000	57.472

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	511	512	920	11255	0	0	0	0	0
N.S.	1	1.00	1.80	22.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.840	6.880	0.441	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	346	476	9399	0	0	0	0	54886
N.S.	1	1.01	1.39	27.40	0.00	0.00	0.00	0.00	160.02
time (sec)	N/A	2.327	6.568	0.216	0.000	0.000	0.000	0.000	63.779

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	219	290	7396	0	31879	0	0	40542
N.S.	1	1.09	1.44	36.80	0.00	158.60	0.00	0.00	201.70
time (sec)	N/A	1.163	2.782	0.160	0.000	61.209	0.000	0.000	38.597

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	161	218	5613	0	7982	0	0	8588
N.S.	1	1.03	1.39	35.75	0.00	50.84	0.00	0.00	54.70
time (sec)	N/A	0.675	1.107	0.127	0.000	1.834	0.000	0.000	18.185

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	309	296	26343	0	0	0	0	0
N.S.	1	1.18	1.13	100.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.101	5.350	0.153	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	515	2078	40619	0	0	0	0	0
N.S.	1	1.15	4.65	90.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.078	6.428	0.240	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	585	605	670	13586	0	0	0	0	0
N.S.	1	1.03	1.15	23.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.582	6.930	0.481	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	388	502	11360	0	0	0	0	88684
N.S.	1	1.08	1.40	31.73	0.00	0.00	0.00	0.00	247.72
time (sec)	N/A	2.562	6.634	0.215	0.000	0.000	0.000	0.000	109.692

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	300	300	8963	0	50755	0	0	64641
N.S.	1	1.10	1.10	32.83	0.00	185.92	0.00	0.00	236.78
time (sec)	N/A	1.551	3.162	0.248	0.000	297.231	0.000	0.000	85.534

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	231	223	6788	0	13143	0	0	14163
N.S.	1	1.11	1.07	32.48	0.00	62.89	0.00	0.00	67.77
time (sec)	N/A	1.118	0.984	0.180	0.000	7.154	0.000	0.000	35.921

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	444	1948	45119	0	0	0	0	0
N.S.	1	1.22	5.34	123.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.435	6.395	0.231	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	767	6052	67570	0	0	0	0	0
N.S.	1	1.13	8.91	99.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	7.028	6.841	0.346	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	679	707	1202	0	0	0	0	0	0
N.S.	1	1.04	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.733	10.255	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	505	523	835	0	0	0	0	0	0
N.S.	1	1.04	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.675	9.103	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	381	389	619	0	0	68078	0	0	0
N.S.	1	1.02	1.62	0.00	0.00	178.68	0.00	0.00	0.00
time (sec)	N/A	2.188	7.944	0.000	0.000	161.870	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	287	292	441	0	0	78051	0	0	0
N.S.	1	1.02	1.54	0.00	0.00	271.95	0.00	0.00	0.00
time (sec)	N/A	1.262	4.705	0.000	0.000	105.636	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	300	329	412	0	0	139535	0	0	0
N.S.	1	1.10	1.37	0.00	0.00	465.12	0.00	0.00	0.00
time (sec)	N/A	1.504	5.862	0.000	0.000	198.892	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	370	441	600	0	0	0	0	0	0
N.S.	1	1.19	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.587	7.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	597	693	1109	0	0	0	0	0	0
N.S.	1	1.16	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.499	7.514	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	682	711	1304	0	0	0	0	0	0
N.S.	1	1.04	1.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.861	9.563	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	508	524	867	0	0	0	0	0	0
N.S.	1	1.03	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.662	9.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	393	613	0	0	115434	0	0	0
N.S.	1	1.02	1.60	0.00	0.00	300.61	0.00	0.00	0.00
time (sec)	N/A	2.370	7.860	0.000	0.000	196.391	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	382	420	1664	0	0	206814	0	0	0
N.S.	1	1.10	4.36	0.00	0.00	541.40	0.00	0.00	0.00
time (sec)	N/A	2.659	7.328	0.000	0.000	289.269	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	402	457	519	0	0	0	0	0	0
N.S.	1	1.14	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.896	6.761	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	586	678	3134	0	0	0	0	0	0
N.S.	1	1.16	5.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.669	9.499	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	697	717	1261	0	0	0	0	0	0
N.S.	1	1.03	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.849	9.878	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	505	524	780	0	0	0	0	0	0
N.S.	1	1.04	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.630	9.038	180.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	535	562	1774	0	0	0	0	0	0
N.S.	1	1.05	3.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.478	8.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	545	603	802	0	0	0	0	0	0
N.S.	1	1.11	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.179	7.632	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	590	658	641	0	0	0	0	0	0
N.S.	1	1.12	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.256	7.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F(-1)	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	946	0	10121	0	0	0	0	0	0
N.S.	1	0.00	10.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	56.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	505	524	785	0	0	0	0	0	0
N.S.	1	1.04	1.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.680	8.844	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	392	607	0	0	115594	0	0	0
N.S.	1	1.02	1.58	0.00	0.00	301.81	0.00	0.00	0.00
time (sec)	N/A	2.151	7.764	0.000	0.000	271.990	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	290	294	456	0	0	77916	0	0	0
N.S.	1	1.01	1.57	0.00	0.00	268.68	0.00	0.00	0.00
time (sec)	N/A	1.245	7.042	0.000	0.000	129.202	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	232	362	0	0	96324	0	0	0
N.S.	1	0.97	1.51	0.00	0.00	403.03	0.00	0.00	0.00
time (sec)	N/A	0.686	2.418	0.000	0.000	141.315	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	303	264	0	0	83974	0	0	0
N.S.	1	1.21	1.05	0.00	0.00	334.56	0.00	0.00	0.00
time (sec)	N/A	1.409	2.868	0.000	0.000	274.810	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	375	458	388	0	0	0	0	0	0
N.S.	1	1.22	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.300	6.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	528	555	2245	0	0	0	0	0	0
N.S.	1	1.05	4.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.415	9.615	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	380	417	2141	0	0	0	0	0	0
N.S.	1	1.10	5.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.569	7.679	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	328	403	0	0	139175	0	0	0
N.S.	1	1.10	1.35	0.00	0.00	465.47	0.00	0.00	0.00
time (sec)	N/A	1.502	6.112	0.000	0.000	246.045	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	302	275	0	0	0	0	0	0
N.S.	1	1.20	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.381	3.529	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	466	484	0	0	0	0	0	0
N.S.	1	1.22	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.442	6.874	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	598	704	902	0	0	0	0	0	0
N.S.	1	1.18	1.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.385	7.167	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	549	605	2650	0	0	0	0	0	0
N.S.	1	1.10	4.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.993	9.792	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	407	462	1135	0	0	0	0	0	0
N.S.	1	1.14	2.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.852	7.165	180.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	373	444	609	0	0	0	0	0	0
N.S.	1	1.19	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.500	7.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	379	461	403	0	0	0	0	0	0
N.S.	1	1.22	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.341	5.999	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	651	751	903	0	0	0	0	0	0
N.S.	1	1.15	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.405	7.207	180.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	376	371	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	560	580	1390	0	0	0	0	0	0
N.S.	1	1.04	2.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.412	6.544	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	363	385	505	0	0	0	0	0	0
N.S.	1	1.06	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.930	6.374	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	262	202	0	0	0	0	0	0
N.S.	1	1.06	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.043	3.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	135	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	265	204	0	0	0	0	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.106	1.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	451	563	0	0	0	0	0	0
N.S.	1	1.12	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.170	6.245	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	702	0	2238	0	0	0	0	0	0
N.S.	1	0.00	3.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.325	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [24] had the largest ratio of [.5000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	10	1.00	36	0.278
2	A	6	6	1.00	30	0.200
3	A	6	6	1.00	36	0.167
4	A	10	10	1.05	38	0.263
5	A	9	9	1.05	38	0.237
6	A	12	12	1.03	38	0.316
7	A	14	14	1.02	38	0.368
8	A	17	17	1.02	38	0.447
9	A	13	13	1.08	38	0.342
10	A	8	8	1.00	32	0.250
11	A	8	8	1.00	38	0.211
12	A	9	9	1.03	40	0.225
13	A	9	9	1.03	40	0.225
14	A	12	12	1.03	40	0.300
15	A	14	14	1.03	40	0.350
16	A	17	17	1.01	40	0.425
17	A	10	10	1.00	32	0.312
18	A	10	10	1.00	38	0.263
19	A	13	13	1.02	40	0.325
20	A	12	12	1.02	40	0.300
21	A	13	13	1.02	40	0.325
22	A	15	15	1.05	40	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	19	19	1.07	40	0.475
24	A	20	20	1.04	40	0.500
25	A	14	13	1.13	40	0.325
26	A	12	11	1.07	38	0.289
27	A	5	4	1.12	32	0.125
28	A	6	6	1.00	38	0.158
29	A	8	8	1.02	40	0.200
30	A	10	10	1.08	40	0.250
31	A	14	14	1.12	40	0.350
32	A	15	14	1.08	40	0.350
33	A	12	11	1.09	38	0.289
34	A	6	6	1.10	32	0.188
35	A	8	8	1.10	38	0.211
36	A	10	10	1.18	40	0.250
37	A	12	12	1.16	40	0.300
38	A	18	17	1.10	40	0.425
39	A	14	13	1.14	40	0.325
40	A	11	11	1.14	38	0.289
41	A	8	8	1.12	32	0.250
42	A	10	10	1.13	38	0.263
43	A	13	13	1.20	40	0.325
44	A	15	15	1.16	40	0.375
45	A	9	8	1.03	39	0.205
46	A	9	8	0.96	39	0.205
47	A	9	8	0.96	41	0.195
48	A	9	8	0.96	41	0.195
49	C	5	4	0.79	43	0.093
50	A	13	13	1.05	43	0.302
51	A	11	11	1.05	43	0.256
52	A	8	8	1.08	41	0.195
53	A	6	6	1.00	31	0.194
54	A	10	9	1.04	43	0.209
55	A	9	8	1.06	43	0.186
56	A	8	8	1.08	43	0.186

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	17	17	1.05	45	0.378
58	A	14	14	1.06	45	0.311
59	A	10	10	1.04	43	0.233
60	A	8	8	1.00	33	0.242
61	A	13	12	1.06	45	0.267
62	A	11	10	1.03	45	0.222
63	A	12	11	1.06	45	0.244
64	A	16	16	1.05	45	0.356
65	A	12	12	1.04	43	0.279
66	A	10	10	1.00	33	0.303
67	A	16	15	1.07	45	0.333
68	A	14	13	1.03	45	0.289
69	A	14	13	1.04	45	0.289
70	A	16	15	1.07	45	0.333
71	A	13	12	1.07	45	0.267
72	A	9	8	1.04	43	0.186
73	A	7	6	1.00	33	0.182
74	A	4	4	0.99	45	0.089
75	A	7	7	1.13	45	0.156
76	A	10	10	1.13	45	0.222
77	A	14	13	1.03	45	0.289
78	A	11	10	1.04	45	0.222
79	A	9	8	1.04	43	0.186
80	A	6	6	1.08	33	0.182
81	A	7	7	1.12	45	0.156
82	A	9	9	1.11	45	0.200
83	A	11	11	1.09	45	0.244
84	A	14	13	1.04	45	0.289
85	A	12	11	1.06	45	0.244
86	A	9	9	1.08	43	0.209
87	A	9	9	1.12	33	0.273
88	A	9	9	1.13	45	0.200
89	A	11	11	1.08	45	0.244
90	A	21	20	1.03	47	0.426

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	18	17	1.02	47	0.362
92	A	15	14	0.97	45	0.311
93	A	12	11	0.86	35	0.314
94	A	16	15	0.96	47	0.319
95	A	17	16	1.03	47	0.340
96	A	20	19	1.08	47	0.404
97	A	23	22	1.03	47	0.468
98	A	20	19	1.02	47	0.404
99	A	17	16	0.98	45	0.356
100	A	14	13	0.89	35	0.371
101	A	20	19	1.01	47	0.404
102	A	20	19	1.04	47	0.404
103	A	20	19	1.06	47	0.404
104	A	22	21	1.02	47	0.447
105	A	19	18	0.98	45	0.400
106	A	16	15	0.91	35	0.429
107	A	24	23	1.01	47	0.489
108	F	0	0	N/A	0.000	N/A
109	F	0	0	N/A	0.000	N/A
110	A	19	18	1.03	47	0.383
111	A	16	15	1.00	47	0.319
112	A	13	12	0.95	45	0.267
113	A	10	9	0.84	35	0.257
114	A	13	12	0.93	47	0.255
115	A	17	16	1.09	47	0.340
116	A	19	18	1.00	47	0.383
117	A	16	15	1.01	47	0.319
118	A	12	11	1.09	45	0.244
119	A	10	9	1.03	35	0.257
120	A	17	16	1.18	47	0.340
121	A	20	19	1.15	47	0.404
122	A	19	18	1.03	47	0.383
123	A	15	14	1.08	47	0.298

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	13	12	1.10	45	0.267
125	A	13	12	1.11	35	0.343
126	A	19	18	1.22	47	0.383
127	A	23	22	1.13	47	0.468
128	A	17	16	1.04	49	0.327
129	A	14	13	1.04	49	0.265
130	A	11	10	1.02	49	0.204
131	A	8	7	1.02	49	0.143
132	A	8	7	1.10	49	0.143
133	A	13	12	1.19	49	0.245
134	A	16	15	1.16	49	0.306
135	A	17	16	1.04	49	0.327
136	A	14	13	1.03	49	0.265
137	A	12	11	1.02	49	0.224
138	A	11	10	1.10	49	0.204
139	A	11	10	1.14	49	0.204
140	A	16	15	1.16	49	0.306
141	A	17	16	1.03	49	0.327
142	A	14	13	1.04	49	0.265
143	A	14	13	1.05	49	0.265
144	A	14	13	1.11	49	0.265
145	A	14	13	1.12	49	0.265
146	F	0	0	N/A	0.000	N/A
147	A	14	13	1.04	49	0.265
148	A	11	10	1.02	49	0.204
149	A	8	7	1.01	49	0.143
150	A	5	4	0.97	49	0.082
151	A	10	9	1.21	49	0.184
152	A	13	12	1.22	49	0.245
153	A	14	13	1.05	49	0.265
154	A	11	10	1.10	49	0.204
155	A	8	7	1.10	49	0.143
156	A	10	9	1.20	49	0.184

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
157	A	13	12	1.22	49	0.245
158	A	16	15	1.18	49	0.306
159	A	14	13	1.10	49	0.265
160	A	11	10	1.14	49	0.204
161	A	13	12	1.19	49	0.245
162	A	13	12	1.22	49	0.245
163	A	16	15	1.15	49	0.306
164	A	5	4	0.99	45	0.089
165	A	17	16	1.04	45	0.356
166	A	13	12	1.06	45	0.267
167	A	11	10	1.06	43	0.233
168	A	9	8	1.00	33	0.242
169	A	11	10	1.03	45	0.222
170	A	14	13	1.12	45	0.289
171	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \tan(c+dx)(a+b\tan(c+dx)) (B\tan(c+dx)+C\tan^2($

3.1.1 Optimal result

Integrand size = 36, antiderivative size = 87

$$\begin{aligned}\int \tan(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx \\ = -((aB-bC)x)+\frac{(bB+aC)\log(\cos(c+dx))}{d} \\ +\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(bB+aC)\tan^2(c+dx)}{2d}+\frac{bC\tan^3(c+dx)}{3d}\end{aligned}$$

output
```
-(B*a-C*b)*x+(B*b+C*a)*ln(cos(d*x+c))/d+(B*a-C*b)*tan(d*x+c)/d+1/2*(B*b+C*
a)*tan(d*x+c)^2/d+1/3*b*C*tan(d*x+c)^3/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \tan(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx \\ = \frac{(-6aB+6bC)\arctan(\tan(c+dx))+6(bB+aC)\log(\cos(c+dx))+6(aB-bC)\tan(c+dx)+3(bB+}{6d}\end{aligned}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*
x]^2),x]
```

output
```
((-6*a*B + 6*b*C)*ArcTan[Tan[c + d*x]] + 6*(b*B + a*C)*Log[Cos[c + d*x]] +
 6*(a*B - b*C)*Tan[c + d*x] + 3*(b*B + a*C)*Tan[c + d*x]^2 + 2*b*C*Tan[c +
 d*x]^3)/(6*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4115, 3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4115}$$

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan^2(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{bC\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{bC\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \tan(c+dx)(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{bC\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{bC\tan^3(c+dx)}{3d}$$

$$\downarrow 4008$$

$$-(aC+bB)\int \tan(c+dx)dx+\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

$$\downarrow 3042$$

$$-(aC+bB)\int \tan(c+dx)dx+\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

$$\downarrow 3956$$

$$\frac{(aC+bB)\tan^2(c+dx)}{2d}+\frac{(aB-bC)\tan(c+dx)}{d}+\frac{(aC+bB)\log(\cos(c+dx))}{d}-x(aB-bC)+\frac{bC\tan^3(c+dx)}{3d}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),
x]
```

output
```
-((a*B - b*C)*x) + ((b*B + a*C)*Log[Cos[c + d*x]])/d + ((a*B - b*C)*Tan[c
+ d*x])/d + ((b*B + a*C)*Tan[c + d*x]^2)/(2*d) + (b*C*Tan[c + d*x]^3)/(3*d
)
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

method	result
norman	$(-Ba+Cb)\,x+\frac{(Ba-Cb)\tan(dx+c)}{d}+\frac{(Bb+Ca)\tan(dx+c)^{2}}{2d}+\frac{bC\tan(dx+c)^{3}}{3d}-\frac{(Bb+Ca)\ln\left(1+\tan(dx\right.}{2d}$
parts	$\frac{(Bb+Ca)\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{Ba(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{Cb\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\right.}{d}$
derivativedivides	$\frac{\frac{Cb\tan(dx+c)^{3}}{3}+\frac{Bb\tan(dx+c)^{2}}{2}+\frac{Ca\tan(dx+c)^{2}}{2}+Ba\tan(dx+c)-Cb\tan(dx+c)+\frac{(-Bb-Ca)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Ba+C}{d}$
default	$\frac{\frac{Cb\tan(dx+c)^{3}}{3}+\frac{Bb\tan(dx+c)^{2}}{2}+\frac{Ca\tan(dx+c)^{2}}{2}+Ba\tan(dx+c)-Cb\tan(dx+c)+\frac{(-Bb-Ca)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Ba+C}{d}$
parallelrisch	$-\frac{-2Cb\tan(dx+c)^{3}+6Badx-3Bb\tan(dx+c)^{2}-6Cbdx-3Ca\tan(dx+c)^{2}+3B\ln\left(1+\tan(dx+c)^{2}\right)b-6Ba\tan(dx+c)+}{6d}$
risch	$-iBbx-iCax-Bax+Cbx-\frac{2iBbc}{d}-\frac{2iCac}{d}+\frac{2i\left(-3iBb\,e^{4i(dx+c)}-3iCa\,e^{4i(dx+c)}+3Ba\,e^{4i(dx+c)}\right.}{}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RE
TURNVERBOSE)
```

output
```
(-B*a+C*b)*x+(B*a-C*b)*tan(d*x+c)/d+1/2*(B*b+C*a)*tan(d*x+c)^2/d+1/3*b*C*t
an(d*x+c)^3/d-1/2*(B*b+C*a)/d*ln(1+tan(d*x+c)^2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb\tan\left(dx+c\right)^3-6\,(Ba-Cb)dx+3\,(Ca+Bb)\tan\left(dx+c\right)^2+3\,(Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+6\,(B}{6\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="fricas")
```

output
```
1/6*(2*C*b*tan(d*x + c)^3 - 6*(B*a - C*b)*d*x + 3*(C*a + B*b)*tan(d*x + c)
^2 + 3*(C*a + B*b)*log(1/(tan(d*x + c)^2 + 1)) + 6*(B*a - C*b)*tan(d*x + c
))/d
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.60

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}-Bax+\frac{Ba\tan(c+dx)}{d}-\frac{Bb\log(\tan^2(c+dx)+1)}{2d}+\frac{Bb\tan^2(c+dx)}{2d}-\frac{Ca\log(\tan^2(c+dx)+1)}{2d}+\frac{Ca\tan^2(c+dx)}{2d}+Cbx+\frac{C}{}\\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\tan(c)\end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((-B*a*x + B*a*tan(c + d*x)/d - B*b*log(tan(c + d*x)**2 + 1)/(2*d
) + B*b*tan(c + d*x)**2/(2*d) - C*a*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*t
an(c + d*x)**2/(2*d) + C*b*x + C*b*tan(c + d*x)**3/(3*d) - C*b*tan(c + d*x
)/d, Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2)*tan(c), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \tan(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb\tan\left(dx+c\right)^3+3\,(Ca+Bb)\tan\left(dx+c\right)^2-6\,(Ba-Cb)(dx+c)-3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^2-\right.}{6\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="maxima")
```

output
```
1/6*(2*C*b*tan(d*x + c)^3 + 3*(C*a + B*b)*tan(d*x + c)^2 - 6*(B*a - C*b)*(
d*x + c) - 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) + 6*(B*a - C*b)*tan(d*x +
 c))/d
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. 2(83) = 166.

Time = 0.91 (sec) , antiderivative size = 937, normalized size of antiderivative = 10.77

$$\int \tan(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="giac")
```

output
```
-1/6*(6*B*a*d*x*tan(d*x)^3*tan(c)^3 - 6*C*b*d*x*tan(d*x)^3*tan(c)^3 - 3*C*
a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 3*B*b*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 18*B*a*d*x*tan(d*x)^2*tan(c)^2 + 18*C*
b*d*x*tan(d*x)^2*tan(c)^2 - 3*C*a*tan(d*x)^3*tan(c)^3 - 3*B*b*tan(d*x)^3*t
an(c)^3 + 9*C*a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 9*B*b*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 6*B*a*tan(d*x)^3*tan(c)^
2 - 6*C*b*tan(d*x)^3*tan(c)^2 + 6*B*a*tan(d*x)^2*tan(c)^3 - 6*C*b*tan(d*x)
^2*tan(c)^3 + 18*B*a*d*x*tan(d*x)*tan(c) - 18*C*b*d*x*tan(d*x)*tan(c) - 3*
C*a*tan(d*x)^3*tan(c) - 3*B*b*tan(d*x)^3*tan(c) + 3*C*a*tan(d*x)^2*tan(c)^
2 + 3*B*b*tan(d*x)^2*tan(c)^2 - 3*C*a*tan(d*x)*tan(c)^3 - 3*B*b*tan(d*x)*t
an(c)^3 + 2*C*b*tan(d*x)^3 - 9*C*a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)
*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*t
an(c) - 9*B*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) - 12*B*a*tan(d*
x)^2*tan(c) + 18*C*b*tan(d*x)^2*tan(c) - 12*B*a*tan(d*x)*tan(c)^2 + 18*C*b
*tan(d*x)*tan(c)^2 + 2*C*b*tan(c)^3 - 6*B*a*d*x + 6*C*b*d*x + 3*C*a*tan...
```

3.1.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \tan(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(B\,a-C\,b\right)-\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,b}{2}+\frac{C\,a}{2}\right)+\tan(c+d\,x)^2\,\left(\frac{B\,b}{2}+\frac{C\,a}{2}\right)-d\,x\,(B\,a-C\,b}{d}$$

input
```
int(tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),
x)
```

output
```
(tan(c + d*x)*(B*a - C*b) - log(tan(c + d*x)^2 + 1)*((B*b)/2 + (C*a)/2) +
tan(c + d*x)^2*((B*b)/2 + (C*a)/2) - d*x*(B*a - C*b) + (C*b*tan(c + d*x)^3
)/3)/d
```

3.2 $\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$

3.2.1 Optimal result

Integrand size = 30, antiderivative size = 66

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx = -((bB+aC)x)-\frac{(aB-bC)\log(\cos(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2bd}$$

output
```
-(B*b+C*a)*x-(B*a-C*b)*ln(cos(d*x+c))/d+b*B*tan(d*x+c)/d+1/2*C*(a+b*tan(d*
x+c))^2/b/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx = \frac{-2(bB+aC)\arctan(\tan(c+dx))+2(-aB+bC)\log(\cos(c+dx))+2(bB+aC)\tan(c+dx)+bC\tan^2(c+dx)}{2d}$$

input
```
Integrate[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
(-2*(b*B + a*C)*ArcTan[Tan[c + d*x]] + 2*(-(a*B) + b*C)*Log[Cos[c + d*x]]
+ 2*(b*B + a*C)*Tan[c + d*x] + b*C*Tan[c + d*x]^2)/(2*d)
```

3.2.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4113, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4113}$$

$$\int (a+b\tan(c+dx))(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^2}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^2}{2bd}$$

$$\Big\downarrow \text{4008}$$

$$(aB-bC)\int \tan(c+dx)dx-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$(aB-bC)\int \tan(c+dx)dx-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aB-bC)\log(\cos(c+dx))}{d}-x(aC+bB)+\frac{C(a+b\tan(c+dx))^2}{2bd}+\frac{bB\tan(c+dx)}{d}$$

input
```
Int[(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
-((b*B + a*C)*x) - ((a*B - b*C)*Log[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/
d + (C*(a + b*Tan[c + d*x])^2)/(2*b*d)
```

3.2.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

method	result
norman	$(-Bb-Ca)\,x+\frac{(Bb+Ca)\tan(dx+c)}{d}+\frac{Cb\tan(dx+c)^{2}}{2d}+\frac{(Ba-Cb)\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$
derivativedivides	$\frac{\frac{C\tan(dx+c)^{2}b}{2}+B\tan(dx+c)b+C\tan(dx+c)a+\frac{(Ba-Cb)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Bb-Ca)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{C\tan(dx+c)^{2}b}{2}+B\tan(dx+c)b+C\tan(dx+c)a+\frac{(Ba-Cb)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Bb-Ca)\arctan(\tan(dx+c))}{d}$
parts	$\frac{(Bb+Ca)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{Ba\ln\left(1+\tan(dx+c)^{2}\right)}{2d}+\frac{Cb\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}$
parallelrisch	$\frac{-2Bbdx-2Cadx+C\tan(dx+c)^{2}b+B\ln\left(1+\tan(dx+c)^{2}\right)a+2B\tan(dx+c)b-C\ln\left(1+\tan(dx+c)^{2}\right)b+2C\tan(dx+c)a}{2d}$
risch	$-Bbx-Cax+iBax-iCbx+\frac{2iBac}{d}-\frac{2iCbc}{d}+\frac{2i\left(-iCb\,e^{2i(dx+c)}+Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}+Bb-\right.}{d\left(e^{2i(dx+c)}+1\right)^{2}}$

input
```
int((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBOSE
)
```

output
```
(-B*b-C*a)*x+(B*b+C*a)/d*tan(d*x+c)+1/2*C*b/d*tan(d*x+c)^2+1/2*(B*a-C*b)/d
*ln(1+tan(d*x+c)^2)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$
$$= \frac{Cb\tan{(dx+c)}^2 - 2\,(Ca+Bb)dx - (Ba-Cb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2\,(Ca+Bb)\tan{(dx+c)}}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="fri
cas")
```

output
```
1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*d*x - (B*a - C*b)*log(1/(tan(d*x +
 c)^2 + 1)) + 2*(C*a + B*b)*tan(d*x + c))/d
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.59

$$\int (a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$
$$= \begin{cases} \frac{Ba\log{(\tan^2{(c+dx)}+1)}}{2d} - Bbx + \frac{Bb\tan{(c+dx)}}{d} - Cax + \frac{Ca\tan{(c+dx)}}{d} - \frac{Cb\log{(\tan^2{(c+dx)}+1)}}{2d} + \frac{Cb\tan^2{(c+dx)}}{2d} & \text{for } d \\ x(a+b\tan{(c)})\,(B\tan{(c)}+C\tan^2{(c)}) & \text{other} \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a*log(tan(c + d*x)**2 + 1)/(2*d) - B*b*x + B*b*tan(c + d*x)/d
 - C*a*x + C*a*tan(c + d*x)/d - C*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*b*t
an(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(c)**2
), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb\tan{(dx+c)}^2-2\,(Ca+Bb)(dx+c)+(Ba-Cb)\log\left(\tan{(dx+c)}^2+1\right)+2\,(Ca+Bb)\tan{(dx+c)}}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="max
ima")
```

output
```
1/2*(C*b*tan(d*x + c)^2 - 2*(C*a + B*b)*(d*x + c) + (B*a - C*b)*log(tan(d*
x + c)^2 + 1) + 2*(C*a + B*b)*tan(d*x + c))/d
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(64) = 128.

Time = 0.65 (sec) , antiderivative size = 556, normalized size of antiderivative = 8.42

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{2\,Cadx\tan{(dx)}^2\tan{(c)}^2+2\,Bbdx\tan{(dx)}^2\tan{(c)}^2+Ba\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan{(}}{}$$

input
```
integrate((a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="gia
c")
```

output
```
-1/2*(2*C*a*d*x*tan(d*x)^2*tan(c)^2 + 2*B*b*d*x*tan(d*x)^2*tan(c)^2 + B*a*
log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
 tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - C*b*log(4*(tan(d*x)^2*t
an(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c
)^2 + 1))*tan(d*x)^2*tan(c)^2 - 4*C*a*d*x*tan(d*x)*tan(c) - 4*B*b*d*x*tan(
d*x)*tan(c) - C*b*tan(d*x)^2*tan(c)^2 - 2*B*a*log(4*(tan(d*x)^2*tan(c)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))
*tan(d*x)*tan(c) + 2*C*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + 2*
C*a*tan(d*x)^2*tan(c) + 2*B*b*tan(d*x)^2*tan(c) + 2*C*a*tan(d*x)*tan(c)^2
+ 2*B*b*tan(d*x)*tan(c)^2 + 2*C*a*d*x + 2*B*b*d*x - C*b*tan(d*x)^2 - C*b*t
an(c)^2 + B*a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) - C*b*log(4*(tan(d*x)^2*tan(c)^
2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 +
1)) - 2*C*a*tan(d*x) - 2*B*b*tan(d*x) - 2*C*a*tan(c) - 2*B*b*tan(c) - C*b)
/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 7.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int (a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\tan(c+d\,x)\,\left(B\,b+C\,a\right)+\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a}{2}-\frac{C\,b}{2}\right)-d\,x\,\left(B\,b+C\,a\right)+\frac{C\,b\,\tan(c+d\,x)^2}{2}}{d}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),x)
```

output
```
(tan(c + d*x)*(B*b + C*a) + log(tan(c + d*x)^2 + 1)*((B*a)/2 - (C*b)/2) -
d*x*(B*b + C*a) + (C*b*tan(c + d*x)^2)/2)/d
```

3.3 $\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.3.1 Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = (aB-bC)x-\frac{(bB+aC)\log(\cos(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

output
```
(B*a-C*b)*x-(B*b+C*a)*ln(cos(d*x+c))/d+b*C*tan(d*x+c)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = aBx-\frac{bC\arctan(\tan(c+dx))}{d}-\frac{bB\log(\cos(c+dx))}{d}-\frac{aC\log(\cos(c+dx))}{d}+\frac{bC\tan(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*
x]^2),x]
```

output
```
a*B*x - (b*C*ArcTan[Tan[c + d*x]])/d - (b*B*Log[Cos[c + d*x]])/d - (a*C*Lo
g[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

3.3.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4115, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4115}$$

$$\int (a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4008}$$

$$(aC+bB)\int \tan(c+dx)dx+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$(aC+bB)\int \tan(c+dx)dx+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{(aC+bB)\log(\cos(c+dx))}{d}+x(aB-bC)+\frac{bC\tan(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),
x]
```

output
```
(a*B - b*C)*x - ((b*B + a*C)*Log[Cos[c + d*x]])/d + (b*C*Tan[c + d*x])/d
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.3.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{(Bb+Ca)\ln\left(\sec(dx+c)^2\right)+2Cb\tan(dx+c)+2dx(Ba-Cb)}{2d}$
norman	$(Ba-Cb)\,x+\frac{bC\tan(dx+c)}{d}+\frac{(Bb+Ca)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
derivativedivides	$\frac{-Bb\ln(\cos(dx+c))+Cb(\tan(dx+c)-dx-c)+Ba(dx+c)-Ca\ln(\cos(dx+c))}{d}$
default	$\frac{-Bb\ln(\cos(dx+c))+Cb(\tan(dx+c)-dx-c)+Ba(dx+c)-Ca\ln(\cos(dx+c))}{d}$
risch	$iBbx+iCax+Bax-Cbx+\frac{2iBbc}{d}+\frac{2iCac}{d}+\frac{2iCb}{d\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)Bb}{d}-\frac{\ln\left(e^{2i(dx+c}\right.}{d}$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RE
TURNVERBOSE)
```

output
```
1/2*((B*b+C*a)*ln(sec(d*x+c)^2)+2*C*b*tan(d*x+c)+2*d*x*(B*a-C*b))/d
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,(Ba-Cb)dx+2\,Cb\tan\left(dx+c\right)-(Ca+Bb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="fricas")
```

output
```
1/2*(2*(B*a - C*b)*d*x + 2*C*b*tan(d*x + c) - (C*a + B*b)*log(1/(tan(d*x +
 c)^2 + 1)))/d
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases} Bax+\frac{Bb\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ca\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-Cbx+\frac{Cb\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot\left(c\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a*x + B*b*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*
x)**2 + 1)/(2*d) - C*b*x + C*b*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c)
)*(B*tan(c) + C*tan(c)**2)*cot(c), True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{2\,Cb\tan(dx+c)+2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="maxima")
```

output
```
1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*
x + c)^2 + 1))/d
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{2\,Cb\tan(dx+c)+2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, alg
orithm="giac")
```

output
```
1/2*(2*C*b*tan(d*x + c) + 2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*
x + c)^2 + 1))/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 7.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = B\,a\,x-C\,b\,x+\frac{C\,b\tan(c+d\,x)}{d}+\frac{B\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}+\frac{C\,a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)),
x)
```

output
```
B*a*x - C*b*x + (C*b*tan(c + d*x))/d + (B*b*log(tan(c + d*x)^2 + 1))/(2*d)
 + (C*a*log(tan(c + d*x)^2 + 1))/(2*d)
```

3.3. $\int \cot(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.4 $\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.4.1 Optimal result

Integrand size = 38, antiderivative size = 37

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = (bB+aC)x-\frac{bC\log(\cos(c+dx))}{d}+\frac{aB\log(\sin(c+dx))}{d}$$

output
```
(B*b+C*a)*x-b*C*ln(cos(d*x+c))/d+a*B*ln(sin(d*x+c))/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = bBx+aCx+\frac{aB\log(\cos(c+dx))}{d}-\frac{bC\log(\cos(c+dx))}{d}+\frac{aB\log(\tan(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
b*B*x + a*C*x + (a*B*Log[Cos[c + d*x]])/d - (b*C*Log[Cos[c + d*x]])/d + (a
*B*Log[Tan[c + d*x]])/d
```

3.4.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4115, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(B+C\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4072}$$

$$\int \cot(c+dx)(aB+(bB+aC)\tan(c+dx))dx+bC\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aB+(bB+aC)\tan(c+dx)}{\tan(c+dx)}dx+bC\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{aB+(bB+aC)\tan(c+dx)}{\tan(c+dx)}dx-\frac{bC\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{4014}$$

$$aB\int \cot(c+dx)dx+x(aC+bB)-\frac{bC\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$aB\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+x(aC+bB)-\frac{bC\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-aB\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+x(aC+bB)-\frac{bC\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3956}$$

$$x(aC+bB)+\frac{aB\log(-\sin(c+dx))}{d}-\frac{bC\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(b*B + a*C)*x - (b*C*Log[Cos[c + d*x]])/d + (a*B*Log[-Sin[c + d*x]])/d
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{Bb(dx+c)-Cb\ln(\cos(dx+c))+Ba\ln(\sin(dx+c))+Ca(dx+c)}{d}$	43
default	$\frac{Bb(dx+c)-Cb\ln(\cos(dx+c))+Ba\ln(\sin(dx+c))+Ca(dx+c)}{d}$	43
parallelrisch	$\frac{(-Ba+Cb)\ln\left(\sec(dx+c)^{2}\right)+2Ba\ln(\tan(dx+c))+2x(Bb+Ca)d}{2d}$	47
norman	$(Bb+Ca)\,x+\frac{Ba\ln(\tan(dx+c))}{d}-\frac{(Ba-Cb)\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$	48
risch	$Bbx+Cax-iBax+iCbx+\frac{2iCbc}{d}-\frac{2iBac}{d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)Cb}{d}+\frac{Ba\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$	77

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(B*b*(d*x+c)-C*b*ln(cos(d*x+c))+B*a*ln(sin(d*x+c))+C*a*(d*x+c))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,(Ca+Bb)dx+Ba\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-Cb\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
1/2*(2*(C*a + B*b)*d*x + B*a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - C*
b*log(1/(tan(d*x + c)^2 + 1)))/d
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(34) = 68$.

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \begin{cases} -\frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Ba\log\left(\tan(c+dx)\right)}{d}+Bbx+Cax+\frac{Cb\log\left(\tan^2(c+dx)+1\right)}{2d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((-B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d +
 B*b*x + C*a*x + C*b*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(a + b*
tan(c))*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{2\,Ba\log\left(\tan\left(dx+c\right)\right)+2\left(Ca+Bb\right)\left(dx+c\right)-\left(Ba-Cb\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/2*(2*B*a*log(tan(d*x + c)) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(t
an(d*x + c)^2 + 1))/d
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Ba\log\left(|\tan\left(dx+c\right)|\right)+2\,(Ca+Bb)(dx+c)-(Ba-Cb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
1/2*(2*B*a*log(abs(tan(d*x + c))) + 2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*
log(tan(d*x + c)^2 + 1))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.86

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{B\,a\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d) - (log(tan(c + d*x
) - 1i)*(B + C*1i)*(a + b*1i))/(2*d) + (B*a*log(tan(c + d*x)))/d
```

3.5 $\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.5.1 Optimal result

Integrand size = 38, antiderivative size = 43

$$\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -((aB-bC)x)-\frac{aB\cot(c+dx)}{d}+\frac{(bB+aC)\log(\sin(c+dx))}{d}$$

output
```
-(B*a-C*b)*x-a*B*cot(d*x+c)/d+(B*b+C*a)*ln(sin(d*x+c))/d
```

3.5.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx &= bCx-\frac{aB\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}\\ &\quad+\frac{bB\log(\cos(c+dx))}{d}+\frac{aC\log(\cos(c+dx))}{d}\\ &\quad+\frac{bB\log(\tan(c+dx))}{d}+\frac{aC\log(\tan(c+dx))}{d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
b*C*x - (a*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]
)/d + (b*B*Log[Cos[c + d*x]])/d + (a*C*Log[Cos[c + d*x]])/d + (b*B*Log[Tan
[c + d*x]])/d + (a*C*Log[Tan[c + d*x]])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4115, 3042, 4074, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(B+C\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \cot(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{aB\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)}dx-\frac{aB\cot(c+dx)}{d}$$

$$\Big\downarrow \text{4014}$$

$$(aC+bB)\int \cot(c+dx)dx-(x(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$(aC+bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-(x(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-(aC+bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-(x(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(aC+bB)\log(-\sin(c+dx))}{d}-(x(aB-bC))-\frac{aB\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
-((a*B - b*C)*x) - (a*B*Cot[c + d*x])/d + ((b*B + a*C)*Log[-Sin[c + d*x]])
/d
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{Bb\ln(\sin(dx+c))+Cb(dx+c)+Ba(-\cot(dx+c)-dx-c)+Ca\ln(\sin(dx+c))}{d}$
default	$\frac{Bb\ln(\sin(dx+c))+Cb(dx+c)+Ba(-\cot(dx+c)-dx-c)+Ca\ln(\sin(dx+c))}{d}$
parallelrisch	$\frac{(-Bb-Ca)\ln\left(\sec(dx+c)^2\right)+(2Bb+2Ca)\ln(\tan(dx+c))-2B\cot(dx+c)a-2dx(Ba-Cb)}{2d}$
norman	$\frac{(-Ba+Cb)x\tan(dx+c)^2-\frac{Ba\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{(Bb+Ca)\ln(\tan(dx+c))}{d}-\frac{(Bb+Ca)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
risch	$-iBbx-iCax-Bax+Cbx-\frac{2iBbc}{d}-\frac{2iCac}{d}-\frac{2iBa}{d\left(e^{2i(dx+c)}-1\right)}+\frac{\ln\left(e^{2i(dx+c)}-1\right)Bb}{d}+\frac{\ln(e^{2i(dx}$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(B*b*ln(sin(d*x+c))+C*b*(d*x+c)+B*a*(-cot(d*x+c)-d*x-c)+C*a*ln(sin(d*x
+c)))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \cot^3(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$
$$= -\frac{2\,(Ba-Cb)dx\tan\left(dx+c\right) - (Ca+Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right) + 2\,Ba}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
-1/2*(2*(B*a - C*b)*d*x*tan(d*x + c) - (C*a + B*b)*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1))*tan(d*x + c) + 2*B*a)/(d*tan(d*x + c))
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.

Time = 0.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.70

$$\int \cot^3(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$
$$= \begin{cases} \text{NaN} \\ x(a+b\tan\left(c\right))\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^3\left(c\right) \\ \text{NaN} \\ -Bax - \frac{Ba}{d\tan\left(c+dx\right)} - \frac{Bb\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Bb\log\left(\tan\left(c+dx\right)\right)}{d} - \frac{Ca\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Ca\log\left(\tan\left(c+dx\right)\right)}{d} + Cbx \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(
c)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a*x - B*a/(d*tan(c +
d*x)) - B*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*b*log(tan(c + d*x))/d - C*a
*log(tan(c + d*x)**2 + 1)/(2*d) + C*a*log(tan(c + d*x))/d + C*b*x, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \cot^3(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx = -\frac{2\,(Ba-Cb)(dx+c)+(Ca+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ca+Bb)\log\left(\tan\left(dx+c\right)\right)+\frac{2\,Ba}{\tan(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(B*a - C*b)*(d*x + c) + (C*a + B*b)*log(tan(d*x + c)^2 + 1) - 2*(C
*a + B*b)*log(tan(d*x + c)) + 2*B*a/tan(d*x + c))/d
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(43) = 86.

Time = 1.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.77

$$\int \cot^3(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx = \frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,(Ba-Cb)(dx+c)-2\,(Ca+Bb)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)+2\,(Ca+Bb)\log}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
1/2*(B*a*tan(1/2*d*x + 1/2*c) - 2*(B*a - C*b)*(d*x + c) - 2*(C*a + B*b)*lo
g(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c)
)) - (2*C*a*tan(1/2*d*x + 1/2*c) + 2*B*b*tan(1/2*d*x + 1/2*c) + B*a)/tan(1
/2*d*x + 1/2*c))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \cot^3(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx = \frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,b+C\,a\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)}{2\,d} -\frac{B\,a\cot(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x))*(B*b + C*a))/d + (log(tan(c + d*x) - 1i)*(B + C*1i)*(a
+ b*1i)*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b))/(2*d) -
 (B*a*cot(c + d*x))/d
```

3.6 $\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2\right.$

3.6.1 Optimal result

Integrand size = 38, antiderivative size = 66

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=-((bB+aC)x)-\frac{(bB+aC)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)}{2d}-\frac{(aB-bC)\log(\sin(c+dx))}{d}$$

output
```
-(B*b+C*a)*x-(B*b+C*a)*cot(d*x+c)/d-1/2*a*B*cot(d*x+c)^2/d-(B*a-C*b)*ln(si
n(d*x+c))/d
```

3.6.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=$$
$$-\frac{aB\cot^2(c+dx)+2(bB+aC)\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)+2(aB-b}{2d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
-1/2*(a*B*Cot[c + d*x]^2 + 2*(b*B + a*C)*Cot[c + d*x]*Hypergeometric2F1[-1
/2, 1, 1/2, -Tan[c + d*x]^2] + 2*(a*B - b*C)*(Log[Cos[c + d*x]] + Log[Tan[
c + d*x]]))/d
```

3.6.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4115, 3042, 4074, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(B+C\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \cot^2(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4012}$$

$$\int -\cot(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{aB-bC+(bB+aC)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4014}$$

$$-(aB-bC)\int \cot(c+dx)dx-\frac{(aC+bB)\cot(c+dx)}{d}-x(aC+bB)-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-(aB-bC)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aC+bB)\cot(c+dx)}{d}-x(aC+bB)-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$(aB-bC)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aC+bB)\cot(c+dx)}{d}-x(aC+bB)-\frac{aB\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aC+bB)\cot(c+dx)}{d}-\frac{(aB-bC)\log(-\sin(c+dx))}{d}-x(aC+bB)-\frac{aB\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
-((b*B + a*C)*x) - ((b*B + a*C)*Cot[c + d*x])/d - (a*B*Cot[c + d*x]^2)/(2*
d) - ((a*B - b*C)*Log[-Sin[c + d*x]])/d
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.6. $\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.6.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{Bb(-\cot(dx+c)-dx-c)+Cb\ln(\sin(dx+c))+Ba\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Ca(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{Bb(-\cot(dx+c)-dx-c)+Cb\ln(\sin(dx+c))+Ba\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Ca(-\cot(dx+c)-dx-c)}{d}$
parallelrisch	$\frac{-Ba\cot(dx+c)^2-2Bbdx-2Cadx-2Bb\cot(dx+c)-2Ba\ln(\tan(dx+c))+B\ln\left(\sec(dx+c)^2\right)a-2Ca\cot(dx+c)+2C\ln(t}{2d}$
norman	$\frac{(-Bb-Ca)x\tan(dx+c)^3-\frac{(Bb+Ca)\tan(dx+c)^2}{d}-\frac{Ba\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(Ba-Cb)\ln(\tan(dx+c))}{d}+\frac{(Ba-Cb)\ln\left(1+\tan(dx\right.}{2d}$
risch	$-Bbx-Cax+iBax-iCbx+\frac{2iBac}{d}-\frac{2iCbc}{d}-\frac{2i\left(iBa\,e^{2i(dx+c)}+Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+c)}-Bb-\right.}{d\left(e^{2i(dx+c)}-1\right)^2}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(B*b*(-cot(d*x+c)-d*x-c)+C*b*ln(sin(d*x+c))+B*a*(-1/2*cot(d*x+c)^2-ln(
sin(d*x+c)))+C*a*(-cot(d*x+c)-d*x-c))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{(Ba-Cb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+(2(Ca+Bb)dx+Ba)\tan(dx+c)^2+Ba+2(Ca+Bb)}{2\,d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
-1/2*((B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2
+ (2*(C*a + B*b)*d*x + B*a)*tan(d*x + c)^2 + B*a + 2*(C*a + B*b)*tan(d*x +
 c))/(d*tan(d*x + c)^2)
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(56) = 112$.

Time = 1.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.17

$$\int \cot^4(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$
$$= \begin{cases} \text{NaN} \\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot^4(c) \\ \text{NaN} \\ \frac{Ba\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba\log(\tan(c+dx))}{d} - \frac{Ba}{2d\tan^2(c+dx)} - Bbx - \frac{Bb}{d\tan(c+dx)} - Cax - \frac{Ca}{d\tan(c+dx)} - \frac{Cb\log(\tan^2(\cdots}{2d} \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(
c)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a*log(tan(c + d*x)**2
+ 1)/(2*d) - B*a*log(tan(c + d*x))/d - B*a/(2*d*tan(c + d*x)**2) - B*b*x -
 B*b/(d*tan(c + d*x)) - C*a*x - C*a/(d*tan(c + d*x)) - C*b*log(tan(c + d*x
)**2 + 1)/(2*d) + C*b*log(tan(c + d*x))/d, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \cot^4(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx =$$
$$-\frac{2\,(Ca+Bb)(dx+c) - (Ba-Cb)\log\left(\tan(dx+c)^2+1\right) + 2\,(Ba-Cb)\log\left(\tan(dx+c)\right) + \frac{Ba+2\,(Ca+\cdots}{\tan(\cdots}}{2\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(C*a + B*b)*(d*x + c) - (B*a - C*b)*log(tan(d*x + c)^2 + 1) + 2*(B
*a - C*b)*log(tan(d*x + c)) + (B*a + 2*(C*a + B*b)*tan(d*x + c))/tan(d*x +
 c)^2)/d
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(64) = 128.

Time = 1.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.71

$$\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,(Ca+Bb)(dx+c)-8\,(Ba-$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
-1/8*(B*a*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x + 1/2*c) - 4*B*b*tan(
1/2*d*x + 1/2*c) + 8*(C*a + B*b)*(d*x + c) - 8*(B*a - C*b)*log(tan(1/2*d*x
 + 1/2*c)^2 + 1) + 8*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a*
tan(1/2*d*x + 1/2*c)^2 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 4*C*a*tan(1/2*d*x
 + 1/2*c) - 4*B*b*tan(1/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\ = -\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a-C\,b\right)}{d}-\frac{\cot(c+d\,x)^2\,\left(\frac{B\,a}{2}+\tan(c+d\,x)\,\left(B\,b+C\,a\right)\right)}{d}\\ +\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}\\ -\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}\end{aligned}$$

input
```
int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i))/(2*d) - (cot(c + d*x)^2*((B
*a)/2 + tan(c + d*x)*(B*b + C*a)))/d - (log(tan(c + d*x))*(B*a - C*b))/d -
 (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d)
```

3.7 $\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.7.1 Optimal result

Integrand size = 38, antiderivative size = 87

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = (aB-bC)x+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(bB+aC)\cot^2(c+dx)}{2d}-\frac{aB\cot^3(c+dx)}{3d}-\frac{(bB+aC)\log(\sin(c+dx))}{d}$$

output
```
(B*a-C*b)*x+(B*a-C*b)*cot(d*x+c)/d-1/2*(B*b+C*a)*cot(d*x+c)^2/d-1/3*a*B*co
t(d*x+c)^3/d-(B*b+C*a)*ln(sin(d*x+c))/d
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -\frac{2aB\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)+6bC\cot(c+dx)\,\text{Hypergeometric2}}{6a}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
-1/6*(2*a*B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^
2] + 6*b*C*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] +
 3*(b*B + a*C)*(Cot[c + d*x]^2 + 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])
))/d
```

3.7.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4115, 3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \ 4115$$

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))(B+C\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \ 4074$$

$$\int \cot^3(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \ 4012$$

$$\int -\cot^2(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \ 25$$

$$-\int \cot^2(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{aB-bC+(bB+aC)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4012}$$

$$-\int \cot(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4014}$$

$$-(aC+bB)\int \cot(c+dx)dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-(aC+bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$(aC+bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aC+bB)\cot^2(c+dx)}{2d}+\frac{(aB-bC)\cot(c+dx)}{d}-\frac{(aC+bB)\log(-\sin(c+dx))}{d}+x(aB-bC)-\frac{aB\cot^3(c+dx)}{3d}$$

3.7. $\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(a*B - b*C)*x + ((a*B - b*C)*Cot[c + d*x])/d - ((b*B + a*C)*Cot[c + d*x]^2
)/(2*d) - (a*B*Cot[c + d*x]^3)/(3*d) - ((b*B + a*C)*Log[-Sin[c + d*x]])/d
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{Bb\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Cb(-\cot(dx+c)-dx-c)+Ba\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+Ca\left(-\frac{\cot(dx+c}{2}\right.}{d}$
default	$\frac{Bb\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Cb(-\cot(dx+c)-dx-c)+Ba\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+Ca\left(-\frac{\cot(dx+c}{2}\right.}{d}$
parallelrisch	$\frac{-2Ba\cot(dx+c)^3-3Bb\cot(dx+c)^2+6Badx-3Ca\cot(dx+c)^2-6Cbdx+6B\cot(dx+c)a-6B\ln(\tan(dx+c))b+3B\ln\left(\mathrm{se}\right.}{6d}$
norman	$\frac{\frac{(Ba-Cb)\tan(dx+c)^3}{d}+(Ba-Cb)x\tan(dx+c)^4-\frac{(Bb+Ca)\tan(dx+c)^2}{2d}-\frac{Ba\tan(dx+c)}{3d}}{\tan(dx+c)^4}-\frac{(Bb+Ca)\ln(\tan(dx+c))}{d}+\frac{(Bb}{}$
risch	$iBbx+iCax+Bax-Cbx+\frac{2iBbc}{d}+\frac{2iCac}{d}-\frac{2i(3iBb\,e^{4i(dx+c)}+3iCa\,e^{4i(dx+c)}-6Ba\,e^{4i(dx+c)}+3Cb}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(B*b*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*b*(-cot(d*x+c)-d*x-c)+B*a*(-
1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+C*a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{3\,(Ca+Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3-3\,(2\,(Ba-Cb)dx-Ca-Bb)\tan\left(dx+c\right)^3-6\,(Ba-C}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
-1/6*(3*(C*a + B*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
3 - 3*(2*(B*a - C*b)*d*x - C*a - B*b)*tan(d*x + c)^3 - 6*(B*a - C*b)*tan(d
*x + c)^2 + 2*B*a + 3*(C*a + B*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(75) = 150.

Time = 1.88 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.99

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$=\begin{cases}\text{NaN} \\ x(a+b\tan(c))\left(B\tan(c)+C\tan^2(c)\right)\cot^5(c) \\ \text{NaN} \\ Bax+\frac{Ba}{d\tan(c+dx)}-\frac{Ba}{3d\tan^3(c+dx)}+\frac{Bb\log(\tan^2(c+dx)+1)}{2d}-\frac{Bb\log(\tan(c+dx))}{d}-\frac{Bb}{2d\tan^2(c+dx)}+\frac{Ca\log(\tan^2(c+dx)+}{2d}\end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(
c)**2)*cot(c)**5, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a*x + B*a/(d*tan(c + d
*x)) - B*a/(3*d*tan(c + d*x)**3) + B*b*log(tan(c + d*x)**2 + 1)/(2*d) - B*
b*log(tan(c + d*x))/d - B*b/(2*d*tan(c + d*x)**2) + C*a*log(tan(c + d*x)**
2 + 1)/(2*d) - C*a*log(tan(c + d*x))/d - C*a/(2*d*tan(c + d*x)**2) - C*b*x
 - C*b/(d*tan(c + d*x)), True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \cot^5(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{6\,(Ba-Cb)(dx+c)+3\,(Ca+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ca+Bb)\log\left(\tan\left(dx+c\right)\right)+\frac{6\,(Ba-Cb)\,\mathrm{ta}}{}}{6\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/6*(6*(B*a - C*b)*(d*x + c) + 3*(C*a + B*b)*log(tan(d*x + c)^2 + 1) - 6*(
C*a + B*b)*log(tan(d*x + c)) + (6*(B*a - C*b)*tan(d*x + c)^2 - 2*B*a - 3*(
C*a + B*b)*tan(d*x + c))/tan(d*x + c)^3)/d
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(83) = 166.

Time = 1.61 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.72

$$\int \cot^5(c+dx)(a+b\tan(c+dx)) \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Cb\,\mathrm{t}}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
1/24*(B*a*tan(1/2*d*x + 1/2*c)^3 - 3*C*a*tan(1/2*d*x + 1/2*c)^2 - 3*B*b*ta
n(1/2*d*x + 1/2*c)^2 - 15*B*a*tan(1/2*d*x + 1/2*c) + 12*C*b*tan(1/2*d*x +
1/2*c) + 24*(B*a - C*b)*(d*x + c) + 24*(C*a + B*b)*log(tan(1/2*d*x + 1/2*c
)^2 + 1) - 24*(C*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a*tan(1/2
*d*x + 1/2*c)^3 + 44*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*B*a*tan(1/2*d*x + 1/2
*c)^2 - 12*C*b*tan(1/2*d*x + 1/2*c)^2 - 3*C*a*tan(1/2*d*x + 1/2*c) - 3*B*b
*tan(1/2*d*x + 1/2*c) - B*a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int \cot^5(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= -\frac{\cot(c+d\,x)^3\,\left((C\,b-B\,a)\,\tan(c+d\,x)^2+\left(\frac{B\,b}{2}+\frac{C\,a}{2}\right)\,\tan(c+d\,x)+\frac{B\,a}{3}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)\right)\,(B\,b+C\,a)}{d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(B+C\,1\mathrm{i})\,(a+b\,1\mathrm{i})\,1\mathrm{i}}{2\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(B-C\,1\mathrm{i})\,(b+a\,1\mathrm{i})}{2\,d}$$

input
```
int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b))/(2*d) - (log(tan(c + d*x))*
(B*b + C*a))/d - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i)*1i)/(2*d) -
 (cot(c + d*x)^3*((B*a)/3 + tan(c + d*x)*((B*b)/2 + (C*a)/2) - tan(c + d*x
)^2*(B*a - C*b)))/d
```

3.8 $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.8.1 Optimal result

Integrand size = 38, antiderivative size = 108

$$\begin{aligned}\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\ =(bB+aC)x+\frac{(bB+aC)\cot(c+dx)}{d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}\\ -\frac{(bB+aC)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}+\frac{(aB-bC)\log(\sin(c+dx))}{d}\end{aligned}$$

output
```
(B*b+C*a)*x+(B*b+C*a)*cot(d*x+c)/d+1/2*(B*a-C*b)*cot(d*x+c)^2/d-1/3*(B*b+C
*a)*cot(d*x+c)^3/d-1/4*a*B*cot(d*x+c)^4/d+(B*a-C*b)*ln(sin(d*x+c))/d
```

3.8.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -\frac{4(bB+aC)\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)+3((-2aB+2bC)\cot^2(c+}{12d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
-1/12*(4*(b*B + a*C)*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[
c + d*x]^2] + 3*((-2*a*B + 2*b*C)*Cot[c + d*x]^2 + a*B*Cot[c + d*x]^4 - 4*
(a*B - b*C)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/d
```

3.8.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.447$, Rules used = {3042, 4115, 3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(B+C\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \cot^4(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)^4}dx-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4012}$$

$$\int -\cot^3(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^3(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{aB-bC+(bB+aC)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4012}$$

$$-\int \cot^2(c+dx)(bB+aC-(aB-bC)\tan(c+dx))dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{bB+aC-(aB-bC)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4012}$$

$$-\int -\cot(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+dx)(aB-bC+(bB+aC)\tan(c+dx))dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aB-bC+(bB+aC)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4014}$$

$$(aB-bC)\int \cot(c+dx)dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+x(aC+bB)-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

3.8. $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$(aB-bC)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+x(aC+bB)-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow 25$$

$$-(aB-bC)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+x(aC+bB)-\frac{aB\cot^4(c+dx)}{4d}$$

$$\Big\downarrow 3956$$

$$-\frac{(aC+bB)\cot^3(c+dx)}{3d}+\frac{(aB-bC)\cot^2(c+dx)}{2d}+\frac{(aC+bB)\cot(c+dx)}{d}+\frac{(aB-bC)\log(-\sin(c+dx))}{d}+x(aC+bB)-\frac{aB\cot^4(c+dx)}{4d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(b*B + a*C)*x + ((b*B + a*C)*Cot[c + d*x])/d + ((a*B - b*C)*Cot[c + d*x]^2
)/(2*d) - ((b*B + a*C)*Cot[c + d*x]^3)/(3*d) - (a*B*Cot[c + d*x]^4)/(4*d)
+ ((a*B - b*C)*Log[-Sin[c + d*x]])/d
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.8. $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.8.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{Bb\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+Cb\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Ba\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx-}{d}$
default	$\frac{Bb\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+Cb\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+Ba\left(-\frac{\cot(dx+c)^4}{4}+\frac{\cot(dx+c)^2}{2}+\ln(\sin(dx-}{d}$
norman	$\frac{\frac{(Bb+Ca)\tan(dx+c)^4}{d}+(Bb+Ca)x\tan(dx+c)^5-\frac{(Bb+Ca)\tan(dx+c)^2}{3d}+\frac{(Ba-Cb)\tan(dx+c)^3}{2d}-\frac{Ba\tan(dx+c)}{4d}}{\tan(dx+c)^5}+\frac{(Ba-Cb)\,\mathrm{l}}{}$
parallelrisch	$\frac{-3Ba\cot(dx+c)^4-4Bb\cot(dx+c)^3-4Ca\cot(dx+c)^3+6Ba\cot(dx+c)^2+12Bbdx-6Cb\cot(dx+c)^2+12Cadx+12Bb\,\mathrm{co}}{12d}$
risch	$Bbx+Cax-iBax+iCbx-\frac{2iBac}{d}+\frac{2iCbc}{d}-\frac{2\left(-6iBb\,\mathrm{e}^{6i(dx+c)}-6iCa\,\mathrm{e}^{6i(dx+c)}+6Ba\,\mathrm{e}^{6i(dx+c)}-3C\right.}{}$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(B*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+C*b*(-1/2*cot(d*x+c)^2-ln(si
n(d*x+c)))+B*a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+C*a*(-1
/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{6\,(Ba-Cb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,(4\,(Ca+Bb)dx+3\,Ba-2\,Cb)\tan\left(dx+c\right)^4+12\,(Ca+}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
1/12*(6*(B*a - C*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
4 + 3*(4*(C*a + B*b)*d*x + 3*B*a - 2*C*b)*tan(d*x + c)^4 + 12*(C*a + B*b)*
tan(d*x + c)^3 + 6*(B*a - C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(
d*x + c))/(d*tan(d*x + c)^4)
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(95) = 190$.

Time = 2.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.89

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \begin{cases} \text{NaN} \\ x(a+b\tan{(c)})\left(B\tan{(c)}+C\tan^2{(c)}\right)\cot^6{(c)} \\ \text{NaN} \\ -\frac{Ba\log{(\tan^2{(c+dx)}+1)}}{2d}+\frac{Ba\log{(\tan{(c+dx)})}}{d}+\frac{Ba}{2d\tan^2{(c+dx)}}-\frac{Ba}{4d\tan^4{(c+dx)}}+Bbx+\frac{Bb}{d\tan{(c+dx)}}-\frac{Bb}{3d\tan^3{(c+dx)}}+ \end{cases}$$

input
```
integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))*(B*tan(c) + C*tan(
c)**2)*cot(c)**6, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a*log(tan(c + d*x)**2
 + 1)/(2*d) + B*a*log(tan(c + d*x))/d + B*a/(2*d*tan(c + d*x)**2) - B*a/(4
*d*tan(c + d*x)**4) + B*b*x + B*b/(d*tan(c + d*x)) - B*b/(3*d*tan(c + d*x)
**3) + C*a*x + C*a/(d*tan(c + d*x)) - C*a/(3*d*tan(c + d*x)**3) + C*b*log(
tan(c + d*x)**2 + 1)/(2*d) - C*b*log(tan(c + d*x))/d - C*b/(2*d*tan(c + d*
x)**2), True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \frac{12\,(Ca+Bb)(dx+c)-6\,(Ba-Cb)\log\left(\tan{(dx+c)}^2+1\right)+12\,(Ba-Cb)\log\left(\tan{(dx+c)}\right)+\frac{12\,(Ca+B}{}}{12\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/12*(12*(C*a + B*b)*(d*x + c) - 6*(B*a - C*b)*log(tan(d*x + c)^2 + 1) + 1
2*(B*a - C*b)*log(tan(d*x + c)) + (12*(C*a + B*b)*tan(d*x + c)^3 + 6*(B*a
- C*b)*tan(d*x + c)^2 - 3*B*a - 4*(C*a + B*b)*tan(d*x + c))/tan(d*x + c)^4
)/d
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(102) = 204.

Time = 1.34 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.77

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,Ca\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
-1/192*(3*B*a*tan(1/2*d*x + 1/2*c)^4 - 8*C*a*tan(1/2*d*x + 1/2*c)^3 - 8*B*
b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 + 24*C*b*tan(1/2*
d*x + 1/2*c)^2 + 120*C*a*tan(1/2*d*x + 1/2*c) + 120*B*b*tan(1/2*d*x + 1/2*
c) - 192*(C*a + B*b)*(d*x + c) + 192*(B*a - C*b)*log(tan(1/2*d*x + 1/2*c)^
2 + 1) - 192*(B*a - C*b)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a*tan(1/2
*d*x + 1/2*c)^4 - 400*C*b*tan(1/2*d*x + 1/2*c)^4 - 120*C*a*tan(1/2*d*x + 1
/2*c)^3 - 120*B*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*tan(1/2*d*x + 1/2*c)^2 +
 24*C*b*tan(1/2*d*x + 1/2*c)^2 + 8*C*a*tan(1/2*d*x + 1/2*c) + 8*B*b*tan(1/
2*d*x + 1/2*c) + 3*B*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a-C\,b\right)}{d}$$

$$-\frac{\cot(c+d\,x)^4\,\left(\left(-B\,b-C\,a\right)\,\tan(c+d\,x)^3+\left(\frac{C\,b}{2}-\frac{B\,a}{2}\right)\,\tan(c+d\,x)^2+\left(\frac{B\,b}{3}+\frac{C\,a}{3}\right)\,\tan(c+d\,x)+\frac{B\,a}{4}\right.}{d}$$

$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}$$

$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x))*(B*a - C*b))/d - (cot(c + d*x)^4*((B*a)/4 + tan(c + d*x
)*((B*b)/3 + (C*a)/3) - tan(c + d*x)^3*(B*b + C*a) - tan(c + d*x)^2*((B*a)
/2 - (C*b)/2)))/d - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i))/(2*d) +
 (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)*1i)/(2*d)
```

3.8. $\int \cot^6(c+dx)(a+b\tan(c+dx))\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.9 $\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2\right.$

3.9.1 Optimal result

Integrand size = 38, antiderivative size = 148

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= -\left(\left(a^2B-b^2B-2abC\right)x\right)+\frac{(2abB+a^2C-b^2C)\log(\cos(c+dx))}{d} - \frac{b(bB+aC)\tan(c+dx)}{d}-\frac{C(a+b\tan(c+dx))^2}{2d} + \frac{(4bB-aC)(a+b\tan(c+dx))^3}{12b^2d}+\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}$$

output
```
-(B*a^2-B*b^2-2*C*a*b)*x+(2*B*a*b+C*a^2-C*b^2)*ln(cos(d*x+c))/d-b*(B*b+C*a
)*tan(d*x+c)/d-1/2*C*(a+b*tan(d*x+c))^2/d+1/12*(4*B*b-C*a)*(a+b*tan(d*x+c)
)^3/b^2/d+1/4*C*tan(d*x+c)*(a+b*tan(d*x+c))^3/b/d
```

3.9.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.49

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd} + \frac{\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}+\frac{2\left((bB-aC)\left(i(a+ib)^2\log(i-\tan(c+dx))-i(a-ib)^2\log(i+\tan(c+dx))-2b^2\tan(c+dx)\right)-C\left((ia-b)^3\log(i-\text{ta}\right.\right.}{d}}{4b}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
(C*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) + (((4*b*B - a*C)*(a + b*T
an[c + d*x])^3)/(3*b*d) + (2*((b*B - a*C)*(I*(a + I*b)^2*Log[I - Tan[c + d
*x]] - I*(a - I*b)^2*Log[I + Tan[c + d*x]] - 2*b^2*Tan[c + d*x]) - C*((I*a
 - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b^
2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)))/d)/(4*b)
```

3.9.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 4115, 3042, 4090, 25, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \; 4115$$

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \; 4090$$

$$\frac{\int -(a+b\tan(c+dx))^2\left(-((4bB-aC)\tan^2(c+dx))+4bC\tan(c+dx)+aC\right)dx}{4b}+\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}$$

$$\Big\downarrow \; 25$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))^2\left(-\left((4bB-aC)\tan^2(c+dx)\right)+4bC\tan(c+dx)+aC\right)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))^2\left(-\left((4bB-aC)\tan(c+dx)^2\right)+4bC\tan(c+dx)+aC\right)dx}{4b}$$

$$\Big\downarrow \text{4113}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))^2(4bB+4bC\tan(c+dx))dx-\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))^2(4bB+4bC\tan(c+dx))dx-\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}}{4b}$$

$$\Big\downarrow \text{4011}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))(4b(aB-bC)+4b(bB+aC)\tan(c+dx))dx-\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}+\frac{2bC(a+b\tan(c+dx))^2}{d}}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int(a+b\tan(c+dx))(4b(aB-bC)+4b(bB+aC)\tan(c+dx))dx-\frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd}+\frac{2bC(a+b\tan(c+dx))^2}{d}}{4b}$$

$$\Big\downarrow \text{4008}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{4b\left(a^2C+2abB-b^2C\right)\int\tan(c+dx)dx+4bx\left(a^2B-2abC-b^2B\right)+\frac{4b^2(aC+bB)\tan(c+dx)}{d}-\frac{(4bB-aC)(a+b\tan(c+dx}{3bd}}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{4b\left(a^2C+2abB-b^2C\right)\int\tan(c+dx)dx+4bx\left(a^2B-2abC-b^2B\right)+\frac{4b^2(aC+bB)\tan(c+dx)}{d}-\frac{(4bB-aC)(a+b\tan(c+dx}{3bd}}{4b}$$

3.9. $\int\tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\Big\downarrow \; 3956$$

$$\frac{C\tan(c+dx)(a+b\tan(c+dx))^3}{4bd} - \frac{-\frac{4b(a^2C+2abB-b^2C)\log(\cos(c+dx))}{d} + 4bx\left(a^2B-2abC-b^2B\right) + \frac{4b^2(aC+bB)\tan(c+dx)}{d} - \frac{(4bB-aC)(a+b\tan(c+dx))^3}{3bd} + 2bC(}{4b}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(C*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) - (4*b*(a^2*B - b^2*B - 2*
a*b*C)*x - (4*b*(2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x]])/d + (4*b^2*(b
*B + a*C)*Tan[c + d*x])/d + (2*b*C*(a + b*Tan[c + d*x])^2)/d - ((4*b*B - a
*C)*(a + b*Tan[c + d*x])^3)/(3*b*d))/(4*b)
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.9.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

method	result
parts	$\frac{(B\,b^2+2Cab)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{(2Bab+C\,a^2)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+$
norman	$(-B\,a^2+B\,b^2+2Cab)\,x+\frac{(B\,a^2-B\,b^2-2Cab)\tan(dx+c)}{d}+\frac{(2Bab+C\,a^2-C\,b^2)\tan(dx+c)^2}{2d}+\frac{C\,b^2\tan}{}$
derivativedivides	$\frac{\frac{C\,b^2\tan(dx+c)^4}{4}+\frac{B\,b^2\tan(dx+c)^3}{3}+\frac{2Cab\tan(dx+c)^3}{3}+Bab\tan(dx+c)^2+\frac{C\,a^2\tan(dx+c)^2}{2}-\frac{C\,b^2\tan(dx+c)^2}{2}+B\,a^2\tan(dx+}{}$
default	$\frac{\frac{C\,b^2\tan(dx+c)^4}{4}+\frac{B\,b^2\tan(dx+c)^3}{3}+\frac{2Cab\tan(dx+c)^3}{3}+Bab\tan(dx+c)^2+\frac{C\,a^2\tan(dx+c)^2}{2}-\frac{C\,b^2\tan(dx+c)^2}{2}+B\,a^2\tan(dx+}{}$
parallelrisch	$-\frac{-3C\,b^2\tan(dx+c)^4-4B\,b^2\tan(dx+c)^3-8Cab\tan(dx+c)^3+12B\,a^2dx-12B\,b^2dx-12Bab\tan(dx+c)^2-24Cabdx-6C}{}$
risch	$-B\,a^2x+B\,b^2x+2Cabx-iC\,a^2x+iC\,b^2x+\frac{2i\left(6iC\,b^2e^{4i(dx+c)}-6iBab\,e^{6i(dx+c)}+6iC\,b^2e^{6i(dx+c)}\right.}{}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
(B*b^2+2*C*a*b)/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))+(2*B*a*
b+C*a^2)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+B*a^2/d*(tan(d*x+c)-a
rctan(tan(d*x+c)))+C*b^2/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan
(d*x+c)^2))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{3\,Cb^2\tan{(dx+c)}^4+4\,(2\,Cab+Bb^2)\tan{(dx+c)}^3-12\,(Ba^2-2\,Cab-Bb^2)dx+6\,(Ca^2+2\,Bab-Cb^2}{12\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 - 12*(B*
a^2 - 2*C*a*b - B*b^2)*d*x + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 +
6*(C*a^2 + 2*B*a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 12*(B*a^2 - 2*C*
a*b - B*b^2)*tan(d*x + c))/d
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.69

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \begin{cases} -Ba^2x + \frac{Ba^2\tan(c+dx)}{d} - \frac{Bab\log(\tan^2(c+dx)+1)}{d} + \frac{Bab\tan^2(c+dx)}{d} + Bb^2x + \frac{Bb^2\tan^3(c+dx)}{3d} - \frac{Bb^2\tan(c+dx)}{d} - \frac{Ca \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\tan(c) \end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((-B*a**2*x + B*a**2*tan(c + d*x)/d - B*a*b*log(tan(c + d*x)**2 +
 1)/d + B*a*b*tan(c + d*x)**2/d + B*b**2*x + B*b**2*tan(c + d*x)**3/(3*d)
- B*b**2*tan(c + d*x)/d - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*t
an(c + d*x)**2/(2*d) + 2*C*a*b*x + 2*C*a*b*tan(c + d*x)**3/(3*d) - 2*C*a*b
*tan(c + d*x)/d + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c + d
*x)**4/(4*d) - C*b**2*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))*
*2*(B*tan(c) + C*tan(c)**2)*tan(c), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \frac{3\,Cb^2\tan(dx+c)^4 + 4\,(2\,Cab+Bb^2)\tan(dx+c)^3 + 6\,(Ca^2+2\,Bab-Cb^2)\tan(dx+c)^2 - 12\,(Ba^2-2}{}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/12*(3*C*b^2*tan(d*x + c)^4 + 4*(2*C*a*b + B*b^2)*tan(d*x + c)^3 + 6*(C*a
^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^2 - 12*(B*a^2 - 2*C*a*b - B*b^2)*(d*x +
 c) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*
C*a*b - B*b^2)*tan(d*x + c))/d
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(141) = 282$.

Time = 1.79 (sec) , antiderivative size = 2078, normalized size of antiderivative = 14.04

$$\int \tan(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
-1/12*(12*B*a^2*d*x*tan(d*x)^4*tan(c)^4 - 24*C*a*b*d*x*tan(d*x)^4*tan(c)^4
 - 12*B*b^2*d*x*tan(d*x)^4*tan(c)^4 - 6*C*a^2*log(4*(tan(d*x)^2*tan(c)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))
*tan(d*x)^4*tan(c)^4 - 12*B*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*ta
n(c)^4 + 6*C*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 48*B*a
^2*d*x*tan(d*x)^3*tan(c)^3 + 96*C*a*b*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^2*d
*x*tan(d*x)^3*tan(c)^3 - 6*C*a^2*tan(d*x)^4*tan(c)^4 - 12*B*a*b*tan(d*x)^4
*tan(c)^4 + 9*C*b^2*tan(d*x)^4*tan(c)^4 + 24*C*a^2*log(4*(tan(d*x)^2*tan(c
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
+ 1))*tan(d*x)^3*tan(c)^3 + 48*B*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*
x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)
^3*tan(c)^3 - 24*C*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 +
12*B*a^2*tan(d*x)^4*tan(c)^3 - 24*C*a*b*tan(d*x)^4*tan(c)^3 - 12*B*b^2*tan
(d*x)^4*tan(c)^3 + 12*B*a^2*tan(d*x)^3*tan(c)^4 - 24*C*a*b*tan(d*x)^3*tan(
c)^4 - 12*B*b^2*tan(d*x)^3*tan(c)^4 + 72*B*a^2*d*x*tan(d*x)^2*tan(c)^2 - 1
44*C*a*b*d*x*tan(d*x)^2*tan(c)^2 - 72*B*b^2*d*x*tan(d*x)^2*tan(c)^2 - 6*C*
a^2*tan(d*x)^4*tan(c)^2 - 12*B*a*b*tan(d*x)^4*tan(c)^2 + 6*C*b^2*tan(d*...
```

3.9.9 Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

$$\int \tan(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=x\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)+\frac{\tan(c+d\,x)^3\left(\frac{B\,b^2}{3}+\frac{2\,C\,a\,b}{3}\right)}{d}$$
$$-\frac{\tan(c+d\,x)\,\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)}{d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}$$
$$+\frac{\tan(c+d\,x)^2\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}+\frac{C\,b^2\tan(c+d\,x)^4}{4\,d}$$

input
```
int(tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^
2,x)
```

output
```
x*(B*b^2 - B*a^2 + 2*C*a*b) + (tan(c + d*x)^3*((B*b^2)/3 + (2*C*a*b)/3))/d
 - (tan(c + d*x)*(B*b^2 - B*a^2 + 2*C*a*b))/d - (log(tan(c + d*x)^2 + 1)*(
(C*a^2)/2 - (C*b^2)/2 + B*a*b))/d + (tan(c + d*x)^2*((C*a^2)/2 - (C*b^2)/2
 + B*a*b))/d + (C*b^2*tan(c + d*x)^4)/(4*d)
```

3.10 $\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)$

3.10.1 Optimal result

Integrand size = 32, antiderivative size = 112

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -\left(\left(2abB+a^2C-b^2C\right)x\right)-\frac{(a^2B-b^2B-2abC)\log(\cos(c+dx))}{d} + \frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

output
```
-(2*B*a*b+C*a^2-C*b^2)*x-(B*a^2-B*b^2-2*C*a*b)*ln(cos(d*x+c))/d+b*(B*a-C*b
)*tan(d*x+c)/d+1/2*B*(a+b*tan(d*x+c))^2/d+1/3*C*(a+b*tan(d*x+c))^3/b/d
```

3.10.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{2C(a+b\tan(c+dx))^3+3(aB+bC)\left(i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx)))\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
(2*C*(a + b*Tan[c + d*x])^3 + 3*(a*B + b*C)*(I*((a + I*b)^2*Log[I - Tan[c
+ d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) + 3*B*(
(I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(6*b*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan(c+dx)^2\right) dx$$

$$\Big\downarrow \text{4113}$$

$$\int (a+b\tan(c+dx))^2(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{4008}$$

$$\left(a^2B-2abC-b^2B\right)\int \tan(c+dx)dx-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2B-2abC-b^2B\right)\int \tan(c+dx)dx-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(\cos(c+dx))}{d}-x\left(a^2C+2abB-b^2C\right)+\frac{b(aB-bC)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3bd}$$

input
```
Int[(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
-((2*a*b*B + a^2*C - b^2*C)*x) - ((a^2*B - b^2*B - 2*a*b*C)*Log[Cos[c + d*
x]])/d + (b*(a*B - b*C)*Tan[c + d*x])/d + (B*(a + b*Tan[c + d*x])^2)/(2*d)
 + (C*(a + b*Tan[c + d*x])^3)/(3*b*d)
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
norman	$(-2Bab - C a^2 + C b^2) x + \frac{(2Bab+C a^2-C b^2)\tan(dx+c)}{d} + \frac{C b^2 \tan(dx+c)^3}{3d} + \frac{b(Bb+2Ca)\tan(dx+c)}{2d}$
parts	$\frac{(B b^2+2Cab)\left(\frac{\tan(dx+c)^2}{2} - \frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d} + \frac{(2Bab+C a^2)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d} + \frac{B a^2 \ln\left(1+\tan\right.}{2d}$
derivativedivides	$\frac{\frac{C b^2 \tan(dx+c)^3}{3} + \frac{B b^2 \tan(dx+c)^2}{2} + Cab\tan(dx+c)^2 + 2Bab\tan(dx+c) + C a^2 \tan(dx+c) - C b^2 \tan(dx+c) + \frac{(B a^2 - B b^2 -}{}}{d}$
default	$\frac{\frac{C b^2 \tan(dx+c)^3}{3} + \frac{B b^2 \tan(dx+c)^2}{2} + Cab\tan(dx+c)^2 + 2Bab\tan(dx+c) + C a^2 \tan(dx+c) - C b^2 \tan(dx+c) + \frac{(B a^2 - B b^2 -}{}}{d}$
parallelrisch	$\frac{2C b^2 \tan(dx+c)^3 - 12Babdx + 3B b^2 \tan(dx+c)^2 - 6C a^2 dx + 6C b^2 dx + 6Cab\tan(dx+c)^2 + 3B\ln\left(1+\tan(dx+c)^2\right) a^2 - 3B}{6d}$
risch	$-iB b^2 x + \frac{2iB a^2 c}{d} + \frac{2i(-3iB b^2 e^{4i(dx+c)} - 6iCab e^{4i(dx+c)} + 6Bab e^{4i(dx+c)} + 3C a^2 e^{4i(dx+c)} - 6C b^2 e^{4i(dx+c)} - 3}{3}$

input
```
int((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBO
SE)
```

output
```
(-2*B*a*b-C*a^2+C*b^2)*x+(2*B*a*b+C*a^2-C*b^2)/d*tan(d*x+c)+1/3*C*b^2/d*ta
n(d*x+c)^3+1/2*b*(B*b+2*C*a)/d*tan(d*x+c)^2+1/2*(B*a^2-B*b^2-2*C*a*b)/d*ln
(1+tan(d*x+c)^2)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int (a + b\tan(c + dx))^2 \left(B\tan(c + dx) + C\tan^2(c + dx)\right)\,dx$$
$$= \frac{2\,Cb^2\tan\left(dx + c\right)^3 - 6\left(Ca^2 + 2\,Bab - Cb^2\right)dx + 3\left(2\,Cab + Bb^2\right)\tan\left(dx + c\right)^2 - 3\left(Ba^2 - 2\,Cab - Bb^2\right)}{6\,d}$$

input
```
integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
1/6*(2*C*b^2*tan(d*x + c)^3 - 6*(C*a^2 + 2*B*a*b - C*b^2)*d*x + 3*(2*C*a*b
 + B*b^2)*tan(d*x + c)^2 - 3*(B*a^2 - 2*C*a*b - B*b^2)*log(1/(tan(d*x + c)
^2 + 1)) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.73

$$\int (a + b\tan(c + dx))^2 \left(B\tan(c + dx) + C\tan^2(c + dx)\right)\,dx$$
$$= \begin{cases} \frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 2Babx + \frac{2Bab\tan\left(c+dx\right)}{d} - \frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Bb^2\tan^2\left(c+dx\right)}{2d} - Ca^2x + \frac{Ca^2\tan\left(c+dx\right)}{d} \\ x(a + b\tan\left(c\right))^2\left(B\tan\left(c\right) + C\tan^2\left(c\right)\right) \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*B*a*b*x + 2*B*a*b*tan
(c + d*x)/d - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c + d*x)*
*2/(2*d) - C*a**2*x + C*a**2*tan(c + d*x)/d - C*a*b*log(tan(c + d*x)**2 +
1)/d + C*a*b*tan(c + d*x)**2/d + C*b**2*x + C*b**2*tan(c + d*x)**3/(3*d) -
 C*b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(
c)**2), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int (a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^2\tan\left(dx+c\right)^3+3\,(2\,Cab+Bb^2)\tan\left(dx+c\right)^2-6\,(Ca^2+2\,Bab-Cb^2)(dx+c)+3\,(Ba^2-2\,Cab-}{6\,d}$$

input
```
integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="m
axima")
```

output
```
1/6*(2*C*b^2*tan(d*x + c)^3 + 3*(2*C*a*b + B*b^2)*tan(d*x + c)^2 - 6*(C*a^
2 + 2*B*a*b - C*b^2)*(d*x + c) + 3*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x +
 c)^2 + 1) + 6*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c))/d
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. 2(108) = 216.

Time = 1.19 (sec) , antiderivative size = 1389, normalized size of antiderivative = 12.40

$$\int (a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="g
iac")
```

output
```
-1/6*(6*C*a^2*d*x*tan(d*x)^3*tan(c)^3 + 12*B*a*b*d*x*tan(d*x)^3*tan(c)^3 -
 6*C*b^2*d*x*tan(d*x)^3*tan(c)^3 + 3*B*a^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*
tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*ta
n(d*x)^3*tan(c)^3 - 6*C*a*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)
^3 - 3*B*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 - 18*C*a^2*d
*x*tan(d*x)^2*tan(c)^2 - 36*B*a*b*d*x*tan(d*x)^2*tan(c)^2 + 18*C*b^2*d*x*t
an(d*x)^2*tan(c)^2 - 6*C*a*b*tan(d*x)^3*tan(c)^3 - 3*B*b^2*tan(d*x)^3*tan(
c)^3 - 9*B*a^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 18*C*a*b
*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 9*B*b^2*log(4*(tan(d*x
)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 6*C*a^2*tan(d*x)^3*tan(c)^2 + 12*B*a*
b*tan(d*x)^3*tan(c)^2 - 6*C*b^2*tan(d*x)^3*tan(c)^2 + 6*C*a^2*tan(d*x)^2*t
an(c)^3 + 12*B*a*b*tan(d*x)^2*tan(c)^3 - 6*C*b^2*tan(d*x)^2*tan(c)^3 + 18*
C*a^2*d*x*tan(d*x)*tan(c) + 36*B*a*b*d*x*tan(d*x)*tan(c) - 18*C*b^2*d*x*ta
n(d*x)*tan(c) - 6*C*a*b*tan(d*x)^3*tan(c) - 3*B*b^2*tan(d*x)^3*tan(c) + 6*
C*a*b*tan(d*x)^2*tan(c)^2 + 3*B*b^2*tan(d*x)^2*tan(c)^2 - 6*C*a*b*tan(d...
```

3.10.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\tan(c+d\,x)^2\left(\frac{B\,b^2}{2}+C\,a\,b\right)}{d}-x\left(C\,a^2+2\,B\,a\,b-C\,b^2\right)$$
$$+\frac{\tan(c+d\,x)\,\left(C\,a^2+2\,B\,a\,b-C\,b^2\right)}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(-\frac{B\,a^2}{2}+C\,a\,b+\frac{B\,b^2}{2}\right)}{d}+\frac{C\,b^2\tan(c+d\,x)^3}{3\,d}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^2,x)
```

output
```
(tan(c + d*x)^2*((B*b^2)/2 + C*a*b))/d - x*(C*a^2 - C*b^2 + 2*B*a*b) + (ta
n(c + d*x)*(C*a^2 - C*b^2 + 2*B*a*b))/d - (log(tan(c + d*x)^2 + 1)*((B*b^2
)/2 - (B*a^2)/2 + C*a*b))/d + (C*b^2*tan(c + d*x)^3)/(3*d)
```

3.11 $\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan\right.$

3.11.1 Optimal result

Integrand size = 38, antiderivative size = 87

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \left(a^2B-b^2B-2abC\right)x-\frac{\left(2abB+a^2C-b^2C\right)\log(\cos(c+dx))}{d} + \frac{b(bB+aC)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

output
```
(B*a^2-B*b^2-2*C*a*b)*x-(2*B*a*b+C*a^2-C*b^2)*ln(cos(d*x+c))/d+b*(B*b+C*a)
*tan(d*x+c)/d+1/2*C*(a+b*tan(d*x+c))^2/d
```

3.11.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{(a+ib)^2(-iB+C)\log(i-\tan(c+dx))+(a-ib)^2(iB+C)\log(i+\tan(c+dx))+2b(bB+2aC)\tan(c}{2d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
((a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d*x]] + (a - I*b)^2*(I*B + C)*Lo
g[I + Tan[c + d*x]] + 2*b*(b*B + 2*a*C)*Tan[c + d*x] + b^2*C*Tan[c + d*x]^
2)/(2*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4115, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4115}$$

$$\int (a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{C(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{C(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4008}$$

$$\left(a^2C+2abB-b^2C\right)\int \tan(c+dx)dx+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2C+2abB-b^2C\right)\int \tan(c+dx)dx+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3956$$

$$-\frac{\left(a^2C+2abB-b^2C\right)\log(\cos(c+dx))}{d}+x\left(a^2B-2abC-b^2B\right)+\frac{b(aC+bB)\tan(c+dx)}{d}+\frac{C(a+b\tan(c+dx))^2}{2d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(a^2*B - b^2*B - 2*a*b*C)*x - ((2*a*b*B + a^2*C - b^2*C)*Log[Cos[c + d*x]]
)/d + (b*(b*B + a*C)*Tan[c + d*x])/d + (C*(a + b*Tan[c + d*x])^2)/(2*d)
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{(2Bab+C a^2-C b^2)\ln\left(\sec(dx+c)^2\right)+C b^2\tan(dx+c)^2+(2B b^2+4Cab)\tan(dx+c)+2dx(B a^2-B b^2-2Cab)}{2d}$
norman	$(B a^2-B b^2-2Cab)\,x+\frac{b(Bb+2Ca)\tan(dx+c)}{d}+\frac{C b^2\tan(dx+c)^2}{2d}+\frac{(2Bab+C a^2-C b^2)\ln\left(1+\tan(dx\right.}{2d}$
derivativedivides	$-\frac{\frac{\left(-2Bab-C a^2+C b^2\right)\ln\left(\cot(dx+c)^2+1\right)}{2}+(B a^2-B b^2-2Cab)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)+(2Bab+C a^2-C b^2)\ln(\cot(dx}{d}$
default	$-\frac{\frac{\left(-2Bab-C a^2+C b^2\right)\ln\left(\cot(dx+c)^2+1\right)}{2}+(B a^2-B b^2-2Cab)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)+(2Bab+C a^2-C b^2)\ln(\cot(dx}{d}$
risch	$B a^2x-B b^2x-2Cabx-\frac{2iC b^2c}{d}-iC b^2x+\frac{2iC a^2c}{d}+iC a^2x+\frac{4iBabc}{d}+2iBabx+\frac{2ib(-}{}$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/2*((2*B*a*b+C*a^2-C*b^2)*ln(sec(d*x+c)^2)+C*b^2*tan(d*x+c)^2+(2*B*b^2+4*
C*a*b)*tan(d*x+c)+2*d*x*(B*a^2-B*b^2-2*C*a*b))/d
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int\cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^2\tan(dx+c)^2+2\,(Ba^2-2\,Cab-Bb^2)dx-(Ca^2+2\,Bab-Cb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+2\,(2\,Cab+Bb}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x - (C*a^2 + 2*B
*a*b - C*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 2*(2*C*a*b + B*b^2)*tan(d*x +
c))/d
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.74

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases} Ba^2x+\frac{Bab\log\left(\tan^2(c+dx)+1\right)}{d}-Bb^2x+\frac{Bb^2\tan(c+dx)}{d}+\frac{Ca^2\log\left(\tan^2(c+dx)+1\right)}{2d}-2Cabx+\frac{2Cab\tan(c+dx)}{d}-\frac{Cb^2}{} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot(c) \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a**2*x + B*a*b*log(tan(c + d*x)**2 + 1)/d - B*b**2*x + B*b**2
*tan(c + d*x)/d + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*C*a*b*x + 2*C*
a*b*tan(c + d*x)/d - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*tan(c
+ d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*
cot(c), True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^2\tan(dx+c)^2+2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)+2\,(2}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/2*(C*b^2*tan(d*x + c)^2 + 2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2
 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1) + 2*(2*C*a*b + B*b^2)*tan(d*x
+ c))/d
```

3.11.8 Giac [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^2\tan{(dx+c)}^2+4\,Cab\tan{(dx+c)}+2\,Bb^2\tan{(dx+c)}+2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
1/2*(C*b^2*tan(d*x + c)^2 + 4*C*a*b*tan(d*x + c) + 2*B*b^2*tan(d*x + c) +
2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*log(tan(
d*x + c)^2 + 1))/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{C\,a^2}{2}+B\,a\,b-\frac{C\,b^2}{2}\right)}{d}-x\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)$$
$$+\frac{\tan(c+d\,x)\,\left(B\,b^2+2\,C\,a\,b\right)}{d}+\frac{C\,b^2\tan(c+d\,x)^2}{2\,d}$$

input
```
int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^
2,x)
```

output
```
(log(tan(c + d*x)^2 + 1)*((C*a^2)/2 - (C*b^2)/2 + B*a*b))/d - x*(B*b^2 - B
*a^2 + 2*C*a*b) + (tan(c + d*x)*(B*b^2 + 2*C*a*b))/d + (C*b^2*tan(c + d*x)
^2)/(2*d)
```

3.12 $\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx) + C\,\mathrm{ta}\right.$

3.12.1 Optimal result

Integrand size = 40, antiderivative size = 70

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\left(2abB+a^2C-b^2C\right)x-\frac{b(bB+2aC)\log(\cos(c+dx))}{d}$$
$$+\frac{a^2B\log(\sin(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

output
```
(2*B*a*b+C*a^2-C*b^2)*x-b*(B*b+2*C*a)*ln(cos(d*x+c))/d+a^2*B*ln(sin(d*x+c)
)/d+b^2*C*tan(d*x+c)/d
```

3.12.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{(a+ib)^2(B+iC)\log(i-\tan(c+dx))-2a^2B\log(\tan(c+dx))+(a-ib)^2(B-iC)\log(i+\tan(c+d}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
-1/2*((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] - 2*a^2*B*Log[Tan[c + d*
x]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]] - 2*b^2*C*Tan[c + d*x])/
d
```

3.12.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {3042, 4115, 3042, 4089, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(B+C\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4089}$$

$$\int \cot(c+dx)\left(Ba^2+b(bB+2aC)\tan^2(c+dx)+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{b^2C\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ba^2+b(bB+2aC)\tan(c+dx)^2+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2C\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4107}$$

$$a^2B\int \cot(c+dx)dx+b(2aC+bB)\int \tan(c+dx)dx+x\left(a^2C+2abB-b^2C\right)+\frac{b^2C\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2B\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b(2aC+bB)\int \tan(c+dx)dx+x\left(a^2C+2abB-b^2C\right)+\frac{b^2C\tan(c+dx)}{d}$$

$\Big\downarrow$ 25

$$a^2(-B)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b(2aC+bB)\int \tan(c+dx)dx+x\left(a^2C+2abB-b^2C\right)+\frac{b^2C\tan(c+dx)}{d}$$

$\Big\downarrow$ 3956

$$x\left(a^2C+2abB-b^2C\right)+\frac{a^2B\log(-\sin(c+dx))}{d}-\frac{b(2aC+bB)\log(\cos(c+dx))}{d}+\frac{b^2C\tan(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
(2*a*b*B + a^2*C - b^2*C)*x - (b*(b*B + 2*a*C)*Log[Cos[c + d*x]])/d + (a^2
*B*Log[-Sin[c + d*x]])/d + (b^2*C*Tan[c + d*x])/d
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4089
```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Simp[1/d   Int[(a^2*A*d - b^2*B*c + (2*a*A*b
+ B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x
]^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{\left(-B a^2+B b^2+2 C a b\right) \ln\left(\sec(dx+c)^2\right)+2 B a^2 \ln(\tan(dx+c))+2 C b^2 \tan(dx+c)+4 d x\left(B a b+\frac{1}{2} C a^2-\frac{1}{2} C b^2\right)}{2 d}$
derivativedivides	$-\frac{\frac{\left(B a^2-B b^2-2 C a b\right) \ln\left(\cot(dx+c)^2+1\right)}{2}+\left(2 B a b+C a^2-C b^2\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{C b^2}{\cot(dx+c)}+b(Bb+2Ca) \ln(\cot(d}{d}$
default	$-\frac{\frac{\left(B a^2-B b^2-2 C a b\right) \ln\left(\cot(dx+c)^2+1\right)}{2}+\left(2 B a b+C a^2-C b^2\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{C b^2}{\cot(dx+c)}+b(Bb+2Ca) \ln(\cot(d}{d}$
norman	$\frac{\left(2 B a b+C a^2-C b^2\right) x \tan(dx+c)+\frac{C b^2 \tan(dx+c)^2}{d}}{\tan(dx+c)}+\frac{B a^2 \ln(\tan(dx+c))}{d}-\frac{\left(B a^2-B b^2-2 C a b\right) \ln\left(1+\tan(dx+c)^2\right)}{2 d}$
risch	$i B b^2 x+\frac{4 i C a b c}{d}+\frac{2 i C b^2}{d\left(e^{2 i(dx+c)}+1\right)}+2 B a b x+C a^2 x-C b^2 x+2 i C a b x-\frac{2 i B a^2 c}{d}+\frac{2 i B b^2 c}{d}-$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*((-B*a^2+B*b^2+2*C*a*b)*ln(sec(d*x+c)^2)+2*B*a^2*ln(tan(d*x+c))+2*C*b^
2*tan(d*x+c)+4*d*x*(B*a*b+1/2*C*a^2-1/2*C*b^2))/d
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Ba^2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,Cb^2\tan\left(dx+c\right)+2\left(Ca^2+2\,Bab-Cb^2\right)dx-\left(2\,Cab+Bb^2\right)\log\left(\frac{1}{\tan(dx+c)^2+1}\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/2*(B*a^2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 2*C*b^2*tan(d*x + c)
 + 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x - (2*C*a*b + B*b^2)*log(1/(tan(d*x + c)
^2 + 1)))/d
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(66) = 132.

Time = 0.75 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}-\frac{Ba^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba^2\log\left(\tan\left(c+dx\right)\right)}{d}+2Babx+\frac{Bb^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+Ca^2x+\frac{Cab\log\left(\tan^2\left(c+dx\right)+1\right)}{d}-\\ x(a+b\tan\left(c\right))^2\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^2\left(c\right)\end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((-B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x
))/d + 2*B*a*b*x + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*x + C*a*
b*log(tan(c + d*x)**2 + 1)/d - C*b**2*x + C*b**2*tan(c + d*x)/d, Ne(d, 0))
, (x*(a + b*tan(c))**2*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{2\,Ba^2\log\left(\tan\left(dx+c\right)\right)+2\,Cb^2\tan\left(dx+c\right)+2\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-\left(Ba^2-2\,Cab-Bb^2\right)\mathrm{lo}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/2*(2*B*a^2*log(tan(d*x + c)) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*B*a*b
 - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{2\,Ba^2\log\left(\left|\tan\left(dx+c\right)\right|\right)+2\,Cb^2\tan\left(dx+c\right)+2\left(Ca^2+2\,Bab-Cb^2\right)(dx+c)-\left(Ba^2-2\,Cab-Bb^2\right)\mathrm{l}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/2*(2*B*a^2*log(abs(tan(d*x + c))) + 2*C*b^2*tan(d*x + c) + 2*(C*a^2 + 2*
B*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 +
1))/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 8.59 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{B\,a^2\,\ln\left(\tan(c+d\,x)\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d}$$
$$+\frac{C\,b^2\tan(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$

input
```
int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^2,x)
```

output
```
(B*a^2*log(tan(c + d*x)))/d + (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b
)^2)/(2*d) + (C*b^2*tan(c + d*x))/d + (log(tan(c + d*x) - 1i)*(B + C*1i)*(
a*1i - b)^2)/(2*d)
```

3.13 $\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.13.1 Optimal result

Integrand size = 40, antiderivative size = 72

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=-\left(\left(a^2B-b^2B-2abC\right)x\right)-\frac{a^2B\cot(c+dx)}{d}-\frac{b^2C\log(\cos(c+dx))}{d}+\frac{a(2bB+aC)\log(\sin(c+dx))}{d}$$

output
```
-(B*a^2-B*b^2-2*C*a*b)*x-a^2*B*cot(d*x+c)/d-b^2*C*ln(cos(d*x+c))/d+a*(2*B*
b+C*a)*ln(sin(d*x+c))/d
```

3.13.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{-2a^2B\cot(c+dx)+i(a+ib)^2(B+iC)\log(i-\tan(c+dx))+2a(2bB+aC)\log(\tan(c+dx))-(a-i\cdots}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-2*a^2*B*Cot[c + d*x] + I*(a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + 2
*a*(2*b*B + a*C)*Log[Tan[c + d*x]] - (a - I*b)^2*(I*B + C)*Log[I + Tan[c +
 d*x]])/(2*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 4115, 3042, 4087, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(B+C\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \cot(c+dx)\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{a^2B\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2C\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)}{\tan(c+dx)}dx-\frac{a^2B\cot(c+dx)}{d}$$

$$\Big\downarrow \text{4107}$$

$$a(aC+2bB)\int \cot(c+dx)dx+b^2C\int \tan(c+dx)dx-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(aC+2bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^2C\int \tan(c+dx)dx-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-a(aC+2bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b^2C\int \tan(c+dx)dx-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}$$

$$\Big\downarrow \; 3956$$

$$-x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot(c+dx)}{d}+\frac{a(aC+2bB)\log(-\sin(c+dx))}{d}-\frac{b^2C\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
-((a^2*B - b^2*B - 2*a*b*C)*x) - (a^2*B*Cot[c + d*x])/d - (b^2*C*Log[Cos[c
 + d*x]])/d + (a*(2*b*B + a*C)*Log[-Sin[c + d*x]])/d
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.13.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{B\,b^2(dx+c)-C\,b^2\ln(\cos(dx+c))+2Bab\ln(\sin(dx+c))+2Cab(dx+c)+B\,a^2(-\cot(dx+c)-dx-c)+C\,a^2\ln(\sin(dx+c))}{d}$
default	$\frac{B\,b^2(dx+c)-C\,b^2\ln(\cos(dx+c))+2Bab\ln(\sin(dx+c))+2Cab(dx+c)+B\,a^2(-\cot(dx+c)-dx-c)+C\,a^2\ln(\sin(dx+c))}{d}$
parallelrisch	$\frac{(-2Bab-C\,a^2+C\,b^2)\ln\left(\sec(dx+c)^2\right)+(4Bab+2C\,a^2)\ln(\tan(dx+c))-2B\,a^2\cot(dx+c)-2dx(B\,a^2-B\,b^2-2Cab)}{2d}$
norman	$\frac{(-B\,a^2+B\,b^2+2Cab)x\tan(dx+c)^2-\frac{B\,a^2\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{a(2Bb+Ca)\ln(\tan(dx+c))}{d}-\frac{(2Bab+C\,a^2-C\,b^2)\ln\left(1+\tan(\right.}{2d}$
risch	$-B\,a^2x+B\,b^2x+2Cabx-\frac{2iC\,a^2c}{d}+iC\,b^2x-iC\,a^2x-\frac{2iB\,a^2}{d\left(e^{2i(dx+c)}-1\right)}-2iBabx-\frac{4iBab}{d}$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/d*(B*b^2*(d*x+c)-C*b^2*ln(cos(d*x+c))+2*B*a*b*ln(sin(d*x+c))+2*C*a*b*(d*
x+c)+B*a^2*(-cot(d*x+c)-d*x-c)+C*a^2*ln(sin(d*x+c)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -\frac{Cb^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)+2\left(Ba^2-2\,Cab-Bb^2\right)dx\tan(dx+c)+2\,Ba^2-\left(Ca^2+2\,Bab\right)\mathrm{l}}{2\,d\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
-1/2*(C*b^2*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(B*a^2 - 2*C*a*b
- B*b^2)*d*x*tan(d*x + c) + 2*B*a^2 - (C*a^2 + 2*B*a*b)*log(tan(d*x + c)^2
/(tan(d*x + c)^2 + 1))*tan(d*x + c))/(d*tan(d*x + c))
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(66) = 132$.

Time = 1.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \begin{cases} \mathrm{NaN} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^3(c) \\ \mathrm{NaN} \\ -Ba^2x-\frac{Ba^2}{d\tan(c+dx)}-\frac{Bab\log(\tan^2(c+dx)+1)}{d}+\frac{2Bab\log(\tan(c+dx))}{d}+Bb^2x-\frac{Ca^2\log(\tan^2(c+dx)+1)}{2d}+\frac{Ca^2\log(\tan}{d} \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*t
an(c)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**2*x - B*a**2/(d
*tan(c + d*x)) - B*a*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a*b*log(tan(c + d*
x))/d + B*b**2*x - C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**2*log(tan(
c + d*x))/d + 2*C*a*b*x + C*b**2*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx =$$
$$-\frac{2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ca^2+2\,Bab)\log\left(\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
-1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*lo
g(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(tan(d*x + c)) + 2*B*a^2/ta
n(d*x + c))/d
```

3.13.8 Giac [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx =$$
$$-\frac{2\,(Ba^2-2\,Cab-Bb^2)(dx+c)+(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ca^2+2\,Bab)\log\left(\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
-1/2*(2*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + (C*a^2 + 2*B*a*b - C*b^2)*lo
g(tan(d*x + c)^2 + 1) - 2*(C*a^2 + 2*B*a*b)*log(abs(tan(d*x + c))) + 2*(C*
a^2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) + B*a^2)/tan(d*x + c))/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(C\,a^2+2\,B\,b\,a\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d}-\frac{B\,a^2\cot(c+d\,x)}{d}$$

input
```
int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(tan(c + d*x))*(C*a^2 + 2*B*a*b))/d - (log(tan(c + d*x) - 1i)*(B*1i -
C)*(a*1i - b)^2)/(2*d) + (log(tan(c + d*x) + 1i)*(B*1i + C)*(a*1i + b)^2)/
(2*d) - (B*a^2*cot(c + d*x))/d
```

3.14 $\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.14.1 Optimal result

Integrand size = 40, antiderivative size = 88

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \left(b^2C-a(2bB+aC)\right)x-\frac{a(2bB+aC)\cot(c+dx)}{d}-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{(a^2B-b^2B-2abC)\log(\sin(c+dx))}{d}$$

output
```
(C*b^2-a*(2*B*b+C*a))*x-a*(2*B*b+C*a)*cot(d*x+c)/d-1/2*a^2*B*cot(d*x+c)^2/
d-(B*a^2-B*b^2-2*C*a*b)*ln(sin(d*x+c))/d
```

3.14.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \frac{-2a(2bB+aC)\cot(c+dx)-a^2B\cot^2(c+dx)+(a+ib)^2(B+iC)\log(i-\tan(c+dx))-2(a^2B-b^2B}{2d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-2*a*(2*b*B + a*C)*Cot[c + d*x] - a^2*B*Cot[c + d*x]^2 + (a + I*b)^2*(B +
 I*C)*Log[I - Tan[c + d*x]] - 2*(a^2*B - b^2*B - 2*a*b*C)*Log[Tan[c + d*x]
] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)
```

3.14.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4115, 3042, 4087, 3042, 4111, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(B+C\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \cot^2(c+dx)\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{a^2B\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2C\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)}{\tan(c+dx)^2}dx-\frac{a^2B\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4111}$$

$$\int -\cot(c+dx)\left(Ba^2-2bCa-b^2B-\left(b^2C-a(2bB+aC)\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{4014}$$

$$-\left(a^2B-2abC-b^2B\right)\int \cot(c+dx)dx-x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2B-2abC-b^2B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\left(a^2B-2abC-b^2B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\left(a^2B-2abC-b^2B\right)\log(-\sin(c+dx))}{d}-x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^2(c+dx)}{2d}-\frac{a(aC+2bB)\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
-((2*a*b*B + a^2*C - b^2*C)*x) - (a*(2*b*B + a*C)*Cot[c + d*x])/d - (a^2*B
*Cot[c + d*x]^2)/(2*d) - ((a^2*B - b^2*B - 2*a*b*C)*Log[-Sin[c + d*x]])/d
```

3.14. $\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{B\,b^2\ln(\sin(dx+c))+C\,b^2(dx+c)+2Bab(-\cot(dx+c)-dx-c)+2Cab\ln(\sin(dx+c))+B\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right.}{d}$
default	$\frac{B\,b^2\ln(\sin(dx+c))+C\,b^2(dx+c)+2Bab(-\cot(dx+c)-dx-c)+2Cab\ln(\sin(dx+c))+B\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right.}{d}$
parallelrisch	$\frac{(B\,a^2-B\,b^2-2Cab)\ln\left(\sec(dx+c)^2\right)+(-2B\,a^2+2B\,b^2+4Cab)\ln(\tan(dx+c))-B\,a^2\cot(dx+c)^2+(-4Bab-2C\,a^2)\cot(}{2d}$
norman	$\frac{(-2Bab-C\,a^2+C\,b^2)x\tan(dx+c)^3-\frac{B\,a^2\tan(dx+c)}{2d}-\frac{a(2Bb+Ca)\tan(dx+c)^2}{d}}{\tan(dx+c)^3}-\frac{(B\,a^2-B\,b^2-2Cab)\ln(\tan(dx+c))}{d}+$
risch	$-iB\,b^2x-\frac{2iB\,b^2c}{d}+\frac{2iB\,a^2c}{d}-2Babx-C\,a^2x+C\,b^2x-\frac{4iCabc}{d}-\frac{2ia(2Bb\,e^{2i(dx+c)}+Ca\,e^{2i(dx+}}{d(e^{2i(dx}}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/d*(B*b^2*ln(sin(d*x+c))+C*b^2*(d*x+c)+2*B*a*b*(-cot(d*x+c)-d*x-c)+2*C*a*
b*ln(sin(d*x+c))+B*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*a^2*(-cot(d*x+
c)-d*x-c))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{(Ba^2-2\,Cab-Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Ba^2+(Ba^2+2\,(Ca^2+2\,Bab-Cb^2)dx)\tan(}{2\,d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
-1/2*((B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*t
an(d*x + c)^2 + B*a^2 + (B*a^2 + 2*(C*a^2 + 2*B*a*b - C*b^2)*d*x)*tan(d*x
+ c)^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(78) = 156$.

Time = 1.76 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.34

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \begin{cases} \text{NaN} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^4(c) \\ \text{NaN} \\ \frac{Ba^2\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^2\log(\tan(c+dx))}{d} - \frac{Ba^2}{2d\tan^2(c+dx)} - 2Babx - \frac{2Bab}{d\tan(c+dx)} - \frac{Bb^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^2\log}{} \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*t
an(c)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**2*log(tan(c + d*
x)**2 + 1)/(2*d) - B*a**2*log(tan(c + d*x))/d - B*a**2/(2*d*tan(c + d*x)**
2) - 2*B*a*b*x - 2*B*a*b/(d*tan(c + d*x)) - B*b**2*log(tan(c + d*x)**2 + 1
)/(2*d) + B*b**2*log(tan(c + d*x))/d - C*a**2*x - C*a**2/(d*tan(c + d*x))
- C*a*b*log(tan(c + d*x)**2 + 1)/d + 2*C*a*b*log(tan(c + d*x))/d + C*b**2*
x, True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{2\,(Ca^2+2\,Bab-Cb^2)(dx+c)-(Ba^2-2\,Cab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(Ba^2-2\,Cab-Bb}{2\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
-1/2*(2*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - (B*a^2 - 2*C*a*b - B*b^2)*lo
g(tan(d*x + c)^2 + 1) + 2*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)) + (B
*a^2 + 2*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x + c)^2)/d
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(86) = 172.

Time = 0.87 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.69

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,(Ca^2+2\,Bab-Cb^2)(dx+}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
-1/8*(B*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a^2*tan(1/2*d*x + 1/2*c) - 8*B*a*
b*tan(1/2*d*x + 1/2*c) + 8*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 8*(B*a^2
- 2*C*a*b - B*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 8*(B*a^2 - 2*C*a*b -
B*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (12*B*a^2*tan(1/2*d*x + 1/2*c)^2 -
 24*C*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*C*a
^2*tan(1/2*d*x + 1/2*c) - 8*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*
x + 1/2*c)^2)/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\ln(\tan(c+d\,x))\,(-B\,a^2+2\,C\,a\,b+B\,b^2)}{d}-\frac{\cot(c+d\,x)^2\left(\frac{B\,a^2}{2}+\tan(c+d\,x)\,(C\,a^2+2\,B\,b\,a)\right)}{d}-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(B-C\,1\mathrm{i})\,(b+a\,1\mathrm{i})^2}{2\,d}-\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(B+C\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^2}{2\,d}$$

input
```
int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(tan(c + d*x))*(B*b^2 - B*a^2 + 2*C*a*b))/d - (cot(c + d*x)^2*((B*a^2)
/2 + tan(c + d*x)*(C*a^2 + 2*B*a*b)))/d - (log(tan(c + d*x) + 1i)*(B - C*1
i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*1i - b)^2)/
(2*d)
```

3.15 $\int \cot^5(c+dx)(a+b\tan(c+dx))^2 (B\tan(c+dx)+C\tan$

3.15.1 Optimal result

Integrand size = 40, antiderivative size = 118

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \left(a^2B-b^2B-2abC\right)x+\frac{(a^2B-b^2B-2abC)\cot(c+dx)}{d}-\frac{a(2bB+aC)\cot^2(c+dx)}{2d}$$
$$-\frac{a^2B\cot^3(c+dx)}{3d}+\frac{(b^2C-a(2bB+aC))\log(\sin(c+dx))}{d}$$

output
```
(B*a^2-B*b^2-2*C*a*b)*x+(B*a^2-B*b^2-2*C*a*b)*cot(d*x+c)/d-1/2*a*(2*B*b+C*
a)*cot(d*x+c)^2/d-1/3*a^2*B*cot(d*x+c)^3/d+(C*b^2-a*(2*B*b+C*a))*ln(sin(d*
x+c))/d
```

3.15.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.29

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \frac{6(a^2B-b^2B-2abC)\cot(c+dx)-3a(2bB+aC)\cot^2(c+dx)-2a^2B\cot^3(c+dx)+3(a+ib)^2(-iB-}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(6*(a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x] - 3*a*(2*b*B + a*C)*Cot[c + d*x]
^2 - 2*a^2*B*Cot[c + d*x]^3 + 3*(a + I*b)^2*((-I)*B + C)*Log[I - Tan[c + d
*x]] - 6*(2*a*b*B + a^2*C - b^2*C)*Log[Tan[c + d*x]] + 3*(a - I*b)^2*(I*B
+ C)*Log[I + Tan[c + d*x]])/(6*d)
```

3.15.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4115, 3042, 4087, 3042, 4111, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \ 4115$$

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^2(B+C\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \ 4087$$

$$\int \cot^3(c+dx)\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{a^2B\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{b^2C\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)}{\tan(c+dx)^3}dx-\frac{a^2B\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \ 4111$$

$$\int -\cot^2(c+dx)\left(Ba^2-2bCa-b^2B-\left(b^2C-a(2bB+aC)\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^2(c+dx)\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4012}$$

$$-\int \cot(c+dx)\left(Ca^2+2bBa-b^2C-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Ca^2+2bBa-b^2C-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4014}$$

$$-\left(a^2C+2abB-b^2C\right)\int \cot(c+dx)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2C+2abB-b^2C\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$\left(a^2C+2abB-b^2C\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B-2abC-b^2B\right)-\frac{a^2B\cot^3(c+dx)}{3d}-\frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

3.15. $\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\downarrow \text{3956}$$

$$\frac{(a^2B - 2abC - b^2B)\cot(c+dx)}{d} - \frac{(a^2C + 2abB - b^2C)\log(-\sin(c+dx))}{d} + x(a^2B - 2abC - b^2B) - \frac{a^2B\cot^3(c+dx)}{3d} - \frac{a(aC+2bB)\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
(a^2*B - b^2*B - 2*a*b*C)*x + ((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x])/d -
 (a*(2*b*B + a*C)*Cot[c + d*x]^2)/(2*d) - (a^2*B*Cot[c + d*x]^3)/(3*d) - (
(2*a*b*B + a^2*C - b^2*C)*Log[-Sin[c + d*x]])/d
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.15.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{B b^2(-\cot(dx+c)-dx-c)+C b^2\ln(\sin(dx+c))+2Bab\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2Cab(-\cot(dx+c)-dx-c)+B}{d}$
default	$\frac{B b^2(-\cot(dx+c)-dx-c)+C b^2\ln(\sin(dx+c))+2Bab\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2Cab(-\cot(dx+c)-dx-c)+B}{d}$
parallelrisch	$\frac{3(2Bab+C a^2-C b^2)\ln\left(\sec(dx+c)^2\right)+6(-2Bab-C a^2+C b^2)\ln(\tan(dx+c))-2B a^2\cot(dx+c)^3+3(-2Bab-C a^2)}{6d}$
norman	$\frac{\frac{\left(B a^2-B b^2-2Cab\right)\tan(dx+c)^3}{d}+(B a^2-B b^2-2Cab)x\tan(dx+c)^4-\frac{B a^2\tan(dx+c)}{3d}-\frac{a(2Bb+Ca)\tan(dx+c)^2}{2d}}{\tan(dx+c)^4}-$
risch	$B a^2x-B b^2x-2Cabx+\frac{4iBabc}{d}-iC b^2x+iC a^2x-\frac{2i\left(6iBab\,e^{4i(dx+c)}+3iC a^2e^{4i(dx+c)}-6B a^2\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/d*(B*b^2*(-cot(d*x+c)-d*x-c)+C*b^2*ln(sin(d*x+c))+2*B*a*b*(-1/2*cot(d*x+
c)^2-ln(sin(d*x+c)))+2*C*a*b*(-cot(d*x+c)-d*x-c)+B*a^2*(-1/3*cot(d*x+c)^3+
cot(d*x+c)+d*x+c)+C*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{3\,(Ca^2+2\,Bab-Cb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+3\,(Ca^2+2\,Bab-2\,(Ba^2-2\,Cab-Bb^2)dx)\,t}{6\,d\tan\left(dx+\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
-1/6*(3*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
*tan(d*x + c)^3 + 3*(C*a^2 + 2*B*a*b - 2*(B*a^2 - 2*C*a*b - B*b^2)*d*x)*ta
n(d*x + c)^3 + 2*B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C
*a^2 + 2*B*a*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(107) = 214.

Time = 2.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.14

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}\text{NaN} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^5(c) \\ \text{NaN} \\ Ba^2x+\frac{Ba^2}{d\tan(c+dx)}-\frac{Ba^2}{3d\tan^3(c+dx)}+\frac{Bab\log(\tan^2(c+dx)+1)}{d}-\frac{2Bab\log(\tan(c+dx))}{d}-\frac{Bab}{d\tan^2(c+dx)}-Bb^2x-\frac{Bb}{d\tan(c}\end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*t
an(c)**2)*cot(c)**5, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**2*x + B*a**2/(d*
tan(c + d*x)) - B*a**2/(3*d*tan(c + d*x)**3) + B*a*b*log(tan(c + d*x)**2 +
 1)/d - 2*B*a*b*log(tan(c + d*x))/d - B*a*b/(d*tan(c + d*x)**2) - B*b**2*x
 - B*b**2/(d*tan(c + d*x)) + C*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - C*a**
2*log(tan(c + d*x))/d - C*a**2/(2*d*tan(c + d*x)**2) - 2*C*a*b*x - 2*C*a*b
/(d*tan(c + d*x)) - C*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + C*b**2*log(tan
(c + d*x))/d, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{6\,(Ba^2-2\,Cab-Bb^2)(dx+c)+3\,(Ca^2+2\,Bab-Cb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ca^2+2\,Bab-Cb^2}{6\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/6*(6*(B*a^2 - 2*C*a*b - B*b^2)*(d*x + c) + 3*(C*a^2 + 2*B*a*b - C*b^2)*l
og(tan(d*x + c)^2 + 1) - 6*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(d*x + c)) - (
2*B*a^2 - 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 + 3*(C*a^2 + 2*B*a*b)
*tan(d*x + c))/tan(d*x + c)^3)/d
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(114) = 228$.

Time = 0.91 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.83

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/24*(B*a^2*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*
a*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^2*tan(1/2*d*x + 1/2*c) + 24*C*a*b*tan(
1/2*d*x + 1/2*c) + 12*B*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^2 - 2*C*a*b - B
*b^2)*(d*x + c) + 24*(C*a^2 + 2*B*a*b - C*b^2)*log(tan(1/2*d*x + 1/2*c)^2
+ 1) - 24*(C*a^2 + 2*B*a*b - C*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C
*a^2*tan(1/2*d*x + 1/2*c)^3 + 88*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 44*C*b^2*t
an(1/2*d*x + 1/2*c)^3 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*C*a*b*tan(1/2
*d*x + 1/2*c)^2 - 12*B*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*a^2*tan(1/2*d*x +
1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) - B*a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.32

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{\cot(c+d\,x)^3\left(\frac{B\,a^2}{3}+\tan(c+d\,x)^2\,(-B\,a^2+2\,C\,a\,b+B\,b^2)+\tan(c+d\,x)\,\left(\frac{C\,a^2}{2}+B\,b\,a\right)\right)}{d}$$
$$-\frac{\ln(\tan(c+d\,x))\,(C\,a^2+2\,B\,a\,b-C\,b^2)}{d}$$
$$+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(-C+B\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^2}{2\,d}$$
$$-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(C+B\,1\mathrm{i})\,(b+a\,1\mathrm{i})^2}{2\,d}$$

input
```
int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(tan(c + d*x) - 1i)*(B*1i - C)*(a*1i - b)^2)/(2*d) - (log(tan(c + d*x)
)*(C*a^2 - C*b^2 + 2*B*a*b))/d - (cot(c + d*x)^3*((B*a^2)/3 + tan(c + d*x)
^2*(B*b^2 - B*a^2 + 2*C*a*b) + tan(c + d*x)*((C*a^2)/2 + B*a*b)))/d - (log
(tan(c + d*x) + 1i)*(B*1i + C)*(a*1i + b)^2)/(2*d)
```

3.16 $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.16.1 Optimal result

Integrand size = 40, antiderivative size = 151

$$\begin{aligned}\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx &= \left(2abB+a^2C-b^2C\right)x-\frac{(b^2C-a(2bB+aC))\cot(c+dx)}{d}\\ &\quad+\frac{(a^2B-b^2B-2abC)\cot^2(c+dx)}{2d}-\frac{a(2bB+aC)\cot^3(c+dx)}{3d}\\ &\quad-\frac{a^2B\cot^4(c+dx)}{4d}+\frac{(a^2B-b^2B-2abC)\log(\sin(c+dx))}{d}\end{aligned}$$

output
```
(2*B*a*b+C*a^2-C*b^2)*x-(C*b^2-a*(2*B*b+C*a))*cot(d*x+c)/d+1/2*(B*a^2-B*b^
2-2*C*a*b)*cot(d*x+c)^2/d-1/3*a*(2*B*b+C*a)*cot(d*x+c)^3/d-1/4*a^2*B*cot(d
*x+c)^4/d+(B*a^2-B*b^2-2*C*a*b)*ln(sin(d*x+c))/d
```

3.16.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\begin{aligned}&\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx\\ &=\frac{12(2abB+a^2C-b^2C)\cot(c+dx)+6(a^2B-b^2B-2abC)\cot^2(c+dx)-4a(2bB+aC)\cot^3(c+dx)-}{}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(12*(2*a*b*B + a^2*C - b^2*C)*Cot[c + d*x] + 6*(a^2*B - b^2*B - 2*a*b*C)*C
ot[c + d*x]^2 - 4*a*(2*b*B + a*C)*Cot[c + d*x]^3 - 3*a^2*B*Cot[c + d*x]^4
- 6*((a + I*b)^2*(B + I*C)*Log[I - Tan[c + d*x]] + (-2*a^2*B + 2*b^2*B + 4
*a*b*C)*Log[Tan[c + d*x]] + (a - I*b)^2*(B - I*C)*Log[I + Tan[c + d*x]]))/
(12*d)
```

3.16.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.425$, Rules used = {3042, 4115, 3042, 4087, 3042, 4111, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \; 4115$$

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(B+C\tan(c+dx))dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))^2(B+C\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \; 4087$$

$$\int \cot^4(c+dx)\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{a^2B\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{b^2C\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)}{\tan(c+dx)^4}dx-\frac{a^2B\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 4111

$$\int -\cot^3(c+dx)\left(Ba^2-2bCa-b^2B-\left(b^2C-a(2bB+aC)\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 25

$$-\int \cot^3(c+dx)\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4012

$$-\int \cot^2(c+dx)\left(Ca^2+2bBa-b^2C-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{Ca^2+2bBa-b^2C-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)}{\tan(c+dx)^2}dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4012

$$-\int -\cot(c+dx)\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 25

3.16. $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\int \cot(c+dx)\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4014}$$

$$\left(a^2B-2abC-b^2B\right)\int \cot(c+dx)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2B-2abC-b^2B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$-\left(a^2B-2abC-b^2B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\left(a^2B-2abC-b^2B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2C+2abB-b^2C\right)\cot(c+dx)}{d}+\frac{\left(a^2B-2abC-b^2B\right)\log(-\sin(c+dx))}{d}+x\left(a^2C+2abB-b^2C\right)-\frac{a^2B\cot^4(c+dx)}{4d}-\frac{a(aC+2bB)\cot^3(c+dx)}{3d}$$

3.16. $\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

input
```
Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^2*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
(2*a*b*B + a^2*C - b^2*C)*x + ((2*a*b*B + a^2*C - b^2*C)*Cot[c + d*x])/d +
 ((a^2*B - b^2*B - 2*a*b*C)*Cot[c + d*x]^2)/(2*d) - (a*(2*b*B + a*C)*Cot[c
 + d*x]^3)/(3*d) - (a^2*B*Cot[c + d*x]^4)/(4*d) + ((a^2*B - b^2*B - 2*a*b*
C)*Log[-Sin[c + d*x]])/d
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{B\,b^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+C\,b^2(-\cot(dx+c)-dx-c)+2Bab\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2Cab\left(-\frac{c}{}\right.}{}$
default	$\frac{B\,b^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+C\,b^2(-\cot(dx+c)-dx-c)+2Bab\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2Cab\left(-\frac{c}{}\right.}{}$
parallelrisch	$\frac{6(-B\,a^2+B\,b^2+2Cab)\ln\left(\sec(dx+c)^2\right)+12(B\,a^2-B\,b^2-2Cab)\ln(\tan(dx+c))-3B\,a^2\cot(dx+c)^4+4(-2Bab-C\,a^2)\,c}{12d}$
norman	$\frac{\frac{(2Bab+C\,a^2-C\,b^2)\tan(dx+c)^4}{d}+(2Bab+C\,a^2-C\,b^2)x\tan(dx+c)^5+\frac{(B\,a^2-B\,b^2-2Cab)\tan(dx+c)^3}{2d}-\frac{B\,a^2\tan(dx+c)}{4d}-\frac{a(2}{}}{\tan(dx+c)^5}$
risch	$iB\,b^2x+\frac{2iB\,b^2c}{d}-\frac{2iB\,a^2c}{d}+2Babx+C\,a^2x-C\,b^2x+2iCabx+\frac{4iCabc}{d}-iB\,a^2x+\frac{20iC}{}$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/d*(B*b^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+C*b^2*(-cot(d*x+c)-d*x-c)+2*
B*a*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*C*a*b*(-1/2*cot(d*x+c)^2-ln(s
in(d*x+c)))+B*a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+C*a^
2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.26

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{6\,(Ba^2-2\,Cab-Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^4+3\,(3\,Ba^2-4\,Cab-2\,Bb^2+4\,(Ca^2+2\,Bab-C}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/12*(6*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
*tan(d*x + c)^4 + 3*(3*B*a^2 - 4*C*a*b - 2*B*b^2 + 4*(C*a^2 + 2*B*a*b - C*
b^2)*d*x)*tan(d*x + c)^4 + 12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3
*B*a^2 + 6*(B*a^2 - 2*C*a*b - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*
tan(d*x + c))/(d*tan(d*x + c)^4)
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(136) = 272$.

Time = 4.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.01

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases} \text{NaN} \\ x(a+b\tan(c))^2\left(B\tan(c)+C\tan^2(c)\right)\cot^6(c) \\ \text{NaN} \\ -\frac{Ba^2\log(\tan^2(c+dx)+1)}{2d}+\frac{Ba^2\log(\tan(c+dx))}{d}+\frac{Ba^2}{2d\tan^2(c+dx)}-\frac{Ba^2}{4d\tan^4(c+dx)}+2Babx+\frac{2Bab}{d\tan(c+dx)}-\frac{2Bab}{3d\tan^3(c+dx)} \end{cases}$$

input
```
integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**2*(B*tan(c) + C*t
an(c)**2)*cot(c)**6, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**2*log(tan(c + d
*x)**2 + 1)/(2*d) + B*a**2*log(tan(c + d*x))/d + B*a**2/(2*d*tan(c + d*x)*
*2) - B*a**2/(4*d*tan(c + d*x)**4) + 2*B*a*b*x + 2*B*a*b/(d*tan(c + d*x))
- 2*B*a*b/(3*d*tan(c + d*x)**3) + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) -
B*b**2*log(tan(c + d*x))/d - B*b**2/(2*d*tan(c + d*x)**2) + C*a**2*x + C*a
**2/(d*tan(c + d*x)) - C*a**2/(3*d*tan(c + d*x)**3) + C*a*b*log(tan(c + d*
x)**2 + 1)/d - 2*C*a*b*log(tan(c + d*x))/d - C*a*b/(d*tan(c + d*x)**2) - C
*b**2*x - C*b**2/(d*tan(c + d*x)), True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.16

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{12\,(Ca^2+2\,Bab-Cb^2)(dx+c)-6\,(Ba^2-2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)+12\,(Ba^2-2\,Cab-B}{12\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/12*(12*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) - 6*(B*a^2 - 2*C*a*b - B*b^2)
*log(tan(d*x + c)^2 + 1) + 12*(B*a^2 - 2*C*a*b - B*b^2)*log(tan(d*x + c))
+ (12*(C*a^2 + 2*B*a*b - C*b^2)*tan(d*x + c)^3 - 3*B*a^2 + 6*(B*a^2 - 2*C*
a*b - B*b^2)*tan(d*x + c)^2 - 4*(C*a^2 + 2*B*a*b)*tan(d*x + c))/tan(d*x +
c)^4)/d
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(145) = 290$.

Time = 0.98 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.88

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,Ca^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^2*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
-1/192*(3*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^2*tan(1/2*d*x + 1/2*c)^3 -
16*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a
*b*tan(1/2*d*x + 1/2*c)^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*a^2*ta
n(1/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*d*x + 1/2*c) - 96*C*b^2*tan(1/2*d*x
 + 1/2*c) - 192*(C*a^2 + 2*B*a*b - C*b^2)*(d*x + c) + 192*(B*a^2 - 2*C*a*b
 - B*b^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^2 - 2*C*a*b - B*b^2)*
log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 800*C
*a*b*tan(1/2*d*x + 1/2*c)^4 - 400*B*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^2
*tan(1/2*d*x + 1/2*c)^3 - 240*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 96*C*b^2*tan(
1/2*d*x + 1/2*c)^3 - 36*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 48*C*a*b*tan(1/2*d*
x + 1/2*c)^2 + 24*B*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*a^2*tan(1/2*d*x + 1/2
*c) + 16*B*a*b*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^2 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{\cot(c+d\,x)^4\left(\frac{B\,a^2}{4}+\tan(c+d\,x)^2\left(-\frac{B\,a^2}{2}+C\,a\,b+\frac{B\,b^2}{2}\right)-\tan(c+d\,x)^3\left(C\,a^2+2\,B\,a\,b-C\,b^2\right)+\mathrm{ta}}{d}$$

$$-\frac{\ln\left(\tan(c+d\,x)\right)\left(-B\,a^2+2\,C\,a\,b+B\,b^2\right)}{d}$$

$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\left(B-C\,1\mathrm{i}\right)\left(b+a\,1\mathrm{i}\right)^2}{2\,d}$$

$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\left(B+C\,1\mathrm{i}\right)\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$

input
```
int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x)
)*(B*b^2 - B*a^2 + 2*C*a*b))/d - (cot(c + d*x)^4*((B*a^2)/4 + tan(c + d*x)
^2*((B*b^2)/2 - (B*a^2)/2 + C*a*b) - tan(c + d*x)^3*(C*a^2 - C*b^2 + 2*B*a
*b) + tan(c + d*x)*((C*a^2)/3 + (2*B*a*b)/3)))/d + (log(tan(c + d*x) - 1i)
*(B + C*1i)*(a*1i - b)^2)/(2*d)
```

3.17 $\int (a+b\tan(c+dx))^3 (B\tan(c+dx) + C\tan^2(c+dx))$

3.17.1 Optimal result

Integrand size = 32, antiderivative size = 165

$$\begin{aligned}\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\ = -\left(\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x\right)-\frac{(a^3B-3ab^2B-3a^2bC+b^3C)\log(\cos(c+dx))}{d}\\ +\frac{b(a^2B-b^2B-2abC)\tan(c+dx)}{d}+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}\\ +\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}\end{aligned}$$

output
```
-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(
cos(d*x+c))/d+b*(B*a^2-B*b^2-2*C*a*b)*tan(d*x+c)/d+1/2*(B*a-C*b)*(a+b*tan(
d*x+c))^2/d+1/3*B*(a+b*tan(d*x+c))^3/d+1/4*C*(a+b*tan(d*x+c))^4/b/d
```

3.17.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.27

$$\begin{aligned}\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\ = \frac{-6i(a+ib)^4B\log(i-\tan(c+dx))+6i(a-ib)^4B\log(i+\tan(c+dx))-12b^2(-6a^2+b^2)\,B\tan(c+dx)}{}\end{aligned}$$

input
```
Integrate[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
((-6*I)*(a + I*b)^4*B*Log[I - Tan[c + d*x]] + (6*I)*(a - I*b)^4*B*Log[I +
Tan[c + d*x]] - 12*b^2*(-6*a^2 + b^2)*B*Tan[c + d*x] + 24*a*b^3*B*Tan[c +
d*x]^2 + 4*b^4*B*Tan[c + d*x]^3 + 3*C*(a + b*Tan[c + d*x])^4 - 6*(a*B + b*
C)*((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]]
+ 6*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(12*b*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan(c+dx)^2\right)dx$$

$$\Big\downarrow \text{4113}$$

$$\int (a+b\tan(c+dx))^3(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3(B\tan(c+dx)-C)dx+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(-bB-aC+(aB-bC)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 4011$$

$$\int (a+b\tan(c+dx))\left(-Ca^2-2bBa+b^2C+\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(c+dx))\left(-Ca^2-2bBa+b^2C+\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 4008$$

$$\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\int \tan(c+dx)dx+\frac{b\left(a^2B-2abC-b^2B\right)\tan(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\int \tan(c+dx)dx+\frac{b\left(a^2B-2abC-b^2B\right)\tan(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3956$$

$$\frac{b\left(a^2B-2abC-b^2B\right)\tan(c+dx)}{d}-\frac{\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\log(\cos(c+dx))}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{(aB-bC)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{C(a+b\tan(c+dx))^4}{4bd}$$

input
```
Int[(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2),x]
```

output
```
-((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - ((a^3*B - 3*a*b^2*B - 3*a^2
*b*C + b^3*C)*Log[Cos[c + d*x]])/d + (b*(a^2*B - b^2*B - 2*a*b*C)*Tan[c +
d*x])/d + ((a*B - b*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (B*(a + b*Tan[c + d
*x])^3)/(3*d) + (C*(a + b*Tan[c + d*x])^4)/(4*b*d)
```

3.17. $\int (a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.17.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.17.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.09

method	result
norman	$\left(-3B\,a^{2}b+B\,b^{3}-C\,a^{3}+3Ca\,b^{2}\right)x+\frac{\left(3B\,a^{2}b-B\,b^{3}+C\,a^{3}-3Ca\,b^{2}\right)\tan(dx+c)}{d}+\frac{C\,b^{3}\tan(dx+c)^{4}}{4d}$
parts	$\frac{\left(B\,b^{3}+3Ca\,b^{2}\right)\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{\left(3Ba\,b^{2}+3C\,a^{2}b\right)\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}$
derivativedivides	$\frac{\frac{C\,b^{3}\tan(dx+c)^{4}}{4}+\frac{B\,b^{3}\tan(dx+c)^{3}}{3}+Ca\,b^{2}\tan(dx+c)^{3}+\frac{3Ba\,b^{2}\tan(dx+c)^{2}}{2}+\frac{3C\,a^{2}b\tan(dx+c)^{2}}{2}-\frac{C\,b^{3}\tan(dx+c)^{2}}{2}+3B\,a^{2}b\,\mathrm{ta}}{}$
default	$\frac{\frac{C\,b^{3}\tan(dx+c)^{4}}{4}+\frac{B\,b^{3}\tan(dx+c)^{3}}{3}+Ca\,b^{2}\tan(dx+c)^{3}+\frac{3Ba\,b^{2}\tan(dx+c)^{2}}{2}+\frac{3C\,a^{2}b\tan(dx+c)^{2}}{2}-\frac{C\,b^{3}\tan(dx+c)^{2}}{2}+3B\,a^{2}b\,\mathrm{ta}}{}$
parallelrisch	$\frac{3C\,b^{3}\tan(dx+c)^{4}+4B\,b^{3}\tan(dx+c)^{3}+12Ca\,b^{2}\tan(dx+c)^{3}-36B\,a^{2}bdx+12B\,b^{3}dx+18Ba\,b^{2}\tan(dx+c)^{2}-12C\,a^{3}dx+3}{}$
risch	$-3iBa\,b^{2}x+iB\,a^{3}x+iC\,b^{3}x-\frac{6iBa\,b^{2}c}{d}-3B\,a^{2}bx+B\,b^{3}x-C\,a^{3}x+3Ca\,b^{2}x-\frac{6iC\,a^{2}}{d}$

input
```
int((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_RETURNVERBO
SE)
```

output
```
(-3*B*a^2*b+B*b^3-C*a^3+3*C*a*b^2)*x+(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)/d*t
an(d*x+c)+1/4*C*b^3/d*tan(d*x+c)^4+1/2*b*(3*B*a*b+3*C*a^2-C*b^2)/d*tan(d*x
+c)^2+1/3*b^2*(B*b+3*C*a)/d*tan(d*x+c)^3+1/2*(B*a^3-3*B*a*b^2-3*C*a^2*b+C*
b^3)/d*ln(1+tan(d*x+c)^2)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{3\,Cb^{3}\tan\left(dx+c\right)^{4}+4\left(3\,Cab^{2}+Bb^{3}\right)\tan\left(dx+c\right)^{3}-12\left(Ca^{3}+3\,Ba^{2}b-3\,Cab^{2}-Bb^{3}\right)dx+6\left(3\,Ca^{2}b\right.}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 - 12*(
C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x + 6*(3*C*a^2*b + 3*B*a*b^2 - C*
b^3)*tan(d*x + c)^2 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(1/(tan
(d*x + c)^2 + 1)) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c
))/d
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(151) = 302$.

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.90

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}\frac{Ba^3\log\left(\tan^2(c+dx)+1\right)}{2d}-3Ba^2bx+\frac{3Ba^2b\tan(c+dx)}{d}-\frac{3Bab^2\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{3Bab^2\tan^2(c+dx)}{2d}+Bb^3x+\frac{Bb^3\tan\ldots}{\ldots}\\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a**2*b*x + 3*B*a**2
*b*tan(c + d*x)/d - 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2
*tan(c + d*x)**2/(2*d) + B*b**3*x + B*b**3*tan(c + d*x)**3/(3*d) - B*b**3*
tan(c + d*x)/d - C*a**3*x + C*a**3*tan(c + d*x)/d - 3*C*a**2*b*log(tan(c +
 d*x)**2 + 1)/(2*d) + 3*C*a**2*b*tan(c + d*x)**2/(2*d) + 3*C*a*b**2*x + C*
a*b**2*tan(c + d*x)**3/d - 3*C*a*b**2*tan(c + d*x)/d + C*b**3*log(tan(c +
d*x)**2 + 1)/(2*d) + C*b**3*tan(c + d*x)**4/(4*d) - C*b**3*tan(c + d*x)**2
/(2*d), Ne(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*tan(c)**2), True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{3\,Cb^3\tan(dx+c)^4+4\,(3\,Cab^2+Bb^3)\tan(dx+c)^3+6\,(3\,Ca^2b+3\,Bab^2-Cb^3)\tan(dx+c)^2-12\,(Ca\ldots}{\ldots}$$

input
```
integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="m
axima")
```

output
```
1/12*(3*C*b^3*tan(d*x + c)^4 + 4*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^3 + 6*(3
*C*a^2*b + 3*B*a*b^2 - C*b^3)*tan(d*x + c)^2 - 12*(C*a^3 + 3*B*a^2*b - 3*C
*a*b^2 - B*b^3)*(d*x + c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(
tan(d*x + c)^2 + 1) + 12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x +
 c))/d
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2670 vs. $2(159) = 318$.

Time = 2.37 (sec) , antiderivative size = 2670, normalized size of antiderivative = 16.18

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, algorithm="g
iac")
```

output
```
-1/12*(12*C*a^3*d*x*tan(d*x)^4*tan(c)^4 + 36*B*a^2*b*d*x*tan(d*x)^4*tan(c)
^4 - 36*C*a*b^2*d*x*tan(d*x)^4*tan(c)^4 - 12*B*b^3*d*x*tan(d*x)^4*tan(c)^4
 + 6*B*a^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 18*C*a^2*b*l
og(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 18*B*a*b^2*log(4*(tan(d*
x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
 tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 6*C*b^3*log(4*(tan(d*x)^2*tan(c)^2 -
 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))
*tan(d*x)^4*tan(c)^4 - 48*C*a^3*d*x*tan(d*x)^3*tan(c)^3 - 144*B*a^2*b*d*x*
tan(d*x)^3*tan(c)^3 + 144*C*a*b^2*d*x*tan(d*x)^3*tan(c)^3 + 48*B*b^3*d*x*t
an(d*x)^3*tan(c)^3 - 18*C*a^2*b*tan(d*x)^4*tan(c)^4 - 18*B*a*b^2*tan(d*x)^
4*tan(c)^4 + 9*C*b^3*tan(d*x)^4*tan(c)^4 - 24*B*a^3*log(4*(tan(d*x)^2*tan(
c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
 + 1))*tan(d*x)^3*tan(c)^3 + 72*C*a^2*b*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan
(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d
*x)^3*tan(c)^3 + 72*B*a*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c)
 + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)
^3 - 24*C*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 12*C*a...
```

3.17.9 Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\int (a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= x\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)-\frac{\tan(c+d\,x)^2\left(\frac{C\,b^3}{2}-\frac{3\,a\,b\,(B\,b+C\,a)}{2}\right)}{d}$$
$$-\frac{\tan(c+d\,x)\,\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a^3}{2}-\frac{3\,C\,a^2\,b}{2}-\frac{3\,B\,a\,b^2}{2}+\frac{C\,b^3}{2}\right)}{d}$$
$$+\frac{\tan(c+d\,x)^3\,\left(\frac{B\,b^3}{3}+C\,a\,b^2\right)}{d}+\frac{C\,b^3\,\tan(c+d\,x)^4}{4\,d}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^3,x)
```

output
```
x*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2) - (tan(c + d*x)^2*((C*b^3)/2 - (
3*a*b*(B*b + C*a))/2))/d - (tan(c + d*x)*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*
a*b^2))/d + (log(tan(c + d*x)^2 + 1)*((B*a^3)/2 + (C*b^3)/2 - (3*B*a*b^2)/
2 - (3*C*a^2*b)/2))/d + (tan(c + d*x)^3*((B*b^3)/3 + C*a*b^2))/d + (C*b^3*
tan(c + d*x)^4)/(4*d)
```

3.18 $\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx) + C\tan$

3.18.1 Optimal result

Integrand size = 38, antiderivative size = 140

$$\begin{aligned}&\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\&=\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x-\frac{(3a^2bB-b^3B+a^3C-3ab^2C)\log(\cos(c+dx))}{d}\\&\quad+\frac{b(2abB+a^2C-b^2C)\tan(c+dx)}{d}\\&\quad+\frac{(bB+aC)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}\end{aligned}$$

output
```
(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(c
os(d*x+c))/d+b*(2*B*a*b+C*a^2-C*b^2)*tan(d*x+c)/d+1/2*(B*b+C*a)*(a+b*tan(d
*x+c))^2/d+1/3*C*(a+b*tan(d*x+c))^3/d
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{3(a+ib)^3(-iB+C)\log(i-\tan(c+dx))+3(a-ib)^3(iB+C)\log(i+\tan(c+dx))+6b(3abB+3a^2C}{6d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c +
d*x]^2),x]
```

output
```
(3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan[c + d*x]] + 3*(a - I*b)^3*(I*B + C
)*Log[I + Tan[c + d*x]] + 6*b*(3*a*b*B + 3*a^2*C - b^2*C)*Tan[c + d*x] + 3
*b^2*(b*B + 3*a*C)*Tan[c + d*x]^2 + 2*b^3*C*Tan[c + d*x]^3)/(6*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4115, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4115}$$

$$\int (a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(aB-bC+(bB+aC)\tan(c+dx))dx+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))\left(Ba^2-2bCa-b^2B+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4008}$$

$$\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int \tan(c+dx)dx+\frac{b\left(a^2C+2abB-b^2C\right)\tan(c+dx)}{d}+x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int \tan(c+dx)dx+\frac{b\left(a^2C+2abB-b^2C\right)\tan(c+dx)}{d}+x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{b\left(a^2C+2abB-b^2C\right)\tan(c+dx)}{d}-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\log(\cos(c+dx))}{d}+x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{(aC+bB)(a+b\tan(c+dx))^2}{2d}+\frac{C(a+b\tan(c+dx))^3}{3d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2
),x]
```

output
```
(a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x - ((3*a^2*b*B - b^3*B + a^3*C -
3*a*b^2*C)*Log[Cos[c + d*x]])/d + (b*(2*a*b*B + a^2*C - b^2*C)*Tan[c + d*x
])/d + ((b*B + a*C)*(a + b*Tan[c + d*x])^2)/(2*d) + (C*(a + b*Tan[c + d*x]
)^3)/(3*d)
```

3.18. $\int \cot(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{3(3Ba^2b-Bb^3+Ca^3-3Cab^2)\ln\left(\sec(dx+c)^2\right)+2Cb^3\tan(dx+c)^3+3(Bb^3+3Cab^2)\tan(dx+c)^2+6(3Bab^2+3Ca^2b-\cdots}{6d}$
norman	$(Ba^3-3Bab^2-3Ca^2b+Cb^3)x+\frac{b(3Bab+3Ca^2-Cb^2)\tan(dx+c)}{d}+\frac{Cb^3\tan(dx+c)^3}{3d}+\frac{b^2(Bb+\cdots}{}$
derivativedivides	$\frac{\frac{Cb^3\tan(dx+c)^3}{3}+\frac{Bb^3\tan(dx+c)^2}{2}+\frac{3Cab^2\tan(dx+c)^2}{2}+3Bab^2\tan(dx+c)+3Ca^2b\tan(dx+c)-Cb^3\tan(dx+c)+\frac{(3Ba^2b-\cdots}{}}{d}$
default	$\frac{\frac{Cb^3\tan(dx+c)^3}{3}+\frac{Bb^3\tan(dx+c)^2}{2}+\frac{3Cab^2\tan(dx+c)^2}{2}+3Bab^2\tan(dx+c)+3Ca^2b\tan(dx+c)-Cb^3\tan(dx+c)+\frac{(3Ba^2b-\cdots}{}}{d}$
risch	$Ba^3x-3Bab^2x-3Ca^2bx+Cb^3x-\frac{2iBb^3c}{d}+\frac{2iCa^3c}{d}-iBb^3x+iCa^3x+\frac{2ib(-3iBb^2e^{4\cdots}}{}$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method=_
RETURNVERBOSE)
```

output
```
1/6*(3*(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(sec(d*x+c)^2)+2*C*b^3*tan(d*x+
c)^3+3*(B*b^3+3*C*a*b^2)*tan(d*x+c)^2+6*(3*B*a*b^2+3*C*a^2*b-C*b^3)*tan(d*
x+c)+6*d*x*(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3))/d
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \cot(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^3\tan\left(dx+c\right)^3+6\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)dx+3\,(3\,Cab^2+Bb^3)\tan\left(dx+c\right)^2-3\,(Ca^3+3\cdots}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="fricas")
```

output
```
1/6*(2*C*b^3*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*
x + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 - 3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^
2 - B*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^3)
*tan(d*x + c))/d
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.77

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \begin{cases} Ba^3x + \frac{3Ba^2b\log(\tan^2(c+dx)+1)}{2d} - 3Bab^2x + \frac{3Bab^2\tan(c+dx)}{d} - \frac{Bb^3\log(\tan^2(c+dx)+1)}{2d} + \frac{Bb^3\tan^2(c+dx)}{2d} + \frac{Ca^3\log(t}{} \\ x(a+b\tan(c))^3 (B\tan(c)+C\tan^2(c))\cot(c) \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2),x)
```

output
```
Piecewise((B*a**3*x + 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a*b*
*2*x + 3*B*a*b**2*tan(c + d*x)/d - B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) +
 B*b**3*tan(c + d*x)**2/(2*d) + C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*
C*a**2*b*x + 3*C*a**2*b*tan(c + d*x)/d - 3*C*a*b**2*log(tan(c + d*x)**2 +
1)/(2*d) + 3*C*a*b**2*tan(c + d*x)**2/(2*d) + C*b**3*x + C*b**3*tan(c + d*
x)**3/(3*d) - C*b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(a + b*tan(c))**3*(B*ta
n(c) + C*tan(c)**2)*cot(c), True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \frac{2\,Cb^3\tan(dx+c)^3 + 3\,(3\,Cab^2+Bb^3)\tan(dx+c)^2 + 6\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c) + 3\,(Ca}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="maxima")
```

output
```
1/6*(2*C*b^3*tan(d*x + c)^3 + 3*(3*C*a*b^2 + B*b^3)*tan(d*x + c)^2 + 6*(B*
a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*a^2*b - 3*
C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) + 6*(3*C*a^2*b + 3*B*a*b^2 - C*b^
3)*tan(d*x + c))/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^3\tan\left(dx+c\right)^3+9\,Cab^2\tan\left(dx+c\right)^2+3\,Bb^3\tan\left(dx+c\right)^2+18\,Ca^2b\tan\left(dx+c\right)+18\,Bab^2\tan\left(dx\right.}{}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x, a
lgorithm="giac")
```

output
```
1/6*(2*C*b^3*tan(d*x + c)^3 + 9*C*a*b^2*tan(d*x + c)^2 + 3*B*b^3*tan(d*x +
 c)^2 + 18*C*a^2*b*tan(d*x + c) + 18*B*a*b^2*tan(d*x + c) - 6*C*b^3*tan(d*
x + c) + 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 +
3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1))/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int \cot(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=x\left(B\,a^3-3\,C\,a^2\,b-3\,B\,a\,b^2+C\,b^3\right)-\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(-\frac{C\,a^3}{2}-\frac{3\,B\,a^2\,b}{2}+\frac{3\,C\,a\,b^2}{2}+\frac{B\,b^3}{2}\right)}{d}+\frac{\tan(c+d\,x)^2\,\left(\frac{B\,b^3}{2}+\frac{3\,C\,a\,b^2}{2}\right)}{d}-\frac{\tan(c+d\,x)\,\left(C\,b^3-3\,a\,b\,(B\,b+C\,a)\right)}{d}+\frac{C\,b^3\tan(c+d\,x)^3}{3\,d}$$

input
```
int(cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x))^
3,x)
```

output
```
x*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2*b) - (log(tan(c + d*x)^2 + 1)*((B*b
^3)/2 - (C*a^3)/2 - (3*B*a^2*b)/2 + (3*C*a*b^2)/2))/d + (tan(c + d*x)^2*((
B*b^3)/2 + (3*C*a*b^2)/2))/d - (tan(c + d*x)*(C*b^3 - 3*a*b*(B*b + C*a)))/
d + (C*b^3*tan(c + d*x)^3)/(3*d)
```

3.19 $\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\,\mathrm{ta}\right.$

3.19.1 Optimal result

Integrand size = 40, antiderivative size = 117

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x-\frac{b(3abB+3a^2C-b^2C)\log(\cos(c+dx))}{d}$$
$$+\frac{a^3B\log(\sin(c+dx))}{d}+\frac{b^2(bB+2aC)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

output
```
(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-b*(3*B*a*b+3*C*a^2-C*b^2)*ln(cos(d*x+c
))/d+a^3*B*ln(sin(d*x+c))/d+b^2*(B*b+2*C*a)*tan(d*x+c)/d+1/2*b*C*(a+b*tan(
d*x+c))^2/d
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{-(a+ib)^3(B+iC)\log(i-\tan(c+dx))+2a^3B\log(\tan(c+dx))-(a-ib)^3(B-iC)\log(i+\tan(c+d}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-((a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]]) + 2*a^3*B*Log[Tan[c + d*x]
] - (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]] + 2*b^2*(b*B + 3*a*C)*Tan[
c + d*x] + b^3*C*Tan[c + d*x]^2)/(2*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 4115, 3042, 4090, 27, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{1}{2}\int 2\cot(c+dx)(a+b\tan(c+dx))\left(Ba^2+b(bB+2aC)\tan^2(c+dx)+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(Ba^2+b(bB+2aC)\tan^2(c+dx)+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)dx+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(Ba^2+b(bB+2aC)\tan(c+dx)^2+\left(Ca^2+2bBa-b^2C\right)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 4120

$$-\int -\cot(c+dx)\left(Ba^3+b\left(3Ca^2+3bBa-b^2C\right)\tan^2(c+dx)+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$\int \cot(c+dx)\left(Ba^3+b\left(3Ca^2+3bBa-b^2C\right)\tan^2(c+dx)+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3042

$$\int \frac{Ba^3+b\left(3Ca^2+3bBa-b^2C\right)\tan(c+dx)^2+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 4107

$$a^3B\int \cot(c+dx)dx+b\left(3a^2C+3abB-b^2C\right)\int \tan(c+dx)dx+x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3042

$$a^3B\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b\left(3a^2C+3abB-b^2C\right)\int \tan(c+dx)dx+x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$a^3(-B)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b\left(3a^2C+3abB-b^2C\right)\int \tan(c+dx)dx+x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)+\frac{b^2(2aC+bB)\tan(c+dx)}{d}+\frac{bC(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3956

3.19. $\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

$$\frac{a^3 B \log(-\sin(c+dx))}{d} - \frac{b(3a^2C + 3abB - b^2C)\log(\cos(c+dx))}{d} + x(a^3C + 3a^2bB - 3ab^2C - b^3B) + \frac{b^2(2aC + bB)\tan(c+dx)}{d} + \frac{bC(a + b\tan(c+dx))^2}{2d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x - (b*(3*a*b*B + 3*a^2*C - b^2*C)
*Log[Cos[c + d*x]])/d + (a^3*B*Log[-Sin[c + d*x]])/d + (b^2*(b*B + 2*a*C)*
Tan[c + d*x])/d + (b*C*(a + b*Tan[c + d*x])^2)/(2*d)
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.19.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{\left(-B\,a^{3}+3Ba\,b^{2}+3C\,a^{2}b-C\,b^{3}\right)\ln\left(\sec(dx+c)^{2}\right)+2B\,a^{3}\ln(\tan(dx+c))+C\,b^{3}\tan(dx+c)^{2}+\left(2B\,b^{3}+6Ca\,b^{2}\right)\tan(dx+c)}{2d}$
derivativedivides	$\frac{\frac{C\,b^{3}\tan(dx+c)^{2}}{2}+B\,b^{3}\tan(dx+c)+3Ca\,b^{2}\tan(dx+c)+B\,a^{3}\ln(\tan(dx+c))+\frac{\left(-B\,a^{3}+3Ba\,b^{2}+3C\,a^{2}b-C\,b^{3}\right)\ln\left(1+\tan(dx+c\right.}{2}}{d}$
default	$\frac{\frac{C\,b^{3}\tan(dx+c)^{2}}{2}+B\,b^{3}\tan(dx+c)+3Ca\,b^{2}\tan(dx+c)+B\,a^{3}\ln(\tan(dx+c))+\frac{\left(-B\,a^{3}+3Ba\,b^{2}+3C\,a^{2}b-C\,b^{3}\right)\ln\left(1+\tan(dx+c\right.}{2}}{d}$
norman	$\frac{\left(3B\,a^{2}b-B\,b^{3}+C\,a^{3}-3Ca\,b^{2}\right)x\tan(dx+c)+\frac{b^{2}(Bb+3Ca)\tan(dx+c)^{2}}{d}+\frac{C\,b^{3}\tan(dx+c)^{3}}{2d}}{\tan(dx+c)}+\frac{B\,a^{3}\ln(\tan(dx+c))}{d}-\frac{(B\,a^{3}}{}$
risch	$\frac{6iBa\,b^{2}c}{d}-iB\,a^{3}x+3iC\,a^{2}bx+\frac{2i\,b^{2}\left(Bb\,\mathrm{e}^{2i(dx+c)}+3Ca\,\mathrm{e}^{2i(dx+c)}-iCb\,\mathrm{e}^{2i(dx+c)}+Bb+3Ca\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+3B\,a^{2}bx$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*((-B*a^3+3*B*a*b^2+3*C*a^2*b-C*b^3)*ln(sec(d*x+c)^2)+2*B*a^3*ln(tan(d*
x+c))+C*b^3*tan(d*x+c)^2+(2*B*b^3+6*C*a*b^2)*tan(d*x+c)+6*d*(B*a^2*b-1/3*B
*b^3+1/3*C*a^3-C*a*b^2)*x)/d
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^3\tan\left(dx+c\right)^2+Ba^3\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)dx-\left(3\,Ca^2b+3\,Bab^2-C\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/2*(C*b^3*tan(d*x + c)^2 + B*a^3*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
 + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x - (3*C*a^2*b + 3*B*a*b^2
- C*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))
/d
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.80

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}-\frac{Ba^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba^3\log\left(\tan\left(c+dx\right)\right)}{d}+3Ba^2bx+\frac{3Bab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-Bb^3x+\frac{Bb^3\tan\left(c+dx\right)}{d}+Ca^3\\ x(a+b\tan\left(c\right))^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^2\left(c\right)\end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((-B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x
))/d + 3*B*a**2*b*x + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*x
 + B*b**3*tan(c + d*x)/d + C*a**3*x + 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/
(2*d) - 3*C*a*b**2*x + 3*C*a*b**2*tan(c + d*x)/d - C*b**3*log(tan(c + d*x)
**2 + 1)/(2*d) + C*b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tan(c)
)**3*(B*tan(c) + C*tan(c)**2)*cot(c)**2, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^3\tan{(dx+c)}^2+2\,Ba^3\log{(\tan{(dx+c)})}+2\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)(dx+c)-(Ba^3-3\,Ca^2}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(tan(d*x + c)) + 2*(C*a^3 + 3*B*a^2
*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3
)*log(tan(d*x + c)^2 + 1) + 2*(3*C*a*b^2 + B*b^3)*tan(d*x + c))/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{Cb^3\tan{(dx+c)}^2+2\,Ba^3\log{(|\tan{(dx+c)}|)}+6\,Cab^2\tan{(dx+c)}+2\,Bb^3\tan{(dx+c)}+2\,(Ca^3+3\,Ba}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/2*(C*b^3*tan(d*x + c)^2 + 2*B*a^3*log(abs(tan(d*x + c))) + 6*C*a*b^2*tan
(d*x + c) + 2*B*b^3*tan(d*x + c) + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^
3)*(d*x + c) - (B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2
+ 1))/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(B\,b^3+3\,C\,a\,b^2\right)}{d}+\frac{B\,a^3\,\ln\left(\tan(c+d\,x)\right)}{d}$$
$$+\frac{C\,b^3\tan(c+d\,x)^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output
```
(tan(c + d*x)*(B*b^3 + 3*C*a*b^2))/d + (B*a^3*log(tan(c + d*x)))/d - (log(
tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b)^3*1i)/(2*d) - (log(tan(c + d*x) -
 1i)*(B + C*1i)*(a*1i - b)^3*1i)/(2*d) + (C*b^3*tan(c + d*x)^2)/(2*d)
```

3.20 $\int \cot^3(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan\right.$

3.20.1 Optimal result

Integrand size = 40, antiderivative size = 119

$$\begin{aligned}&\int \cot^3(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\&=-\left(\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x\right)\\&\quad-\frac{b^2(bB+3aC)\log(\cos(c+dx))}{d}+\frac{a^2(3bB+aC)\log(\sin(c+dx))}{d}\\&\quad+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}\end{aligned}$$

output
```
-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-b^2*(B*b+3*C*a)*ln(cos(d*x+c))/d+a^2*
(3*B*b+C*a)*ln(sin(d*x+c))/d+b^2*(B*a+C*b)*tan(d*x+c)/d-a*B*cot(d*x+c)*(a+
b*tan(d*x+c))^2/d
```

3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \cot^3(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\&=\frac{-2a^3B\cot(c+dx)+i(a+ib)^3(B+iC)\log(i-\tan(c+dx))+2a^2(3bB+aC)\log(\tan(c+dx))+(ia+}{2d}\end{aligned}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-2*a^3*B*Cot[c + d*x] + I*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 2
*a^2*(3*b*B + a*C)*Log[Tan[c + d*x]] + (I*a + b)^3*(B - I*C)*Log[I + Tan[c
 + d*x]] + 2*b^3*C*Tan[c + d*x])/(2*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4115, 3042, 4088, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \ 4115$$

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \ 4088$$

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(b(aB+bC)\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(3bB+aC)\right)dx-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))\left(b(aB+bC)\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(3bB+aC)\right)}{\tan(c+dx)}dx-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 4120

$$-\int -\cot(c+dx)\left((3bB+aC)a^2+b^2(bB+3aC)\tan^2(c+dx)-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 25

$$\int \cot(c+dx)\left((3bB+aC)a^2+b^2(bB+3aC)\tan^2(c+dx)-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 3042

$$\int \frac{(3bB+aC)a^2+b^2(bB+3aC)\tan(c+dx)^2-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 4107

$$a^2(aC+3bB)\int \cot(c+dx)dx+b^2(3aC+bB)\int \tan(c+dx)dx-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 3042

$$a^2(aC+3bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^2(3aC+bB)\int \tan(c+dx)dx-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 25

$$-\left(a^2(aC+3bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)+b^2(3aC+bB)\int \tan(c+dx)dx-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\Big\downarrow$ 3956

3.20. $\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

$$\frac{a^2(aC+3bB)\log(-\sin(c+dx))}{d}-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)+\frac{b^2(aB+bC)\tan(c+dx)}{d}-\frac{b^2(3aC+bB)\log(\cos(c+dx))}{d}-\frac{aB\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
-((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) - (b^2*(b*B + 3*a*C)*Log[Cos[
c + d*x]])/d + (a^2*(3*b*B + a*C)*Log[-Sin[c + d*x]])/d + (b^2*(a*B + b*C)
*Tan[c + d*x])/d - (a*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{(-3B a^2 b+B b^3-C a^3+3C a b^2)\ln\left(\sec(dx+c)^2\right)+(6B a^2 b+2C a^3)\ln(\tan(dx+c))-2B\cot(dx+c)a^3+2C b^3\tan(dx+c)\cdots}{2d}$
derivativedivides	$\frac{C b^3\tan(dx+c)-\frac{B a^3}{\tan(dx+c)}+a^2(3Bb+Ca)\ln(\tan(dx+c))+\frac{(-3B a^2 b+B b^3-C a^3+3C a b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-B a^3+3\cdots}{d}$
default	$\frac{C b^3\tan(dx+c)-\frac{B a^3}{\tan(dx+c)}+a^2(3Bb+Ca)\ln(\tan(dx+c))+\frac{(-3B a^2 b+B b^3-C a^3+3C a b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-B a^3+3\cdots}{d}$
norman	$\frac{(-B a^3+3B a b^2+3C a^2 b-C b^3)x\tan(dx+c)^2+\frac{C b^3\tan(dx+c)^3}{d}-\frac{B a^3\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{a^2(3Bb+Ca)\ln(\tan(dx+c))}{d}-\frac{(3\cdots}{\cdots}$
risch	$-B a^3 x+3B a b^2 x+3C a^2 b x-C b^3 x-\frac{2iC a^3 c}{d}+iB b^3 x-iC a^3 x+\frac{2iB b^3 c}{d}-\frac{2i(B a^3 e^{2i(\cdots}}{d(e^{\cdots}}$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*((-3*B*a^2*b+B*b^3-C*a^3+3*C*a*b^2)*ln(sec(d*x+c)^2)+(6*B*a^2*b+2*C*a^
3)*ln(tan(d*x+c))-2*B*cot(d*x+c)*a^3+2*C*b^3*tan(d*x+c)-2*d*x*(B*a^3-3*B*a
*b^2-3*C*a^2*b+C*b^3))/d
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^3\tan\left(dx+c\right)^2-2\,Ba^3-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)dx\tan\left(dx+c\right)+(Ca^3+3\,Ba^2b)\log\left(\frac{\text{ta}}{\text{tan}}\right.}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/2*(2*C*b^3*tan(d*x + c)^2 - 2*B*a^3 - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 +
 C*b^3)*d*x*tan(d*x + c) + (C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c) - (3*C*a*b^2 + B*b^3)*log(1/(tan(d*x + c)^2 + 1
))*tan(d*x + c))/(d*tan(d*x + c))
```

3.20.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.80

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}\text{NaN}\\ x(a+b\tan\left(c\right))^3\left(B\tan\left(c\right)+C\tan^2\left(c\right)\right)\cot^3\left(c\right)\\ \text{NaN}\\ -Ba^3x-\frac{Ba^3}{d\tan\left(c+dx\right)}-\frac{3Ba^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3Ba^2b\log\left(\tan\left(c+dx\right)\right)}{d}+3Bab^2x+\frac{Bb^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-\frac{Ca^3\text{lo}}{}\end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*t
an(c)**2)*cot(c)**3, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**3*x - B*a**3/(d
*tan(c + d*x)) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a**2*b*lo
g(tan(c + d*x))/d + 3*B*a*b**2*x + B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) -
 C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + C*a**3*log(tan(c + d*x))/d + 3*C*
a**2*b*x + 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - C*b**3*x + C*b**3*t
an(c + d*x)/d, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^3\tan\left(dx+c\right)-\frac{2\,Ba^3}{\tan(dx+c)}-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c)-(Ca^3+3\,Ba^2b-3\,Cab^2-B}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/2*(2*C*b^3*tan(d*x + c) - 2*B*a^3/tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b -
3*B*a*b^2 + C*b^3)*(d*x + c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log
(tan(d*x + c)^2 + 1) + 2*(C*a^3 + 3*B*a^2*b)*log(tan(d*x + c)))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.28

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\,Cb^3\tan\left(dx+c\right)-2\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c)-(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\tan\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/2*(2*C*b^3*tan(d*x + c) - 2*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x
 + c) - (C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) +
2*(C*a^3 + 3*B*a^2*b)*log(abs(tan(d*x + c))) - 2*(C*a^3*tan(d*x + c) + 3*B
*a^2*b*tan(d*x + c) + B*a^3)/tan(d*x + c))/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(C\,a^3+3\,B\,b\,a^2\right)}{d}-\frac{B\,a^3\cot(c+d\,x)}{d}+\frac{C\,b^3\tan(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(tan(c + d*x))*(C*a^3 + 3*B*a^2*b))/d + (log(tan(c + d*x) - 1i)*(B + C
*1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i
)^3*1i)/(2*d) - (B*a^3*cot(c + d*x))/d + (C*b^3*tan(c + d*x))/d
```

3.21 $\int \cot^4(c+dx)(a+b\tan(c+dx))^3 (B\tan(c+dx)+C\,\mathrm{ta}$

3.21.1 Optimal result

Integrand size = 40, antiderivative size = 127

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= -\left(\left(3a^2bB-b^3B+a^3C-3ab^2C\right)x\right)-\frac{a^2(2bB+aC)\cot(c+dx)}{d}-\frac{b^3C\log(\cos(c+dx))}{d}$$
$$-\frac{a(a^2B-3b^2B-3abC)\log(\sin(c+dx))}{d}-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

output
```
-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x-a^2*(2*B*b+C*a)*cot(d*x+c)/d-b^3*C*ln
(cos(d*x+c))/d-a*(B*a^2-3*B*b^2-3*C*a*b)*ln(sin(d*x+c))/d-1/2*a*B*cot(d*x+
c)^2*(a+b*tan(d*x+c))^2/d
```

3.21.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{-2a^2(3bB+aC)\cot(c+dx)-a^3B\cot^2(c+dx)+(a+ib)^3(B+iC)\log(i-\tan(c+dx))-2a(a^2B-3}{2d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-2*a^2*(3*b*B + a*C)*Cot[c + d*x] - a^3*B*Cot[c + d*x]^2 + (a + I*b)^3*(B
 + I*C)*Log[I - Tan[c + d*x]] - 2*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log[Tan[c
+ d*x]] + (a - I*b)^3*(B - I*C)*Log[I + Tan[c + d*x]])/(2*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 4115, 3042, 4088, 27, 3042, 4118, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{2}\int 2\cot^2(c+dx)(a+b\tan(c+dx))\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(b^2C\tan^2(c+dx)-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)dx-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(b^2C\tan(c+dx)^2-\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(2bB+aC)\right)}{\tan(c+dx)^2}dx-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 4118

$$\int -\cot(c+dx)\left(-C\tan^2(c+dx)b^3+a\left(Ba^2-3bCa-3b^2B\right)+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$-\int \cot(c+dx)\left(-C\tan^2(c+dx)b^3+a\left(Ba^2-3bCa-3b^2B\right)+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3042

$$-\int \frac{-C\tan(c+dx)^2b^3+a\left(Ba^2-3bCa-3b^2B\right)+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 4107

$$-a\left(a^2B-3abC-3b^2B\right)\int \cot(c+dx)dx+b^3C\int \tan(c+dx)dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3042

$$-a\left(a^2B-3abC-3b^2B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^3C\int \tan(c+dx)dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$a\left(a^2B-3abC-3b^2B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b^3C\int \tan(c+dx)dx-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-x\left(a^3C+3a^2bB-3ab^2C-b^3B\right)-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

3.21. $\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\Big\downarrow \text{3956}$$

$$-\frac{a(a^2B-3abC-3b^2B)\log(-\sin(c+dx))}{d}-\frac{a^2(aC+2bB)\cot(c+dx)}{d}-x(a^3C+3a^2bB-3ab^2C-b^3B)-\frac{aB\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{b^3C\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
-((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*x) - (a^2*(2*b*B + a*C)*Cot[c +
d*x])/d - (b^3*C*Log[Cos[c + d*x]])/d - (a*(a^2*B - 3*b^2*B - 3*a*b*C)*Log
[-Sin[c + d*x]])/d - (a*B*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d)
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.21. $\int \cot^4(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) \, dx$

3.21.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{(B a^3-3Ba b^2-3C a^2b+C b^3)\ln\left(\sec(dx+c)^2\right)+(-2B a^3+6Ba b^2+6C a^2b)\ln(\tan(dx+c))-B a^3\cot(dx+c)^2+(-6B a}{2d}$
derivativedivides	$\frac{-\frac{B a^3}{2\tan(dx+c)^2}-\frac{a^2(3Bb+Ca)}{\tan(dx+c)}-a(B a^2-3B b^2-3Cab)\ln(\tan(dx+c))+\frac{\left(B a^3-3Ba b^2-3C a^2b+C b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-}{d}$
default	$\frac{-\frac{B a^3}{2\tan(dx+c)^2}-\frac{a^2(3Bb+Ca)}{\tan(dx+c)}-a(B a^2-3B b^2-3Cab)\ln(\tan(dx+c))+\frac{\left(B a^3-3Ba b^2-3C a^2b+C b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-}{d}$
norman	$\frac{(-3B a^2b+B b^3-C a^3+3Ca b^2)x\tan(dx+c)^3-\frac{B a^3\tan(dx+c)}{2d}-\frac{a^2(3Bb+Ca)\tan(dx+c)^2}{d}}{\tan(dx+c)^3}+\frac{(B a^3-3Ba b^2-3C a^2b+C}{2d}$
risch	$iB a^3x-3iC a^2bx-3iBa b^2x+\frac{2iC b^3c}{d}-3B a^2bx+B b^3x-C a^3x+3Ca b^2x-\frac{2ia^2(3}{}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/2*((B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(sec(d*x+c)^2)+(-2*B*a^3+6*B*a*b^
2+6*C*a^2*b)*ln(tan(d*x+c))-B*a^3*cot(d*x+c)^2+(-6*B*a^2*b-2*C*a^3)*cot(d*
x+c)-6*d*(B*a^2*b-1/3*B*b^3+1/3*C*a^3-C*a*b^2)*x)/d
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{Cb^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Ba^3+(Ba^3-3Ca^2b-3Bab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2}{2d\tan(dx+}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
-1/2*(C*b^3*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + B*a^3 + (B*a^3 -
3*C*a^2*b - 3*B*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x +
c)^2 + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c
)^2 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(121) = 242$.

Time = 2.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.99

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases} \text{NaN} \\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^4(c) \\ \text{NaN} \\ \frac{Ba^3\log(\tan^2(c+dx)+1)}{2d}-\frac{Ba^3\log(\tan(c+dx))}{d}-\frac{Ba^3}{2d\tan^2(c+dx)}-3Ba^2bx-\frac{3Ba^2b}{d\tan(c+dx)}-\frac{3Bab^2\log(\tan^2(c+dx)+1)}{2d}+\frac{3B \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*t
an(c)**2)*cot(c)**4, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**3*log(tan(c + d*
x)**2 + 1)/(2*d) - B*a**3*log(tan(c + d*x))/d - B*a**3/(2*d*tan(c + d*x)**
2) - 3*B*a**2*b*x - 3*B*a**2*b/(d*tan(c + d*x)) - 3*B*a*b**2*log(tan(c + d
*x)**2 + 1)/(2*d) + 3*B*a*b**2*log(tan(c + d*x))/d + B*b**3*x - C*a**3*x -
 C*a**3/(d*tan(c + d*x)) - 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*C
*a**2*b*log(tan(c + d*x))/d + 3*C*a*b**2*x + C*b**3*log(tan(c + d*x)**2 +
1)/(2*d), True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$
$$-\frac{2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+}{2\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
-1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a
^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b -
 3*B*a*b^2)*log(tan(d*x + c)) + (B*a^3 + 2*(C*a^3 + 3*B*a^2*b)*tan(d*x + c
))/tan(d*x + c)^2)/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = -\frac{2\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
-1/2*(2*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - (B*a^3 - 3*C*a
^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 2*(B*a^3 - 3*C*a^2*b -
 3*B*a*b^2)*log(abs(tan(d*x + c))) - (3*B*a^3*tan(d*x + c)^2 - 9*C*a^2*b*t
an(d*x + c)^2 - 9*B*a*b^2*tan(d*x + c)^2 - 2*C*a^3*tan(d*x + c) - 6*B*a^2*
b*tan(d*x + c) - B*a^3)/tan(d*x + c)^2)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \cot^4(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx\\&=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B\,a^3+3\,C\,a^2\,b+3\,B\,a\,b^2\right)}{d}\\&\quad-\frac{\cot(c+d\,x)^2\,\left(\tan(c+d\,x)\,\left(C\,a^3+3\,B\,b\,a^2\right)+\frac{B\,a^3}{2}\right)}{d}\\&\quad+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(B-C\,1{}\mathrm{i}\right)\,{\left(b+a\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}\\&\quad+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1{}\mathrm{i}\right)\,{\left(-b+a\,1{}\mathrm{i}\right)}^3\,1{}\mathrm{i}}{2\,d}\end{aligned}$$

input
```
int(cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(tan(c + d*x))*(3*B*a*b^2 - B*a^3 + 3*C*a^2*b))/d - (cot(c + d*x)^2*(t
an(c + d*x)*(C*a^3 + 3*B*a^2*b) + (B*a^3)/2))/d + (log(tan(c + d*x) + 1i)*
(B - C*1i)*(a*1i + b)^3*1i)/(2*d) + (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*
1i - b)^3*1i)/(2*d)
```

3.21. $\int \cot^4(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$

3.22 $\int \cot^5(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2\right.$

3.22.1 Optimal result

Integrand size = 40, antiderivative size = 154

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x+\frac{a(3a^2B-8b^2B-9abC)\cot(c+dx)}{3d}$$
$$-\frac{a^2(5bB+3aC)\cot^2(c+dx)}{6d}-\frac{(3a^2bB-b^3B+a^3C-3ab^2C)\log(\sin(c+dx))}{d}$$
$$-\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

output
```
(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x+1/3*a*(3*B*a^2-8*B*b^2-9*C*a*b)*cot(d*
x+c)/d-1/6*a^2*(5*B*b+3*C*a)*cot(d*x+c)^2/d-(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b
^2)*ln(sin(d*x+c))/d-1/3*a*B*cot(d*x+c)^3*(a+b*tan(d*x+c))^2/d
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{6a(a^2B-3b^2B-3abC)\cot(c+dx)-3a^2(3bB+aC)\cot^2(c+dx)-2a^3B\cot^3(c+dx)+3(a+ib)^3(-i}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(6*a*(a^2*B - 3*b^2*B - 3*a*b*C)*Cot[c + d*x] - 3*a^2*(3*b*B + a*C)*Cot[c
+ d*x]^2 - 2*a^3*B*Cot[c + d*x]^3 + 3*(a + I*b)^3*((-I)*B + C)*Log[I - Tan
[c + d*x]] - 6*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] +
 3*(a - I*b)^3*(I*B + C)*Log[I + Tan[c + d*x]])/(6*d)
```

3.22.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 4115, 3042, 4088, 3042, 4118, 25, 3042, 4111, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \ 4115$$

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \ 4088$$

$$\frac{1}{3}\int \cot^3(c+dx)(a+b\tan(c+dx))\left(-b(aB-3bC)\tan^2(c+dx)-3\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(5bB+3aC)\right)dx-\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{(a+b\tan(c+dx))\left(-b(aB-3bC)\tan(c+dx)^2-3\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(5bB+3aC)\right)}{\tan(c+dx)^3}dx-$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4118}$$

$$\frac{1}{3}\left(\int -\cot^2(c+dx)\left(b^2(aB-3bC)\tan^2(c+dx)+3\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(3Ba^2-9bC\right.\right.\right.$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-\int \cot^2(c+dx)\left(b^2(aB-3bC)\tan^2(c+dx)+3\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(3Ba^2-9bC\right.\right.\right.$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\int \frac{b^2(aB-3bC)\tan(c+dx)^2+3\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(3Ba^2-9bCa-8b^2B\right)}{\tan(c+dx)^2}dx\right.$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{3}\left(-\int 3\cot(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a\left(3a^2B\right.}{}\right.$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-3\int \cot(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a\left(3a^2B\right.}{}\right.$$

$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

3.22. $\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\frac{1}{3}\left(-3\int \frac{Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{a\left(3a^2B-9abC-8b^2\right.}{d}\right.$$
$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\big\downarrow$ 4014

$$\frac{1}{3}\left(-3\left(\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int\cot(c+dx)dx-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\right)+\frac{a\left(3a^2B-9abC\right.}{}\right.$$
$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(-3\left(\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\right)+\frac{a\left(3a^2B\right.}{}\right.$$
$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\big\downarrow$ 25

$$\frac{1}{3}\left(-3\left(-\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(x\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\right)\right)+\right.$$
$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\big\downarrow$ 3956

$$\frac{1}{3}\left(\frac{a\left(3a^2B-9abC-8b^2B\right)\cot(c+dx)}{d}-\frac{a^2(3aC+5bB)\cot^2(c+dx)}{2d}-3\left(\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\log}{d}\right.\right.$$
$$\frac{aB\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
((a*(3*a^2*B - 8*b^2*B - 9*a*b*C)*Cot[c + d*x])/d - (a^2*(5*b*B + 3*a*C)*C
ot[c + d*x]^2)/(2*d) - 3*(-((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*x) + (
(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[-Sin[c + d*x]])/d))/3 - (a*B*C
ot[c + d*x]^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

3.22. $\int\cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.22.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{3(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2)\ln\left(\sec(dx+c)^2\right)+6(-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2)\ln(\tan(dx+c))-2B\,a^3\cot(dx+c)^3+}{6d}$
derivativedivides	$\frac{(-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2)\ln(\tan(dx+c))-\frac{B\,a^3}{3\tan(dx+c)^3}-\frac{a^2(3Bb+Ca)}{2\tan(dx+c)^2}+\frac{a\left(B\,a^2-3B\,b^2-3Cab\right)}{\tan(dx+c)}+\frac{(3B\,a^2b-B\,b^3+C\,a^3}{}}{d}$
default	$\frac{(-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2)\ln(\tan(dx+c))-\frac{B\,a^3}{3\tan(dx+c)^3}-\frac{a^2(3Bb+Ca)}{2\tan(dx+c)^2}+\frac{a\left(B\,a^2-3B\,b^2-3Cab\right)}{\tan(dx+c)}+\frac{(3B\,a^2b-B\,b^3+C\,a^3}{}}{d}$
norman	$\frac{(B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3)x\tan(dx+c)^4+\frac{a\left(B\,a^2-3B\,b^2-3Cab\right)\tan(dx+c)^3}{d}-\frac{B\,a^3\tan(dx+c)}{3d}-\frac{a^2(3Bb+Ca)\tan(dx+c)^2}{2d}}{\tan(dx+c)^4}$
risch	$B\,a^3x-3Ba\,b^2x-3C\,a^2bx+C\,b^3x-\frac{6iCa\,b^2c}{d}+\frac{6iB\,a^2bc}{d}-iB\,b^3x+iC\,a^3x-\frac{2ia(9iBab\,e}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/6*(3*(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(sec(d*x+c)^2)+6*(-3*B*a^2*b+B*
b^3-C*a^3+3*C*a*b^2)*ln(tan(d*x+c))-2*B*a^3*cot(d*x+c)^3+3*(-3*B*a^2*b-C*a
^3)*cot(d*x+c)^2+6*a*cot(d*x+c)*(B*a^2-3*B*b^2-3*C*a*b)+6*d*x*(B*a^3-3*B*a
*b^2-3*C*a^2*b+C*b^3))/d
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{3\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+2\,Ba^3+3\left(Ca^3+3\,Ba^2b-2\left(Ba^3\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
-1/6*(3*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^3 + 2*B*a^3 + 3*(C*a^3 + 3*B*a^2*b - 2*(B*a^3
- 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*
b - 3*B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan
(d*x + c)^3)
```

3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(150) = 300$.

Time = 4.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.10

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \begin{cases} \text{NaN} \\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^5(c) \\ \text{NaN} \\ Ba^3x + \frac{Ba^3}{d\tan(c+dx)} - \frac{Ba^3}{3d\tan^3(c+dx)} + \frac{3Ba^2b\log(\tan^2(c+dx)+1)}{2d} - \frac{3Ba^2b\log(\tan(c+dx))}{d} - \frac{3Ba^2b}{2d\tan^2(c+dx)} - 3Bab^2x - \end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*t
an(c)**2)*cot(c)**5, Eq(d, 0)), (nan, Eq(c, -d*x)), (B*a**3*x + B*a**3/(d*
tan(c + d*x)) - B*a**3/(3*d*tan(c + d*x)**3) + 3*B*a**2*b*log(tan(c + d*x)
**2 + 1)/(2*d) - 3*B*a**2*b*log(tan(c + d*x))/d - 3*B*a**2*b/(2*d*tan(c +
d*x)**2) - 3*B*a*b**2*x - 3*B*a*b**2/(d*tan(c + d*x)) - B*b**3*log(tan(c +
 d*x)**2 + 1)/(2*d) + B*b**3*log(tan(c + d*x))/d + C*a**3*log(tan(c + d*x)
**2 + 1)/(2*d) - C*a**3*log(tan(c + d*x))/d - C*a**3/(2*d*tan(c + d*x)**2)
 - 3*C*a**2*b*x - 3*C*a**2*b/(d*tan(c + d*x)) - 3*C*a*b**2*log(tan(c + d*x
)**2 + 1)/(2*d) + 3*C*a*b**2*log(tan(c + d*x))/d + C*b**3*x, True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \frac{6\,(Ba^3 - 3\,Ca^2b - 3\,Bab^2 + Cb^3)(dx+c) + 3\,(Ca^3 + 3\,Ba^2b - 3\,Cab^2 - Bb^3)\log\left(\tan(dx+c)^2+1\right) - 6}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/6*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 3*(C*a^3 + 3*B*
a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 6*(C*a^3 + 3*B*a^2*b
- 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) - (2*B*a^3 - 6*(B*a^3 - 3*C*a^2*b -
 3*B*a*b^2)*tan(d*x + c)^2 + 3*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x +
 c)^3)/d
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(148) = 296$.

Time = 1.43 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.53

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \frac{Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\cdots}{\cdots}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/24*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 3*C*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*B*
a^2*b*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c) + 36*C*a^2*b*
tan(1/2*d*x + 1/2*c) + 36*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*(B*a^3 - 3*C*a
^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 -
B*b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 24*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2
 - B*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (44*C*a^3*tan(1/2*d*x + 1/2*c)^
3 + 132*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 132*C*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 44*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 36
*C*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 36*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*C*
a^3*tan(1/2*d*x + 1/2*c) - 9*B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^3)/tan(1/2
*d*x + 1/2*c)^3)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\ln(\tan(c+d\,x))\,\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)}{d}$$
$$-\frac{\cot(c+d\,x)^3\left(\tan(c+d\,x)\,\left(\frac{C\,a^3}{2}+\frac{3\,B\,b\,a^2}{2}\right)+\frac{B\,a^3}{3}+\tan(c+d\,x)^2\,\left(-B\,a^3+3\,C\,a^2\,b+3\,B\,a\,b^2\right)\right)}{d}$$
$$-\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(B+C\,1\mathrm{i})\,(a+b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$
$$+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(B-C\,1\mathrm{i})\,(a-b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^5*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(tan(c + d*x))*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2))/d - (cot(c + d
*x)^3*(tan(c + d*x)*((C*a^3)/2 + (3*B*a^2*b)/2) + (B*a^3)/3 + tan(c + d*x)
^2*(3*B*a*b^2 - B*a^3 + 3*C*a^2*b)))/d - (log(tan(c + d*x) - 1i)*(B + C*1i
)*(a + b*1i)^3*1i)/(2*d) + (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i)^3
*1i)/(2*d)
```

3.23 $\int \cot^6(c+dx)(a+b\tan(c+dx))^3 (B\tan(c+dx)+C\tan$

3.23.1 Optimal result

Integrand size = 40, antiderivative size = 191

$$\begin{aligned}
&\int \cot^6(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx \\
&= \left(3a^2bB-b^3B+a^3C-3ab^2C\right)x+\frac{(3a^2bB-b^3B+a^3C-3ab^2C)\cot(c+dx)}{d} \\
&\quad+\frac{a(2a^2B-5b^2B-6abC)\cot^2(c+dx)}{4d}-\frac{a^2(3bB+2aC)\cot^3(c+dx)}{6d} \\
&\quad+\frac{(a^3B-3ab^2B-3a^2bC+b^3C)\log(\sin(c+dx))}{d} \\
&\quad-\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}
\end{aligned}$$

output
```
(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*x+(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*cot(
d*x+c)/d+1/4*a*(2*B*a^2-5*B*b^2-6*C*a*b)*cot(d*x+c)^2/d-1/6*a^2*(3*B*b+2*C
*a)*cot(d*x+c)^3/d+(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(sin(d*x+c))/d-1/4*
a*B*cot(d*x+c)^4*(a+b*tan(d*x+c))^2/d
```

3.23.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \frac{12(3a^2bB-b^3B+a^3C-3ab^2C)\cot(c+dx)+6a(a^2B-3b^2B-3abC)\cot^2(c+dx)-4a^2(3bB+aC)\cot}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(12*(3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x] + 6*a*(a^2*B - 3*
b^2*B - 3*a*b*C)*Cot[c + d*x]^2 - 4*a^2*(3*b*B + a*C)*Cot[c + d*x]^3 - 3*a
^3*B*Cot[c + d*x]^4 - 6*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 12*(
a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Log[Tan[c + d*x]] - 6*(a - I*b)^3*(
B - I*C)*Log[I + Tan[c + d*x]])/(12*d)
```

3.23.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.475, Rules used = {3042, 4115, 3042, 4088, 27, 3042, 4118, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \ 4088$$

$$\frac{1}{4}\int 2\cot^4(c+dx)(a+b\tan(c+dx))\left(-b(aB-2bC)\tan^2(c+dx)-2\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(3bB+2aC)\right)dx-\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \cot^4(c+dx)(a+b\tan(c+dx))\left(-b(aB-2bC)\tan^2(c+dx)-2\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(3bB+2aC)\right)dx-\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{(a+b\tan(c+dx))\left(-b(aB-2bC)\tan(c+dx)^2-2\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(3bB+2aC)\right)}{\tan(c+dx)^4}dx-\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 4118$$

$$\frac{1}{2}\left(\int -\cot^3(c+dx)\left(b^2(aB-2bC)\tan^2(c+dx)+2\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(2Ba^2-6bC\right.\right.\right.$$
$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\int \cot^3(c+dx)\left(b^2(aB-2bC)\tan^2(c+dx)+2\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(2Ba^2-6bC\right.\right.\right.$$
$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(-\int \frac{b^2(aB-2bC)\tan(c+dx)^2+2\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(2Ba^2-6bCa-5b^2B\right)}{\tan(c+dx)^3}dx\right.$$
$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \ 4111$$

3.23. $\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)dx$

$$\frac{1}{2}\left(-\int 2\cot^2(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a(2a^2B$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-2\int \cot^2(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a(2a^2B$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\int \frac{Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)}{\tan(c+dx)^2}dx+\frac{a(2a^2B-6abC-5b^2}{2d}$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 4012

$$\frac{1}{2}\left(-2\left(\int -\cot(c+dx)\left(Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{(a^3C$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(-2\left(-\int \cot(c+dx)\left(Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{(a^3C$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(-\int \frac{Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(a^3C+3a^2bB-3$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 4014

3.23. $\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\frac{1}{2}\left(-2\left(-\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\int\cot(c+dx)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot(c+dx)}{d}-(x(a^3\right.\right.$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(-\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot(c+dx)}{d}\right.\right.$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(-2\left(\left(a^3B-3a^2bC-3ab^2B+b^3C\right)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot(c+dx)}{d}\right.\right.$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\downarrow$ 3956

$$\frac{1}{2}\left(\frac{a\left(2a^2B-6abC-5b^2B\right)\cot^2(c+dx)}{2d}-\frac{a^2(2aC+3bB)\cot^3(c+dx)}{3d}-2\left(-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)c}{d}\right.\right.$$

$$\frac{aB\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
((a*(2*a^2*B - 5*b^2*B - 6*a*b*C)*Cot[c + d*x]^2)/(2*d) - (a^2*(3*b*B + 2*
a*C)*Cot[c + d*x]^3)/(3*d) - 2*(-((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*
x) - ((3*a^2*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x])/d - ((a^3*B -
3*a*b^2*B - 3*a^2*b*C + b^3*C)*Log[-Sin[c + d*x]])/d))/2 - (a*B*Cot[c + d*
x]^4*(a + b*Tan[c + d*x])^2)/(4*d)
```

3.23. $\int\cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{6(-B a^3+3Ba b^2+3C a^2b-C b^3) \ln\left(\sec(dx+c)^2\right)+12(B a^3-3Ba b^2-3C a^2b+C b^3) \ln(\tan(dx+c))-3B a^3 \cot(dx+c)^4}{}$
derivativedivides	$\frac{-\frac{-3B a^2b+B b^3-C a^3+3Ca b^2}{\tan(dx+c)}+(B a^3-3Ba b^2-3C a^2b+C b^3) \ln(\tan(dx+c))-\frac{B a^3}{4\tan(dx+c)^4}-\frac{a^2(3Bb+Ca)}{3\tan(dx+c)^3}+\frac{a\left(B a^2-3B b\right.}{2\tan(dx}}{d}$
default	$\frac{-\frac{-3B a^2b+B b^3-C a^3+3Ca b^2}{\tan(dx+c)}+(B a^3-3Ba b^2-3C a^2b+C b^3) \ln(\tan(dx+c))-\frac{B a^3}{4\tan(dx+c)^4}-\frac{a^2(3Bb+Ca)}{3\tan(dx+c)^3}+\frac{a\left(B a^2-3B b\right.}{2\tan(dx}}{d}$
norman	$\frac{\frac{(3B a^2b-B b^3+C a^3-3Ca b^2)\tan(dx+c)^4}{d}+(3B a^2b-B b^3+C a^3-3Ca b^2)x\tan(dx+c)^5-\frac{B a^3\tan(dx+c)}{4d}+\frac{a\left(B a^2-3B b^2-3\right.}{2}}{\tan(dx+c)^5}$
risch	$\frac{6iC a^2bc}{d}+\frac{6iBa b^2c}{d}-iB a^3x+3iC a^2bx+3B a^2bx-B b^3x+C a^3x-3Ca b^2x-\frac{2iC b^3c}{d}$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/12*(6*(-B*a^3+3*B*a*b^2+3*C*a^2*b-C*b^3)*ln(sec(d*x+c)^2)+12*(B*a^3-3*B*
a*b^2-3*C*a^2*b+C*b^3)*ln(tan(d*x+c))-3*B*a^3*cot(d*x+c)^4+4*(-3*B*a^2*b-C
*a^3)*cot(d*x+c)^3+6*a*cot(d*x+c)^2*(B*a^2-3*B*b^2-3*C*a*b)+12*cot(d*x+c)*
(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)+36*d*(B*a^2*b-1/3*B*b^3+1/3*C*a^3-C*a*b^
2)*x)/d
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \frac{6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\left(3\,Ba^3-6\,Ca^2b-6\,Bab^2+4\left(Ca^3\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/12*(6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^4 + 3*(3*B*a^3 - 6*C*a^2*b - 6*B*a*b^2 + 4*(C*
a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c)^4 - 3*B*a^3 + 12*(C
*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 6*(B*a^3 - 3*C*a^2*
b - 3*B*a*b^2)*tan(d*x + c)^2 - 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*tan
(d*x + c)^4)
```

3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(187) = 374$.

Time = 5.46 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.05

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\begin{cases}\text{NaN}\\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^6(c)\\ \text{NaN}\\ -\frac{Ba^3\log(\tan^2(c+dx)+1)}{2d}+\frac{Ba^3\log(\tan(c+dx))}{d}+\frac{Ba^3}{2d\tan^2(c+dx)}-\frac{Ba^3}{4d\tan^4(c+dx)}+3Ba^2bx+\frac{3Ba^2b}{d\tan(c+dx)}-\frac{Ba^2b}{d\tan^3(c+dx}\end{cases}$$

input
```
integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*t
an(c)**2)*cot(c)**6, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**3*log(tan(c + d
*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x))/d + B*a**3/(2*d*tan(c + d*x)*
*2) - B*a**3/(4*d*tan(c + d*x)**4) + 3*B*a**2*b*x + 3*B*a**2*b/(d*tan(c +
d*x)) - B*a**2*b/(d*tan(c + d*x)**3) + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)
/(2*d) - 3*B*a*b**2*log(tan(c + d*x))/d - 3*B*a*b**2/(2*d*tan(c + d*x)**2)
 - B*b**3*x - B*b**3/(d*tan(c + d*x)) + C*a**3*x + C*a**3/(d*tan(c + d*x))
 - C*a**3/(3*d*tan(c + d*x)**3) + 3*C*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d
) - 3*C*a**2*b*log(tan(c + d*x))/d - 3*C*a**2*b/(2*d*tan(c + d*x)**2) - 3*
C*a*b**2*x - 3*C*a*b**2/(d*tan(c + d*x)) - C*b**3*log(tan(c + d*x)**2 + 1)
/(2*d) + C*b**3*log(tan(c + d*x))/d, True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.13

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{12\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)(dx+c)-6\left(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
1/12*(12*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) - 6*(B*a^3 - 3*
C*a^2*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)^2 + 1) + 12*(B*a^3 - 3*C*a^2
*b - 3*B*a*b^2 + C*b^3)*log(tan(d*x + c)) - (3*B*a^3 - 12*(C*a^3 + 3*B*a^2
*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 - 6*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)
*tan(d*x + c)^2 + 4*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(185) = 370.

Time = 1.48 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.76

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx=$$
$$-\frac{3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
-1/192*(3*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*C*a^3*tan(1/2*d*x + 1/2*c)^3 -
24*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*C
*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*C*
a^3*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) - 288*C*a*b^2*
tan(1/2*d*x + 1/2*c) - 96*B*b^3*tan(1/2*d*x + 1/2*c) - 192*(C*a^3 + 3*B*a^
2*b - 3*C*a*b^2 - B*b^3)*(d*x + c) + 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 +
C*b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 192*(B*a^3 - 3*C*a^2*b - 3*B*a*b^
2 + C*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (400*B*a^3*tan(1/2*d*x + 1/2*c
)^4 - 1200*C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1200*B*a*b^2*tan(1/2*d*x + 1/2
*c)^4 + 400*C*b^3*tan(1/2*d*x + 1/2*c)^4 - 120*C*a^3*tan(1/2*d*x + 1/2*c)^
3 - 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 288*C*a*b^2*tan(1/2*d*x + 1/2*c)^
3 + 96*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^2 + 72
*C*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*C*
a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^2*b*tan(1/2*d*x + 1/2*c) + 3*B*a^3)/tan(
1/2*d*x + 1/2*c)^4)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \frac{\ln(\tan(c+dx))\,\left(B\,a^3-3\,C\,a^2\,b-3\,B\,a\,b^2+C\,b^3\right)}{d}$$

$$- \frac{\cot(c+dx)^4\left(\tan(c+dx)\,\left(\frac{C\,a^3}{3}+B\,b\,a^2\right)+\frac{B\,a^3}{4}+\tan(c+dx)^2\left(-\frac{B\,a^3}{2}+\frac{3\,C\,a^2\,b}{2}+\frac{3\,B\,a\,b^2}{2}\right)+\tan(c+\right.}{d}$$

$$- \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B-C\,1\mathrm{i})\,(b+a\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

$$- \frac{\ln(\tan(c+dx)-\mathrm{i})\,(B+C\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^6*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(tan(c + d*x))*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2*b))/d - (cot(c + d
*x)^4*(tan(c + d*x)*((C*a^3)/3 + B*a^2*b) + (B*a^3)/4 + tan(c + d*x)^2*((3
*B*a*b^2)/2 - (B*a^3)/2 + (3*C*a^2*b)/2) + tan(c + d*x)^3*(B*b^3 - C*a^3 -
 3*B*a^2*b + 3*C*a*b^2)))/d - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a*1i + b
)^3*1i)/(2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i)*(a*1i - b)^3*1i)/(2*d)
```

3.24 $\int \cot^7(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$

3.24.1 Optimal result

Integrand size = 40, antiderivative size = 233

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3 \left(B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= -\left(\left(a^3B-3ab^2B-3a^2bC+b^3C\right)x\right)-\frac{\left(a^3B-3ab^2B-3a^2bC+b^3C\right)\cot(c+dx)}{d}$$
$$+\frac{\left(3a^2bB-b^3B+a^3C-3ab^2C\right)\cot^2(c+dx)}{2d}+\frac{a\left(5a^2B-12b^2B-15abC\right)\cot^3(c+dx)}{15d}$$
$$-\frac{a^2(7bB+5aC)\cot^4(c+dx)}{20d}+\frac{\left(3a^2bB-b^3B+a^3C-3ab^2C\right)\log(\sin(c+dx))}{d}$$
$$-\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

output
```
-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*x-(B*a^3-3*B*a*b^2-3*C*a^2*b+C*b^3)*cot
(d*x+c)/d+1/2*(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*cot(d*x+c)^2/d+1/15*a*(5*B
*a^2-12*B*b^2-15*C*a*b)*cot(d*x+c)^3/d-1/20*a^2*(7*B*b+5*C*a)*cot(d*x+c)^4
/d+(3*B*a^2*b-B*b^3+C*a^3-3*C*a*b^2)*ln(sin(d*x+c))/d-1/5*a*B*cot(d*x+c)^5
*(a+b*tan(d*x+c))^2/d
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$= \frac{-60(a^3B-3ab^2B-3a^2bC+b^3C)\cot(c+dx)+30(3a^2bB-b^3B+a^3C-3ab^2C)\cot^2(c+dx)+20a(a}{}$$

input
```
Integrate[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(-60*(a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Cot[c + d*x] + 30*(3*a^2*b*B
- b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x]^2 + 20*a*(a^2*B - 3*b^2*B - 3*a*
b*C)*Cot[c + d*x]^3 - 15*a^2*(3*b*B + a*C)*Cot[c + d*x]^4 - 12*a^3*B*Cot[c
 + d*x]^5 + (30*I)*(a + I*b)^3*(B + I*C)*Log[I - Tan[c + d*x]] + 60*(3*a^2
*b*B - b^3*B + a^3*C - 3*a*b^2*C)*Log[Tan[c + d*x]] + 30*(I*a + b)^3*(B -
I*C)*Log[I + Tan[c + d*x]])/(60*d)
```

3.24.3 Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4115, 3042, 4088, 3042, 4118, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{\tan(c+dx)^7}dx$$

$$\Big\downarrow \text{ 4115}$$

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(B+C\tan(c+dx))dx$$

$$\Big\downarrow 3042$$

$$\int \frac{(a+b\tan(c+dx))^3(B+C\tan(c+dx))}{\tan(c+dx)^6}dx$$

$$\Big\downarrow 4088$$

$$\frac{1}{5}\int \cot^5(c+dx)(a+b\tan(c+dx))\left(-b(3aB-5bC)\tan^2(c+dx)-5\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(7bB+5aC)\right)dx-\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(-b(3aB-5bC)\tan(c+dx)^2-5\left(Ba^2-2bCa-b^2B\right)\tan(c+dx)+a(7bB+5aC)\right)}{\tan(c+dx)^5}dx \cdots \frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow 4118$$

$$\frac{1}{5}\left(\int -\cot^4(c+dx)\left(b^2(3aB-5bC)\tan^2(c+dx)+5\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(5Ba^2-15 \cdots\right.\right.\right. \frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow 25$$

$$\frac{1}{5}\left(-\int \cot^4(c+dx)\left(b^2(3aB-5bC)\tan^2(c+dx)+5\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(5Ba^2-15 \cdots\right.\right.\right. \frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(-\int \frac{b^2(3aB-5bC)\tan(c+dx)^2+5\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)+a\left(5Ba^2-15bCa-12b^2B \cdots\right.}{\tan(c+dx)^4}\right. \frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow 4111$$

$$\frac{1}{5}\left(-\int 5\cot^3(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a(5a^2 \cdots}{\cdots}\right. \frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

3.24. $\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$\downarrow$ 27

$$\frac{1}{5}\left(-5\int \cot^3(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx+\frac{a\left(5a^2B\right.}{}\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\int\frac{Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)}{\tan(c+dx)^3}dx+\frac{a\left(5a^2B-15abC-12\right.}{3d}\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 4012

$$\frac{1}{5}\left(-5\left(\int-\cot^2(c+dx)\left(Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{(a^3}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-5\left(-\int\cot^2(c+dx)\left(Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)\right)dx-\frac{(a^3}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\left(-\int\frac{Ba^3-3bCa^2-3b^2Ba+b^3C+\left(Ca^3+3bBa^2-3b^2Ca-b^3B\right)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{\left(a^3C+3a^2bB-3\right.}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 4012

$$\frac{1}{5}\left(-5\left(-\int\cot(c+dx)\left(Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)\right)dx-\frac{(a^3C}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

3.24. $\int\cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

$$\frac{1}{5}\left(-5\left(-\int \frac{Ca^3+3bBa^2-3b^2Ca-b^3B-\left(Ba^3-3bCa^2-3b^2Ba+b^3C\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{\left(a^3C+3a^2bB-3\right.}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 4014

$$\frac{1}{5}\left(-5\left(-\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int \cot(c+dx)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot^2(c+dx)}{2d}+\frac{\left(a^3\right.}{}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-5\left(-\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot^2(c+dx}{2d}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(-5\left(\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3B\right)\cot^2(c+a}{2d}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3956

$$\frac{1}{5}\left(\frac{a\left(5a^2B-15abC-12b^2B\right)\cot^3(c+dx)}{3d}-\frac{a^2(5aC+7bB)\cot^4(c+dx)}{4d}-5\left(-\frac{\left(a^3C+3a^2bB-3ab^2C-b^3E\right.}{2d}\right.\right.$$
$$\frac{aB\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

input
```
Int[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^3*(B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

3.24. $\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

output
```
((a*(5*a^2*B - 12*b^2*B - 15*a*b*C)*Cot[c + d*x]^3)/(3*d) - (a^2*(7*b*B +
5*a*C)*Cot[c + d*x]^4)/(4*d) - 5*((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*
x + ((a^3*B - 3*a*b^2*B - 3*a^2*b*C + b^3*C)*Cot[c + d*x])/d - ((3*a^2*b*B
 - b^3*B + a^3*C - 3*a*b^2*C)*Cot[c + d*x]^2)/(2*d) - ((3*a^2*b*B - b^3*B
+ a^3*C - 3*a*b^2*C)*Log[-Sin[c + d*x]])/d))/5 - (a*B*Cot[c + d*x]^5*(a +
b*Tan[c + d*x])^2)/(5*d)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.24. $\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.24.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{(-90B\,a^2b+30B\,b^3-30C\,a^3+90Ca\,b^2)\ln\left(\sec(dx+c)^2\right)+(180B\,a^2b-60B\,b^3+60C\,a^3-180Ca\,b^2)\ln(\tan(dx+c))-12B}{}$
derivativedivides	$\frac{-\frac{-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2}{2\tan(dx+c)^2}+(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2)\ln(\tan(dx+c))-\frac{B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3}{\tan(dx+c)}-\frac{B\,a^3}{5\tan(dx+c)^5}}{}$
default	$\frac{-\frac{-3B\,a^2b+B\,b^3-C\,a^3+3Ca\,b^2}{2\tan(dx+c)^2}+(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2)\ln(\tan(dx+c))-\frac{B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3}{\tan(dx+c)}-\frac{B\,a^3}{5\tan(dx+c)^5}}{}$
norman	$\frac{(-B\,a^3+3Ba\,b^2+3C\,a^2b-C\,b^3)x\tan(dx+c)^6+\frac{(3B\,a^2b-B\,b^3+C\,a^3-3Ca\,b^2)\tan(dx+c)^4}{2d}-\frac{(B\,a^3-3Ba\,b^2-3C\,a^2b+C\,b^3)\tan}{d}}{\tan(dx+c)^6}$
risch	$-B\,a^3x+3Ba\,b^2x+3C\,a^2bx-C\,b^3x-\frac{6iB\,a^2bc}{d}-\frac{2i(-60C\,a^2b-60Ba\,b^2+15C\,b^3+23B\,a^3-70B\,a^3}{}$

input
```
int(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,method
=_RETURNVERBOSE)
```

output
```
1/60*((-90*B*a^2*b+30*B*b^3-30*C*a^3+90*C*a*b^2)*ln(sec(d*x+c)^2)+(180*B*a
^2*b-60*B*b^3+60*C*a^3-180*C*a*b^2)*ln(tan(d*x+c))-12*B*a^3*cot(d*x+c)^5+(
-45*B*a^2*b-15*C*a^3)*cot(d*x+c)^4+20*a*cot(d*x+c)^3*(B*a^2-3*B*b^2-3*C*a*
b)+(90*B*a^2*b-30*B*b^3+30*C*a^3-90*C*a*b^2)*cot(d*x+c)^2+(-60*B*a^3+180*B
*a*b^2+180*C*a^2*b-60*C*b^3)*cot(d*x+c)-60*d*x*(B*a^3-3*B*a*b^2-3*C*a^2*b+
C*b^3))/d
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.14

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{30\left(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\left(3\,Ca^3+9\,Ba^2b-6\,Cab^2-2\,Bb\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
1/60*(30*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d
*x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*C*a^3 + 9*B*a^2*b - 6*C*a*b^2 - 2*B
*b^3 - 4*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*d*x)*tan(d*x + c)^5 - 60*
(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*tan(d*x + c)^4 - 12*B*a^3 + 30*(C*
a^3 + 3*B*a^2*b - 3*C*a*b^2 - B*b^3)*tan(d*x + c)^3 + 20*(B*a^3 - 3*C*a^2*
b - 3*B*a*b^2)*tan(d*x + c)^2 - 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/(d*ta
n(d*x + c)^5)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 12.36 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.98

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$= \begin{cases} \text{NaN} \\ x(a+b\tan(c))^3\left(B\tan(c)+C\tan^2(c)\right)\cot^7(c) \\ \text{NaN} \\ -Ba^3x - \frac{Ba^3}{d\tan(c+dx)} + \frac{Ba^3}{3d\tan^3(c+dx)} - \frac{Ba^3}{5d\tan^5(c+dx)} - \frac{3Ba^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3Ba^2b\log(\tan(c+dx))}{d} + \frac{3Ba^2b}{2d\tan^2(c+} \end{cases}$$

input
```
integrate(cot(d*x+c)**7*(a+b*tan(d*x+c))**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Piecewise((nan, Eq(c, 0) & Eq(d, 0)), (x*(a + b*tan(c))**3*(B*tan(c) + C*t
an(c)**2)*cot(c)**7, Eq(d, 0)), (nan, Eq(c, -d*x)), (-B*a**3*x - B*a**3/(d
*tan(c + d*x)) + B*a**3/(3*d*tan(c + d*x)**3) - B*a**3/(5*d*tan(c + d*x)**
5) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a**2*b*log(tan(c + d*
x))/d + 3*B*a**2*b/(2*d*tan(c + d*x)**2) - 3*B*a**2*b/(4*d*tan(c + d*x)**4
) + 3*B*a*b**2*x + 3*B*a*b**2/(d*tan(c + d*x)) - B*a*b**2/(d*tan(c + d*x)*
*3) + B*b**3*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*log(tan(c + d*x))/d -
 B*b**3/(2*d*tan(c + d*x)**2) - C*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + C*
a**3*log(tan(c + d*x))/d + C*a**3/(2*d*tan(c + d*x)**2) - C*a**3/(4*d*tan(
c + d*x)**4) + 3*C*a**2*b*x + 3*C*a**2*b/(d*tan(c + d*x)) - C*a**2*b/(d*ta
n(c + d*x)**3) + 3*C*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - 3*C*a*b**2*lo
g(tan(c + d*x))/d - 3*C*a*b**2/(2*d*tan(c + d*x)**2) - C*b**3*x - C*b**3/(
d*tan(c + d*x)), True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.07

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{60\,(Ba^3-3\,Ca^2b-3\,Bab^2+Cb^3)(dx+c)+30\,(Ca^3+3\,Ba^2b-3\,Cab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
-1/60*(60*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) + 30*(C*a^3 +
3*B*a^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1) - 60*(C*a^3 + 3*B*a
^2*b - 3*C*a*b^2 - B*b^3)*log(tan(d*x + c)) + (60*(B*a^3 - 3*C*a^2*b - 3*B
*a*b^2 + C*b^3)*tan(d*x + c)^4 + 12*B*a^3 - 30*(C*a^3 + 3*B*a^2*b - 3*C*a*
b^2 - B*b^3)*tan(d*x + c)^3 - 20*(B*a^3 - 3*C*a^2*b - 3*B*a*b^2)*tan(d*x +
 c)^2 + 15*(C*a^3 + 3*B*a^2*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(225) = 450$.

Time = 1.57 (sec) , antiderivative size = 670, normalized size of antiderivative = 2.88

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$=\frac{6\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,Ca^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-45\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^3*(B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
1/960*(6*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 15*C*a^3*tan(1/2*d*x + 1/2*c)^4 -
45*B*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 120*
C*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 180*
C*a^3*tan(1/2*d*x + 1/2*c)^2 + 540*B*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 360*C*
a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*B*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*B*a^
3*tan(1/2*d*x + 1/2*c) - 1800*C*a^2*b*tan(1/2*d*x + 1/2*c) - 1800*B*a*b^2*
tan(1/2*d*x + 1/2*c) + 480*C*b^3*tan(1/2*d*x + 1/2*c) - 960*(B*a^3 - 3*C*a
^2*b - 3*B*a*b^2 + C*b^3)*(d*x + c) - 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b^2 -
 B*b^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) + 960*(C*a^3 + 3*B*a^2*b - 3*C*a*b
^2 - B*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - (2192*C*a^3*tan(1/2*d*x + 1/2
*c)^5 + 6576*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 6576*C*a*b^2*tan(1/2*d*x + 1
/2*c)^5 - 2192*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 660*B*a^3*tan(1/2*d*x + 1/2*
c)^4 - 1800*C*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 1800*B*a*b^2*tan(1/2*d*x + 1/
2*c)^4 + 480*C*b^3*tan(1/2*d*x + 1/2*c)^4 - 180*C*a^3*tan(1/2*d*x + 1/2*c)
^3 - 540*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 360*C*a*b^2*tan(1/2*d*x + 1/2*c)
^3 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 70*B*a^3*tan(1/2*d*x + 1/2*c)^2 +
120*C*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 120*B*a*b^2*tan(1/2*d*x + 1/2*c)^2 +
15*C*a^3*tan(1/2*d*x + 1/2*c) + 45*B*a^2*b*tan(1/2*d*x + 1/2*c) + 6*B*a^3)
/tan(1/2*d*x + 1/2*c)^5)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx =$$

$$-\frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)\,\left(\frac{C\,a^3}{4}+\frac{3\,B\,b\,a^2}{4}\right)+\frac{B\,a^3}{5}+\tan(c+d\,x)^2\left(-\frac{B\,a^3}{3}+C\,a^2\,b+B\,a\,b^2\right)+\tan(c+\cdots}{d}$$

$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(-C\,a^3-3\,B\,a^2\,b+3\,C\,a\,b^2+B\,b^3\right)}{d}$$

$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^7*(B*tan(c + d*x) + C*tan(c + d*x)^2)*(a + b*tan(c + d*x)
)^3,x)
```

output

```
(log(tan(c + d*x) - 1i)*(B + C*1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d
*x))*(B*b^3 - C*a^3 - 3*B*a^2*b + 3*C*a*b^2))/d - (cot(c + d*x)^5*(tan(c +
 d*x)*((C*a^3)/4 + (3*B*a^2*b)/4) + (B*a^3)/5 + tan(c + d*x)^2*(B*a*b^2 -
(B*a^3)/3 + C*a^2*b) + tan(c + d*x)^4*(B*a^3 + C*b^3 - 3*B*a*b^2 - 3*C*a^2
*b) + tan(c + d*x)^3*((B*b^3)/2 - (C*a^3)/2 - (3*B*a^2*b)/2 + (3*C*a*b^2)/
2)))/d - (log(tan(c + d*x) + 1i)*(B - C*1i)*(a - b*1i)^3*1i)/(2*d)
```

3.24. $\int \cot^7(c+dx)(a+b\tan(c+dx))^3\left(B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.25 $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

3.25.1 Optimal result

Integrand size = 40, antiderivative size = 127

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{(bB-aC)x}{a^2+b^2}+\frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d}$$
$$-\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)\,d}+\frac{(bB-aC)\tan(c+dx)}{b^2d}+\frac{C\tan^2(c+dx)}{2bd}$$

output
```
-(B*b-C*a)*x/(a^2+b^2)+(B*a+C*b)*ln(cos(d*x+c))/(a^2+b^2)/d-a^3*(B*b-C*a)*
ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)/d+(B*b-C*a)*tan(d*x+c)/b^2/d+1/2*C*tan(d*
x+c)^2/b/d
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-\frac{b(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{b(B-iC)\log(i+\tan(c+dx))}{a-ib}+\frac{2a^3(-bB+aC)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}+\frac{2(bB-aC)\tan(c+dx)}{b}+C\tan^2(}{2bd}$$

input
```
Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x]),x]
```

output
```
(-((b*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)) - (b*(B - I*C)*Log[I + T
an[c + d*x]])/(a - I*b) + (2*a^3*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(
b^2*(a^2 + b^2)) + (2*(b*B - a*C)*Tan[c + d*x])/b + C*Tan[c + d*x]^2)/(2*b
*d)
```

3.25.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 4115, 3042, 4090, 27, 3042, 4130, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\frac{2\tan(c+dx)\left(-\left((bB-aC)\tan^2(c+dx)\right)+bC\tan(c+dx)+aC\right)}{a+b\tan(c+dx)}dx}{2b}+\frac{C\tan^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\int \frac{\tan(c+dx)\left(-\left((bB-aC)\tan^2(c+dx)\right)+bC\tan(c+dx)+aC\right)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\int \frac{\tan(c+dx)(-((bB-aC)\tan(c+dx)^2)+bC\tan(c+dx)+aC)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \textbf{4130}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\int \frac{B\tan(c+dx)b^2+\left(-Ca^2+bBa+b^2C\right)\tan^2(c+dx)+a(bB-aC)}{a+b\tan(c+dx)}dx}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\int \frac{B\tan(c+dx)b^2+\left(-Ca^2+bBa+b^2C\right)\tan(c+dx)^2+a(bB-aC)}{a+b\tan(c+dx)}dx}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{4109}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{b^2(aB+bC)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^3(bB-aC)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(bB-aC)}{a^2+b^2}}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{b^2(aB+bC)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^3(bB-aC)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(bB-aC)}{a^2+b^2}}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{3956}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{a^3(bB-aC)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(bB-aC)}{a^2+b^2}}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{4100}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{a^3(bB-aC)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}-\frac{b^2(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(bB-aC)}{a^2+b^2}}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{C\tan^2(c+dx)}{2bd}-\frac{\frac{-\frac{b^2(aB+bC)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(bB-aC)}{a^2+b^2}+\frac{a^3(bB-aC)\log(a+b\tan(c+dx))}{bd\left(a^2+b^2\right)}}{b}-\frac{(bB-aC)\tan(c+dx)}{bd}}{b}$$

input
```
Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x]),x]
```

3.25. $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

output
```
(C*Tan[c + d*x]^2)/(2*b*d) - (((b^2*(b*B - a*C)*x)/(a^2 + b^2) - (b^2*(a*B
 + b*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^3*(b*B - a*C)*Log[a + b*Ta
n[c + d*x]])/(b*(a^2 + b^2)*d))/b - ((b*B - a*C)*Tan[c + d*x])/(b*d))/b
```

3.25.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.25.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{C\tan(dx+c)^2 b}{2}+B\tan(dx+c)b-C\tan(dx+c)a}{b^2}+\frac{\frac{(-Ba-Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Bb-Ca)\ln(a+}{b^3\left(a^2+\right.}}{d}$
default	$\frac{\frac{\frac{C\tan(dx+c)^2 b}{2}+B\tan(dx+c)b-C\tan(dx+c)a}{b^2}+\frac{\frac{(-Ba-Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Bb+Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Bb-Ca)\ln(a+}{b^3\left(a^2+\right.}}{d}$
norman	$\frac{(Bb-Ca)\tan(dx+c)}{b^2d}-\frac{(Bb-Ca)x}{a^2+b^2}+\frac{C\tan(dx+c)^2}{2bd}-\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{a^3(Bb-Ca)\ln(a+b\tan(}{b^3(a^2+b^2)d}$
parallelrisch	$-\frac{2Bx\,b^4d-2Cxa\,b^3d-C\tan(dx+c)^2a^2b^2-C\tan(dx+c)^2b^4+B\ln\left(1+\tan(dx+c)^2\right)a\,b^3+2B\ln(a+b\tan(dx+c))a^3b-2}{2}$
risch	$\frac{2iC\,a^2c}{b^3d}-\frac{xC}{ib-a}-\frac{2iBac}{b^2d}-\frac{2iCc}{bd}-\frac{2iCx}{b}-\frac{2iBax}{b^2}-\frac{ixB}{ib-a}-\frac{2ia^4Cx}{(a^2+b^2)b^3}+\frac{2iC\,a^2x}{b^3}+\frac{2ia^3Bc}{(a^2+b^2)b^2d}-\frac{}{(a}$

input
```
int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(1/b^2*(1/2*C*tan(d*x+c)^2*b+B*tan(d*x+c)*b-C*tan(d*x+c)*a)+1/(a^2+b^2
)*(1/2*(-B*a-C*b)*ln(1+tan(d*x+c)^2)+(-B*b+C*a)*arctan(tan(d*x+c)))-1/b^3*
a^3*(B*b-C*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2\left(Cab^3-Bb^4\right)dx+\left(Ca^2b^2+Cb^4\right)\tan(dx+c)^2+\left(Ca^4-Ba^3b\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(Ca}{2\left(a^2b^3+b^5\right)d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="fricas")
```

output
```
1/2*(2*(C*a*b^3 - B*b^4)*d*x + (C*a^2*b^2 + C*b^4)*tan(d*x + c)^2 + (C*a^4
 - B*a^3*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1)) - (C*a^4 - B*a^3*b - B*a*b^3 - C*b^4)*log(1/(tan(d*x + c)^2 +
1)) - 2*(C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*tan(d*x + c))/((a^2*b^3 +
b^5)*d)
```

3.25.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 1306, normalized size of antiderivative = 10.28

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*tan(c), Eq(a, 0) & Eq(b, 0) & Eq
(d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(2*d) + C
*x + C*tan(c + d*x)**3/(3*d) - C*tan(c + d*x)/d)/a, Eq(b, 0)), (-3*B*d*x*t
an(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B*d*x/(2*b*d*tan(c + d*x)
 - 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x
) - 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) +
 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*B/(2*b*d*tan(c + d
*x) - 2*I*b*d) - 3*I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3
*C*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - 2*C*log(tan(c + d*x)**2 + 1)*tan(c
 + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*C*log(tan(c + d*x)**2 + 1)/(2
*b*d*tan(c + d*x) - 2*I*b*d) + C*tan(c + d*x)**3/(2*b*d*tan(c + d*x) - 2*I
*b*d) + I*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*C/(2*b*d*
tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-3*B*d*x*tan(c + d*x)/(2*b*d*tan(c
 + d*x) + 2*I*b*d) - 3*I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(ta
n(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan
(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*
b*d*tan(c + d*x) + 2*I*b*d) + 3*B/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*I*C*d
*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*C*d*x/(2*b*d*tan(c + d*
x) + 2*I*b*d) - 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d
*x) + 2*I*b*d) - 2*I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2...
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2} + \frac{2\,(Ca^4-Ba^3b)\log(b\tan(dx+c)+a)}{a^2b^3+b^5} - \frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{Cb\tan(dx+c)^2-2\,(Ca-Bb)\tan(dx+c)}{b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="maxima")
```

output
```
1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(b*tan(d
*x + c) + a)/(a^2*b^3 + b^5) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) + (C*b*tan(d*x + c)^2 - 2*(C*a - B*b)*tan(d*x + c))/b^2)/d
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2} - \frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{2\,(Ca^4-Ba^3b)\log(|b\tan(dx+c)+a|)}{a^2b^3+b^5} + \frac{Cb\tan(dx+c)^2-2\,Ca\tan(dx+c)+2\,Bb\tan(d}{b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="giac")
```

output
```
1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a^4 - B*a^3*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b
^3 + b^5) + (C*b*tan(d*x + c)^2 - 2*C*a*tan(d*x + c) + 2*B*b*tan(d*x + c))
/b^2)/d
```

3.25.9 Mupad [B] (verification not implemented)

Time = 8.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(\frac{B}{b}-\frac{C\,a}{b^2}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$
$$+ \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^4-B\,a^3\,b\right)}{d\,\left(a^2\,b^3+b^5\right)}$$
$$- \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} + \frac{C\tan(c+d\,x)^2}{2\,b\,d}$$

input
```
int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x)),x)
```

output
```
(tan(c + d*x)*(B/b - (C*a)/b^2))/d - (log(tan(c + d*x) - 1i)*(B*1i - C))/(
2*d*(a*1i - b)) + (log(a + b*tan(c + d*x))*(C*a^4 - B*a^3*b))/(d*(b^5 + a^
2*b^3)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i)) + (C*tan(c
+ d*x)^2)/(2*b*d)
```

3.25. $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

3.26 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

3.26.1 Optimal result

Integrand size = 38, antiderivative size = 101

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = -\frac{(aB+bC)x}{a^2+b^2}-\frac{(bB-aC)\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)\,d}+\frac{C\tan(c+dx)}{bd}$$

output
```
-(B*a+C*b)*x/(a^2+b^2)-(B*b-C*a)*ln(cos(d*x+c))/(a^2+b^2)/d+a^2*(B*b-C*a)*
ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)/d+C*tan(d*x+c)/b/d
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{\frac{i(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{(iB+C)\log(i+\tan(c+dx))}{a-ib}+\frac{2a^2(bB-aC)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}+\frac{2C\tan(c+dx)}{b}}{2d}$$

input
```
Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x]),x]
```

output
```
((I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - ((I*B + C)*Log[I + Tan[c
+ d*x]])/(a - I*b) + (2*a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2
 + b^2)) + (2*C*Tan[c + d*x])/b)/(2*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4115, 3042, 4089, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4089}$$

$$\frac{\int -\frac{-((bB-aC)\tan^2(c+dx))+bC\tan(c+dx)+aC}{a+b\tan(c+dx)}dx}{b}+\frac{C\tan(c+dx)}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{\int \frac{-((bB-aC)\tan^2(c+dx))+bC\tan(c+dx)+aC}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{\int\frac{-((bB-aC)\tan(c+dx)^2)+bC\tan(c+dx)+aC}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{4109}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{-\frac{a^2(bB-aC)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(bB-aC)\int\tan(c+dx)dx}{a^2+b^2}+\frac{bx(aB+bC)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{-\frac{a^2(bB-aC)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(bB-aC)\int\tan(c+dx)dx}{a^2+b^2}+\frac{bx(aB+bC)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{-\frac{a^2(bB-aC)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b(bB-aC)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aB+bC)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{4100}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{-\frac{a^2(bB-aC)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(bB-aC)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aB+bC)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{C\tan(c+dx)}{bd}-\frac{-\frac{a^2(bB-aC)\log(a+b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(bB-aC)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aB+bC)}{a^2+b^2}}{b}$$

input
```
Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
),x]
```

output
```
-(((b*(a*B + b*C)*x)/(a^2 + b^2) + (b*(b*B - a*C)*Log[Cos[c + d*x]])/((a^2
 + b^2)*d) - (a^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/
b) + (C*Tan[c + d*x])/(b*d)
```

3.26. $\int\frac{\tan(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

3.26.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4089
```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Simp[1/d   Int[(a^2*A*d - b^2*B*c + (2*a*A*b
+ B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x
]^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

3.26. $\int \frac{\tan(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.26.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)C}{b}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ba-Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
default	$\frac{\frac{\tan(dx+c)C}{b}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ba-Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
norman	$\frac{C\tan(dx+c)}{bd}-\frac{(Ba+Cb)x}{a^2+b^2}+\frac{a^2(Bb-Ca)\ln(a+b\tan(dx+c))}{b^2(a^2+b^2)d}+\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$
parallelrisch	$\frac{-2Ba\,b^2dx-2C\,b^3dx+B\ln\left(1+\tan(dx+c)^2\right)b^3+2B\ln(a+b\tan(dx+c))a^2b-C\ln\left(1+\tan(dx+c)^2\right)a\,b^2-2C\ln(a+b\tan(}{2d(a^2+b^2)b^2}$
risch	$\frac{xB}{ib-a}-\frac{ixC}{ib-a}-\frac{2ia^2Bx}{b(a^2+b^2)}-\frac{2ia^2Bc}{bd(a^2+b^2)}+\frac{2ia^3Cx}{b^2(a^2+b^2)}+\frac{2ia^3Cc}{b^2d(a^2+b^2)}+\frac{2iBx}{b}+\frac{2iBc}{bd}-\frac{2iCax}{b^2}-\frac{2iCac}{b^2d}+$

input
```
int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*C/b+1/(a^2+b^2)*(1/2*(B*b-C*a)*ln(1+tan(d*x+c)^2)+(-B*a-C*
b)*arctan(tan(d*x+c)))+1/b^2*a^2*(B*b-C*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{2\left(Bab^2+Cb^3\right)dx+\left(Ca^3-Ba^2b\right)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-\left(Ca^3-Ba^2b+Cab^2-Bb^3\right)\log}{2\left(a^2b^2+b^4\right)d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="fricas")
```

output
```
-1/2*(2*(B*a*b^2 + C*b^3)*d*x + (C*a^3 - B*a^2*b)*log((b^2*tan(d*x + c)^2
+ 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (C*a^3 - B*a^2*b + C*a
*b^2 - B*b^3)*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^2*b + C*b^3)*tan(d*x +
c))/((a^2*b^2 + b^4)*d)
```

3.26.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 1020, normalized size of antiderivative = 10.10

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2), Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, ((-B*x + B*tan(c + d*x)/d - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*tan(c +
 d*x)**2/(2*d))/a, Eq(b, 0)), (I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) -
2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*log(tan(c + d*x)**2 +
1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*log(tan(c + d*x)**2 +
 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B/(2*b*d*tan(c + d*x) - 2*I*b*d) -
3*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*C*d*x/(2*b*d*tan
(c + d*x) - 2*I*b*d) + I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*ta
n(c + d*x) - 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2
*I*b*d) + 2*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*C/(2*b*d*
tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)/(2*b*d*tan(c
 + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*log(tan(c +
 d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) + 2
*I*b*d) - 3*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*C*d*x/
(2*b*d*tan(c + d*x) + 2*I*b*d) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)
/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c
+ d*x) + 2*I*b*d) + 2*C*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3
*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2
)*tan(c)/(a + b*tan(c)), Eq(d, 0)), (2*B*a**2*b*log(a/b + tan(c + d*x))...
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2\,(Ca^3-Ba^2b)\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="maxima")
```

output
```
-1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(b*tan(
d*x + c) + a)/(a^2*b^2 + b^4) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) - 2*C*tan(d*x + c)/b)/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ca^3-Ba^2b)\log(|b\tan(dx+c)+a|)}{a^2b^2+b^4}-\frac{2\,C\tan(dx+c)}{b}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="giac")
```

output
```
-1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
 + 1)/(a^2 + b^2) + 2*(C*a^3 - B*a^2*b)*log(abs(b*tan(d*x + c) + a))/(a^2*
b^2 + b^4) - 2*C*tan(d*x + c)/b)/d
```

3.26.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{C\tan(c+d\,x)}{b\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^3-B\,a^2\,b\right)}{d\,\left(a^2\,b^2+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
),x)
```

output
```
(log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*1i + b)) - (log(a + b*tan(c +
d*x))*(C*a^3 - B*a^2*b))/(d*(b^4 + a^2*b^2)) + (C*tan(c + d*x))/(b*d) + (l
og(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a + b*1i))
```

3.27 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$

3.27.1 Optimal result

Integrand size = 32, antiderivative size = 85

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(bB-aC)x}{a^2+b^2} - \frac{(aB+bC)\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{a(bB-aC)\log(a+b\tan(c+dx))}{b\,(a^2+b^2)\,d}$$

output
```
(B*b-C*a)*x/(a^2+b^2)-(B*a+C*b)*ln(cos(d*x+c))/(a^2+b^2)/d-a*(B*b-C*a)*ln(
a+b*tan(d*x+c))/b/(a^2+b^2)/d
```

3.27.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a-ib)b(B+iC)\log(i-\tan(c+dx))+(a+ib)b(B-iC)\log(i+\tan(c+dx))+2a(-bB+aC)\log(a}{2b\,(a^2+b^2)\,d}$$

input
```
Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

output
```
((a - I*b)*b*(B + I*C)*Log[I - Tan[c + d*x]] + (a + I*b)*b*(B - I*C)*Log[I
 + Tan[c + d*x]] + 2*a*(-(b*B) + a*C)*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 +
 b^2)*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4853, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4853}$$

$$\frac{\int \frac{\tan(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\int \left(\frac{a(aC-bB)}{(a^2+b^2)(a+b\tan(c+dx))}+\frac{bB-aC+(aB+bC)\tan(c+dx)}{(a^2+b^2)(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(bB-aC)\arctan(\tan(c+dx))}{a^2+b^2}+\frac{(aB+bC)\log(\tan^2(c+dx)+1)}{2(a^2+b^2)}-\frac{a(bB-aC)\log(a+b\tan(c+dx))}{b(a^2+b^2)}}{d}$$

input
```
Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x]),x]
```

output
```
(((b*B - a*C)*ArcTan[Tan[c + d*x]])/(a^2 + b^2) - (a*(b*B - a*C)*Log[a + b
*Tan[c + d*x]])/(b*(a^2 + b^2)) + ((a*B + b*C)*Log[1 + Tan[c + d*x]^2])/(2
*(a^2 + b^2)))/d
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

3.27.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
default	$\frac{\frac{\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
norman	$\frac{(Bb-Ca)x}{a^2+b^2}+\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{a(Bb-Ca)\ln(a+b\tan(dx+c))}{b(a^2+b^2)d}$
parallelrisch	$\frac{2B\,b^2dx-2Cabdx+B\ln\left(1+\tan(dx+c)^2\right)ab-2B\ln(a+b\tan(dx+c))ab+C\ln\left(1+\tan(dx+c)^2\right)b^2+2C\ln(a+b\tan(dx+c)}{2(a^2+b^2)bd}$
risch	$\frac{ixB}{ib-a}+\frac{xC}{ib-a}+\frac{2iCx}{b}+\frac{2iCc}{bd}+\frac{2iaBx}{a^2+b^2}+\frac{2iaBc}{(a^2+b^2)d}-\frac{2ia^2Cx}{(a^2+b^2)b}-\frac{2ia^2Cc}{(a^2+b^2)bd}-\frac{\ln\left(e^{2i(dx+c)}+1\right)C}{bd}-\frac{a\ln}{}$

input
```
int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)*(1/2*(B*a+C*b)*ln(1+tan(d*x+c)^2)+(B*b-C*a)*arctan(tan(d*
x+c)))-a*(B*b-C*a)/(a^2+b^2)/b*ln(a+b*tan(d*x+c)))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\,(Cab-Bb^2)dx-(Ca^2-Bab)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(Ca^2+Cb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,(a^2b+b^3)d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(2*(C*a*b - B*b^2)*d*x - (C*a^2 - B*a*b)*log((b^2*tan(d*x + c)^2 + 2*
a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (C*a^2 + C*b^2)*log(1/(tan
(d*x + c)^2 + 1)))/((a^2*b + b^3)*d)
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.36

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x(B\tan(c)+C\tan^2(c))}{\tan(c)} \\ \frac{\frac{B\log(\tan^2(c+dx)+1)}{2d}-Cx+\frac{C\tan(c+dx)}{d}}{a} \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}-\frac{iBdx}{2bd\tan(c+dx)-2ibd}-\frac{B}{2bd\tan(c+dx)-2ibd}+\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd}+\frac{Cdx}{2bd\tan(c+dx)-2ibd}+\frac{C\log(\tan^2(c\cdots}{2bd\tan\cdots} \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{iBdx}{2bd\tan(c+dx)+2ibd}-\frac{B}{2bd\tan(c+dx)+2ibd}-\frac{iCdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd}+\frac{Cdx}{2bd\tan(c+dx)+2ibd}+\frac{C\log(\tan^2(c\cdots}{2bd\tan\cdots} \\ \frac{x(B\tan(c)+C\tan^2(c))}{a+b\tan(c)} \\ -\frac{2Bab\log(\frac{a}{b}+\tan(c+dx))}{2a^2bd+2b^3d}+\frac{Bab\log(\tan^2(c+dx)+1)}{2a^2bd+2b^3d}+\frac{2Bb^2dx}{2a^2bd+2b^3d}+\frac{2Ca^2\log(\frac{a}{b}+\tan(c+dx))}{2a^2bd+2b^3d}-\frac{2Cabdx}{2a^2bd+2b^3d}+\frac{Cb^2\log(\tan^2\cdots}{2a^2bd+\cdots} \end{cases}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq
(d, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - C*x + C*tan(c + d*x)/d)/a, E
q(b, 0)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*d*x/(2*
b*d*tan(c + d*x) - 2*I*b*d) - B/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*C*d*x*t
an(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*x/(2*b*d*tan(c + d*x) - 2
*I*b*d) + C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*
I*b*d) - I*C*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*C
/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (B*d*x*tan(c + d*x)/(2*b*d*
tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - B/(2*b*
d*tan(c + d*x) + 2*I*b*d) - I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I
*b*d) + C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*C*log(tan(c + d*x)**2 + 1)
/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a
, I*b)), (x*(B*tan(c) + C*tan(c)**2)/(a + b*tan(c)), Eq(d, 0)), (-2*B*a*b*
log(a/b + tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) + B*a*b*log(tan(c + d*x)**
2 + 1)/(2*a**2*b*d + 2*b**3*d) + 2*B*b**2*d*x/(2*a**2*b*d + 2*b**3*d) + 2*
C*a**2*log(a/b + tan(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*C*a*b*d*x/(2*a*
*2*b*d + 2*b**3*d) + C*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*
d), True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{2\,(Ca^2-Bab)\log(b\tan(dx+c)+a)}{a^2b+b^3}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(b*tan(d*
x + c) + a)/(a^2*b + b^3) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2
))/d
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2} - \frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{2\,(Ca^2-Bab)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
 + 1)/(a^2 + b^2) - 2*(C*a^2 - B*a*b)*log(abs(b*tan(d*x + c) + a))/(a^2*b
+ b^3))/d
```

3.27.9 Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{b\,d\,\left(a^2+b^2\right)}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*1i - b)) + (log(tan(c + d*x) +
 1i)*(B - C*1i))/(2*d*(a - b*1i)) - (a*log(a + b*tan(c + d*x))*(B*b - C*a)
)/(b*d*(a^2 + b^2))
```

3.28 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

3.28.1 Optimal result

Integrand size = 38, antiderivative size = 58

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{(aB+bC)x}{a^2+b^2}+\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
(B*a+C*b)*x/(a^2+b^2)+(B*b-C*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

3.28.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{-2(aB+bC)\arctan(\cot(c+dx))+(bB-aC)\left(2\log(b+a\cot(c+dx))-\log\left(\csc^2(c+dx)\right)\right)}{2\left(a^2+b^2\right)d}$$

input
```
Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x]),x]
```

output
```
(-2*(a*B + b*C)*ArcTan[Cot[c + d*x]] + (b*B - a*C)*(2*Log[b + a*Cot[c + d*
x]] - Log[Csc[c + d*x]^2]))/(2*(a^2 + b^2)*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 4115, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{B+C\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4014}$$

$$\frac{(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(aB+bC)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(aB+bC)}{a^2+b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x(aB+bC)}{a^2+b^2}$$

input
```
Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
),x]
```

output
```
((a*B + b*C)*x)/(a^2 + b^2) + ((b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c +
d*x]])/((a^2 + b^2)*d)
```

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.28.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{(2Bb-2Ca)\ln(a+b\tan(dx+c))+(-Bb+Ca)\ln\left(\sec(dx+c)^2\right)+2dx(Ba+Cb)}{2d(a^2+b^2)}$
derivativedivides	$\frac{\frac{\frac{(-Bb+Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ba+Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(-Bb+Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ba+Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{(Ba+Cb)x}{a^2+b^2}+\frac{(Bb-Ca)\ln(a+b\tan(dx+c))}{d(a^2+b^2)}-\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$
risch	$-\frac{xB}{ib-a}+\frac{ixC}{ib-a}-\frac{2iBbx}{a^2+b^2}+\frac{2iCax}{a^2+b^2}-\frac{2iBbc}{d(a^2+b^2)}+\frac{2iCac}{d(a^2+b^2)}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)Bb}{d(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+}{ib-}\right.}{d(a^2+b^2)}$

input
```
int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_RE
TURNVERBOSE)
```

output
```
1/2*((2*B*b-2*C*a)*ln(a+b*tan(d*x+c))+(-B*b+C*a)*ln(sec(d*x+c)^2)+2*d*x*(B
*a+C*b))/d/(a^2+b^2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{2\,(Ba+Cb)dx-(Ca-Bb)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^2+b^2)d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="fricas")
```

output
```
1/2*(2*(B*a + C*b)*d*x - (C*a - B*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d
*x + c) + a^2)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 541, normalized size of antiderivative = 9.33

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}x(B\tan{(c)}+C\tan^2{(c)})\cot{(c)}}{\tan{(c)}} \\ \frac{Bx+\frac{C\log{(\tan^2{(c+dx)}+1)}}{2d}}{a} \\ \frac{iBdx\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd}+\frac{Bdx}{2bd\tan{(c+dx)}-2ibd}+\frac{iB}{2bd\tan{(c+dx)}-2ibd}+\frac{Cdx\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd}-\frac{iCdx}{2bd\tan{(c+dx)}-2ibd}-\frac{C}{2bd\tan{(c+dx)} \\ -\frac{iBdx\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd}+\frac{Bdx}{2bd\tan{(c+dx)}+2ibd}-\frac{iB}{2bd\tan{(c+dx)}+2ibd}+\frac{Cdx\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd}+\frac{iCdx}{2bd\tan{(c+dx)}+2ibd}-\frac{C}{2bd\tan{(c+} \\ \frac{x(B\tan{(c)}+C\tan^2{(c)})\cot{(c)}}{a+b\tan{(c)}} \\ \frac{2Badx}{2a^2d+2b^2d}+\frac{2Bb\log{(\frac{a}{b}+\tan{(c+dx)})}}{2a^2d+2b^2d}-\frac{Bb\log{(\tan^2{(c+dx)}+1)}}{2a^2d+2b^2d}-\frac{2Ca\log{(\frac{a}{b}+\tan{(c+dx)})}}{2a^2d+2b^2d}+\frac{Ca\log{(\tan^2{(c+dx)}+1)}}{2a^2d+2b^2d}+\frac{2Cbdx}{2a^2d+2b^2} \end{cases}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)/tan(c), Eq(a, 0) & Eq(b,
0) & Eq(d, 0)), ((B*x + C*log(tan(c + d*x)**2 + 1)/(2*d))/a, Eq(b, 0)), (I
*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*tan(c +
d*x) - 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*x*tan(c + d*x)/
(2*b*d*tan(c + d*x) - 2*I*b*d) - I*C*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) -
C/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)/(2*
b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B/(
2*b*d*tan(c + d*x) + 2*I*b*d) + C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2
*I*b*d) + I*C*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - C/(2*b*d*tan(c + d*x) +
 2*I*b*d), Eq(a, I*b)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)/(a + b*tan(c)),
 Eq(d, 0)), (2*B*a*d*x/(2*a**2*d + 2*b**2*d) + 2*B*b*log(a/b + tan(c + d*x
))/(2*a**2*d + 2*b**2*d) - B*b*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2
*d) - 2*C*a*log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) + C*a*log(tan(c
+ d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*C*b*d*x/(2*a**2*d + 2*b**2*d), Tr
ue))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{\cot(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2} - \frac{2\,(Ca-Bb)\log(b\tan(dx+c)+a)}{a^2+b^2} + \frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="maxima")
```

output
```
1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a - B*b)*log(b*tan(d*x + c
) + a)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int \frac{\cot(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,(Ba+Cb)(dx+c)}{a^2+b^2} + \frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{2\,(Cab-Bb^2)\log(|b\tan(dx+c)+a|)}{a^2b+b^3}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, alg
orithm="giac")
```

output
```
1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) - 2*(C*a*b - B*b^2)*log(abs(b*tan(d*x + c) + a))/(a^2*b +
 b^3))/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{d\,\left(a^2+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
),x)
```

output
```
(log(a + b*tan(c + d*x))*(B*b - C*a))/(d*(a^2 + b^2)) - (log(tan(c + d*x)
+ 1i)*(B - C*1i))/(2*d*(a*1i + b)) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(
2*d*(a + b*1i))
```

3.29 $\int \frac{\cot^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

3.29.1 Optimal result

Integrand size = 40, antiderivative size = 80

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(bB-aC)x}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad}-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a\,(a^2+b^2)\,d}$$

output
```
-(B*b-C*a)*x/(a^2+b^2)+B*ln(sin(d*x+c))/a/d-b*(B*b-C*a)*ln(a*cos(d*x+c)+b*
sin(d*x+c))/a/(a^2+b^2)/d
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{2B\log(\tan(c+dx))}{a}+\frac{(B-iC)\log(i+\tan(c+dx))}{a-ib}+\frac{2b(bB-aC)\log(a+b\tan(c+dx))}{a(a^2+b^2)}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x]),x]
```

output
```
-1/2*(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) - (2*B*Log[Tan[c + d*x]]
)/a + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b*(b*B - a*C)*Log[a
 + b*Tan[c + d*x]])/(a*(a^2 + b^2)))/d
```

3.29.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4115, 3042, 4094, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4115}$$

$$\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4094}$$

$$-\frac{b(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{B\int \cot(c+dx)dx}{a}-\frac{x(bB-aC)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{B\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{x(bB-aC)}{a^2+b^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{B\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{x(bB-aC)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{b(bB-aC)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{B\log(-\sin(c+dx))}{ad}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{b(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{x(bB-aC)}{a^2+b^2}+\frac{B\log(-\sin(c+dx))}{ad}$$

input
```
Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x]),x]
```

output
```
-(((b*B - a*C)*x)/(a^2 + b^2)) + (B*Log[-Sin[c + d*x]])/(a*d) - (b*(b*B -
a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4094
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(B*(b*
c + a*d) + A*(a*c - b*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[b*((A*b
 - a*B)/((b*c - a*d)*(a^2 + b^2)))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
+ f*x]), x], x] + Simp[d*((B*c - A*d)/((b*c - a*d)*(c^2 + d^2)))   Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.29.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{\left(-2 B \,b^{2}+2 C a b \right) \ln \left(a +b \tan \left(d x +c \right)\right)+\left(-B \,a^{2}-C a b \right) \ln \left(\sec \left(d x +c \right)^{2}\right)+2 B \left(a^{2}+b^{2}\right) \ln \left(\tan \left(d x +c \right)\right)-2 a d x \left(B b -C a \right)}{2 \left(a^{2}+b^{2}\right) a d}$
derivativedivides	$\frac{\frac{B \ln \left(\tan \left(d x +c \right)\right)}{a}+\frac{\frac{\left(-B a -C b \right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2}+\left(-B b +C a \right) \arctan \left(\tan \left(d x +c \right)\right)}{a^{2}+b^{2}}-\frac{\left(B b -C a \right) b \ln \left(a +b \tan \left(d x +c \right)\right)}{\left(a^{2}+b^{2}\right) a}}{d}$
default	$\frac{\frac{B \ln \left(\tan \left(d x +c \right)\right)}{a}+\frac{\frac{\left(-B a -C b \right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2}+\left(-B b +C a \right) \arctan \left(\tan \left(d x +c \right)\right)}{a^{2}+b^{2}}-\frac{\left(B b -C a \right) b \ln \left(a +b \tan \left(d x +c \right)\right)}{\left(a^{2}+b^{2}\right) a}}{d}$
norman	$-\frac{\left(B b -C a \right) x}{a^{2}+b^{2}}+\frac{B \ln \left(\tan \left(d x +c \right)\right)}{a d}-\frac{\left(B a +C b \right) \ln \left(1+\tan \left(d x +c \right)^{2}\right)}{2 d \left(a^{2}+b^{2}\right)}-\frac{\left(B b -C a \right) b \ln \left(a +b \tan \left(d x +c \right)\right)}{\left(a^{2}+b^{2}\right) a d}$
risch	$-\frac{i x B}{i b -a}-\frac{x C}{i b -a}+\frac{2 i b^{2} B x}{\left(a^{2}+b^{2}\right) a}+\frac{2 i b^{2} B c}{\left(a^{2}+b^{2}\right) a d}-\frac{2 i b C x}{a^{2}+b^{2}}-\frac{2 i b C c}{\left(a^{2}+b^{2}\right) d}-\frac{2 i B x}{a}-\frac{2 i B c}{a d}-\frac{b^{2} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}-\frac{i b +a}{i b -a}\right)}{\left(a^{2}+b^{2}\right) a d}$

input
```
int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_
RETURNVERBOSE)
```

output
```
1/2*((-2*B*b^2+2*C*a*b)*ln(a+b*tan(d*x+c))+(-B*a^2-C*a*b)*ln(sec(d*x+c)^2)
+2*B*(a^2+b^2)*ln(tan(d*x+c))-2*a*d*x*(B*b-C*a))/(a^2+b^2)/a/d
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2\,(Ca^2-Bab)dx+(Ba^2+Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+(Cab-Bb^2)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^3+ab^2)d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="fricas")
```

output
```
1/2*(2*(C*a^2 - B*a*b)*d*x + (B*a^2 + B*b^2)*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1)) + (C*a*b - B*b^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
 + a^2)/(tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)
```

3.29.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 966, normalized size of antiderivative = 12.08

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/tan(c), Eq(a, 0) & Eq(
b, 0) & Eq(d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*x
))/d + C*x)/a, Eq(b, 0)), ((-B*x - B/(d*tan(c + d*x)) - C*log(tan(c + d*x)
**2 + 1)/(2*d) + C*log(tan(c + d*x))/d)/b, Eq(a, 0)), (B*d*x*tan(c + d*x)/
(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) -
I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) -
 B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*B*log(tan
(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*B*log(tan(c + d
*x))/(2*b*d*tan(c + d*x) - 2*I*b*d) + B/(2*b*d*tan(c + d*x) - 2*I*b*d) + I
*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + C*d*x/(2*b*d*tan(c +
d*x) - 2*I*b*d) + I*C/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (B*d*x
*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x)
 + 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x
) + 2*I*b*d) - B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) -
 2*I*B*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B
*log(tan(c + d*x))/(2*b*d*tan(c + d*x) + 2*I*b*d) + B/(2*b*d*tan(c + d*x)
+ 2*I*b*d) - I*C*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + C*d*x/(
2*b*d*tan(c + d*x) + 2*I*b*d) - I*C/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a,
I*b)), (x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/(a + b*tan(c)), Eq(d, 0)), (-
B*a**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d) + 2*B*a**2*log(...
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}+\frac{2\,(Cab-Bb^2)\log(b\tan(dx+c)+a)}{a^3+ab^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,B\log(\tan(dx+c))}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="maxima")
```

output
```
1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b - B*b^2)*log(b*tan(d*x
 + c) + a)/(a^3 + a*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2)
 + 2*B*log(tan(d*x + c))/a)/d
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Cab^2-Bb^3)\log(|b\tan(dx+c)+a|)}{a^3b+ab^3}+\frac{2\,B\log(|\tan(dx+c)|)}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="giac")
```

output
```
1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
+ 1)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^3*b
 + a*b^3) + 2*B*log(abs(tan(d*x + c)))/a)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \frac{\cot^2(c+dx) (B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)} \, dx$$
$$= \frac{B\ln(\tan(c+dx))}{a\,d} - \frac{\ln(\tan(c+dx)-\mathrm{i})\,(-C+B\,1\mathrm{i})}{2\,d\,(-b+a\,1\mathrm{i})} - \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B-C\,1\mathrm{i})}{2\,d\,(a-b\,1\mathrm{i})} - \frac{b\ln(a+b\tan(c+dx))\,(B\,b-C\,a)}{a\,d\,(a^2+b^2)}$$

input
```
int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x)),x)
```

output
```
(B*log(tan(c + d*x)))/(a*d) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*
1i - b)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i)) - (b*log(a
 + b*tan(c + d*x))*(B*b - C*a))/(a*d*(a^2 + b^2))
```

3.30 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

3.30.1 Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = -\frac{(aB+bC)x}{a^2+b^2}-\frac{B\cot(c+dx)}{ad}-\frac{(bB-aC)\log(\sin(c+dx))}{a^2d}+\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)d}$$

output
```
-(B*a+C*b)*x/(a^2+b^2)-B*cot(d*x+c)/a/d-(B*b-C*a)*ln(sin(d*x+c))/a^2/d+b^2
*(B*b-C*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)/d
```

3.30.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \frac{-\frac{2B\cot(c+dx)}{a}+\frac{i(B+iC)\log(i-\tan(c+dx))}{a+ib}+\frac{2(-bB+aC)\log(\tan(c+dx))}{a^2}-\frac{(iB+C)\log(i+\tan(c+dx))}{a-ib}+\frac{2b^2(bB-aC)\log(a+b\tan(c+dx))}{a^2(a^2+b^2)}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x]),x]
```

output
```
((-2*B*Cot[c + d*x])/a + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b) + (
2*(-(b*B) + a*C)*Log[Tan[c + d*x]])/a^2 - ((I*B + C)*Log[I + Tan[c + d*x]]
)/(a - I*b) + (2*b^2*(b*B - a*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)
))/(2*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4115, 3042, 4092, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot(c+dx)\left(bB\tan^2(c+dx)+aB\tan(c+dx)+bB-aC\right)}{a+b\tan(c+dx)}dx}{a}-\frac{B\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{bB\tan(c+dx)^2+aB\tan(c+dx)+bB-aC}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{B\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{4134}$$

$$-\frac{-\frac{b^2(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(bB-aC)\int\cot(c+dx)dx}{a}+\frac{ax(aB+bC)}{a^2+b^2}}{a}-\frac{B\cot(c+dx)}{ad}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{b^2(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(bB-aC)\int-\tan(c+dx+\frac{\pi}{2})dx}{a}+\frac{ax(aB+bC)}{a^2+b^2}}{a}-\frac{B\cot(c+dx)}{ad}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{b^2(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{(bB-aC)\int\tan(\frac{1}{2}(2c+\pi)+dx)dx}{a}+\frac{ax(aB+bC)}{a^2+b^2}}{a}-\frac{B\cot(c+dx)}{ad}$$

$\Big\downarrow$ 3956

$$-\frac{-\frac{b^2(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{ax(aB+bC)}{a^2+b^2}+\frac{(bB-aC)\log(-\sin(c+dx))}{ad}}{a}-\frac{B\cot(c+dx)}{ad}$$

$\Big\downarrow$ 4013

$$-\frac{-\frac{b^2(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}+\frac{ax(aB+bC)}{a^2+b^2}+\frac{(bB-aC)\log(-\sin(c+dx))}{ad}}{a}-\frac{B\cot(c+dx)}{ad}$$

input
```
Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x]),x]
```

output
```
-((B*Cot[c + d*x])/(a*d)) - ((a*(a*B + b*C)*x)/(a^2 + b^2) + ((b*B - a*C)*
Log[-Sin[c + d*x]])/(a*d) - (b^2*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/(a*(a^2 + b^2)*d))/a
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.30. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

3.30.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{(2Bb^3-2Cab^2)\ln(a+b\tan(dx+c))+(Ba^2b-Ca^3)\ln\left(\sec(dx+c)^2\right)-2(a^2+b^2)(Bb-Ca)\ln(\tan(dx+c))-2a\left(B\left(a^2+b^2\right.\right.}{2a^2d(a^2+b^2)}$
derivativedivides	$\frac{-\frac{B}{a\tan(dx+c)}+\frac{(-Bb+Ca)\ln(\tan(dx+c))}{a^2}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ba-Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Bb-Ca)b^2\ln(a+b\tan(}{\left(a^2+b^2\right)a^2}}{d}$
default	$\frac{-\frac{B}{a\tan(dx+c)}+\frac{(-Bb+Ca)\ln(\tan(dx+c))}{a^2}+\frac{\frac{(Bb-Ca)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ba-Cb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Bb-Ca)b^2\ln(a+b\tan(}{\left(a^2+b^2\right)a^2}}{d}$
norman	$\frac{-\frac{B\tan(dx+c)}{ad}-\frac{(Ba+Cb)x\tan(dx+c)^2}{a^2+b^2}}{\tan(dx+c)^2}+\frac{(Bb-Ca)b^2\ln(a+b\tan(dx+c))}{a^2d(a^2+b^2)}-\frac{(Bb-Ca)\ln(\tan(dx+c))}{a^2d}+\frac{(Bb-Ca)\ln(}{2d(}$
risch	$\frac{xB}{ib-a}-\frac{ixC}{ib-a}+\frac{2iBbx}{a^2}+\frac{2iBbc}{a^2d}-\frac{2iCx}{a}-\frac{2iCc}{ad}-\frac{2ib^3Bx}{a^2(a^2+b^2)}-\frac{2ib^3Bc}{a^2d(a^2+b^2)}+\frac{2ib^2Cx}{a(a^2+b^2)}+\frac{2ib^2Cc}{ad(a^2+b^2)}$

input
```
int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_
RETURNVERBOSE)
```

output
```
1/2*((2*B*b^3-2*C*a*b^2)*ln(a+b*tan(d*x+c))+(B*a^2*b-C*a^3)*ln(sec(d*x+c)^
2)-2*(a^2+b^2)*(B*b-C*a)*ln(tan(d*x+c))-2*a*(B*(a^2+b^2)*cot(d*x+c)+a*d*x*
(B*a+C*b)))/a^2/d/(a^2+b^2)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{2\,Ba^3+2\,Bab^2+2\,(Ba^3+Ca^2b)dx\tan(dx+c)-(Ca^3-Ba^2b+Cab^2-Bb^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan}{2\,(a^4+a^2b^2)d\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="fricas")
```

output
```
-1/2*(2*B*a^3 + 2*B*a*b^2 + 2*(B*a^3 + C*a^2*b)*d*x*tan(d*x + c) - (C*a^3
- B*a^2*b + C*a*b^2 - B*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(
d*x + c) + (C*a*b^2 - B*b^3)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c))/((a^4 + a^2*b^2)*d*tan(d*x + c)
)
```

3.30.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.72 (sec) , antiderivative size = 2067, normalized size of antiderivative = 20.07

$$\int \frac{\cot^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((-B*x - B/(d*
tan(c + d*x)) - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*log(tan(c + d*x))/d)/
a, Eq(b, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d -
 B/(2*d*tan(c + d*x)**2) - C*x - C/(d*tan(c + d*x)))/b, Eq(a, 0)), (-3*B*d
*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*I*B*
d*x*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - I*B*log(
tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(
c + d*x)) + B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a*d*tan(c + d*x)**2
 + 2*I*a*d*tan(c + d*x)) + 2*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*a*d*
tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 2*B*log(tan(c + d*x))*tan(c + d*
x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*B*tan(c + d*x)/(2*a*
d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 2*I*B/(2*a*d*tan(c + d*x)**2 +
 2*I*a*d*tan(c + d*x)) + I*C*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 +
2*I*a*d*tan(c + d*x)) - C*d*x*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*
d*tan(c + d*x)) - C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c
+ d*x)**2 + 2*I*a*d*tan(c + d*x)) - I*C*log(tan(c + d*x)**2 + 1)*tan(c + d
*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) + 2*C*log(tan(c + d*x))
*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) + 2*I*C*lo
g(tan(c + d*x))*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)
) + I*C*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)), Eq...
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{2(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{2(Cab^2-Bb^3)\log(b\tan(dx+c)+a)}{a^4+a^2b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2(Ca-Bb)\log(\tan(dx+c))}{a^2}+\frac{2B}{a\tan(dx+c)}}{2d}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + 2*(C*a*b^2 - B*b^3)*log(b*tan(
d*x + c) + a)/(a^4 + a^2*b^2) + (C*a - B*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) - 2*(C*a - B*b)*log(tan(d*x + c))/a^2 + 2*B/(a*tan(d*x + c)))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{2(Ba+Cb)(dx+c)}{a^2+b^2}+\frac{(Ca-Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2(Cab^3-Bb^4)\log(|b\tan(dx+c)+a|)}{a^4b+a^2b^3}-\frac{2(Ca-Bb)\log(|\tan(dx+c)|)}{a^2}+\cdots}{2d}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="giac")
```

output
```
-1/2*(2*(B*a + C*b)*(d*x + c)/(a^2 + b^2) + (C*a - B*b)*log(tan(d*x + c)^2
 + 1)/(a^2 + b^2) + 2*(C*a*b^3 - B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^4*
b + a^2*b^3) - 2*(C*a - B*b)*log(abs(tan(d*x + c)))/a^2 + 2*(C*a*tan(d*x +
 c) - B*b*tan(d*x + c) + B*a)/(a^2*tan(d*x + c)))/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.36

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,b^3-C\,a\,b^2\right)}{d\,\left(a^4+a^2\,b^2\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,b-C\,a\right)}{a^2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{B\cot(c+d\,x)}{a\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x)),x)
```

output
```
(log(a + b*tan(c + d*x))*(B*b^3 - C*a*b^2))/(d*(a^4 + a^2*b^2)) - (log(tan
(c + d*x))*(B*b - C*a))/(a^2*d) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d
*(a*1i + b)) - (B*cot(c + d*x))/(a*d) + (log(tan(c + d*x) - 1i)*(B*1i - C)
)/(2*d*(a + b*1i))
```

3.31 $\int \frac{\cot^4(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx$

3.31.1 Optimal result

Integrand size = 40, antiderivative size = 137

$$\begin{aligned}\int \frac{\cot^4(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx &= \frac{(bB-aC)x}{a^2+b^2}+\frac{(bB-aC)\cot(c+dx)}{a^2d}-\frac{B\cot^2(c+dx)}{2ad}\\ &\quad-\frac{(a^2B-b^2B+abC)\log(\sin(c+dx))}{a^3d}-\frac{b^3(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3(a^2+b^2)d}\end{aligned}$$

output
```
(B*b-C*a)*x/(a^2+b^2)+(B*b-C*a)*cot(d*x+c)/a^2/d-1/2*B*cot(d*x+c)^2/a/d-(B
*a^2-B*b^2+C*a*b)*ln(sin(d*x+c))/a^3/d-b^3*(B*b-C*a)*ln(a*cos(d*x+c)+b*sin
(d*x+c))/a^3/(a^2+b^2)/d
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cot^4(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{2(bB-aC)\cot(c+dx)}{a^2}-\frac{B\cot^2(c+dx)}{a}+\frac{(B+iC)\log(i-\tan(c+dx))}{a+ib}-\frac{2(a^2B-b^2B+abC)\log(\tan(c+dx))}{a^3}+\frac{(B-iC)\log(i+\tan(c+dx))}{a-ib}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x]),x]
```

output
```
((2*(b*B - a*C)*Cot[c + d*x])/a^2 - (B*Cot[c + d*x]^2)/a + ((B + I*C)*Log[
I - Tan[c + d*x]])/(a + I*b) - (2*(a^2*B - b^2*B + a*b*C)*Log[Tan[c + d*x]
])/a^3 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b^3*(-(b*B) + a*
C)*Log[a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)))/(2*d)
```

3.31.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4115, 3042, 4092, 27, 3042, 4132, 25, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^4(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\cot^3(c+dx)(B+C\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{2\cot^2(c+dx)\left(bB\tan^2(c+dx)+aB\tan(c+dx)+bB-aC\right)}{a+b\tan(c+dx)}dx}{2a}-\frac{B\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot^2(c+dx)\left(bB\tan^2(c+dx)+aB\tan(c+dx)+bB-aC\right)}{a+b\tan(c+dx)}dx}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{bB\tan(c+dx)^2+aB\tan(c+dx)+bB-aC}{\tan(c+dx)^2(a+b\tan(c+dx))}dx}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 4132

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(Ba^2+C\tan(c+dx)a^2+bCa-b(bB-aC)\tan^2(c+dx)-b^2B\right)}{a+b\tan(c+dx)}dx}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(Ba^2+C\tan(c+dx)a^2+bCa-b(bB-aC)\tan^2(c+dx)-b^2B\right)}{a+b\tan(c+dx)}dx}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{Ba^2+C\tan(c+dx)a^2+bCa-b(bB-aC)\tan(c+dx)^2-b^2B}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\left(a^2B+abC-b^2B\right)\int\cot(c+dx)dx}{a}+\frac{b^3(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(bB-aC)}{a^2+b^2}}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^2B+abC-b^2B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^3(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(bB-aC)}{a^2+b^2}}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{\left(a^2B+abC-b^2B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^3(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(bB-aC)}{a^2+b^2}}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{\frac{b^3(bB-aC)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2B+abC-b^2B\right)\log(-\sin(c+dx))}{ad}-\frac{a^2x(bB-aC)}{a^2+b^2}}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 4013

3.31. $\displaystyle\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx$

$$-\frac{\frac{\frac{(a^2B+abC-b^2B)\log(-\sin(c+dx))}{ad}-\frac{a^2x(bB-aC)}{a^2+b^2}+\frac{b^3(bB-aC)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}}{a}-\frac{(bB-aC)\cot(c+dx)}{ad}}{a}-\frac{B\cot^2(c+dx)}{2ad}$$

input
```
Int[(Cot[c + d*x]^4*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x]),x]
```

output
```
-1/2*(B*Cot[c + d*x]^2)/(a*d) - (-(((b*B - a*C)*Cot[c + d*x])/(a*d)) + (-(
(a^2*(b*B - a*C)*x)/(a^2 + b^2)) + ((a^2*B - b^2*B + a*b*C)*Log[-Sin[c + d
*x]])/(a*d) + (b^3*(b*B - a*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a
^2 + b^2)*d))/a)/a
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.31. $\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$

3.31.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{B}{2a\tan(dx+c)^2}-\frac{-Bb+Ca}{a^2\tan(dx+c)}+\frac{\left(-B\,a^2+B\,b^2-Cab\right)\ln(\tan(dx+c))}{a^3}+\frac{\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c)}{a^2+b^2}}{d}$
default	$\frac{-\frac{B}{2a\tan(dx+c)^2}-\frac{-Bb+Ca}{a^2\tan(dx+c)}+\frac{\left(-B\,a^2+B\,b^2-Cab\right)\ln(\tan(dx+c))}{a^3}+\frac{\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Bb-Ca)\arctan(\tan(dx+c)}{a^2+b^2}}{d}$
parallelrisch	$\frac{(-2B\,b^4+2Ca\,b^3)\ln(a+b\tan(dx+c))+(B\,a^4+C\,a^3b)\ln\left(\sec(dx+c)^2\right)+(-2B\,a^4+2B\,b^4-2C\,a^3b-2Ca\,b^3)\ln(\tan(dx+}{2(a^2+b^2)a^3d}$
norman	$\frac{\frac{(Bb-Ca)\tan(dx+c)^2}{a^2d}+\frac{(Bb-Ca)x\tan(dx+c)^3}{a^2+b^2}-\frac{B\tan(dx+c)}{2ad}}{\tan(dx+c)^3}+\frac{(Ba+Cb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{(B\,a^2-B\,b^2+Cab)\ln(t}{a^3d}$
risch	$-\frac{2iB\,b^2c}{a^3d}+\frac{xC}{ib-a}-\frac{2ib^3Cx}{(a^2+b^2)a^2}+\frac{ixB}{ib-a}-\frac{2iB\,b^2x}{a^3}+\frac{2iCbc}{a^2d}+\frac{2iBx}{a}+\frac{2ib^4Bx}{(a^2+b^2)a^3}+\frac{2iBc}{ad}-\frac{2i(iBa\,e^{2i(d}}{}$

input
```
int(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x,method=_
RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*B/tan(d*x+c)^2-(-B*b+C*a)/a^2/tan(d*x+c)+1/a^3*(-B*a^2+B*b^2-C
*a*b)*ln(tan(d*x+c))+1/(a^2+b^2)*(1/2*(B*a+C*b)*ln(1+tan(d*x+c)^2)+(B*b-C*
a)*arctan(tan(d*x+c)))-(B*b-C*a)*b^3/(a^2+b^2)/a^3*ln(a+b*tan(d*x+c)))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.71

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{Ba^4+Ba^2b^2+(Ba^4+Ca^3b+Cab^3-Bb^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2-(Cab^3-Bb^4)\log\left(\frac{b^2\tan}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="fricas")
```

output
```
-1/2*(B*a^4 + B*a^2*b^2 + (B*a^4 + C*a^3*b + C*a*b^3 - B*b^4)*log(tan(d*x
+ c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (C*a*b^3 - B*b^4)*log((b^2*t
an(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x +
c)^2 + (B*a^4 + B*a^2*b^2 + 2*(C*a^4 - B*a^3*b)*d*x)*tan(d*x + c)^2 + 2*(C
*a^4 - B*a^3*b + C*a^2*b^2 - B*a*b^3)*tan(d*x + c))/((a^5 + a^3*b^2)*d*tan
(d*x + c)^2)
```

3.31.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 2596, normalized size of antiderivative = 18.95

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**4*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((B*log(tan(c
+ d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2) - C
*x - C/(d*tan(c + d*x)))/a, Eq(b, 0)), ((B*x + B/(d*tan(c + d*x)) - B/(3*d
*tan(c + d*x)**3) + C*log(tan(c + d*x)**2 + 1)/(2*d) - C*log(tan(c + d*x))
/d - C/(2*d*tan(c + d*x)**2))/b, Eq(a, 0)), (-3*I*B*d*x*tan(c + d*x)**3/(2
*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + 3*B*d*x*tan(c + d*x)**2/
(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + 2*B*log(tan(c + d*x)**
2 + 1)*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) +
 2*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2
*I*a*d*tan(c + d*x)**2) - 4*B*log(tan(c + d*x))*tan(c + d*x)**3/(2*a*d*tan
(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 4*I*B*log(tan(c + d*x))*tan(c +
d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*I*B*tan(c +
d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + B*tan(c + d*x)
/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - I*B/(2*a*d*tan(c + d*
x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*C*d*x*tan(c + d*x)**3/(2*a*d*tan(c +
d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*I*C*d*x*tan(c + d*x)**2/(2*a*d*tan(
c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - I*C*log(tan(c + d*x)**2 + 1)*tan(
c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + C*log(tan(
c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c +
d*x)**2) + 2*I*C*log(tan(c + d*x))*tan(c + d*x)**3/(2*a*d*tan(c + d*x)*...
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{2\,(Cab^3-Bb^4)\log(b\tan(dx+c)+a)}{a^5+a^3b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Ba^2+Cab-Bb^2)\log(\tan(dx+c))}{a^3}+\frac{B\cdots}{\cdots}}{2\,d}$$

input
```
integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - 2*(C*a*b^3 - B*b^4)*log(b*tan(
d*x + c) + a)/(a^5 + a^3*b^2) - (B*a + C*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) + 2*(B*a^2 + C*a*b - B*b^2)*log(tan(d*x + c))/a^3 + (B*a + 2*(C*a -
B*b)*tan(d*x + c))/(a^2*tan(d*x + c)^2))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2\,(Ca-Bb)(dx+c)}{a^2+b^2}-\frac{(Ba+Cb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,(Cab^4-Bb^5)\log(|b\tan(dx+c)+a|)}{a^5b+a^3b^3}+\frac{2\,(Ba^2+Cab-Bb^2)\log(|\tan(dx+c)|)}{a^3}-\cdots}{2\,d}$$

input
```
integrate(cot(d*x+c)^4*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c)),x, a
lgorithm="giac")
```

output
```
-1/2*(2*(C*a - B*b)*(d*x + c)/(a^2 + b^2) - (B*a + C*b)*log(tan(d*x + c)^2
 + 1)/(a^2 + b^2) - 2*(C*a*b^4 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^5*
b + a^3*b^3) + 2*(B*a^2 + C*a*b - B*b^2)*log(abs(tan(d*x + c)))/a^3 - (3*B
*a^2*tan(d*x + c)^2 + 3*C*a*b*tan(d*x + c)^2 - 3*B*b^2*tan(d*x + c)^2 - 2*
C*a^2*tan(d*x + c) + 2*B*a*b*tan(d*x + c) - B*a^2)/(a^3*tan(d*x + c)^2))/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int \frac{\cot^4(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)^2\left(\frac{B}{2a}-\frac{\tan(c+dx)\,(Bb-Ca)}{a^2}\right)}{d}+\frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\,(-C+B\,1\mathrm{i})}{2\,d\,(-b+a\,1\mathrm{i})}$$
$$-\frac{\ln\left(\tan(c+dx)\right)\,(B\,a^2+C\,a\,b-B\,b^2)}{a^3\,d}$$
$$-\frac{\ln\left(a+b\tan(c+dx)\right)\,(B\,b^4-C\,a\,b^3)}{d\,(a^5+a^3\,b^2)}+\frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)\,(B-C\,1\mathrm{i})}{2\,d\,(a-b\,1\mathrm{i})}$$

input
```
int((cot(c + d*x)^4*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x)),x)
```

output
```
(log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*1i - b)) - (cot(c + d*x)^2*(B/
(2*a) - (tan(c + d*x)*(B*b - C*a))/a^2))/d - (log(tan(c + d*x))*(B*a^2 - B
*b^2 + C*a*b))/(a^3*d) - (log(a + b*tan(c + d*x))*(B*b^4 - C*a*b^3))/(d*(a
^5 + a^3*b^2)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a - b*1i))
```

3.32 $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

3.32.1 Optimal result

Integrand size = 40, antiderivative size = 208

$$\begin{aligned}
&\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx \\
&= -\frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}+\frac{(a^2B-b^2B+2abC)\log(\cos(c+dx))}{(a^2+b^2)^2\,d} \\
&\quad+\frac{a^2(a^2bB+3b^3B-2a^3C-4ab^2C)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(abB-2a^2C-b^2C)\tan(c+dx)}{b^2\,(a^2+b^2)\,d}+\frac{a(bB-aC)\tan^2(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-(2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2+(B*a^2-B*b^2+2*C*a*b)*ln(cos(d*x+c))/
(a^2+b^2)^2/d+a^2*(B*a^2*b+3*B*b^3-2*C*a^3-4*C*a*b^2)*ln(a+b*tan(d*x+c))/b
^3/(a^2+b^2)^2/d-(B*a*b-2*C*a^2-C*b^2)*tan(d*x+c)/b^2/(a^2+b^2)/d+a*(B*b-C
*a)*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.32.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2a^2(-a^2bB-3b^3B+2a^3C+4ab^2C)\log(a+b\tan(c+dx))}{b^3(a^2+b^2)^2}+\frac{2a^2(-abB+2a^2C}{b^3(a^2+b^2)(a+b\tan}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^2,x]
```

output
```
-1/2*(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((B - I*C)*Log[I + T
an[c + d*x]])/(a - I*b)^2 + (2*a^2*(-(a^2*b*B) - 3*b^3*B + 2*a^3*C + 4*a*b
^2*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2) + (2*a^2*(-(a*b*B) + 2*
a^2*C + b^2*C))/(b^3*(a^2 + b^2)*(a + b*Tan[c + d*x])) - (2*C*Tan[c + d*x]
^2)/(b*(a + b*Tan[c + d*x])))/d
```

3.32.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4115, 3042, 4088, 25, 3042, 4130, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$\Big\downarrow$ 4088

$$\frac{\int -\frac{\tan(c+dx)\left(\left(-2Ca^2+bBa-b^2C\right)\tan^2(c+dx)-b(bB-aC)\tan(c+dx)+2a(bB-aC)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}+\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 25

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\int \frac{\tan(c+dx)\left(\left(-2Ca^2+bBa-b^2C\right)\tan^2(c+dx)-b(bB-aC)\tan(c+dx)+2a(bB-aC)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\int \frac{\tan(c+dx)\left(\left(-2Ca^2+bBa-b^2C\right)\tan(c+dx)^2-b(bB-aC)\tan(c+dx)+2a(bB-aC)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4130

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\int -\frac{-\left((aB+bC)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(bB-2aC)\tan^2(c+dx)+a\left(-2Ca^2+bBa-b^2C\right)}{a+b\tan(c+dx)}dx}{b}+\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left((aB+bC)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(bB-2aC)\tan^2(c+dx)+a\left(-2Ca^2+bBa-b^2C\right)}{a+b\tan(c+dx)}dx}{b}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left((aB+bC)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(bB-2aC)\tan(c+dx)^2+a\left(-2Ca^2+bBa-b^2C\right)}{a+b\tan(c+dx)}dx}{b}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4109

3.32. $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{-\frac{b^2\left(a^2B+2abC-b^2B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-2a^3C+a^2bB-4ab^2C+3b^3B\right)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2x\left(a^2(-C)+2abB+b\right.}{a^2+b^2}}{b}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{-\frac{b^2\left(a^2B+2abC-b^2B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-2a^3C+a^2bB-4ab^2C+3b^3B\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2x\left(a^2(-C)+2abB+b\right.}{a^2+b^2}}{b}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3956

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-2a^3C+a^2bB-4ab^2C+3b^3B\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2\left(a^2B+2abC-b^2B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{b^2x\left(a^2(-C)+2abB+b^2\right.}{a^2+b^2}}{b}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 4100

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-2a^3C+a^2bB-4ab^2C+3b^3B\right)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}+\frac{b^2\left(a^2B+2abC-b^2B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{b^2x\left(a^2(-C\right.}{a^2}}{b}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 16

$$\frac{a(bB-aC)\tan^2(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\left(-2a^2C+abB-b^2C\right)\tan(c+dx)}{bd}-\frac{\frac{b^2\left(a^2B+2abC-b^2B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}-\frac{b^2x\left(a^2(-C)+2abB+b^2C\right)}{a^2+b^2}+\frac{a^2\left(-2a^3C+a^2bB-4ab^2C+3b^3B\right)\log(a+b\tan(c+dx\right.}{bd\left(a^2+b^2\right)}}{b}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^2,x]
```

3.32. $\int\frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

output
```
(a*(b*B - a*C)*Tan[c + d*x]^2)/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (-
((-((b^2*(2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)) + (b^2*(a^2*B - b^2*B +
 2*a*b*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^2*(a^2*b*B + 3*b^3*B - 2
*a^3*C - 4*a*b^2*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/b) + ((a*b
*B - 2*a^2*C - b^2*C)*Tan[c + d*x])/(b*d))/(b*(a^2 + b^2))
```

3.32.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

3.32. $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.32.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)C}{b^2}+\frac{\frac{(-B\,a^2+B\,b^2-2Cab)\ln(1+\tan(dx+c)^2)}{2}+(-2Bab+C\,a^2-C\,b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a^2(B\,a^2b+3B\,b^3-2C\,a^3-4C}{b^3(a^2+b}}{d}$
default	$\frac{\frac{\tan(dx+c)C}{b^2}+\frac{\frac{(-B\,a^2+B\,b^2-2Cab)\ln(1+\tan(dx+c)^2)}{2}+(-2Bab+C\,a^2-C\,b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a^2(B\,a^2b+3B\,b^3-2C\,a^3-4C}{b^3(a^2+b}}{d}$
norman	$\frac{\frac{C\tan(dx+c)^2}{bd}+\frac{(B\,a^2b-2C\,a^3-Ca\,b^2)a}{d\,b^3(a^2+b^2)}-\frac{a(2Bab-C\,a^2+C\,b^2)x}{a^4+2a^2b^2+b^4}-\frac{b(2Bab-C\,a^2+C\,b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{a^2(B\,a^2b+3B\,b^3-2C}{(a^4}$
parallelrisch	$-\frac{4C\,a^6+2C\,a^2b^4-2B\,a^3b^3+6C\,a^4b^2-2B\,a^5b+B\ln(1+\tan(dx+c)^2)\tan(dx+c)a^2b^4-2B\ln(a+b\tan(dx+c))\tan(dx+}{}$
risch	$\frac{2i(-B\,a^3b\,e^{2i(dx+c)}+2C\,a^4e^{2i(dx+c)}-C\,b^4e^{2i(dx+c)}-2iC\,a^3b\,e^{2i(dx+c)}-2iCa\,b^3e^{2i(dx+c)}-B\,a^3b+2C\,a^4+2C\,a^2b^2+C}{(e^{2i(dx+c)}+1)(ib+a)(-ib+a)^2(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a)b^2d}$

input
```
int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*C/b^2+1/(a^2+b^2)^2*(1/2*(-B*a^2+B*b^2-2*C*a*b)*ln(1+tan(d
*x+c)^2)+(-2*B*a*b+C*a^2-C*b^2)*arctan(tan(d*x+c)))+1/b^3*a^2*(B*a^2*b+3*B
*b^3-2*C*a^3-4*C*a*b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/b^3*a^3*(B*b-C*a)
/(a^2+b^2)/(a+b*tan(d*x+c)))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(209) = 418$.

Time = 0.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.09

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,Ca^4b^2-2\,Ba^3b^3-2\left(Ca^3b^3-2\,Ba^2b^4-Cab^5\right)dx-2\left(Ca^4b^2+2\,Ca^2b^4+Cb^6\right)\tan\left(dx+c\right)^2+(2\,Ca}{}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="fricas")
```

output
```
-1/2*(2*C*a^4*b^2 - 2*B*a^3*b^3 - 2*(C*a^3*b^3 - 2*B*a^2*b^4 - C*a*b^5)*d*
x - 2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*tan(d*x + c)^2 + (2*C*a^6 - B*a^5*
b + 4*C*a^4*b^2 - 3*B*a^3*b^3 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B
*a^2*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2
)/(tan(d*x + c)^2 + 1)) - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 +
 2*C*a^2*b^4 - B*a*b^5 + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^
4 + 2*C*a*b^5 - B*b^6)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(2*C*
a^5*b - B*a^4*b^2 + 2*C*a^3*b^3 + C*a*b^5 + (C*a^2*b^4 - 2*B*a*b^5 - C*b^6
)*d*x)*tan(d*x + c))/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*tan(d*x + c) + (a^5*b^
3 + 2*a^3*b^5 + a*b^7)*d)
```

3.32.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 4541, normalized size of antiderivative = 21.83

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2
,x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2), Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(2*d) + C*x + C*
tan(c + d*x)**3/(3*d) - C*tan(c + d*x)/d)/a**2, Eq(b, 0)), (3*I*B*d*x*tan(
c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d
) + 6*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) - 3*I*B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c +
d*x) - 4*b**2*d) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*
tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*B*log(tan(c +
d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d
*x) - 4*b**2*d) - 2*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 -
 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*B*tan(c + d*x)/(4*b**2*d*tan(c
+ d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B/(4*b**2*d*tan(c + d*
x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 9*C*d*x*tan(c + d*x)**2/(4*b
**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 18*I*C*d*x*t
an(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d
) + 9*C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d
) + 4*I*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)*
*2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*C*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
 - 4*I*C*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*...
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{2\left(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3\right)\log(b\tan(dx+c)+a)}{a^4b^3+2\,a^2b^5+b^7} - \frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{\cdots}{a^3b^3\cdots}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="maxima")
```

output
```
1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*
C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(b*tan(d*x + c) + a)/(a^4*
b^3 + 2*a^2*b^5 + b^7) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)
/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*a^4 - B*a^3*b)/(a^3*b^3 + a*b^5 + (a^2*b^4
 + b^6)*tan(d*x + c)) + 2*C*tan(d*x + c)/b^2)/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.39

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(2\,Ca^5-Ba^4b+4\,Ca^3b^2-3\,Ba^2b^3)\log(|b\tan(dx+c)+a|)}{a^4b^3+2\,a^2b^5+b^7} + \frac{2\,C}{}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="giac")
```

output
```
1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^
2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(
2*C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3)*log(abs(b*tan(d*x + c) + a)
)/(a^4*b^3 + 2*a^2*b^5 + b^7) + 2*C*tan(d*x + c)/b^2 + 2*(2*C*a^5*b*tan(d*
x + c) - B*a^4*b^2*tan(d*x + c) + 4*C*a^3*b^3*tan(d*x + c) - 3*B*a^2*b^4*t
an(d*x + c) + C*a^6 + 3*C*a^4*b^2 - 2*B*a^3*b^3)/((a^4*b^3 + 2*a^2*b^5 + b
^7)*(b*tan(d*x + c) + a)))/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{C\tan(c+d\,x)}{b^2\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(2\,C\,a^5-B\,a^4\,b+4\,C\,a^3\,b^2-3\,B\,a^2\,b^3\right)}{d\,\left(a^4\,b^3+2\,a^2\,b^5+b^7\right)}$$
$$- \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$
$$- \frac{a^2\,\left(C\,a^2-B\,a\,b\right)}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)\,\left(a^2+b^2\right)}$$

input
```
int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^2,x)
```

output
```
(C*tan(c + d*x))/(b^2*d) - (log(a + b*tan(c + d*x))*(2*C*a^5 - 3*B*a^2*b^3
 + 4*C*a^3*b^2 - B*a^4*b))/(d*(b^7 + 2*a^2*b^5 + a^4*b^3)) - (log(tan(c +
d*x) - 1i)*(B + C*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i
)*(B*1i + C))/(2*d*(2*a*b + a^2*1i - b^2*1i)) - (a^2*(C*a^2 - B*a*b))/(b*d
*(a*b^2 + b^3*tan(c + d*x))*(a^2 + b^2))
```

3.32. $\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.33 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.33.1 Optimal result

Integrand size = 38, antiderivative size = 157

$$\begin{aligned}\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx &= -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}-\frac{(2abB-a^2C+b^2C)\log(\cos(c+dx))}{(a^2+b^2)^2\,d}\\ &\quad -\frac{a(2b^3B-a^3C-3ab^2C)\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)^2\,d}-\frac{a^2(bB-aC)}{b^2\,(a^2+b^2)\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2-(2*B*a*b-C*a^2+C*b^2)*ln(cos(d*x+c))/
(a^2+b^2)^2/d-a*(2*B*b^3-C*a^3-3*C*a*b^2)*ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)
^2/d-a^2*(B*b-C*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.33.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{i(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2a\left(\left(-2bB+3aC+\frac{a^3C}{b^2}\right)\log(a+b\tan(c+dx))+\frac{a\left(a^2+b^2\right)(-bB+aC)}{b^2(a+b\tan(c+dx))}\right)}{\left(a^2+b^2\right)^2}}{2d}$$

input
```
Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x])^2,x]
```

output
```
((I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*(B - I*C)*Log[I + Ta
n[c + d*x]])/(a - I*b)^2 + (2*a*((-2*b*B + 3*a*C + (a^3*C)/b^2)*Log[a + b*
Tan[c + d*x]] + (a*(a^2 + b^2)*(-(b*B) + a*C))/(b^2*(a + b*Tan[c + d*x])))
)/(a^2 + b^2)^2)/(2*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4115, 3042, 4087, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{\int -\frac{-((a^2+b^2)C\tan^2(c+dx))-b(bB-aC)\tan(c+dx)+a(bB-aC)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-((a^2+b^2)C\tan^2(c+dx))-b(bB-aC)\tan(c+dx)+a(bB-aC)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-((a^2+b^2)C\tan(c+dx)^2)-b(bB-aC)\tan(c+dx)+a(bB-aC)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 4109

$$-\frac{-\frac{b(a^2(-C)+2abB+b^2C)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a(a^3(-C)-3ab^2C+2b^3B)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx(a^2B+2abC-b^2B)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{b(a^2(-C)+2abB+b^2C)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a(a^3(-C)-3ab^2C+2b^3B)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx(a^2B+2abC-b^2B)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 3956

$$-\frac{\frac{a(a^3(-C)-3ab^2C+2b^3B)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b(a^2(-C)+2abB+b^2C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2B+2abC-b^2B)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 4100

$$-\frac{\frac{a(a^3(-C)-3ab^2C+2b^3B)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(a^2(-C)+2abB+b^2C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2B+2abC-b^2B)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 16

$$-\frac{a^2(bB-aC)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{b(a^2(-C)+2abB+b^2C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2B+2abC-b^2B)}{a^2+b^2}+\frac{a(a^3(-C)-3ab^2C+2b^3B)\log(a+b\tan(c+dx))}{bd(a^2+b^2)}}{b(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
)^2,x]
```

3.33. $\int \frac{\tan(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}dx$

output
```
-(((b*(a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2) + (b*(2*a*b*B - a^2*C + b^2
*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a*(2*b^3*B - a^3*C - 3*a*b^2*C)*
Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/(b*(a^2 + b^2))) - (a^2*(b*B -
 a*C))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

3.33.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

3.33. $\int \frac{\tan(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\frac{(2Bab-C\,a^2+C\,b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-B\,a^2+B\,b^2-2Cab\right)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{a^2(Bb-Ca)}{b^2(a^2+b^2)(a+b\tan(dx+c))}-\frac{a\left(2B\,b^3-C\,a^3\right.}{}}{d}$
default	$\frac{\frac{\frac{(2Bab-C\,a^2+C\,b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-B\,a^2+B\,b^2-2Cab\right)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{a^2(Bb-Ca)}{b^2(a^2+b^2)(a+b\tan(dx+c))}-\frac{a\left(2B\,b^3-C\,a^3\right.}{}}{d}$
norman	$\frac{-\frac{a(B\,a^2-B\,b^2+2Cab)x}{a^4+2a^2b^2+b^4}-\frac{b(B\,a^2-B\,b^2+2Cab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(Bab-C\,a^2)a}{d\,b^2(a^2+b^2)}}{a+b\tan(dx+c)}+\frac{(2Bab-C\,a^2+C\,b^2)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^4+2a^2b^2+b^4)}-$
parallelrisch	$2C\,a^5-2B\,a^4b+2B\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)a\,b^4-4B\ln(a+b\tan(dx+c))\tan(dx+c)a\,b^4-C\ln\left(1+\tan(dx+c)^2\right)\tan$
risch	$\frac{xB}{2iba-a^2+b^2}+\frac{4iabBx}{a^4+2a^2b^2+b^4}-\frac{6ia^2Cx}{a^4+2a^2b^2+b^4}+\frac{2iCc}{b^2d}+\frac{2ia^2B}{(ib+a)d(-ib+a)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}-$

input
```
int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_
RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(2*B*a*b-C*a^2+C*b^2)*ln(1+tan(d*x+c)^2)+(-B*a^2+B
*b^2-2*C*a*b)*arctan(tan(d*x+c)))-a^2*(B*b-C*a)/b^2/(a^2+b^2)/(a+b*tan(d*x
+c))-a*(2*B*b^3-C*a^3-3*C*a*b^2)/(a^2+b^2)^2/b^2*ln(a+b*tan(d*x+c)))
```

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(153) = 306$.

Time = 0.29 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.98

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,Ca^3b^2 - 2\,Ba^2b^3 - 2\,(Ba^3b^2 + 2\,Ca^2b^3 - Bab^4)dx + (Ca^5 + 3\,Ca^3b^2 - 2\,Ba^2b^3 + (Ca^4b + 3\,Ca^2b^3 - 2\ldots}{}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="fricas")
```

output
```
1/2*(2*C*a^3*b^2 - 2*B*a^2*b^3 - 2*(B*a^3*b^2 + 2*C*a^2*b^3 - B*a*b^4)*d*x
 + (C*a^5 + 3*C*a^3*b^2 - 2*B*a^2*b^3 + (C*a^4*b + 3*C*a^2*b^3 - 2*B*a*b^4
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) - (C*a^5 + 2*C*a^3*b^2 + C*a*b^4 + (C*a^4*b + 2*C*a^2*b^3
+ C*b^5)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(C*a^4*b - B*a^3*b^
2 + (B*a^2*b^3 + 2*C*a*b^4 - B*b^5)*d*x)*tan(d*x + c))/((a^4*b^3 + 2*a^2*b
^5 + b^7)*d*tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d)
```

3.33.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 3497, normalized size of antiderivative = 22.27

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq
(d, 0)), ((-B*x + B*tan(c + d*x)/d - C*log(tan(c + d*x)**2 + 1)/(2*d) + C*
tan(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (B*d*x*tan(c + d*x)**2/(4*b**2*d*t
an(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B*d*x*tan(c + d
*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*d*
x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*B*ta
n(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
 + 2*I*B/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
 3*I*C*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c +
d*x) - 4*b**2*d) + 6*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b*
*2*d*tan(c + d*x) - 4*b**2*d) - 3*I*C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*
b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x
)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I
*C*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) - 2*C*log(tan(c + d*x)**2 + 1)/(4*b**2*d*ta
n(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*C*tan(c + d*x)/(
4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*C/(4*b*
*2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)),
(B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) + 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b...
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^4+3\,Ca^2b^2-2\,Bab^3)\log(b\tan(dx+c)+a)}{a^4b^2+2\,a^2b^4+b^6}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2}{a^3b^2+ab^4}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C
*a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^2 + 2*a^2*b
^4 + b^6) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2
*b^2 + b^4) - 2*(C*a^3 - B*a^2*b)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d
*x + c)))/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.55

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ca^4+3\,Ca^2b^2-2\,Bab^3)\log(|b\tan(dx+c)+a|)}{a^4b^2+2\,a^2b^4+b^6}+\frac{2\,(Ca^4\tan}{}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="giac")
```

output
```
-1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a
^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*
(C*a^4 + 3*C*a^2*b^2 - 2*B*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b^2 +
2*a^2*b^4 + b^6) + 2*(C*a^4*tan(d*x + c) + 3*C*a^2*b^2*tan(d*x + c) - 2*B*
a*b^3*tan(d*x + c) + B*a^4 + 2*C*a^3*b - B*a^2*b^2)/((a^4*b + 2*a^2*b^3 +
b^5)*(b*tan(d*x + c) + a)))/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$
$$-\frac{a^2\,\left(B\,b-C\,a\right)}{b^2\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)}$$
$$+\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^3+3\,C\,a\,b^2-2\,B\,b^3\right)}{b^2\,d\,{\left(a^2+b^2\right)}^2}$$

input
```
int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(a*b*2i - a^2 + b^2)) + (log(tan(
c + d*x) - 1i)*(B + C*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2*(B*b - C
*a))/(b^2*d*(a^2 + b^2)*(a + b*tan(c + d*x))) + (a*log(a + b*tan(c + d*x))
*(C*a^3 - 2*B*b^3 + 3*C*a*b^2))/(b^2*d*(a^2 + b^2)^2)
```

3.34 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.34.1 Optimal result

Integrand size = 32, antiderivative size = 115

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}-\frac{(a^2B-b^2B+2abC)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} + \frac{a(bB-aC)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2-(B*a^2-B*b^2+2*C*a*b)*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^2/d+a*(B*b-C*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.34.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2\left((-a^2B+b^2B-2abC)\log(a+b\tan(c+dx))-\frac{a(a^2+b^2)(-bB+aC)}{b(a+b\tan(c+dx))}\right)}{(a^2+b^2)^2}}{2d}$$

input
```
Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

output
```
(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((B - I*C)*Log[I + Tan[c
+ d*x]])/(a - I*b)^2 + (2*((-(a^2*B) + b^2*B - 2*a*b*C)*Log[a + b*Tan[c +
d*x]] - (a*(a^2 + b^2)*(-(b*B) + a*C))/(b*(a + b*Tan[c + d*x]))))/(a^2 + b
^2)^2)/(2*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4111, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{x\left(a^2(-C)+2abB+b^2C\right)}{a^2+b^2}-\frac{\left(a^2B+2abC-b^2B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x\left(a^2(-C)+2abB+b^2C\right)}{a^2+b^2}-\frac{\left(a^2B+2abC-b^2B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a(bB-aC)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{a(bB-aC)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}+\frac{\frac{x\left(a^2(-C)+2abB+b^2C\right)}{a^2+b^2}-\frac{\left(a^2B+2abC-b^2B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}}{a^2+b^2}$$

input
```
Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2,x]
```

output
```
(((2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2) - ((a^2*B - b^2*B + 2*a*b*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) + (a*(b*B
 - a*C))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.34.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{(B a^2-B b^2+2Cab)\ln(1+\tan(dx+c)^2)}{2}+(2Bab-C a^2+C b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a(Bb-Ca)}{(a^2+b^2)b(a+b\tan(dx+c))}-\frac{(B a^2-B b^2+2Ca}{(a^2}}{d}$
default	$\frac{\frac{\frac{(B a^2-B b^2+2Cab)\ln(1+\tan(dx+c)^2)}{2}+(2Bab-C a^2+C b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a(Bb-Ca)}{(a^2+b^2)b(a+b\tan(dx+c))}-\frac{(B a^2-B b^2+2Ca}{(a^2}}{d}$
norman	$\frac{\frac{a(2Bab-C a^2+C b^2)x}{a^4+2a^2b^2+b^4}+\frac{b(2Bab-C a^2+C b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{a(Bb-Ca)}{(a^2+b^2)bd}}{a+b\tan(dx+c)}+\frac{(B a^2-B b^2+2Cab)\ln(1+\tan(dx+c)^2)}{2d(a^4+2a^2b^2+b^4)}-(B$
parallelrisch	$2B a^3b+2Ba b^3-2C a^2b^2-2C a^4+2Cxa b^3d+4Bx\tan(dx+c)a b^3d-2Cx\tan(dx+c)a^2b^2d-B\ln(1+\tan(dx+c)^2)a b^3-$
risch	$\frac{ixB}{2iba-a^2+b^2}+\frac{xC}{2iba-a^2+b^2}+\frac{2ia^2Bx}{a^4+2a^2b^2+b^4}-\frac{2iB b^2x}{a^4+2a^2b^2+b^4}+\frac{4iCabx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Bc}{(a^4+2a^2b^2+b^4)d}-\frac{2i}{d(a^4+2}$

input
```
int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(B*a^2-B*b^2+2*C*a*b)*ln(1+tan(d*x+c)^2)+(2*B*a*b-
C*a^2+C*b^2)*arctan(tan(d*x+c)))+a*(B*b-C*a)/(a^2+b^2)/b/(a+b*tan(d*x+c))-
(B*a^2-B*b^2+2*C*a*b)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,Ca^2b-2\,Bab^2+2\,(Ca^3-2\,Ba^2b-Cab^2)dx+(Ba^3+2\,Ca^2b-Bab^2+(Ba^2b+2\,Cab^2-Bb^3)\tan}{2\,((a^4b+2\,a^2b^3+b^5)d\tan(dx}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/2*(2*C*a^2*b - 2*B*a*b^2 + 2*(C*a^3 - 2*B*a^2*b - C*a*b^2)*d*x + (B*a^3
 + 2*C*a^2*b - B*a*b^2 + (B*a^2*b + 2*C*a*b^2 - B*b^3)*tan(d*x + c))*log((
b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(
C*a^3 - B*a^2*b - (C*a^2*b - 2*B*a*b^2 - C*b^3)*d*x)*tan(d*x + c))/((a^4*b
 + 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

3.34.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 2995, normalized size of antiderivative = 26.04

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)/tan(c)**2, Eq(a, 0) & Eq(b, 0) &
 Eq(d, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - C*x + C*tan(c + d*x)/d)/a
**2, Eq(b, 0)), (I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*
x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*d*x/(4*b**2*d*tan(c + d*
x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*tan(c + d*x)/(4*b**2*d*t
an(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + C*d*x*tan(c + d*x)*
*2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*C
*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*
b**2*d) - C*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b*
*2*d) - 3*C*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) + 2*I*C/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d), Eq(a, -I*b)), (-I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*
x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2
*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*d*x/(4*b**2
*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*tan(c + d*x
)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + C*d*x*
tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b*
*2*d) + 2*I*C*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(
c + d*x) - 4*b**2*d) - C*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan...
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^2+2\,Cab-Bb^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ca^2}{a^3b+ab^3+(a^2b^2}}{2\,d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B
*a^2 + 2*C*a*b - B*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) -
(B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4)
+ 2*(C*a^2 - B*a*b)/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)))/d
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(117) = 234.

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.10

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba^2b+2\,Cab^2-Bb^3)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{2\,(Ba^2b^2\tan}{}}{2\,d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a
^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*
(B*a^2*b + 2*C*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*
b^3 + b^5) - 2*(B*a^2*b^2*tan(d*x + c) + 2*C*a*b^3*tan(d*x + c) - B*b^4*ta
n(d*x + c) - C*a^4 + 2*B*a^3*b + C*a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*
tan(d*x + c) + a)))/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{a\,(B\,b-C\,a)}{b\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{B}{a^2+b^2}-\frac{2\,b\,(B\,b-C\,a)}{\left(a^2+b^2\right)^2}\right)}{d}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(a +
b*tan(c + d*x))*(B/(a^2 + b^2) - (2*b*(B*b - C*a))/(a^2 + b^2)^2))/d + (lo
g(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(2*a*b + a^2*1i - b^2*1i)) + (a*(B*b
 - C*a))/(b*d*(a^2 + b^2)*(a + b*tan(c + d*x)))
```

3.35 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.35.1 Optimal result

Integrand size = 38, antiderivative size = 111

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}+\frac{(2abB-a^2C+b^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{bB-aC}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2+(2*B*a*b-C*a^2+C*b^2)*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^2/d+(-B*b+C*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.35.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{C((-ia-b)\log(i-\tan(c+dx))+i(a+ib)\log(i+\tan(c+dx))+2b\log(a+b\tan(c+dx)))}{a^2+b^2}-(bB-aC)\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+}{(a-ib)^2}\right.}{2bd}$$

input
```
Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x])^2,x]
```

output
```
((C*(((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]
] + 2*b*Log[a + b*Tan[c + d*x]]))/(a^2 + b^2) - (b*B - a*C)*((I*Log[I - Ta
n[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(-
2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d*x])))/(a^2 + b^
2)^2))/(2*b*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4115, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{bB-aC}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{bB-aC}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{(a^2(-C)+2abB+b^2C)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2B+2abC-b^2B)}{a^2+b^2}}{a^2+b^2}-\frac{bB-aC}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2(-C)+2abB+b^2C)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2B+2abC-b^2B)}{a^2+b^2}}{a^2+b^2}-\frac{bB-aC}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{\frac{(a^2(-C)+2abB+b^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{x(a^2B+2abC-b^2B)}{a^2+b^2}}{a^2+b^2}-\frac{bB-aC}{d(a^2+b^2)(a+b\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
)^2,x]
```

output
```
(((a^2*B - b^2*B + 2*a*b*C)*x)/(a^2 + b^2) + ((2*a*b*B - a^2*C + b^2*C)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) - (b*B -
a*C)/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{\left(-2Bab+C\,a^2-C\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(B\,a^2-B\,b^2+2Cab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{Bb-Ca}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(2Bab-C\,a^2+C\,b\right.}{\left(a^2\right.}}{d}$
default	$\frac{\frac{\frac{\left(-2Bab+C\,a^2-C\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(B\,a^2-B\,b^2+2Cab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{Bb-Ca}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(2Bab-C\,a^2+C\,b\right.}{\left(a^2\right.}}{d}$
parallelrisch	$\frac{2a\left(Bab-\frac{1}{2}C\,a^2+\frac{1}{2}C\,b^2\right)(a+b\tan(dx+c))\ln(a+b\tan(dx+c))-a\left(Bab-\frac{1}{2}C\,a^2+\frac{1}{2}C\,b^2\right)(a+b\tan(dx+c))\ln\left(\sec(dx+c)^2\right.}{(a+b\tan(dx+c))da(a^2+b^2}$
norman	$\frac{\frac{a\left(B\,a^2-B\,b^2+2Cab\right)x}{a^4+2a^2b^2+b^4}+\frac{b\left(B\,a^2-B\,b^2+2Cab\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{(Bb-Ca)b\tan(dx+c)}{ad\left(a^2+b^2\right)}}{a+b\tan(dx+c)}+\frac{\left(2Bab-C\,a^2+C\,b^2\right)\ln(a+b\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}$
risch	$-\frac{xB}{2iba-a^2+b^2}+\frac{ixC}{2iba-a^2+b^2}-\frac{4iabBx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Cx}{a^4+2a^2b^2+b^4}-\frac{2iC\,b^2x}{a^4+2a^2b^2+b^4}-\frac{4iabBc}{d(a^4+2a^2b^2+b^4)}+\frac{}{d(a^4}$

input
```
int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method=_
RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(-2*B*a*b+C*a^2-C*b^2)*ln(1+tan(d*x+c)^2)+(B*a^2-B
*b^2+2*C*a*b)*arctan(tan(d*x+c)))-(B*b-C*a)/(a^2+b^2)/(a+b*tan(d*x+c))+(2*
B*a*b-C*a^2+C*b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.00

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,Cab^2 - 2\,Bb^3 + 2\,(Ba^3 + 2\,Ca^2b - Bab^2)dx - (Ca^3 - 2\,Ba^2b - Cab^2 + (Ca^2b - 2\,Bab^2 - Cb^3)\tan(dx}{2\,((a^4b + 2\,a^2b^3 + b^5)d\tan(dx +}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="fricas")
```

output
```
1/2*(2*C*a*b^2 - 2*B*b^3 + 2*(B*a^3 + 2*C*a^2*b - B*a*b^2)*d*x - (C*a^3 -
2*B*a^2*b - C*a*b^2 + (C*a^2*b - 2*B*a*b^2 - C*b^3)*tan(d*x + c))*log((b^2
*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a
^2*b - B*a*b^2 - (B*a^2*b + 2*C*a*b^2 - B*b^3)*d*x)*tan(d*x + c))/((a^4*b
+ 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

3.35.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 2895, normalized size of antiderivative = 26.08

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)/tan(c)**2, Eq(a, 0) & Eq(
b, 0) & Eq(d, 0)), ((B*x + C*log(tan(c + d*x)**2 + 1)/(2*d))/a**2, Eq(b, 0
)), (-B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c +
 d*x) - 4*b**2*d) + 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I
*b**2*d*tan(c + d*x) - 4*b**2*d) + B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) - B*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2
- 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*B/(4*b**2*d*tan(c + d*x)**2 -
8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*C*d*x*tan(c + d*x)**2/(4*b**2*d*ta
n(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*d*x*tan(c + d*x)
/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*C*d*x
/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*C*tan
(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d),
 Eq(a, -I*b)), (-B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**
2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*
x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + B*d*x/(4*b**2*d*tan(c + d*x)
**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*tan(c + d*x)/(4*b**2*d*tan(c
 + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B/(4*b**2*d*tan(c +
 d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*C*d*x*tan(c + d*x)**2/(
4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*C*d*x*t
an(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**...
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^2-2\,Bab-Cb^2\right)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ca-Bb}{a^3+ab^2+(a^2b+b^3)}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="maxima")
```

output
```
1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(C*
a^2 - 2*B*a*b - C*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) + (
C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
 2*(C*a - B*b)/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(111) = 222$.

Time = 0.87 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.11

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\left(Ca^2b-2\,Bab^2-Cb^3\right)\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}+\frac{2\left(Ca^2b\tan(dx-\right.}{}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x, a
lgorithm="giac")
```

output
```
1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a^
2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(
C*a^2*b - 2*B*a*b^2 - C*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b
^3 + b^5) + 2*(C*a^2*b*tan(d*x + c) - 2*B*a*b^2*tan(d*x + c) - C*b^3*tan(d
*x + c) + 2*C*a^3 - 3*B*a^2*b - B*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x
 + c) + a)))/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(-C\,a^2+2\,B\,a\,b+C\,b^2\right)}{d\,\left(a^2+b^2\right)^2}-\frac{B\,b-C\,a}{d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
)^2,x)
```

output
```
(log(a + b*tan(c + d*x))*(C*b^2 - C*a^2 + 2*B*a*b))/(d*(a^2 + b^2)^2) - (B
*b - C*a)/(d*(a^2 + b^2)*(a + b*tan(c + d*x))) - (log(tan(c + d*x) + 1i)*(
B*1i + C))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(c + d*x) - 1i)*(B + C*1i)
)/(2*d*(2*a*b - a^2*1i + b^2*1i))
```

3.36 $\int \frac{\cot^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

3.36.1 Optimal result

Integrand size = 40, antiderivative size = 137

$$\begin{aligned}\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx\\ = -\frac{(2abB-a^2C+b^2C)\,x}{(a^2+b^2)^2}+\frac{B\log(\sin(c+dx))}{a^2d}\\ -\frac{b(3a^2bB+b^3B-2a^3C)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\left(a^2+b^2\right)^2d}\\ +\frac{b(bB-aC)}{a\left(a^2+b^2\right)d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(2*B*a*b-C*a^2+C*b^2)*x/(a^2+b^2)^2+B*ln(sin(d*x+c))/a^2/d-b*(3*B*a^2*b+B
*b^3-2*C*a^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)^2/d+b*(B*b-C*a)/
a/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.36.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{2B\log(\tan(c+dx))}{a^2}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b(3a^2bB+b^3B-2a^3C)\log(a+b\tan(c+dx))}{a^2(a^2+b^2)^2}+\frac{\cdots}{a(a^2+\cdots}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^2,x]
```

output
```
-1/2*(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (2*B*Log[Tan[c + d*x
]])/a^2 + ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(3*a^2*b*B
+ b^3*B - 2*a^3*C)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)^2) + (2*b*(-(
b*B) + a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

3.36.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4115, 3042, 4092, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^2(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{\cot(c+dx)\left(b(bB-aC)\tan^2(c+dx)-a(bB-aC)\tan(c+dx)+\left(a^2+b^2\right)B\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(bB-aC)\tan(c+dx)^2-a(bB-aC)\tan(c+dx)+\left(a^2+b^2\right)B}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4134}$$

$$\frac{\frac{B(a^2+b^2)\int \cot(c+dx)dx}{a}-\frac{b(-2a^3C+3a^2bB+b^3B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-C)+2abB+b^2C)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(bB-aC)}{ad(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{B(a^2+b^2)\int -\tan(c+dx+\frac{\pi}{2})dx}{a}-\frac{b(-2a^3C+3a^2bB+b^3B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-C)+2abB+b^2C)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(bB-aC)}{ad(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{B(a^2+b^2)\int \tan(\frac{1}{2}(2c+\pi)+dx)dx}{a}-\frac{b(-2a^3C+3a^2bB+b^3B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-C)+2abB+b^2C)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(bB-aC)}{ad(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3956}$$

$$\frac{-\frac{b(-2a^3C+3a^2bB+b^3B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{B(a^2+b^2)\log(-\sin(c+dx))}{ad}-\frac{ax(a^2(-C)+2abB+b^2C)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(bB-aC)}{ad(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{b(bB-aC)}{ad(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{B(a^2+b^2)\log(-\sin(c+dx))}{ad}-\frac{ax(a^2(-C)+2abB+b^2C)}{a^2+b^2}-\frac{b(-2a^3C+3a^2bB+b^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}}{a(a^2+b^2)}$$

input
```
Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^2,x]
```

output
```
(-((a*(2*a*b*B - a^2*C + b^2*C)*x)/(a^2 + b^2)) + ((a^2 + b^2)*B*Log[-Sin[
c + d*x]])/(a*d) - (b*(3*a^2*b*B + b^3*B - 2*a^3*C)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(b*B - a*C))/(a*(a
^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

3.36. $\int \frac{\cot^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.36. $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.36.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{B\ln(\tan(dx+c))}{a^2}+\frac{\frac{(-B a^2+B b^2-2Cab)\ln(1+\tan(dx+c)^2)}{2}+(-2Bab+C a^2-C b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{b(3B a^2b+B b^3-2C a^3)\,\mathrm{l}...}{(a^2+b^2)^2...}}{d}$
default	$\frac{\frac{B\ln(\tan(dx+c))}{a^2}+\frac{\frac{(-B a^2+B b^2-2Cab)\ln(1+\tan(dx+c)^2)}{2}+(-2Bab+C a^2-C b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{b(3B a^2b+B b^3-2C a^3)\,\mathrm{l}...}{(a^2+b^2)^2...}}{d}$
parallelrisch	$-6(B a^2b+\frac{1}{3}B b^3-\frac{2}{3}C a^3)b(a+b\tan(dx+c))\ln(a+b\tan(dx+c))-a^2(a+b\tan(dx+c))(B a^2-B b^2+2Cab)\ln(\sec(dx+c)...)$
norman	$\frac{-\frac{a(2Bab-C a^2+C b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{b(2Bab-C a^2+C b^2)x\tan(dx+c)^2}{a^4+2a^2b^2+b^4}-\frac{(B b^2-Cab)b\tan(dx+c)^2}{d a^2(a^2+b^2)}}{\tan(dx+c)(a+b\tan(dx+c))}+\frac{B\ln(\tan(dx+c))}{a^2d}-...$
risch	$-\frac{4iCabx}{a^4+2a^2b^2+b^4}-\frac{xC}{2iba-a^2+b^2}-\frac{ixB}{2iba-a^2+b^2}+\frac{2ib^4Bx}{(a^4+2a^2b^2+b^4)a^2}+\frac{2ib^4Bc}{(a^4+2a^2b^2+b^4)a^2d}-\frac{2iBc}{a^2d}-\frac{4i...}{d(a^4+...)}$

input
```
int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*B*ln(tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(-B*a^2+B*b^2-2*C*a*b)*ln(1
+tan(d*x+c)^2)+(-2*B*a*b+C*a^2-C*b^2)*arctan(tan(d*x+c)))-b*(3*B*a^2*b+B*b
^3-2*C*a^3)/(a^2+b^2)^2/a^2*ln(a+b*tan(d*x+c))+(B*b-C*a)*b/(a^2+b^2)/a/(a+
b*tan(d*x+c)))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(137) = 274$.

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.36

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,Ca^2b^3 - 2\,Bab^4 - 2\,(Ca^5 - 2\,Ba^4b - Ca^3b^2)dx - (Ba^5 + 2\,Ba^3b^2 + Bab^4 + (Ba^4b + 2\,Ba^2b^3 + Bb^5)}{}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="fricas")
```

output
```
-1/2*(2*C*a^2*b^3 - 2*B*a*b^4 - 2*(C*a^5 - 2*B*a^4*b - C*a^3*b^2)*d*x - (B
*a^5 + 2*B*a^3*b^2 + B*a*b^4 + (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*tan(d*x + c
))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - (2*C*a^4*b - 3*B*a^3*b^2 - B
*a*b^4 + (2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))*log((b^2*tan(d*
x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^3*b^2
- B*a^2*b^3 + (C*a^4*b - 2*B*a^3*b^2 - C*a^2*b^3)*d*x)*tan(d*x + c))/((a^6
*b + 2*a^4*b^3 + a^2*b^5)*d*tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)*d)
```

3.36.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 4502, normalized size of antiderivative = 32.86

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2
,x)
```

output
```
Piecewise((zoo*x*(B*tan(c) + C*tan(c)**2)*cot(c)**2/tan(c)**2, Eq(a, 0) &
Eq(b, 0) & Eq(d, 0)), ((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c +
d*x))/d + C*x)/a**2, Eq(b, 0)), ((B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log
(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2) - C*x - C/(d*tan(c + d*x)))/b**
2, Eq(a, 0)), (3*I*B*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a
**2*d*tan(c + d*x) - 4*a**2*d) - 6*B*d*x*tan(c + d*x)/(4*a**2*d*tan(c + d*
x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 3*I*B*d*x/(4*a**2*d*tan(c +
d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 2*B*log(tan(c + d*x)**2 +
1)*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4
*a**2*d) - 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*a**2*d*tan(c + d
*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + 2*B*log(tan(c + d*x)**2 + 1
)/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + 4*B*lo
g(tan(c + d*x))*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan
(c + d*x) - 4*a**2*d) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(4*a**2*d*tan
(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 4*B*log(tan(c + d*x))
/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + 3*I*B*t
an(c + d*x)/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d
) - 4*B/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) +
C*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x)
- 4*a**2*d) + 2*I*C*d*x*tan(c + d*x)/(4*a**2*d*tan(c + d*x)**2 + 8*I*a*...
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.52

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{\frac{2\,(Ca^2-2\,Bab-Cb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(2\,Ca^3b-3\,Ba^2b^2-Bb^4)\log(b\tan(dx+c)+a)}{a^6+2\,a^4b^2+a^2b^4}-\frac{(Ba^2+2\,Cab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(C\cdots}{a^4+a^2b^2+(a^3\cdots}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="maxima")
```

output
```
1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*
C*a^3*b - 3*B*a^2*b^2 - B*b^4)*log(b*tan(d*x + c) + a)/(a^6 + 2*a^4*b^2 +
a^2*b^4) - (B*a^2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*
b^2 + b^4) - 2*(C*a*b - B*b^2)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x +
c)) + 2*B*log(tan(d*x + c))/a^2)/d
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(137) = 274$.

Time = 1.18 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.04

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\left(Ca^2-2\,Bab-Cb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{\left(Ba^2+2\,Cab-Bb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\left(2\,Ca^3b^2-3\,Ba^2b^3-Bb^5\right)\log(|b\tan(dx+c)+a|)}{a^6b+2\,a^4b^3+a^2b^5} + \frac{2\,B\log(|\tan}{a^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="giac")
```

output
```
1/2*(2*(C*a^2 - 2*B*a*b - C*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - (B*a^
2 + 2*C*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(
2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 2
*a^4*b^3 + a^2*b^5) + 2*B*log(abs(tan(d*x + c)))/a^2 - 2*(2*C*a^3*b^2*tan(
d*x + c) - 3*B*a^2*b^3*tan(d*x + c) - B*b^5*tan(d*x + c) + 3*C*a^4*b - 4*B
*a^3*b^2 + C*a^2*b^3 - 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a^2*b^4)*(b*tan(d*x
+ c) + a)))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx &= \frac{B\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)} \\ &\quad - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)} + \frac{B\,b^2-C\,a\,b}{a\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)} \\ &\quad - \frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-2\,C\,a^3+3\,B\,a^2\,b+B\,b^3\right)}{a^2\,d\,{\left(a^2+b^2\right)}^2}\end{aligned}$$

input
```
int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^2,x)
```

output
```
(B*log(tan(c + d*x)))/(a^2*d) - (log(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(
a*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(2*a*b + a
^2*1i - b^2*1i)) + (B*b^2 - C*a*b)/(a*d*(a^2 + b^2)*(a + b*tan(c + d*x)))
- (b*log(a + b*tan(c + d*x))*(B*b^3 - 2*C*a^3 + 3*B*a^2*b))/(a^2*d*(a^2 +
b^2)^2)
```

3.36. $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.37 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.37.1 Optimal result

Integrand size = 40, antiderivative size = 192

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2B-b^2B+2abC)\,x}{(a^2+b^2)^2}-\frac{(2bB-aC)\log(\sin(c+dx))}{a^3d}+\frac{b^2(4a^2bB+2b^3B-3a^3C-ab^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3(a^2+b^2)^2d}-\frac{b(a^2B+2b^2B-abC)}{a^2(a^2+b^2)\,d(a+b\tan(c+dx))}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

output
```
-(B*a^2-B*b^2+2*C*a*b)*x/(a^2+b^2)^2-(2*B*b-C*a)*ln(sin(d*x+c))/a^3/d+b^2*
(4*B*a^2*b+2*B*b^3-3*C*a^3-C*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2
+b^2)^2/d-b*(B*a^2+2*B*b^2-C*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))-B*cot(d
*x+c)/a/d/(a+b*tan(d*x+c))
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.81 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\frac{2B\cot(c+dx)}{a^2} + \frac{i(B+iC)\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{2(-2bB+aC)\log(\tan(c+dx))}{a^3} - \frac{(iB+C)\log(i+\tan(c+dx))}{(a-ib)^2} - \frac{2b^2(-4a^2bB-2b^3B+3a^3}{a^3(}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^2,x]
```

output
```
((-2*B*Cot[c + d*x])/a^2 + (I*(B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^2
 + (2*(-2*b*B + a*C)*Log[Tan[c + d*x]])/a^3 - ((I*B + C)*Log[I + Tan[c + d
*x]])/(a - I*b)^2 - (2*b^2*(-4*a^2*b*B - 2*b^3*B + 3*a^3*C + a*b^2*C)*Log[
a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)^2) + (2*b^2*(-(b*B) + a*C))/(a^2*(a^
2 + b^2)*(a + b*Tan[c + d*x])))/(2*d)
```

3.37.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4115, 3042, 4092, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^3(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \; 4115$$
$$\int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^2}dx$$

$\Big\downarrow$ 4092

$$-\frac{\int \frac{\cot(c+dx)(2bB\tan^2(c+dx)+aB\tan(c+dx)+2bB-aC)}{(a+b\tan(c+dx))^2}dx}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{2bB\tan(c+dx)^2+aB\tan(c+dx)+2bB-aC}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4132

$$-\frac{\frac{\int \frac{\cot(c+dx)\left((aB+bC)\tan(c+dx)a^2+b\left(Ba^2-bCa+2b^2B\right)\tan^2(c+dx)+\left(a^2+b^2\right)(2bB-aC)\right)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{(aB+bC)\tan(c+dx)a^2+b\left(Ba^2-bCa+2b^2B\right)\tan(c+dx)^2+\left(a^2+b^2\right)(2bB-aC)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\left(a^2+b^2\right)(2bB-aC)\int \cot(c+dx)dx}{a}-\frac{b^2\left(-3a^3C+4a^2bB-ab^2C+2b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2B+2abC-b^2B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^2+b^2\right)(2bB-aC)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(-3a^3C+4a^2bB-ab^2C+2b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2B+2abC-b^2B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 25

3.37. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx$

$$-\frac{\frac{-\frac{(a^2+b^2)(2bB-aC)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b^2\left(-3a^3C+4a^2bB-ab^2C+2b^3B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2B+2abC-b^2B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 3956$$

$$-\frac{\frac{-\frac{b^2\left(-3a^3C+4a^2bB-ab^2C+2b^3B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{(a^2+b^2)(2bB-aC)\log(-\sin(c+dx))}{ad}+\frac{a^2x\left(a^2B+2abC-b^2B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 4013$$

$$-\frac{\frac{b(a^2B-abC+2b^2B)}{ad(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{(a^2+b^2)(2bB-aC)\log(-\sin(c+dx))}{ad}+\frac{a^2x\left(a^2B+2abC-b^2B\right)}{a^2+b^2}-\frac{b^2\left(-3a^3C+4a^2bB-ab^2C+2b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}}{a(a^2+b^2)}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^2,x]
```

output
```
-((B*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x]))) - (((a^2*(a^2*B - b^2*B + 2
*a*b*C)*x)/(a^2 + b^2) + ((a^2 + b^2)*(2*b*B - a*C)*Log[-Sin[c + d*x]])/(a
*d) - (b^2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C)*Log[a*Cos[c + d*x] +
b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(a^2*B + 2*b^2*B
- a*b*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/a
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.37. $\int \frac{\cot^3(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{B}{a^2\tan(dx+c)}+\frac{(-2Bb+Ca)\ln(\tan(dx+c))}{a^3}+\frac{\frac{(2Bab-Ca^2+Cb^2)\ln(1+\tan(dx+c)^2)}{2}+(-Ba^2+Bb^2-2Cab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+}{d}$
default	$\frac{-\frac{B}{a^2\tan(dx+c)}+\frac{(-2Bb+Ca)\ln(\tan(dx+c))}{a^3}+\frac{\frac{(2Bab-Ca^2+Cb^2)\ln(1+\tan(dx+c)^2)}{2}+(-Ba^2+Bb^2-2Cab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+}{d}$
parallelrisch	$4b^2(a+b\tan(dx+c))\left(Ba^2b+\frac{1}{2}Bb^3-\frac{3}{4}Ca^3-\frac{1}{4}Cab^2\right)\ln(a+b\tan(dx+c))+a^3\left(Bab-\frac{1}{2}Ca^2+\frac{1}{2}Cb^2\right)(a+b\tan(dx+c))\ln$
norman	$\frac{\frac{(Ba^2b+2Bb^3-Cab^2)b\tan(dx+c)^3}{da^3(a^2+b^2)}-\frac{B\tan(dx+c)}{ad}-\frac{a(Ba^2-Bb^2+2Cab)x\tan(dx+c)^2}{a^4+2a^2b^2+b^4}-\frac{b(Ba^2-Bb^2+2Cab)x\tan(dx+c)^3}{a^4+2a^2b^2+b^4}}{\tan(dx+c)^2(a+b\tan(dx+c))}+$
risch	$\frac{xB}{2iba-a^2+b^2}-\frac{4ib^5Bc}{a^3d(a^4+2a^2b^2+b^4)}+\frac{2ib^4Cx}{a^2(a^4+2a^2b^2+b^4)}-\frac{4ib^5Bx}{a^3(a^4+2a^2b^2+b^4)}-\frac{ixC}{2iba-a^2+b^2}-\frac{2i(-2iBa^3b\mathrm{e}^{2}$

input
```
int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*B/tan(d*x+c)+(-2*B*b+C*a)/a^3*ln(tan(d*x+c))+1/(a^2+b^2)^2*(1/
2*(2*B*a*b-C*a^2+C*b^2)*ln(1+tan(d*x+c)^2)+(-B*a^2+B*b^2-2*C*a*b)*arctan(t
an(d*x+c)))+b^2*(4*B*a^2*b+2*B*b^3-3*C*a^3-C*a*b^2)/(a^2+b^2)^2/a^3*ln(a+b
*tan(d*x+c))-(B*b-C*a)*b^2/(a^2+b^2)/a^2/(a+b*tan(d*x+c)))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(190) = 380$.

Time = 0.32 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.42

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,Ba^6+4\,Ba^4b^2+2\,Ba^2b^4+2\,(Ca^3b^3-Ba^2b^4+(Ba^5b+2\,Ca^4b^2-Ba^3b^3)dx)\tan{(dx+c)}^2-((Ca^5$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="fricas")
```

output
```
-1/2*(2*B*a^6 + 4*B*a^4*b^2 + 2*B*a^2*b^4 + 2*(C*a^3*b^3 - B*a^2*b^4 + (B*
a^5*b + 2*C*a^4*b^2 - B*a^3*b^3)*d*x)*tan(d*x + c)^2 - ((C*a^5*b - 2*B*a^4
*b^2 + 2*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (C*
a^6 - 2*B*a^5*b + 2*C*a^4*b^2 - 4*B*a^3*b^3 + C*a^2*b^4 - 2*B*a*b^5)*tan(d
*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + ((3*C*a^3*b^3 - 4*B*a^
2*b^4 + C*a*b^5 - 2*B*b^6)*tan(d*x + c)^2 + (3*C*a^4*b^2 - 4*B*a^3*b^3 + C
*a^2*b^4 - 2*B*a*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*
x + c) + a^2)/(tan(d*x + c)^2 + 1)) + 2*(B*a^5*b + 2*B*a^3*b^3 - C*a^2*b^4
 + 2*B*a*b^5 + (B*a^6 + 2*C*a^5*b - B*a^4*b^2)*d*x)*tan(d*x + c))/((a^7*b
+ 2*a^5*b^3 + a^3*b^5)*d*tan(d*x + c)^2 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d*ta
n(d*x + c))
```

3.37.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.03 (sec) , antiderivative size = 8143, normalized size of antiderivative = 42.41

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**2
,x)
```

output
```
Piecewise(((-B*x - B/(d*tan(c + d*x)) - C*log(tan(c + d*x)**2 + 1)/(2*d) +
 C*log(tan(c + d*x))/d)/a**2, Eq(b, 0)), ((B*x + B/(d*tan(c + d*x)) - B/(3
*d*tan(c + d*x)**3) + C*log(tan(c + d*x)**2 + 1)/(2*d) - C*log(tan(c + d*x
))/d - C/(2*d*tan(c + d*x)**2))/b**2, Eq(a, 0)), (-9*B*d*x*tan(c + d*x)**3
/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c +
 d*x)) - 18*I*B*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d
*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) + 9*B*d*x*tan(c + d*x)/(4*a**2*d
*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) - 4
*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**3 +
8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) + 8*B*log(tan(c + d*x)
**2 + 1)*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*
x)**2 - 4*a**2*d*tan(c + d*x)) + 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*
x)/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c
 + d*x)) + 8*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)*
*3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) - 16*B*log(tan(c
+ d*x))*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x
)**2 - 4*a**2*d*tan(c + d*x)) - 8*I*B*log(tan(c + d*x))*tan(c + d*x)/(4*a*
*2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x))
 - 9*B*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)
**2 - 4*a**2*d*tan(c + d*x)) - 14*I*B*tan(c + d*x)/(4*a**2*d*tan(c + d*...
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.36

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\left(Ba^2+2\,Cab-Bb^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\left(3\,Ca^3b^2-4\,Ba^2b^3+Cab^4-2\,Bb^5\right)\log(b\tan(dx+c)+a)}{a^7+2\,a^5b^2+a^3b^4}+\frac{\left(Ca^2-2\,Bab-Cb^2\right)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2}{}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="maxima")
```

output
```
-1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3
*C*a^3*b^2 - 4*B*a^2*b^3 + C*a*b^4 - 2*B*b^5)*log(b*tan(d*x + c) + a)/(a^7
 + 2*a^5*b^2 + a^3*b^4) + (C*a^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1
)/(a^4 + 2*a^2*b^2 + b^4) + 2*(B*a^3 + B*a*b^2 + (B*a^2*b - C*a*b^2 + 2*B*
b^3)*tan(d*x + c))/((a^4*b + a^2*b^3)*tan(d*x + c)^2 + (a^5 + a^3*b^2)*tan
(d*x + c)) - 2*(C*a - 2*B*b)*log(tan(d*x + c))/a^3)/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.89

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{2\,(Ba^2+2\,Cab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{(Ca^2-2\,Bab-Cb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,Ca^3b^3-4\,Ba^2b^4+Cab^5-2\,Bb^6)\log(|b\tan(dx+c)+a|)}{a^7b+2\,a^5b^3+a^3b^5}+}{}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^2,x,
 algorithm="giac")
```

output
```
-1/2*(2*(B*a^2 + 2*C*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + (C*a
^2 - 2*B*a*b - C*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*
(3*C*a^3*b^3 - 4*B*a^2*b^4 + C*a*b^5 - 2*B*b^6)*log(abs(b*tan(d*x + c) + a
))/(a^7*b + 2*a^5*b^3 + a^3*b^5) + (C*a^4*b*tan(d*x + c)^2 - 2*B*a^3*b^2*t
an(d*x + c)^2 - C*a^2*b^3*tan(d*x + c)^2 + C*a^5*tan(d*x + c) - 3*C*a^3*b^
2*tan(d*x + c) + 6*B*a^2*b^3*tan(d*x + c) - 2*C*a*b^4*tan(d*x + c) + 4*B*b
^5*tan(d*x + c) + 2*B*a^5 + 4*B*a^3*b^2 + 2*B*a*b^4)/((a^6 + 2*a^4*b^2 + a
^2*b^4)*(b*tan(d*x + c)^2 + a*tan(d*x + c))) - 2*(C*a - 2*B*b)*log(abs(tan
(d*x + c)))/a^3)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20

$$\begin{aligned}
&\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx \\
&= \frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,C\,a^3+4\,B\,a^2\,b-C\,a\,b^2+2\,B\,b^3\right)}{a^3\,d\,{\left(a^2+b^2\right)}^2} \\
&\quad -\frac{\ln\left(\tan(c+d\,x)\right)\,\left(2\,B\,b-C\,a\right)}{a^3\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)} \\
&\quad +\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+C\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}-\frac{\frac{B}{a}+\frac{\tan(c+d\,x)\,\left(B\,a^2\,b-C\,a\,b^2+2\,B\,b^3\right)}{a^2\,\left(a^2+b^2\right)}}{d\,\left(b\tan\left(c+d\,x\right)^2+a\tan\left(c+d\,x\right)\right)}
\end{aligned}$$

input
```
int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(B*1i + C))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(
c + d*x))*(2*B*b - C*a))/(a^3*d) - (B/a + (tan(c + d*x)*(2*B*b^3 + B*a^2*b
 - C*a*b^2))/(a^2*(a^2 + b^2)))/(d*(a*tan(c + d*x) + b*tan(c + d*x)^2)) +
(log(tan(c + d*x) - 1i)*(B + C*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) + (b^2
*log(a + b*tan(c + d*x))*(2*B*b^3 - 3*C*a^3 + 4*B*a^2*b - C*a*b^2))/(a^3*d
*(a^2 + b^2)^2)
```

3.37. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}\,dx$

3.38 $\int \frac{\tan^3(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

3.38.1 Optimal result

Integrand size = 40, antiderivative size = 331

$$\begin{aligned}\int \frac{\tan^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= \frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3}+\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}\\ &+\frac{a^2(a^4bB+3a^2b^3B+6b^5B-3a^5C-9a^3b^2C-10ab^4C)\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)^3\,d}\\ &-\frac{(a^3bB+3ab^3B-3a^4C-6a^2b^2C-b^4C)\tan(c+dx)}{b^3\,(a^2+b^2)^2\,d}\\ &+\frac{a(bB-aC)\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{a(a^2bB+5b^3B-3a^3C-7ab^2C)\tan^2(c+dx)}{2b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3+(3*B*a^2*b-B*b^3-C*a^3+3*C
*a*b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a^2*(B*a^4*b+3*B*a^2*b^3+6*B*b^5-3*C*
a^5-9*C*a^3*b^2-10*C*a*b^4)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^3/d-(B*a^3*b+
3*B*a*b^3-3*C*a^4-6*C*a^2*b^2-C*b^4)*tan(d*x+c)/b^3/(a^2+b^2)^2/d+1/2*a*(B
*b-C*a)*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*(B*a^2*b+5*B*b
^3-3*C*a^3-7*C*a*b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.83

$$\int \frac{\tan^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(-ia+b)^3}+\frac{(B-iC)\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{2a^2(a^4bB+3a^2b^3B+6b^5B-3a^5C-9a^3b^2C-10ab^4C)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^3}+\frac{a}{b^4(}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^3,x]
```

output
```
(((B + I*C)*Log[I - Tan[c + d*x]])/((-I)*a + b)^3 + ((B - I*C)*Log[I + Tan
[c + d*x]])/(I*a + b)^3 + (2*a^2*(a^4*b*B + 3*a^2*b^3*B + 6*b^5*B - 3*a^5*
C - 9*a^3*b^2*C - 10*a*b^4*C)*Log[a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2)^3)
 + (a^3*(-(a*b*B) + 3*a^2*C + 2*b^2*C))/(b^4*(a^2 + b^2)*(a + b*Tan[c + d*
x])^2) + (2*C*Tan[c + d*x]^3)/(b*(a + b*Tan[c + d*x])^2) - (2*a^2*(-2*a^3*
b*B - 4*a*b^3*B + 6*a^4*C + 11*a^2*b^2*C + 3*b^4*C))/(b^4*(a^2 + b^2)^2*(a
 + b*Tan[c + d*x])))/(2*d)
```

3.38.3 Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.425$, Rules used = {3042, 4115, 3042, 4088, 25, 3042, 4128, 27, 3042, 4130, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^3\,(B\tan(c+dx)+C\tan(c+dx)^2)}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^4(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(c+dx)^4(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 4088$$

$$\frac{\int -\frac{\tan^2(c+dx)\left(\left(-3Ca^2+bBa-2b^2C\right)\tan^2(c+dx)-2b(bB-aC)\tan(c+dx)+3a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan^2(c+dx)\left(\left(-3Ca^2+bBa-2b^2C\right)\tan^2(c+dx)-2b(bB-aC)\tan(c+dx)+3a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)^2\left(\left(-3Ca^2+bBa-2b^2C\right)\tan(c+dx)^2-2b(bB-aC)\tan(c+dx)+3a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 4128$$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{2\tan(c+dx)\left(\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)b^2+\left(-3Ca^4+bBa^3-6b^2Ca^2+3b^3Ba-b^4C\right)\tan^2(c+dx)+a\left(-3Ca^3+bBa^2-7b^2Ca+5b^3B\right)\right)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a(-3a^3C+a^2 \ldots}{bd(a^2 \ldots}}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\int \frac{\tan(c+dx)\left(\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)b^2+\left(-3Ca^4+bBa^3-6b^2Ca^2+3b^3Ba-b^4C\right)\tan^2(c+dx)+a\left(-3Ca^3+bBa^2-7b^2Ca+5b^3B\right)\right)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a(-3a^3C+a^2 \ldots}{bd(a^2 \ldots}}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3042$$

3.38. $\int \frac{\tan^3(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\int\frac{\tan(c+dx)\left(\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)b^2+\left(-3Ca^4+bBa^3-6b^2Ca^2+3b^3Ba-b^4C\right)\tan(c+dx)^2+a\left(-3Ca^3+bBa^2-7b^2Ca+5b^3B\right)\right)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a(-3a^3C+a^2\cdots}{bd(a^2\cdots}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4130

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\int-\frac{-\left(\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(bB-3aC)\tan^2(c+dx)+a\left(-3Ca^4+bBa^3-6b^2Ca^2+3b^3Ba-b^4C\right)}{a+b\tan(c+dx)}dx}{b}+\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3\cdots\right.}{bd}\cdots\right.}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{\int\frac{-\left(\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(bB-3aC)\tan^2(c+dx)+a\left(-3Ca^4+bBa^3-6b^2Ca^2+\cdots\right.}{a+b\tan(c+dx)}}{b}\cdots\right.}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{\int\frac{-\left(\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(bB-3aC)\tan(c+dx)^2+a\left(-3Ca^4+bBa^3-6b^2Ca^2+\cdots\right.}{a+b\tan(c+dx)}}{b}\cdots\right.}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4109

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{-\frac{b^3\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-3a^5C+a^4bB-9a^3b^2C+3a^2b^3B-10ab^4C+6b\cdots\right.}{a^2+b^2}}{b}\cdots\right.}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

3.38. $\int\frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}dx$

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{-\frac{b^3\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-3a^5C+a^4bB-9a^3b^2C+3a^2b^3B-10ab^4C+6b\right.}{a^2+b^2}}{b}\right.}{b\left(a^2+b^2\right)}$$

$$2b\left(a^2+b^2\right)$$

$\downarrow$ 3956

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-3a^5C+a^4bB-9a^3b^2C+3a^2b^3B-10ab^4C+6b^5B\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^3\left(a^3(-C)+3a^2bB+3ab^2C\right.}{d\left(a^2+\right.}}{b}\right.}{b\left(a^2+b^2\right)}$$

$$2b\left(a^2+b^2\right)$$

$\downarrow$ 4100

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-3a^5C+a^4bB-9a^3b^2C+3a^2b^3B-10ab^4C+6b^5B\right)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}+\frac{b^3\left(a^3(-C)+3a\right.}{}}{b}\right.}{b\left(a^2+b^2\right)}$$

$$2b\left(a^2+b^2\right)$$

$\downarrow$ 16

$$\frac{a(bB-aC)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(\frac{\left(-3a^4C+a^3bB-6a^2b^2C+3ab^3B-b^4C\right)\tan(c+dx)}{bd}-\frac{\frac{b^3\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}+\frac{a^2\left(-3a^5C+a^4b\right.}{}}{b}\right.}{b\left(a^2+b^2\right)}$$

$$2b\left(a^2+b^2\right)$$

input

```
Int[(Tan[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^3,x]
```

3.38. $\int\frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

output
```
(a*(b*B - a*C)*Tan[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (-((a*(a^2*b*B + 5*b^3*B - 3*a^3*C - 7*a*b^2*C)*Tan[c + d*x]^2)/(b*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]))) + (2*(-(((b^3*(a^3*B - 3*a*b^2*B + 3*a^2*b
*C - b^3*C)*x)/(a^2 + b^2) + (b^3*(3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*
Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^2*(a^4*b*B + 3*a^2*b^3*B + 6*b^5*B
 - 3*a^5*C - 9*a^3*b^2*C - 10*a*b^4*C)*Log[a + b*Tan[c + d*x]])/(b*(a^2 +
b^2)*d))/b) + ((a^3*b*B + 3*a*b^3*B - 3*a^4*C - 6*a^2*b^2*C - b^4*C)*Tan[c
 + d*x])/(b*d)))/(b*(a^2 + b^2)))/(2*b*(a^2 + b^2))
```

3.38.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

3.38. $\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.38. $\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.38.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)C}{b^3}+\frac{\frac{(-3Ba^2b+Bb^3+Ca^3-3Cab^2)\ln(1+\tan(dx+c)^2)}{2}+(Ba^3-3Bab^2+3Ca^2b-Cb^3)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{a^2(Ba^4b+\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{\tan(dx+c)C}{b^3}+\frac{\frac{(-3Ba^2b+Bb^3+Ca^3-3Cab^2)\ln(1+\tan(dx+c)^2)}{2}+(Ba^3-3Bab^2+3Ca^2b-Cb^3)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{a^2(Ba^4b+\cdots}{\cdots}}{\cdots}$
norman	$\frac{\frac{C\tan(dx+c)^3}{bd}+\frac{(Ba^3-3Bab^2+3Ca^2b-Cb^3)a^2x}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{b^2(Ba^3-3Bab^2+3Ca^2b-Cb^3)x\tan(dx+c)^2}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{a(2Ba^4b+4Ba^2b^3-6Ca^5-\cdots}{db^3(a^4+2\cdots}}{(a+b\tan(\cdots}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)*C/b^3+1/(a^2+b^2)^3*(1/2*(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2
)*ln(1+tan(d*x+c)^2)+(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*arctan(tan(d*x+c)))
+1/b^4*a^2*(B*a^4*b+3*B*a^2*b^3+6*B*b^5-3*C*a^5-9*C*a^3*b^2-10*C*a*b^4)/(a
^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2/b^4*a^4*(B*b-C*a)/(a^2+b^2)/(a+b*tan(d*x+
c))^2+1/b^4*a^3*(2*B*a^2*b+4*B*b^3-3*C*a^3-5*C*a*b^2)/(a^2+b^2)^2/(a+b*tan
(d*x+c)))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(328) = 656$.

Time = 0.34 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.69

$$\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{3\,Ca^7b^2 - Ba^6b^3 + 9\,Ca^5b^4 - 7\,Ba^4b^5 - 2\,(Ca^6b^3 + 3\,Ca^4b^5 + 3\,Ca^2b^7 + Cb^9)\tan{(dx+c)}^3 - 2\,(Ba^5b^4}{}$$

input
```
integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="fricas")
```

output
```
-1/2*(3*C*a^7*b^2 - B*a^6*b^3 + 9*C*a^5*b^4 - 7*B*a^4*b^5 - 2*(C*a^6*b^3 +
 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*tan(d*x + c)^3 - 2*(B*a^5*b^4 + 3*C*a^
4*b^5 - 3*B*a^3*b^6 - C*a^2*b^7)*d*x - (9*C*a^7*b^2 - 3*B*a^6*b^3 + 23*C*a
^5*b^4 - 9*B*a^4*b^5 + 12*C*a^3*b^6 + 4*C*a*b^8 + 2*(B*a^3*b^6 + 3*C*a^2*b
^7 - 3*B*a*b^8 - C*b^9)*d*x)*tan(d*x + c)^2 + (3*C*a^9 - B*a^8*b + 9*C*a^7
*b^2 - 3*B*a^6*b^3 + 10*C*a^5*b^4 - 6*B*a^4*b^5 + (3*C*a^7*b^2 - B*a^6*b^3
 + 9*C*a^5*b^4 - 3*B*a^4*b^5 + 10*C*a^3*b^6 - 6*B*a^2*b^7)*tan(d*x + c)^2
+ 2*(3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^5*b^4 + 10*C*a^4*b^5 - 6*
B*a^3*b^6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^
2)/(tan(d*x + c)^2 + 1)) - (3*C*a^9 - B*a^8*b + 9*C*a^7*b^2 - 3*B*a^6*b^3
+ 9*C*a^5*b^4 - 3*B*a^4*b^5 + 3*C*a^3*b^6 - B*a^2*b^7 + (3*C*a^7*b^2 - B*a
^6*b^3 + 9*C*a^5*b^4 - 3*B*a^4*b^5 + 9*C*a^3*b^6 - 3*B*a^2*b^7 + 3*C*a*b^8
 - B*b^9)*tan(d*x + c)^2 + 2*(3*C*a^8*b - B*a^7*b^2 + 9*C*a^6*b^3 - 3*B*a^
5*b^4 + 9*C*a^4*b^5 - 3*B*a^3*b^6 + 3*C*a^2*b^7 - B*a*b^8)*tan(d*x + c))*l
og(1/(tan(d*x + c)^2 + 1)) - 2*(3*C*a^8*b - B*a^7*b^2 + 6*C*a^6*b^3 - 3*B*
a^5*b^4 - 2*C*a^4*b^5 + 4*B*a^3*b^6 + C*a^2*b^7 + 2*(B*a^4*b^5 + 3*C*a^3*b
^6 - 3*B*a^2*b^7 - C*a*b^8)*d*x)*tan(d*x + c))/((a^6*b^6 + 3*a^4*b^8 + 3*a
^2*b^10 + b^12)*d*tan(d*x + c)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*
b^11)*d*tan(d*x + c) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*d)
```

3.38.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3
,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4-6\,Ba^2b^5)\log(b\tan(dx+c)+a)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}} + \frac{(Ca^3-3\,Ba^2b-3\,C}{a^6+3}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="maxima")
```

output
```
1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3 + 10
*C*a^3*b^4 - 6*B*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^4 + 3*a^4*b^6 + 3
*a^2*b^8 + b^10) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (5*C*a^7 - 3*B*a^6*b + 9*C*
a^5*b^2 - 7*B*a^4*b^3 + 2*(3*C*a^6*b - 2*B*a^5*b^2 + 5*C*a^4*b^3 - 4*B*a^3
*b^4)*tan(d*x + c))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a^2*b^8
+ b^10)*tan(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(d*x + c)) + 2
*C*tan(d*x + c)/b^3)/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.53

$$\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(3\,Ca^7-Ba^6b+9\,Ca^5b^2-3\,Ba^4b^3+10\,Ca^3b^4}{a^6b^4+3\,a^4b^6+3\,a^2b}}{}$$

input
```
integrate(tan(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="giac")
```

output
```
1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
 c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(3*C*a^7 - B*a^6*b + 9*
C*a^5*b^2 - 3*B*a^4*b^3 + 10*C*a^3*b^4 - 6*B*a^2*b^5)*log(abs(b*tan(d*x +
c) + a))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + 2*C*tan(d*x + c)/b^3 +
 (9*C*a^7*b^2*tan(d*x + c)^2 - 3*B*a^6*b^3*tan(d*x + c)^2 + 27*C*a^5*b^4*t
an(d*x + c)^2 - 9*B*a^4*b^5*tan(d*x + c)^2 + 30*C*a^3*b^6*tan(d*x + c)^2 -
 18*B*a^2*b^7*tan(d*x + c)^2 + 12*C*a^8*b*tan(d*x + c) - 2*B*a^7*b^2*tan(d
*x + c) + 38*C*a^6*b^3*tan(d*x + c) - 6*B*a^5*b^4*tan(d*x + c) + 50*C*a^4*
b^5*tan(d*x + c) - 28*B*a^3*b^6*tan(d*x + c) + 4*C*a^9 + 13*C*a^7*b^2 + B*
a^6*b^3 + 21*C*a^5*b^4 - 11*B*a^4*b^5)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 +
 b^10)*(b*tan(d*x + c) + a)^2))/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.01

$$\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{C\tan(c+d\,x)}{b^3\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$-\frac{\frac{5\,C\,a^7-3\,B\,a^6\,b+9\,C\,a^5\,b^2-7\,B\,a^4\,b^3}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(3\,C\,a^6-2\,B\,a^5\,b+5\,C\,a^4\,b^2-4\,B\,a^3\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2\,b^3+2\,a\,b^4\tan\left(c+d\,x\right)+b^5\tan\left(c+d\,x\right)^2\right)}$$
$$+\frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,C\,a^5+B\,a^4\,b-9\,C\,a^3\,b^2+3\,B\,a^2\,b^3-10\,C\,a\,b^4+6\,B\,b^5\right)}{b^4\,d\,\left(a^2+b^2\right)^3}$$

input
```
int((tan(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^3,x)
```

output
```
(log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1
i)) - ((5*C*a^7 - 7*B*a^4*b^3 + 9*C*a^5*b^2 - 3*B*a^6*b)/(2*b*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c + d*x)*(3*C*a^6 - 4*B*a^3*b^3 + 5*C*a^4*b^2 - 2*B*a^
5*b))/(a^4 + b^4 + 2*a^2*b^2))/(d*(a^2*b^3 + b^5*tan(c + d*x)^2 + 2*a*b^4*
tan(c + d*x))) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a^
2*b - a^3*1i + b^3)) + (C*tan(c + d*x))/(b^3*d) + (a^2*log(a + b*tan(c + d
*x))*(6*B*b^5 - 3*C*a^5 + 3*B*a^2*b^3 - 9*C*a^3*b^2 + B*a^4*b - 10*C*a*b^4
))/(b^4*d*(a^2 + b^2)^3)
```

3.38. $\int \frac{\tan^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.39 $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

3.39.1 Optimal result

Integrand size = 40, antiderivative size = 250

$$\begin{aligned}\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}+\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}\\&\quad+\frac{a(a^2b^3B-3b^5B+a^5C+3a^3b^2C+6ab^4C)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^3\,d}\\&\quad+\frac{a(bB-aC)\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{a^2(2b^3B-a^3C-3ab^2C)}{b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3+(B*a^3-3*B*a*b^2+3*C*a^2*
b-C*b^3)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a*(B*a^2*b^3-3*B*b^5+C*a^5+3*C*a^3*b
^2+6*C*a*b^4)*ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)^3/d+1/2*a*(B*b-C*a)*tan(d*x
+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-a^2*(2*B*b^3-C*a^3-3*C*a*b^2)/b^3/(
a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

3.39.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= -\frac{(B+iC)\log(i-\tan(c+dx))}{2(a+ib)^3d} - \frac{(B-iC)\log(i+\tan(c+dx))}{2(a-ib)^3d}$$
$$+ \frac{a(a^2b^3B-3b^5B+a^5C+3a^3b^2C+6ab^4C)\log(a+b\tan(c+dx))}{b^3\left(a^2+b^2\right)^3d}$$
$$+ \frac{a^3(bB-aC)}{2b^3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{a^2(a^2bB+3b^3B-2a^3C-4ab^2C)}{b^3\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}$$

input
```
Integrate[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^3,x]
```

output
```
-1/2*((B + I*C)*Log[I - Tan[c + d*x]])/((a + I*b)^3*d) - ((B - I*C)*Log[I
+ Tan[c + d*x]])/(2*(a - I*b)^3*d) + (a*(a^2*b^3*B - 3*b^5*B + a^5*C + 3*a
^3*b^2*C + 6*a*b^4*C)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^3*d) + (a^
3*(b*B - a*C))/(2*b^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (a^2*(a^2*b*
B + 3*b^3*B - 2*a^3*C - 4*a*b^2*C))/(b^3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*
x]))
```

3.39.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 4115, 3042, 4088, 27, 3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^2\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\tan^3(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{2\tan(c+dx)\left(-\left(\left(a^2+b^2\right)C\tan^2(c+dx)\right)-b(bB-aC)\tan(c+dx)+a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(a^2+b^2\right)C\tan^2(c+dx)\right)-b(bB-aC)\tan(c+dx)+a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(a^2+b^2\right)C\tan(c+dx)^2\right)-b(bB-aC)\tan(c+dx)+a(bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4118}$$

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2C\tan^2(c+dx)+a\left(-Ca^3-3b^2Ca+2b^3B\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}+\frac{a^2\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(bB-aC)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2C\tan(c+dx)^2+a\left(-Ca^3-3b^2Ca+2b^3B\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}+\frac{a^2\left(a^3(-C)-3ab^2C+2b^3B\right)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4109}$$

3.39. $\int \frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\frac{a(bB-aC)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b^2(a^3B+3a^2bC-3ab^2B-b^3C)\int\tan(c+dx)dx}{a^2+b^2}-\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-3b^5B)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a^2(a^3}{b^2d(a^2}}{b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{\frac{a(bB-aC)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b^2(a^3B+3a^2bC-3ab^2B-b^3C)\int\tan(c+dx)dx}{a^2+b^2}-\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-3b^5B)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a^2(a^3}{b^2d(a^2}}{b(a^2+b^2)}$$

$\downarrow$ 3956

$$\frac{\frac{a(bB-aC)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-3b^5B)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2(a^3B+3a^2bC-3ab^2B-b^3C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a^2(a}{b^2d(a}}{b(a^2+b^2)}$$

$\downarrow$ 4100

$$\frac{\frac{a(bB-aC)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-3b^5B)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}-\frac{b^2(a^3B+3a^2bC-3ab^2B-b^3C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x(a^3(-C)+3a^2bB+3ab^2C-b^3B}{a^2+b^2}}{b(a^2+b^2)}}{b(a^2+b^2)}$$

$\downarrow$ 16

$$\frac{\frac{a(bB-aC)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{a^2(a^3(-C)-3ab^2C+2b^3B)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}+\frac{-\frac{b^2(a^3B+3a^2bC-3ab^2B-b^3C)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x(a^3(-C)+3a^2bB+3ab^2C-b^3B)}{a^2+b^2}-\frac{a(a^5C+3a^3b^2C+a^2b^3B+6ab^4C-}{bd(a^2+b^2}}{b(a^2+b^2)}}{b(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^3,x]
```

3.39. $\int\frac{\tan^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

output
```
(a*(b*B - a*C)*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (((b^2*(3*a^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2) - (b^2*(a^
3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) -
(a*(a^2*b^3*B - 3*b^5*B + a^5*C + 3*a^3*b^2*C + 6*a*b^4*C)*Log[a + b*Tan[c
 + d*x]])/(b*(a^2 + b^2)*d))/(b*(a^2 + b^2)) + (a^2*(2*b^3*B - a^3*C - 3*a
*b^2*C))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(b*(a^2 + b^2))
```

3.39.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.39.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{\left(-B\,a^{3}+3B a\,b^{2}-3C\,a^{2}b+C\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-3B\,a^{2}b+B\,b^{3}+C\,a^{3}-3C a\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(B\,a^{2}b^{3}-3B\,b^{5}+C\,a^{5}\right.}{}}{d}$
default	$\frac{\frac{\left(-B\,a^{3}+3B a\,b^{2}-3C\,a^{2}b+C\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-3B\,a^{2}b+B\,b^{3}+C\,a^{3}-3C a\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(B\,a^{2}b^{3}-3B\,b^{5}+C\,a^{5}\right.}{}}{d}$
norman	$\frac{-\frac{a^{2}\left(B\,a^{3}b+5B a\,b^{3}-3C\,a^{4}-7C\,a^{2}b^{2}\right)}{2d\,b^{3}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3C a\,b^{2}\right)a^{2}x}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{b^{2}\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3C a\,b^{2}\right)x\tan(dx+c)^{2}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{a\left(B\right.}{}}{(a+b\tan(dx+c))^{2}}$
risch	$-\frac{6ia^{4}Cx}{(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6})b}-\frac{xC}{3ia^{2}b-ib^{3}-a^{3}+3a\,b^{2}}+\frac{2iCc}{d\,b^{3}}+\frac{2iCx}{b^{3}}-\frac{2ia^{3}Bc}{(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6})d}-\frac{2ic}{(a^{6}+3a^{4}b^{2}+}$
parallelrisch	Expression too large to display

input
```
int(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*b^3)*ln(1+tan(d*x+c)
^2)+(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*arctan(tan(d*x+c)))+a*(B*a^2*b^3-3*
B*b^5+C*a^5+3*C*a^3*b^2+6*C*a*b^4)/(a^2+b^2)^3/b^3*ln(a+b*tan(d*x+c))-a^2*
(B*a^2*b+3*B*b^3-2*C*a^3-4*C*a*b^2)/b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+1/2*a
^3*(B*b-C*a)/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^2)
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(243) = 486$.

Time = 0.31 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.66

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{Ca^6b^2+Ba^5b^3+7\,Ca^4b^4-5\,Ba^3b^5+2\,(Ca^5b^3-3\,Ba^4b^4-3\,Ca^3b^5+Ba^2b^6)dx-(3\,Ca^6b^2-Ba^5b^3+9}{}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="fricas")
```

output
```
1/2*(C*a^6*b^2 + B*a^5*b^3 + 7*C*a^4*b^4 - 5*B*a^3*b^5 + 2*(C*a^5*b^3 - 3*
B*a^4*b^4 - 3*C*a^3*b^5 + B*a^2*b^6)*d*x - (3*C*a^6*b^2 - B*a^5*b^3 + 9*C*
a^4*b^4 - 7*B*a^3*b^5 - 2*(C*a^3*b^5 - 3*B*a^2*b^6 - 3*C*a*b^7 + B*b^8)*d*
x)*tan(d*x + c)^2 + (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 + 6*C*a^4*b^4 - 3*B*a
^3*b^5 + (C*a^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 + 6*C*a^2*b^6 - 3*B*a*b^7)*t
an(d*x + c)^2 + 2*(C*a^7*b + 3*C*a^5*b^3 + B*a^4*b^4 + 6*C*a^3*b^5 - 3*B*a
^2*b^6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - (C*a^8 + 3*C*a^6*b^2 + 3*C*a^4*b^4 + C*a^2*b^6 + (
C*a^6*b^2 + 3*C*a^4*b^4 + 3*C*a^2*b^6 + C*b^8)*tan(d*x + c)^2 + 2*(C*a^7*b
 + 3*C*a^5*b^3 + 3*C*a^3*b^5 + C*a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^
2 + 1)) - 2*(C*a^7*b + 3*C*a^5*b^3 - 3*B*a^4*b^4 - 4*C*a^3*b^5 + 3*B*a^2*b
^6 - 2*(C*a^4*b^4 - 3*B*a^3*b^5 - 3*C*a^2*b^6 + B*a*b^7)*d*x)*tan(d*x + c)
)/((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d*tan(d*x + c)^2 + 2*(a^7*b^4
+ 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*tan(d*x + c) + (a^8*b^3 + 3*a^6*b^5 +
3*a^4*b^7 + a^2*b^9)*d)
```

3.39.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3
,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.46

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(Ca^6+3\,Ca^4b^2+Ba^3b^3+6\,Ca^2b^4-3\,Bab^5\right)\log(b\tan(dx+c)+a)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}-\frac{\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)\mathrm{lo}}{a^6+3\,a^4b^2+3\,a^2b}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="maxima")
```

output
```
1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 + B*a^3*b^3 + 6*C*a^2*b^4 - 3*
B*a*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) -
 (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) + (3*C*a^6 - B*a^5*b + 7*C*a^4*b^2 - 5*B*a^3*b^
3 + 2*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a^2*b^4)*tan(d*x + c))/(a
^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2
+ 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)))/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.83

$$\int \frac{\tan^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(Ca^6+3\,Ca^4b^2+Ba^3b^3+6\,Ca^2b^4-3\,Bab^5\right)\mathrm{lo}}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9}}{}$$

input
```
integrate(tan(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="giac")
```

output
```
1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x +
 c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(C*a^6 + 3*C*a^4*b^2 +
B*a^3*b^3 + 6*C*a^2*b^4 - 3*B*a*b^5)*log(abs(b*tan(d*x + c) + a))/(a^6*b^3
 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) - (3*C*a^6*b*tan(d*x + c)^2 + 9*C*a^4*b^3*
tan(d*x + c)^2 + 3*B*a^3*b^4*tan(d*x + c)^2 + 18*C*a^2*b^5*tan(d*x + c)^2
- 9*B*a*b^6*tan(d*x + c)^2 + 2*C*a^7*tan(d*x + c) + 2*B*a^6*b*tan(d*x + c)
 + 6*C*a^5*b^2*tan(d*x + c) + 14*B*a^4*b^3*tan(d*x + c) + 28*C*a^3*b^4*tan
(d*x + c) - 12*B*a^2*b^5*tan(d*x + c) + B*a^7 - C*a^6*b + 9*B*a^5*b^2 + 11
*C*a^4*b^3 - 4*B*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(
d*x + c) + a)^2))/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 8.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.23

$$\int \frac{\tan^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,C\,a^6-B\,a^5\,b+7\,C\,a^4\,b^2-5\,B\,a^3\,b^3}{2\,b^3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{a^2\tan(c+d\,x)\,(-2\,C\,a^3+B\,a^2\,b-4\,C\,a\,b^2+3\,B\,b^3)}{b^2\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$
$$+\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^5+3\,C\,a^3\,b^2+B\,a^2\,b^3+6\,C\,a\,b^4-3\,B\,b^5\right)}{b^3\,d\,\left(a^2+b^2\right)^3}$$

input
```
int((tan(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^3,x)
```

output
```
((3*C*a^6 - 5*B*a^3*b^3 + 7*C*a^4*b^2 - B*a^5*b)/(2*b^3*(a^4 + b^4 + 2*a^2
*b^2)) - (a^2*tan(c + d*x)*(3*B*b^3 - 2*C*a^3 + B*a^2*b - 4*C*a*b^2))/(b^2
*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*
x))) + (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*
1i - b^3)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3*a*b^2 + a^2*b*3i
- a^3 - b^3*1i)) + (a*log(a + b*tan(c + d*x))*(C*a^5 - 3*B*b^5 + B*a^2*b^3
 + 3*C*a^3*b^2 + 6*C*a*b^4))/(b^3*d*(a^2 + b^2)^3)
```

3.40 $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.40.1 Optimal result

Integrand size = 38, antiderivative size = 189

$$\begin{aligned}\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3}\\ &\quad -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}\\ &\quad -\frac{a^2(bB-aC)}{2b^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{a(2b^3B-a^3C-3ab^2C)}{b^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3-(3*B*a^2*b-B*b^3-C*a^3+3*
C*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*a^2*(B*b-C*a)/b^2
/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+a*(2*B*b^3-C*a^3-3*C*a*b^2)/b^2/(a^2+b^2)^
2/d/(a+b*tan(d*x+c))
```

3.40.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.52

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{-\frac{bB+aC}{b(a+b\tan(c+dx))^2}-\frac{2C\tan(c+dx)}{(a+b\tan(c+dx))^2}+C\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}\right.}{}$$

input
```
Integrate[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x])^3,x]
```

output
```
(-((b*B + a*C)/(b*(a + b*Tan[c + d*x])^2)) - (2*C*Tan[c + d*x])/(a + b*Tan
[c + d*x])^2 + C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c
 + d*x]])/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(
a + b*Tan[c + d*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x
]])/(a + I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)
*Log[a + b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x])
)/(a + b*Tan[c + d*x])^2))/(a^2 + b^2)^3))/(2*b*d)
```

3.40.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4115, 3042, 4087, 25, 3042, 4111, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan(c+dx)^2\right)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 4115$$

$$\int \frac{\tan^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\tan(c+dx)^2(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$\Big\downarrow$ 4087

$$\frac{\int -\frac{-((a^2+b^2)C\tan^2(c+dx))-b(bB-aC)\tan(c+dx)+a(bB-aC)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{-((a^2+b^2)C\tan^2(c+dx))-b(bB-aC)\tan(c+dx)+a(bB-aC)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{-((a^2+b^2)C\tan(c+dx)^2)-b(bB-aC)\tan(c+dx)+a(bB-aC)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4111

$$-\frac{\frac{\int \frac{b\left(Ba^2+2bCa-b^2B\right)-b\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a(a^3(-C)-3ab^2C+2b^3B)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{b\left(Ba^2+2bCa-b^2B\right)-b\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a(a^3(-C)-3ab^2C+2b^3B)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4014

$$-\frac{\frac{\frac{b\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a(a^3(-C)-3ab^2C+2b^3B)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

3.40. $\int \frac{\tan(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}dx$

$$-\frac{\frac{\frac{b\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4013$$

$$-\frac{a^2(bB-aC)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{bx\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a\left(a^3(-C)-3ab^2C+2b^3B\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
)^3,x]
```

output
```
-1/2*(a^2*(b*B - a*C))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (((b*(
a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2) + (b*(3*a^2*b*B - b^
3*B - a^3*C + 3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)*d))/(a^2 + b^2) - (a*(2*b^3*B - a^3*C - 3*a*b^2*C))/(b*(a^2 + b^2)*d*(a
+ b*Tan[c + d*x])))/(b*(a^2 + b^2))
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeBQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.40. $\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.40.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\frac{(3Ba^2b-Bb^3-Ca^3+3Cab^2)\ln(1+\tan(dx+c)^2)}{2}+(-Ba^3+3Bab^2-3Ca^2b+Cb^3)\arctan(\tan(dx+c))}{(a^2+b^2)^3}-\frac{a^2(Bb-Ca)}{2b^2(a^2+b^2)(a+b\tan(dx+}}{d}$
default	$\frac{\frac{\frac{(3Ba^2b-Bb^3-Ca^3+3Cab^2)\ln(1+\tan(dx+c)^2)}{2}+(-Ba^3+3Bab^2-3Ca^2b+Cb^3)\arctan(\tan(dx+c))}{(a^2+b^2)^3}-\frac{a^2(Bb-Ca)}{2b^2(a^2+b^2)(a+b\tan(dx+}}{d}$
norman	$\frac{-\frac{(2Bab^3-Ca^4-3Ca^2b^2)\tan(dx+c)^2}{2ad(a^4+2a^2b^2+b^4)}-\frac{a(Ba^3-Bab^2+2Ca^2b)}{2db(a^4+2a^2b^2+b^4)}-\frac{(Ba^3-3Bab^2+3Ca^2b-Cb^3)a^2x}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{b^2(Ba^3-3Bab^2+3Ca^2}{(a^4+2a^2b^2+b}}{(a+b\tan(dx+c))^2}$
risch	$\frac{xB}{3ia^2b-ib^3-a^3+3ab^2}-\frac{ixC}{3ia^2b-ib^3-a^3+3ab^2}+\frac{6iBa^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2iBb^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2iCa^3x}{a^6+3a^4b^2+3a}$
parallelrisch	$-Ca^7+4B\tan(dx+c)a^3b^4+4B\tan(dx+c)ab^6-2C\tan(dx+c)a^6b-8C\tan(dx+c)a^4b^3-6C\tan(dx+c)a^2b^5-B\ln(1+\mathrm{ta}$

input
```
int(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_
RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*ln(1+tan(d*x+c)^
2)+(-B*a^3+3*B*a*b^2-3*C*a^2*b+C*b^3)*arctan(tan(d*x+c)))-1/2*a^2*(B*b-C*a
)/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^2-(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)/(a^2+
b^2)^3*ln(a+b*tan(d*x+c))+a*(2*B*b^3-C*a^3-3*C*a*b^2)/(a^2+b^2)^2/b^2/(a+b
*tan(d*x+c)))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(184) = 368.

Time = 0.27 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.53

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{Ca^5-3Ba^4b-5Ca^3b^2+3Ba^2b^3-2(Ba^5+3Ca^4b-3Ba^3b^2-Ca^2b^3)dx+(Ca^5+Ba^4b+7Ca^3b^2-}{}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="fricas")
```

output
```
1/2*(C*a^5 - 3*B*a^4*b - 5*C*a^3*b^2 + 3*B*a^2*b^3 - 2*(B*a^5 + 3*C*a^4*b
- 3*B*a^3*b^2 - C*a^2*b^3)*d*x + (C*a^5 + B*a^4*b + 7*C*a^3*b^2 - 5*B*a^2*
b^3 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2
+ (C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3
- 3*C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b
^3 + B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)) + 2*(B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2
*b^3 + 2*B*a*b^4 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*
tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 +
2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6)*d)
```

3.40.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="maxima")
```

output
```
-1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(
d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3
*C*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (C*a^5 + B*a^4*b + 5*C*a^3*b^2 - 3*B*a^2*b^3 + 2*(C*a^4*b + 3*C*a^2*
b^3 - 2*B*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 +
 2*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(d*x
 + c)))/d
```

3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(184) = 368$.

Time = 0.68 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.17

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4)\log(|b\tan}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}}{}$$

input
```
integrate(tan(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="giac")
```

output
```
-1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x
+ c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2
 - 3*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7) + (3*C*a^3*b^4*tan(d*x + c)^2 - 9*B*a^2*b^5*tan(d*x + c)^2
- 9*C*a*b^6*tan(d*x + c)^2 + 3*B*b^7*tan(d*x + c)^2 + 2*C*a^6*b*tan(d*x +
c) + 14*C*a^4*b^3*tan(d*x + c) - 22*B*a^3*b^4*tan(d*x + c) - 12*C*a^2*b^5*
tan(d*x + c) + 2*B*a*b^6*tan(d*x + c) + C*a^7 + B*a^6*b + 9*C*a^5*b^2 - 11
*B*a^4*b^3 - 4*C*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*(b*tan(
d*x + c) + a)^2))/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.48

$$\int \frac{\tan(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(C\,a^3-3\,B\,a^2\,b-3\,C\,a\,b^2+B\,b^3\right)}{d\,{\left(a^2+b^2\right)}^3}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$-\frac{\frac{a\,\left(C\,a^4+B\,a^3\,b+5\,C\,a^2\,b^2-3\,B\,a\,b^3\right)}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(C\,a^4+3\,C\,a^2\,b^2-2\,B\,a\,b^3\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}$$

input
```
int((tan(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(a + b*tan(c + d*x))*(B*b^3 + C*a^3 - 3*B*a^2*b - 3*C*a*b^2))/(d*(a^2
+ b^2)^3) - (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i -
 a^3 + b^3*1i)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a
^2*b - a^3*1i + b^3)) - ((a*(C*a^4 + 5*C*a^2*b^2 - 3*B*a*b^3 + B*a^3*b))/(
2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)*(C*a^4 + 3*C*a^2*b^2 - 2*B*
a*b^3))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*
tan(c + d*x)))
```

3.41 $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.41.1 Optimal result

Integrand size = 32, antiderivative size = 179

$$\begin{aligned}&\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx\\&=\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}\\&\quad-\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}\\&\quad+\frac{a(bB-aC)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{a^2B-b^2B+2abC}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3-(B*a^3-3*B*a*b^2+3*C*a^2*b
-C*b^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*(B*b-C*a)/b/(a^2
+b^2)/d/(a+b*tan(d*x+c))^2+(B*a^2-B*b^2+2*C*a*b)/(a^2+b^2)^2/d/(a+b*tan(d*
x+c))
```

3.41.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^3}+\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^3}-\frac{2(a^3B-3ab^2B+3a^2bC-b^3C)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}+\frac{a(bB-aC)}{b(a^2+b^2)(a+b\tan(c+dx))^2}}{2d}$$

input
```
Integrate[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

output
```
(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3 + ((B - I*C)*Log[I + Tan[c
+ d*x]])/(a - I*b)^3 - (2*(a^3*B - 3*a*b^2*B + 3*a^2*b*C - b^3*C)*Log[a +
b*Tan[c + d*x]])/(a^2 + b^2)^3 + (a*(b*B - a*C))/(b*(a^2 + b^2)*(a + b*Tan
[c + d*x])^2) + (2*(a^2*B - b^2*B + 2*a*b*C))/((a^2 + b^2)^2*(a + b*Tan[c
+ d*x])))/(2*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4111, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4111}$$
$$\frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a(bB-aC)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{bB-aC+(aB+bC)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int \frac{-Ca^2+2bBa+b^2C+\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^2B+2abC-b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-Ca^2+2bBa+b^2C+\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^2B+2abC-b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}-\frac{\left(a^3B+3a^2bC-3ab^2B-b^3C\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^2B+2abC-b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}-\frac{\left(a^3B+3a^2bC-3ab^2B-b^3C\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^2B+2abC-b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$\frac{a(bB-aC)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{\frac{a^2B+2abC-b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}-\frac{\left(a^3B+3a^2bC-3ab^2B-b^3C\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(B*Tan[c + d*x] + C*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^3,x]
```

3.41. $\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

output
```
(a*(b*B - a*C))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((3*a^2*b*B
 - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2) - ((a^3*B - 3*a*b^2*B + 3*a^2
*b*C - b^3*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2
+ b^2) + (a^2*B - b^2*B + 2*a*b*C)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(
a^2 + b^2)
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.41.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Bb-Ca)}{2\left(a^2+b^2\right)b(a+b\tan(dx+c)}}{d}$
default	$\frac{\frac{\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Bb-Ca)}{2\left(a^2+b^2\right)b(a+b\tan(dx+c)}}{d}$
norman	$\frac{\frac{\left(B\,a^2b^2-B\,b^4+2Ca\,b^3\right)\tan(dx+c)}{db\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(3B\,a^2b-B\,b^3-C\,a^3+3Ca\,b^2\right)x\tan(dx+c)^2}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{a\left(3B\,a^2\right.}{2}}{(a+b\tan(dx+c))^2}$
risch	$\frac{ixB}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{xC}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{2ia^3Bx}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6ia\,b^2Bx}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{6ia^2bCa}{a^6+3a^4b^2+3a}$
parallelrisch	$2B\,a^3b^4-b^6Ba+3C\,a^2b^5+3B\,a^5b^2+4C\tan(dx+c)a^3b^4+4C\tan(dx+c)a\,b^6-C\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)^2b^7+2C\,l$

input
```
int((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*ln(1+tan(d*x+c)^
2)+(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*arctan(tan(d*x+c)))+1/2*a*(B*b-C*a)/(
a^2+b^2)/b/(a+b*tan(d*x+c))^2+(B*a^2-B*b^2+2*C*a*b)/(a^2+b^2)^2/(a+b*tan(d
*x+c))-(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(176) = 352$.

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.73

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{3\,Ca^4b-5\,Ba^3b^2-3\,Ca^2b^3+Bab^4+2\,(Ca^5-3\,Ba^4b-3\,Ca^3b^2+Ba^2b^3)dx-(Ca^4b-3\,Ba^3b^2-5\,C}{}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/2*(3*C*a^4*b - 5*B*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + 2*(C*a^5 - 3*B*a^4
*b - 3*C*a^3*b^2 + B*a^2*b^3)*d*x - (C*a^4*b - 3*B*a^3*b^2 - 5*C*a^2*b^3 +
 3*B*a*b^4 - 2*(C*a^3*b^2 - 3*B*a^2*b^3 - 3*C*a*b^4 + B*b^5)*d*x)*tan(d*x
+ c)^2 + (B*a^5 + 3*C*a^4*b - 3*B*a^3*b^2 - C*a^2*b^3 + (B*a^3*b^2 + 3*C*a
^2*b^3 - 3*B*a*b^4 - C*b^5)*tan(d*x + c)^2 + 2*(B*a^4*b + 3*C*a^3*b^2 - 3*
B*a^2*b^3 - C*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x
 + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(C*a^5 - 2*B*a^4*b - 3*C*a^3*b^2 +
3*B*a^2*b^3 + 2*C*a*b^4 - B*b^5 - 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 +
 B*a*b^4)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*ta
n(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) +
(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

3.41.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.84

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\big(\tan(dx}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + 2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(b*tan(
d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3
*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (C*a^4 - 3*B*a^3*b - 3*C*a^2*b^2 + B*a*b^3 - 2*(B*a^2*b^2 + 2*C*a*b^
3 - B*b^4)*tan(d*x + c))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b
^5 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)))/
d
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(176) = 352$.

Time = 0.69 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.29

$$\int \frac{B\tan(c+dx) + C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(Ba^3b+3\,Ca^2b^2-3\,Bab^3-Cb^4)\log(|b\tan}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}}{}$$

input
```
integrate((B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x
+ c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(B*a^3*b + 3*C*a^2*b^2
 - 3*B*a*b^3 - C*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7) - (3*B*a^3*b^3*tan(d*x + c)^2 + 9*C*a^2*b^4*tan(d*x + c)^2
- 9*B*a*b^5*tan(d*x + c)^2 - 3*C*b^6*tan(d*x + c)^2 + 8*B*a^4*b^2*tan(d*x
+ c) + 22*C*a^3*b^3*tan(d*x + c) - 18*B*a^2*b^4*tan(d*x + c) - 2*C*a*b^5*t
an(d*x + c) - 2*B*b^6*tan(d*x + c) - C*a^6 + 6*B*a^5*b + 11*C*a^4*b^2 - 7*
B*a^3*b^3 - B*a*b^5)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*tan(d*x + c
) + a)^2))/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.58

$$\int \frac{B\tan(c+dx)+C\tan^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{\frac{\tan(c+d\,x)\,\left(B\,a^2\,b+2\,C\,a\,b^2-B\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{C\,a^4-3\,B\,a^3\,b-3\,C\,a^2\,b^2+B\,a\,b^3}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}$$
$$-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{B\,a+3\,C\,b}{\left(a^2+b^2\right)^2}-\frac{4\,b^2\,\left(B\,a+C\,b\right)}{\left(a^2+b^2\right)^3}\right)}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$

input
```
int((B*tan(c + d*x) + C*tan(c + d*x)^2)/(a + b*tan(c + d*x))^3,x)
```

output
```
((tan(c + d*x)*(B*a^2*b - B*b^3 + 2*C*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (C
*a^4 - 3*C*a^2*b^2 + B*a*b^3 - 3*B*a^3*b)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(
d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) - (log(a + b*tan(c + d*
x))*((B*a + 3*C*b)/(a^2 + b^2)^2 - (4*b^2*(B*a + C*b))/(a^2 + b^2)^3))/d -
 (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b
^3)) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3
- b^3*1i))
```

3.42 $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.42.1 Optimal result

Integrand size = 38, antiderivative size = 175

$$\begin{aligned}\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx &= \frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3} \\ &\quad+\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} \\ &\quad-\frac{bB-aC}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{2abB-a^2C+b^2C}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3+(3*B*a^2*b-B*b^3-C*a^3+3*C
*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*(-B*b+C*a)/(a^2+b^
2)/d/(a+b*tan(d*x+c))^2+(-2*B*a*b+C*a^2-C*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+
c))
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.74 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.39

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{C\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}\right)+(bB-aC)\left(\frac{i\log(i-\tan(c+dx}{(a+ib)^3}\right.}{2bd}$$

input
```
Integrate[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c
+ d*x])^3,x]
```

output
```
-1/2*(C*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])
/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan
[c + d*x])))/(a^2 + b^2)^2) + (b*B - a*C)*((I*Log[I - Tan[c + d*x]])/(a +
I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a +
b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*
Tan[c + d*x])^2))/(a^2 + b^2)^3))/(b*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4115, 3042, 4012, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{B+C\tan(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$\Big\downarrow$ 4012

$$\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{aB+bC-(bB-aC)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int \frac{Ba^2+2bCa-b^2B-\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2(-C)+2abB+b^2C}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{Ba^2+2bCa-b^2B-\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2(-C)+2abB+b^2C}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-C)+2abB+b^2C}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-C)+2abB+b^2C}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

3.42. $\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\frac{\frac{\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-C)+2abB+b^2C}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{bB-aC}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

input
```
Int[(Cot[c + d*x]*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*x]
)^3,x]
```

output
```
-1/2*(b*B - a*C)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((a^3*B - 3*a*
b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2) + ((3*a^2*b*B - b^3*B - a^3*C +
3*a*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b
^2) - (2*a*b*B - a^2*C + b^2*C)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2
 + b^2)
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.42.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{\left(-3B\,a^2b+B\,b^3+C\,a^3-3Ca\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3C\cdots\right.}{\left(a^2+\cdots\right.}}{d}$
default	$\frac{\frac{\frac{\left(-3B\,a^2b+B\,b^3+C\,a^3-3Ca\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\left(3B\,a^2b-B\,b^3-C\,a^3+3C\cdots\right.}{\left(a^2+\cdots\right.}}{d}$
parallelrisch	$\frac{6a\left(B\,a^2b-\frac{1}{3}B\,b^3-\frac{1}{3}C\,a^3+Ca\,b^2\right)(a+b\tan(dx+c))^2\ln(a+b\tan(dx+c))-3a\left(B\,a^2b-\frac{1}{3}B\,b^3-\frac{1}{3}C\,a^3+Ca\,b^2\right)(a+b\tan(dx\cdots\right.}{\cdots}$
norman	$\frac{\frac{\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(B\,a^3-3Ba\,b^2+3C\,a^2b-C\,b^3\right)x\tan(dx+c)^2}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{3B\,a^2b^2+B\,b^4-2C\,a^3b}{2bd\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2Ba\,b^2-C\,a^2b+C\cdots\right.}{2da\left(a^4+2a^2\cdots\right.}}{(a+b\tan(dx+c))^2}$
risch	$-\frac{xB}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{ixC}{3ia^2b-ib^3-a^3+3a\,b^2}-\frac{6iB\,a^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2iB\,b^3x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2iC\,a\cdots}{a^6+3a^4b^2+\cdots}$

input
```
int(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method=_
RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*ln(1+tan(d*x+c)
^2)+(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*arctan(tan(d*x+c)))+(3*B*a^2*b-B*b^3
-C*a^3+3*C*a*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2*(B*b-C*a)/(a^2+b^2)/(
a+b*tan(d*x+c))^2-(2*B*a*b-C*a^2+C*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c)))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(171) = 342$.

Time = 0.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.75

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{5\,Ca^3b^2 - 7\,Ba^2b^3 - Cab^4 - Bb^5 + 2\,(Ba^5 + 3\,Ca^4b - 3\,Ba^3b^2 - Ca^2b^3)dx - (3\,Ca^3b^2 - 5\,Ba^2b^3 - 3\,Ca}{}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="fricas")
```

output
```
1/2*(5*C*a^3*b^2 - 7*B*a^2*b^3 - C*a*b^4 - B*b^5 + 2*(B*a^5 + 3*C*a^4*b -
3*B*a^3*b^2 - C*a^2*b^3)*d*x - (3*C*a^3*b^2 - 5*B*a^2*b^3 - 3*C*a*b^4 + B*
b^5 - 2*(B*a^3*b^2 + 3*C*a^2*b^3 - 3*B*a*b^4 - C*b^5)*d*x)*tan(d*x + c)^2
- (C*a^5 - 3*B*a^4*b - 3*C*a^3*b^2 + B*a^2*b^3 + (C*a^3*b^2 - 3*B*a^2*b^3
- 3*C*a*b^4 + B*b^5)*tan(d*x + c)^2 + 2*(C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b
^3 + B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)) - 2*(2*C*a^4*b - 3*B*a^3*b^2 - 3*C*a^2*b^3 + 3
*B*a*b^4 + C*b^5 - 2*(B*a^4*b + 3*C*a^3*b^2 - 3*B*a^2*b^3 - C*a*b^4)*d*x)*
tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 +
2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6)*d)
```

3.42.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.83

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="maxima")
```

output
```
1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(b*tan(d
*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*
C*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (3*C*a^3 - 5*B*a^2*b - C*a*b^2 - B*b^3 + 2*(C*a^2*b - 2*B*a*b^2 - C*b
^3)*tan(d*x + c))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)
*tan(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c)))/d
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(171) = 342.

Time = 1.25 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.34

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(Ca^3b-3\,Ba^2b^2-3\,Cab^3+Bb^4)\log(|b\tan(d}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}}{}$$

input
```
integrate(cot(d*x+c)*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x, a
lgorithm="giac")
```

output
```
1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
 c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(C*a^3*b - 3*B*a^2*b^2
- 3*C*a*b^3 + B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7) + (3*C*a^3*b^2*tan(d*x + c)^2 - 9*B*a^2*b^3*tan(d*x + c)^2 -
 9*C*a*b^4*tan(d*x + c)^2 + 3*B*b^5*tan(d*x + c)^2 + 8*C*a^4*b*tan(d*x + c
) - 22*B*a^3*b^2*tan(d*x + c) - 18*C*a^2*b^3*tan(d*x + c) + 2*B*a*b^4*tan(
d*x + c) - 2*C*b^5*tan(d*x + c) + 6*C*a^5 - 14*B*a^4*b - 7*C*a^3*b^2 - 3*B
*a^2*b^3 - C*a*b^4 - B*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*
x + c) + a)^2))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.59

$$\int \frac{\cot(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\left(\frac{3\,B\,b-C\,a}{\left(a^2+b^2\right)^2}-\frac{4\,b^2\,(B\,b-C\,a)}{\left(a^2+b^2\right)^3}\right)}{d} - \frac{\frac{-3\,C\,a^3+5\,B\,a^2\,b+C\,a\,b^2+B\,b^3}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,\left(-C\,a^2\,b+2\,B\,a\,b^2+C\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

input
```
int((cot(c + d*x)*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*x)
)^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((3*B*b - C*a)/(a^2 + b^2)^2 - (4*b^2*(B*b - C*a)
)/(a^2 + b^2)^3))/d - ((B*b^3 - 3*C*a^3 + 5*B*a^2*b + C*a*b^2)/(2*(a^4 + b
^4 + 2*a^2*b^2)) + (tan(c + d*x)*(C*b^3 + 2*B*a*b^2 - C*a^2*b))/(a^4 + b^4
 + 2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (log(
tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) +
 (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b
^3))
```

3.43 $\int \frac{\cot^2(c+dx)(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

3.43.1 Optimal result

Integrand size = 40, antiderivative size = 215

$$\begin{aligned}\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(3a^2bB-b^3B-a^3C+3ab^2C)\,x}{(a^2+b^2)^3}+\frac{B\log(\sin(c+dx))}{a^3d}\\ &\quad-\frac{b(6a^4bB+3a^2b^3B+b^5B-3a^5C+a^3b^2C)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\,(a^2+b^2)^3\,d}\\ &\quad+\frac{b(bB-aC)}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{b(3a^2bB+b^3B-2a^3C)}{a^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*x/(a^2+b^2)^3+B*ln(sin(d*x+c))/a^3/d-b*
(6*B*a^4*b+3*B*a^2*b^3+B*b^5-3*C*a^5+C*a^3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+
c))/a^3/(a^2+b^2)^3/d+1/2*b*(B*b-C*a)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+b*(
3*B*a^2*b+B*b^3-2*C*a^3)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.04

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{-\frac{(B+iC)\log(i-\tan(c+dx))}{(a+ib)^3}+\frac{2B\log(\tan(c+dx))}{a^3}-\frac{(B-iC)\log(i+\tan(c+dx))}{(a-ib)^3}-\frac{2b\left(6a^4bB+3a^2b^3B+b^5B-3a^5C+a^3b^2C\right)\log(a+b\tan(c}{a^3(a^2+b^2)^3}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^3,x]
```

output
```
(-(((B + I*C)*Log[I - Tan[c + d*x]])/(a + I*b)^3) + (2*B*Log[Tan[c + d*x]]
)/a^3 - ((B - I*C)*Log[I + Tan[c + d*x]])/(a - I*b)^3 - (2*b*(6*a^4*b*B +
3*a^2*b^3*B + b^5*B - 3*a^5*C + a^3*b^2*C)*Log[a + b*Tan[c + d*x]])/(a^3*(
a^2 + b^2)^3) + (b*(b*B - a*C))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (
2*b*(3*a^2*b*B + b^3*B - 2*a^3*C))/(a^2*(a^2 + b^2)^2*(a + b*Tan[c + d*x])
))/(2*d)
```

3.43.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 4115, 3042, 4092, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\cot(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 4092$$

$$\frac{\int \frac{2\cot(c+dx)\left(b(bB-aC)\tan^2(c+dx)-a(bB-aC)\tan(c+dx)+\left(a^2+b^2\right)B\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\cot(c+dx)\left(b(bB-aC)\tan^2(c+dx)-a(bB-aC)\tan(c+dx)+\left(a^2+b^2\right)B\right)}{(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{b(bB-aC)\tan(c+dx)^2-a(bB-aC)\tan(c+dx)+\left(a^2+b^2\right)B}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)a^2\right)+b\left(-2Ca^3+3bBa^2+b^3B\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2B\right)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b(-2a^3C+3a^2bB+b^3B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{-\left(\left(-Ca^2+2bBa+b^2C\right)\tan(c+dx)a^2\right)+b\left(-2Ca^3+3bBa^2+b^3B\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2B}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(-2a^3C+3a^2bB+b^3B)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4134$$

$$\frac{\frac{\frac{B\left(a^2+b^2\right)^2\int \cot(c+dx)dx}{a}-\frac{b\left(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3C+3a^2bB+b^3}{ad(a^2+b^2)(a+b\tan(c+}}{a\left(a^2+b^2\right)}+\frac{b(bB-aC)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

3.43. $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\frac{\frac{B(a^2+b^2)^2\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^2x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3C+3a^2}{ad(a^2+b^2)(a+b}}{a(a^2+b^2)}$$

$$\frac{b(bB-aC)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\frac{B(a^2+b^2)^2\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b\left(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^2x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3C+}{ad(a^2+b^2)(a}}{a(a^2+b^2)}$$

$$\frac{b(bB-aC)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3956

$$\frac{\frac{-\frac{b\left(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{B(a^2+b^2)^2\log(-\sin(c+dx))}{ad}-\frac{a^2x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3C+3a^2bB}{ad(a^2+b^2)(a+b\tan}}{a(a^2+b^2)}$$

$$\frac{b(bB-aC)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$\frac{b(bB-aC)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}+$$

$$\frac{\frac{b(-2a^3C+3a^2bB+b^3B)}{ad(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{B(a^2+b^2)^2\log(-\sin(c+dx))}{ad}-\frac{a^2x\left(a^3(-C)+3a^2bB+3ab^2C-b^3B\right)}{a^2+b^2}-\frac{b\left(-3a^5C+6a^4bB+a^3b^2C+3a^2b^3B+b^5B\right)\log(a\cos(c+}{ad(a^2+b^2)}}{a(a^2+b^2)}}{a(a^2+b^2)}$$

input
```
Int[(Cot[c + d*x]^2*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^3,x]
```

output
```
(b*(b*B - a*C))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((-((a^2*(3*a
^2*b*B - b^3*B - a^3*C + 3*a*b^2*C)*x)/(a^2 + b^2)) + ((a^2 + b^2)^2*B*Log
[-Sin[c + d*x]])/(a*d) - (b*(6*a^4*b*B + 3*a^2*b^3*B + b^5*B - 3*a^5*C + a
^3*b^2*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2
 + b^2)) + (b*(3*a^2*b*B + b^3*B - 2*a^3*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c
 + d*x])))/(a*(a^2 + b^2))
```

3.43. $\int\frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

3.43. $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.43.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{B\ln(\tan(dx+c))}{a^3}+\frac{\frac{(-B a^3+3Ba b^2-3C a^2 b+C b^3)\ln(1+\tan(dx+c)^2)}{2}+(-3B a^2 b+B b^3+C a^3-3Ca b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{b(\cdots}{(a^2+\cdots}}{d}$
default	$\frac{\frac{B\ln(\tan(dx+c))}{a^3}+\frac{\frac{(-B a^3+3Ba b^2-3C a^2 b+C b^3)\ln(1+\tan(dx+c)^2)}{2}+(-3B a^2 b+B b^3+C a^3-3Ca b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{b(\cdots}{(a^2+\cdots}}{d}$
parallelrisch	$-12b(B a^4 b+\frac{1}{2}B a^2 b^3+\frac{1}{6}B b^5-\frac{1}{2}C a^5+\frac{1}{6}C a^3 b^2)(a+b\tan(dx+c))^2\ln(a+b\tan(dx+c))-a^3(a+b\tan(dx+c))^2(B a^3-3\cdots$
norman	$\frac{-\frac{b^2(3B a^2 b-B b^3-C a^3+3Ca b^2)x\tan(dx+c)^3}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{b(4B a^2 b^2+2B b^4-3C a^3 b-Ca b^3)\tan(dx+c)^2}{d a^2(a^4+2a^2b^2+b^4)}-\frac{b^2(7B a^2 b^2+3B b^4-5C a^3 b-C\cdots}{2d a^3(a^4+2a^2b^2+\cdots}}{\tan(dx+c)(a+b\tan(d\cdots}$
risch	$\frac{2iC b^3 x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{xC}{3ia^2b-ib^3-a^3+3a b^2}+\frac{6ib^4 Bx}{(a^6+3a^4b^2+3a^2b^4+b^6)a}+\frac{2iC b^3 c}{d(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{\cdots}{(a^6+3a^4\cdots}$

input
```
int(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(1/a^3*B*ln(tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-B*a^3+3*B*a*b^2-3*C*a^2*b
+C*b^3)*ln(1+tan(d*x+c)^2)+(-3*B*a^2*b+B*b^3+C*a^3-3*C*a*b^2)*arctan(tan(d
*x+c)))+b*(3*B*a^2*b+B*b^3-2*C*a^3)/(a^2+b^2)^2/a^2/(a+b*tan(d*x+c))-b*(6*
B*a^4*b+3*B*a^2*b^3+B*b^5-3*C*a^5+C*a^3*b^2)/(a^2+b^2)^3/a^3*ln(a+b*tan(d*
x+c))+1/2*(B*b-C*a)*b/(a^2+b^2)/a/(a+b*tan(d*x+c))^2)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(213) = 426$.

Time = 0.33 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.18

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{7\,Ca^5b^3-9\,Ba^4b^4+Ca^3b^5-3\,Ba^2b^6-2\,(Ca^8-3\,Ba^7b-3\,Ca^6b^2+Ba^5b^3)dx-(5\,Ca^5b^3-7\,Ba^4b^4}{}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="fricas")
```

output
```
-1/2*(7*C*a^5*b^3 - 9*B*a^4*b^4 + C*a^3*b^5 - 3*B*a^2*b^6 - 2*(C*a^8 - 3*B
*a^7*b - 3*C*a^6*b^2 + B*a^5*b^3)*d*x - (5*C*a^5*b^3 - 7*B*a^4*b^4 - C*a^3
*b^5 - B*a^2*b^6 + 2*(C*a^6*b^2 - 3*B*a^5*b^3 - 3*C*a^4*b^4 + B*a^3*b^5)*d
*x)*tan(d*x + c)^2 - (B*a^8 + 3*B*a^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6 + (B*a
^6*b^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*tan(d*x + c)^2 + 2*(B*a^7*b +
3*B*a^5*b^3 + 3*B*a^3*b^5 + B*a*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1)) - (3*C*a^7*b - 6*B*a^6*b^2 - C*a^5*b^3 - 3*B*a^4*b^4 - B
*a^2*b^6 + (3*C*a^5*b^3 - 6*B*a^4*b^4 - C*a^3*b^5 - 3*B*a^2*b^6 - B*b^8)*t
an(d*x + c)^2 + 2*(3*C*a^6*b^2 - 6*B*a^5*b^3 - C*a^4*b^4 - 3*B*a^3*b^5 - B
*a*b^7)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - 2*(3*C*a^6*b^2 - 4*B*a^5*b^3 - 3*C*a^4*b^4 + 3*B*a
^3*b^5 + B*a*b^7 + 2*(C*a^7*b - 3*B*a^6*b^2 - 3*C*a^5*b^3 + B*a^4*b^4)*d*x
)*tan(d*x + c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*tan(d*x + c
)^2 + 2*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*tan(d*x + c) + (a^11
+ 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d)
```

3.43.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)**2*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3
,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.73

$$\int \frac{\cot^2(c+dx)\,(B\tan(c+dx)+C\tan^2(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(3\,Ca^5b-6\,Ba^4b^2-Ca^3b^3-3\,Ba^2b^4-Bb^6)\log(b\tan(dx+c)+a)}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6} - \frac{(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)\,\mathrm{lo}}{a^6+3\,a^4b^2+3\,a^2b^4}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="maxima")
```

output
```
1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b - 6*B*a^4*b^2 - C*a^3*b^3 - 3*B*a^2*b^4
- B*b^6)*log(b*tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6) -
 (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) - (5*C*a^4*b - 7*B*a^3*b^2 + C*a^2*b^3 - 3*B*a*
b^4 + 2*(2*C*a^3*b^2 - 3*B*a^2*b^3 - B*b^5)*tan(d*x + c))/(a^8 + 2*a^6*b^2
 + a^4*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2
*a^5*b^3 + a^3*b^5)*tan(d*x + c)) + 2*B*log(tan(d*x + c))/a^3)/d
```

3.43.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(213) = 426$.

Time = 1.24 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.23

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(3\,Ca^5b^2-6\,Ba^4b^3-Ca^3b^4-3\,Ba^2b^5-Bb^7\right)}{a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7}}{}$$

input
```
integrate(cot(d*x+c)^2*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="giac")
```

output
```
1/2*(2*(C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - (B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*log(tan(d*x +
 c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(3*C*a^5*b^2 - 6*B*a^4*
b^3 - C*a^3*b^4 - 3*B*a^2*b^5 - B*b^7)*log(abs(b*tan(d*x + c) + a))/(a^9*b
 + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7) + 2*B*log(abs(tan(d*x + c)))/a^3 - (9*
C*a^5*b^3*tan(d*x + c)^2 - 18*B*a^4*b^4*tan(d*x + c)^2 - 3*C*a^3*b^5*tan(d
*x + c)^2 - 9*B*a^2*b^6*tan(d*x + c)^2 - 3*B*b^8*tan(d*x + c)^2 + 22*C*a^6
*b^2*tan(d*x + c) - 42*B*a^5*b^3*tan(d*x + c) - 2*C*a^4*b^4*tan(d*x + c) -
 26*B*a^3*b^5*tan(d*x + c) - 8*B*a*b^7*tan(d*x + c) + 14*C*a^7*b - 25*B*a^
6*b^2 + 3*C*a^5*b^3 - 19*B*a^4*b^4 + C*a^3*b^5 - 6*B*a^2*b^6)/((a^9 + 3*a^
7*b^2 + 3*a^5*b^4 + a^3*b^6)*(b*tan(d*x + c) + a)^2))/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 10.87 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{-5\,C\,a^3\,b+7\,B\,a^2\,b^2-C\,a\,b^3+3\,B\,b^4}{2\,a\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(-2\,C\,a^3\,b^2+3\,B\,a^2\,b^3+B\,b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{B\,\ln\left(\tan(c+d\,x)\right)}{a^3\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)} - \frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,C\,a^5+6\,B\,a^4\,b+C\,a^3\,b^2+3\,B\,a^2\,b^3+B\,b^5\right)}{a^3\,d\,{\left(a^2+b^2\right)}^3}$$

input
```
int((cot(c + d*x)^2*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^3,x)
```

output
```
((3*B*b^4 + 7*B*a^2*b^2 - C*a*b^3 - 5*C*a^3*b)/(2*a*(a^4 + b^4 + 2*a^2*b^2
)) + (tan(c + d*x)*(B*b^5 + 3*B*a^2*b^3 - 2*C*a^3*b^2))/(a^2*(a^4 + b^4 +
2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (B*log(
tan(c + d*x)))/(a^3*d) + (log(tan(c + d*x) - 1i)*(B*1i - C))/(2*d*(a*b^2*3
i + 3*a^2*b - a^3*1i - b^3)) + (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(3
*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) - (b*log(a + b*tan(c + d*x))*(B*b^5 - 3
*C*a^5 + 3*B*a^2*b^3 + C*a^3*b^2 + 6*B*a^4*b))/(a^3*d*(a^2 + b^2)^3)
```

3.43. $\int \frac{\cot^2(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.44 $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.44.1 Optimal result

Integrand size = 40, antiderivative size = 287

$$\begin{aligned}\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(a^3B-3ab^2B+3a^2bC-b^3C)\,x}{(a^2+b^2)^3}-\frac{(3bB-aC)\log(\sin(c+dx))}{a^4d}\\ &\quad+\frac{b^2(10a^4bB+9a^2b^3B+3b^5B-6a^5C-3a^3b^2C-ab^4C)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4\left(a^2+b^2\right)^3d}\\ &\quad-\frac{b(2a^2B+3b^2B-abC)}{2a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}\\ &\quad-\frac{b(a^4B+6a^2b^2B+3b^4B-3a^3bC-ab^3C)}{a^3\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(B*a^3-3*B*a*b^2+3*C*a^2*b-C*b^3)*x/(a^2+b^2)^3-(3*B*b-C*a)*ln(sin(d*x+c)
)/a^4/d+b^2*(10*B*a^4*b+9*B*a^2*b^3+3*B*b^5-6*C*a^5-3*C*a^3*b^2-C*a*b^4)*l
n(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^3/d-1/2*b*(2*B*a^2+3*B*b^2-C*a*
b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-B*cot(d*x+c)/a/d/(a+b*tan(d*x+c))^2-
b*(B*a^4+6*B*a^2*b^2+3*B*b^4-3*C*a^3*b-C*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*tan
(d*x+c))
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= -\frac{B\cot(c+dx)}{a^3d}+\frac{(B+iC)\log(i-\tan(c+dx))}{2(ia-b)^3d}$$
$$-\frac{(3bB-aC)\log(\tan(c+dx))}{a^4d}-\frac{(iB+C)\log(i+\tan(c+dx))}{2(a-ib)^3d}$$
$$+\frac{b^2(10a^4bB+9a^2b^3B+3b^5B-6a^5C-3a^3b^2C-ab^4C)\log(a+b\tan(c+dx))}{a^4\left(a^2+b^2\right)^3d}$$
$$-\frac{b^2(bB-aC)}{2a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{b^2(4a^2bB+2b^3B-3a^3C-ab^2C)}{a^3\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}$$

input
```
Integrate[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[
c + d*x])^3,x]
```

output
```
-((B*Cot[c + d*x])/(a^3*d)) + ((B + I*C)*Log[I - Tan[c + d*x]])/(2*(I*a -
b)^3*d) - ((3*b*B - a*C)*Log[Tan[c + d*x]])/(a^4*d) - ((I*B + C)*Log[I + T
an[c + d*x]])/(2*(a - I*b)^3*d) + (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B
 - 6*a^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a + b*Tan[c + d*x]])/(a^4*(a^2 + b
^2)^3*d) - (b^2*(b*B - a*C))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (b^2*(4*a^2*b*B + 2*b^3*B - 3*a^3*C - a*b^2*C))/(a^3*(a^2 + b^2)^2*d*(a
+ b*Tan[c + d*x]))
```

3.44.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4115, 3042, 4092, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{B\tan(c+dx)+C\tan(c+dx)^2}{\tan(c+dx)^3(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4115}$$

$$\int \frac{\cot^2(c+dx)(B+C\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+C\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot(c+dx)(3bB\tan^2(c+dx)+aB\tan(c+dx)+3bB-aC)}{(a+b\tan(c+dx))^3}dx}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3bB\tan(c+dx)^2+aB\tan(c+dx)+3bB-aC}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\int \frac{2\cot(c+dx)\left((aB+bC)\tan(c+dx)a^2+b\left(2Ba^2-bCa+3b^2B\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{2a(a^2+b^2)}+\frac{b(2a^2B-abC+3b^2B)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left((aB+bC)\tan(c+dx)a^2+b\left(2Ba^2-bCa+3b^2B\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3bB-aC)\right)}{(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(2a^2B-abC+3b^2B)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{(aB+bC)\tan(c+dx)a^2+b\left(2Ba^2-bCa+3b^2B\right)\tan(c+dx)^2+\left(a^2+b^2\right)(3bB-aC)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(2a^2B-abC+3b^2B)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

3.44. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)a^3+b\left(Ba^4-3bCa^3+6b^2Ba^2-b^3Ca+3b^4B\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(3bB-aC)\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b\left(a^4B-3a^3bC+6a^2b^2B-ab^3\cdots\right.}{ad\left(a^2+b^2\right)(a+b\tan(c+\cdots}}{a(a^2+b^2)}\cdots}{a}\cdots$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{\left(Ba^2+2bCa-b^2B\right)\tan(c+dx)a^3+b\left(Ba^4-3bCa^3+6b^2Ba^2-b^3Ca+3b^4B\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2(3bB-aC)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b\left(a^4B-3a^3bC+6a^2b^2B-ab^3C+3b^4B\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a(a^2+b^2)}+\cdots}{a}\cdots$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^2(3bB-aC)\int\cot(c+dx)dx}{a}-\frac{b^2\left(-6a^5C+10a^4bB-3a^3b^2C+9a^2b^3B-ab^4C+3b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b\left(a^4B-\cdots\right.}{a\cdots}}{a(a^2+b^2)}\cdots}{a}\cdots$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^2(3bB-aC)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(-6a^5C+10a^4bB-3a^3b^2C+9a^2b^3B-ab^4C+3b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\cdots}{a(a^2+b^2)}\cdots}{a}\cdots$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{-\frac{\left(a^2+b^2\right)^2(3bB-aC)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b^2\left(-6a^5C+10a^4bB-3a^3b^2C+9a^2b^3B-ab^4C+3b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{a}\cdots$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3956

3.44. $\displaystyle\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

$$-\frac{\frac{-\frac{b^2\left(-6a^5C+10a^4bB-3a^3b^2C+9a^2b^3B-ab^4C+3b^5B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)^2(3bB-aC)\log(-\sin(c+dx))}{ad}+\frac{a^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b\left(a\cdots\right.}{\cdots}}{a(a^2+b^2)}}{a}$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$-\frac{\frac{b(2a^2B-abC+3b^2B)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{b\left(a^4B-3a^3bC+6a^2b^2B-ab^3C+3b^4B\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\frac{\left(a^2+b^2\right)^2(3bB-aC)\log(-\sin(c+dx))}{ad}+\frac{a^3x\left(a^3B+3a^2bC-3ab^2B-b^3C\right)}{a^2+b^2}-\frac{b^2\cdots}{\cdots}}{a\left(\cdots\right.}}{a(a^2+b^2)}}{a}$$

$$\frac{B\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]^3*(B*Tan[c + d*x] + C*Tan[c + d*x]^2))/(a + b*Tan[c + d*
x])^3,x]
```

output
```
-((B*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2)) - ((b*(2*a^2*B + 3*b^2*B
- a*b*C))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (((a^3*(a^3*B - 3*a
*b^2*B + 3*a^2*b*C - b^3*C)*x)/(a^2 + b^2) + ((a^2 + b^2)^2*(3*b*B - a*C)*
Log[-Sin[c + d*x]])/(a*d) - (b^2*(10*a^4*b*B + 9*a^2*b^3*B + 3*b^5*B - 6*a
^5*C - 3*a^3*b^2*C - a*b^4*C)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^
2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(a^4*B + 6*a^2*b^2*B + 3*b^4*B - 3*a^3*b
*C - a*b^3*C))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a*(a^2 + b^2)))/a
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4115
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[1/b^2   Int[(a + b*Tan[e + f*x])^(m
+ 1)*(c + d*Tan[e + f*x])^n*(b*B - a*C + b*C*Tan[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[A*b^2 -
a*b*B + a^2*C, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.44. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.44.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{B}{a^{3}\tan(dx+c)}+\frac{(-3Bb+Ca)\ln(\tan(dx+c))}{a^{4}}+\frac{\frac{\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3Ca\,b^{2}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-B\,a^{3}+3Ba\,b^{2}-3C\,a^{2}b+C\,b^{3}\right)}{\left(a^{2}+b^{2}\right)^{3}}}{}$
default	$\frac{-\frac{B}{a^{3}\tan(dx+c)}+\frac{(-3Bb+Ca)\ln(\tan(dx+c))}{a^{4}}+\frac{\frac{\left(3B\,a^{2}b-B\,b^{3}-C\,a^{3}+3Ca\,b^{2}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-B\,a^{3}+3Ba\,b^{2}-3C\,a^{2}b+C\,b^{3}\right)}{\left(a^{2}+b^{2}\right)^{3}}}{}$
parallelrisch	$\frac{20b^{2}(a+b\tan(dx+c))^{2}\left(B\,a^{4}b+\frac{9}{10}B\,a^{2}b^{3}+\frac{3}{10}B\,b^{5}-\frac{3}{5}C\,a^{5}-\frac{3}{10}C\,a^{3}b^{2}-\frac{1}{10}Ca\,b^{4}\right)\ln(a+b\tan(dx+c))+3a^{4}\left(B\,a^{2}b-\frac{1}{3}B\,b\right)}{}$
norman	$\frac{\frac{b\left(3B\,a^{4}b+11B\,a^{2}b^{3}+6B\,b^{5}-4C\,a^{3}b^{2}-2Ca\,b^{4}\right)\tan(dx+c)^{3}}{d\,a^{3}\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{B\tan(dx+c)}{ad}+\frac{b^{2}\left(4B\,a^{4}b+17B\,a^{2}b^{3}+9B\,b^{5}-7C\,a^{3}b^{2}-3Ca\,b^{4}\right)\tan(d}{2a^{4}d\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}}{\tan(}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,method
=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*B/tan(d*x+c)+(-3*B*b+C*a)/a^4*ln(tan(d*x+c))+1/(a^2+b^2)^3*(1/
2*(3*B*a^2*b-B*b^3-C*a^3+3*C*a*b^2)*ln(1+tan(d*x+c)^2)+(-B*a^3+3*B*a*b^2-3
*C*a^2*b+C*b^3)*arctan(tan(d*x+c)))-b^2*(4*B*a^2*b+2*B*b^3-3*C*a^3-C*a*b^2
)/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))+b^2*(10*B*a^4*b+9*B*a^2*b^3+3*B*b^5-6*C
*a^5-3*C*a^3*b^2-C*a*b^4)/(a^2+b^2)^3/a^4*ln(a+b*tan(d*x+c))-1/2*(B*b-C*a)
*b^2/(a^2+b^2)/a^2/(a+b*tan(d*x+c))^2)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(283) = 566$.

Time = 0.37 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.20

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = -\frac{2\,Ba^9+6\,Ba^7b^2+6\,Ba^5b^4+2\,Ba^3b^6+\left(7\,Ca^5b^4-9\,Ba^4b^5+Ca^3b^6-3\,Ba^2b^7+2\left(Ba^7b^2+3\,Ca^6b^3-\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="fricas")
```

output
```
-1/2*(2*B*a^9 + 6*B*a^7*b^2 + 6*B*a^5*b^4 + 2*B*a^3*b^6 + (7*C*a^5*b^4 - 9
*B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7 + 2*(B*a^7*b^2 + 3*C*a^6*b^3 - 3*B*a^
5*b^4 - C*a^4*b^5)*d*x)*tan(d*x + c)^3 + 2*(B*a^7*b^2 + 4*C*a^6*b^3 - 2*B*
a^5*b^4 - 3*C*a^4*b^5 + 6*B*a^3*b^6 - C*a^2*b^7 + 3*B*a*b^8 + 2*(B*a^8*b +
 3*C*a^7*b^2 - 3*B*a^6*b^3 - C*a^5*b^4)*d*x)*tan(d*x + c)^2 - ((C*a^7*b^2
- 3*B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b^7 + C*
a*b^8 - 3*B*b^9)*tan(d*x + c)^3 + 2*(C*a^8*b - 3*B*a^7*b^2 + 3*C*a^6*b^3 -
 9*B*a^5*b^4 + 3*C*a^4*b^5 - 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x
+ c)^2 + (C*a^9 - 3*B*a^8*b + 3*C*a^7*b^2 - 9*B*a^6*b^3 + 3*C*a^5*b^4 - 9*
B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1)) + ((6*C*a^5*b^4 - 10*B*a^4*b^5 + 3*C*a^3*b^6 - 9*B*a^2*b
^7 + C*a*b^8 - 3*B*b^9)*tan(d*x + c)^3 + 2*(6*C*a^6*b^3 - 10*B*a^5*b^4 + 3
*C*a^4*b^5 - 9*B*a^3*b^6 + C*a^2*b^7 - 3*B*a*b^8)*tan(d*x + c)^2 + (6*C*a^
7*b^2 - 10*B*a^6*b^3 + 3*C*a^5*b^4 - 9*B*a^4*b^5 + C*a^3*b^6 - 3*B*a^2*b^7
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) + (4*B*a^8*b + 12*B*a^6*b^3 - 9*C*a^5*b^4 + 23*B*a^4*b^5 -
 3*C*a^3*b^6 + 9*B*a^2*b^7 + 2*(B*a^9 + 3*C*a^8*b - 3*B*a^7*b^2 - C*a^6*b^
3)*d*x)*tan(d*x + c))/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(
d*x + c)^3 + 2*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2
 + (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

3.44.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)**3*(B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))**3
,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.58

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\,(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(6\,Ca^5b^2-10\,Ba^4b^3+3\,Ca^3b^4-9\,Ba^2b^5+Cab^6-3\,Bb^7)\log(b\tan(dx+c)+a)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6}+\frac{(Ca^3-3\,Ba^2b-}{a^6}}{}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="maxima")
```

output
```
-1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^2 - 10*B*a^4*b^3 + 3*C*a^3*b^4 - 9*B*a^
2*b^5 + C*a*b^6 - 3*B*b^7)*log(b*tan(d*x + c) + a)/(a^10 + 3*a^8*b^2 + 3*a
^6*b^4 + a^4*b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*B*a^6 + 4*B*a^4*b^2 + 2
*B*a^2*b^4 + 2*(B*a^4*b^2 - 3*C*a^3*b^3 + 6*B*a^2*b^4 - C*a*b^5 + 3*B*b^6)
*tan(d*x + c)^2 + (4*B*a^5*b - 7*C*a^4*b^2 + 17*B*a^3*b^3 - 3*C*a^2*b^4 +
9*B*a*b^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^3 +
 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^2 + (a^9 + 2*a^7*b^2 + a^5*b
^4)*tan(d*x + c)) - 2*(C*a - 3*B*b)*log(tan(d*x + c))/a^4)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.95

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\left(Ba^3+3\,Ca^2b-3\,Bab^2-Cb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(Ca^3-3\,Ba^2b-3\,Cab^2+Bb^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(6\,Ca^5b^3-10\,Ba^4b^4+3\,Ca^3b^5-9\,Ba^2b^6+\right.}{a^{10}b+3\,a^8b^3+3\,a}}{}$$

input
```
integrate(cot(d*x+c)^3*(B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^3,x,
 algorithm="giac")
```

output
```
-1/2*(2*(B*a^3 + 3*C*a^2*b - 3*B*a*b^2 - C*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (C*a^3 - 3*B*a^2*b - 3*C*a*b^2 + B*b^3)*log(tan(d*x
+ c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(6*C*a^5*b^3 - 10*B*a^
4*b^4 + 3*C*a^3*b^5 - 9*B*a^2*b^6 + C*a*b^7 - 3*B*b^8)*log(abs(b*tan(d*x +
 c) + a))/(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7) - (18*C*a^5*b^4*tan(d
*x + c)^2 - 30*B*a^4*b^5*tan(d*x + c)^2 + 9*C*a^3*b^6*tan(d*x + c)^2 - 27*
B*a^2*b^7*tan(d*x + c)^2 + 3*C*a*b^8*tan(d*x + c)^2 - 9*B*b^9*tan(d*x + c)
^2 + 42*C*a^6*b^3*tan(d*x + c) - 68*B*a^5*b^4*tan(d*x + c) + 26*C*a^4*b^5*
tan(d*x + c) - 66*B*a^3*b^6*tan(d*x + c) + 8*C*a^2*b^7*tan(d*x + c) - 22*B
*a*b^8*tan(d*x + c) + 25*C*a^7*b^2 - 39*B*a^6*b^3 + 19*C*a^5*b^4 - 41*B*a^
4*b^5 + 6*C*a^3*b^6 - 14*B*a^2*b^7)/((a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b
^6)*(b*tan(d*x + c) + a)^2) - 2*(C*a - 3*B*b)*log(abs(tan(d*x + c)))/a^4 +
 2*(C*a*tan(d*x + c) - 3*B*b*tan(d*x + c) + B*a)/(a^4*tan(d*x + c)))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-6\,C\,a^5+10\,B\,a^4\,b-3\,C\,a^3\,b^2+9\,B\,a^2\,b^3-C\,a\,b^4+3\,B\,b^5\right)}{a^4\,d\,{\left(a^2+b^2\right)}^3}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(3\,B\,b-C\,a\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B-C\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$-\frac{\frac{B}{a}+\frac{{\tan(c+d\,x)}^2\,\left(B\,a^4\,b^2-3\,C\,a^3\,b^3+6\,B\,a^2\,b^4-C\,a\,b^5+3\,B\,b^6\right)}{a^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(4\,B\,a^4\,b-7\,C\,a^3\,b^2+17\,B\,a^2\,b^3-3\,C\,a\,b^4+9\,B\,b^5\right)}{2\,a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^2+b^2\tan\left(c+d\,x\right)^3\right)}$$

input
```
int((cot(c + d*x)^3*(B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c + d*
x))^3,x)
```

output
```
(b^2*log(a + b*tan(c + d*x))*(3*B*b^5 - 6*C*a^5 + 9*B*a^2*b^3 - 3*C*a^3*b^
2 + 10*B*a^4*b - C*a*b^4))/(a^4*d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)
*(B*1i - C))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c + d*x)
)*(3*B*b - C*a))/(a^4*d) - (log(tan(c + d*x) + 1i)*(B - C*1i))/(2*d*(a*b^2
*3i - 3*a^2*b - a^3*1i + b^3)) - (B/a + (tan(c + d*x)^2*(3*B*b^6 + 6*B*a^2
*b^4 + B*a^4*b^2 - 3*C*a^3*b^3 - C*a*b^5))/(a^3*(a^4 + b^4 + 2*a^2*b^2)) +
 (tan(c + d*x)*(9*B*b^5 + 17*B*a^2*b^3 - 7*C*a^3*b^2 + 4*B*a^4*b - 3*C*a*b
^4))/(2*a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d
*x)^3 + 2*a*b*tan(c + d*x)^2))
```

3.44. $\int \frac{\cot^3(c+dx)\left(B\tan(c+dx)+C\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}\,dx$

3.45 $\int \tan^2(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\right.$

3.45.1 Optimal result

Integrand size = 39, antiderivative size = 132

$$\int \tan^2(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{C(b\tan(c+dx))^{3+n}}{b^3d(3+n)}+\frac{(A-C)\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(c+dx)\right)(b\tan(c+dx))^{3+n}}{b^3d(3+n)}+\frac{B\,\text{Hypergeometric2F1}\left(1,\frac{4+n}{2},\frac{6+n}{2},-\tan^2(c+dx)\right)(b\tan(c+dx))^{4+n}}{b^4d(4+n)}$$

output
```
C*(b*tan(d*x+c))^(3+n)/b^3/d/(3+n)+(A-C)*hypergeom([1, 3/2+1/2*n],[5/2+1/2
*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(3+n)/b^3/d/(3+n)+B*hypergeom([1, 2+1/2*
n],[3+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(4+n)/b^4/d/(4+n)
```

3.45.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \tan^2(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\tan^3(c+dx)(b\tan(c+dx))^n\left(C(4+n)+(A-C)(4+n)\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(c+d\right.\right.}{d(3+n)(4+n)}$$

input
```
Integrate[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(Tan[c + d*x]^3*(b*Tan[c + d*x])^n*(C*(4 + n) + (A - C)*(4 + n)*Hypergeome
tric2F1[1, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2] + B*(3 + n)*Hypergeometr
ic2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(3 + n)*
(4 + n))
```

3.45.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2030, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\, dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\tan(c+dx))^{n+2}\left(C\tan^2(c+dx)+B\tan(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (b\tan(c+dx))^{n+2}\left(C\tan(c+dx)^2+B\tan(c+dx)+A\right)dx}{b^2}$$

$$\Big\downarrow \; 4113$$

$$\frac{\int (b\tan(c+dx))^{n+2}(A-C+B\tan(c+dx))dx+\frac{C(b\tan(c+dx))^{n+3}}{bd(n+3)}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (b\tan(c+dx))^{n+2}(A-C+B\tan(c+dx))dx+\frac{C(b\tan(c+dx))^{n+3}}{bd(n+3)}}{b^2}$$

$$\Big\downarrow \; 4021$$

$$\frac{(A-C)\int (b\tan(c+dx))^{n+2}dx+\frac{B\int (b\tan(c+dx))^{n+3}dx}{b}+\frac{C(b\tan(c+dx))^{n+3}}{bd(n+3)}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A-C)\int (b\tan(c+dx))^{n+2}dx+\frac{B\int (b\tan(c+dx))^{n+3}dx}{b}+\frac{C(b\tan(c+dx))^{n+3}}{bd(n+3)}}{b^2}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{\frac{b(A-C)\int \frac{(b\tan(c+dx))^{n+2}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}+\frac{B\int \frac{(b\tan(c+dx))^{n+3}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}+\frac{C(b\tan(c+dx))^{n+3}}{bd(n+3)}}{b^2}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\frac{(A-C)(b\tan(c+dx))^{n+3}\text{Hypergeometric2F1}\left(1,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(c+dx)\right)}{bd(n+3)}+\frac{B(b\tan(c+dx))^{n+4}\text{Hypergeometric2F1}\left(1,\frac{n+4}{2},\frac{n+6}{2},-\tan^2(c+dx)\right)}{b^2d(n+4)}}{b^2}$$

input
```
Int[Tan[c + d*x]^2*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
((C*(b*Tan[c + d*x])^(3 + n))/(b*d*(3 + n)) + ((A - C)*Hypergeometric2F1[1
, (3 + n)/2, (5 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3 + n))/(b*d*(3
 + n)) + (B*Hypergeometric2F1[1, (4 + n)/2, (6 + n)/2, -Tan[c + d*x]^2]*(b
*Tan[c + d*x])^(4 + n))/(b^2*d*(4 + n)))/b^2
```

3.45.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.45.4 Maple [F]

$$\int \tan(dx+c)^2 (b\tan(dx+c))^n \left(A+B\tan(dx+c)+C\tan(dx+c)^2\right) dx$$

input
```
int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

output
```
int(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

3.45.5 Fricas [F]

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int (C\tan(dx+c)^2 + B\tan(dx+c) + A)(b\tan(dx+c))^n \tan(dx+c)^2 \, dx$$

input
```
integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
integral((C*tan(d*x + c)^4 + B*tan(d*x + c)^3 + A*tan(d*x + c)^2)*(b*tan(d
*x + c))^n, x)
```

3.45.6 Sympy [F]

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int (b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\tan^2(c+dx)\, dx$$

input
```
integrate(tan(d*x+c)**2*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(
c + d*x)**2, x)
```

3.45.7 Maxima [F]

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int \left(C\tan(dx+c)^2+B\tan(dx+c)+A\right)(b\tan(dx+c))^n\tan(dx+c)^2\, dx$$

input
```
integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d
*x + c)^2, x)
```

3.45.8 Giac [F]

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int \left(C\tan(dx+c)^2+B\tan(dx+c)+A\right)(b\tan(dx+c))^n\tan(dx+c)^2\, dx$$

input
```
integrate(tan(d*x+c)^2*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d
*x + c)^2, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \tan^2(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int \tan(c+d\,x)^2\,(b\tan(c+d\,x))^n\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right) dx$$

input
```
int(tan(c + d*x)^2*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)
^2),x)
```

output
```
int(tan(c + d*x)^2*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)
^2), x)
```

3.46 $\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\right.$

3.46.1 Optimal result

Integrand size = 39, antiderivative size = 154

$$\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$=\frac{C\tan^{1+m}(c+dx)(b\tan(c+dx))^n}{d(1+m+n)}$$

$$+\frac{(A-C)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)(b\tan(c+}{d(1+m+n)}$$

$$+\frac{B\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(2+m+n),\frac{1}{2}(4+m+n),-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)(b\tan(c+dx))}{d(2+m+n)}$$

output
```
C*tan(d*x+c)^(1+m)*(b*tan(d*x+c))^n/d/(1+m+n)+(A-C)*hypergeom([1, 1/2+1/2*
m+1/2*n],[3/2+1/2*m+1/2*n],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)*(b*tan(d*x+c))^
n/d/(1+m+n)+B*hypergeom([1, 1+1/2*m+1/2*n],[2+1/2*m+1/2*n],-tan(d*x+c)^2)*
tan(d*x+c)^(2+m)*(b*tan(d*x+c))^n/d/(2+m+n)
```

3.46.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{\tan^{1+m}(c+dx)(b\tan(c+dx))^n\left(\frac{C}{1+m+n}+\frac{(A-C)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(c+dx)\right)}{1+m+n}+\frac{B\,\text{Hyperg}}{}\right.}{d}$$

input
```
Integrate[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c
+ d*x]^2),x]
```

output
```
(Tan[c + d*x]^(1 + m)*(b*Tan[c + d*x])^n*(C/(1 + m + n) + ((A - C)*Hyperge
ometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2])/(1 + m + n)
+ (B*Hypergeometric2F1[1, (2 + m + n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*T
an[c + d*x])/(2 + m + n)))/d
```

3.46.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.205, Rules used = {2034, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\int \tan^{m+n}(c+dx)\left(C\tan^2(c+dx)+B\tan(c+dx)+A\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\int \tan(c+dx)^{m+n}\left(C\tan(c+dx)^2+B\tan(c+dx)+A\right)dx$$

$$\Big\downarrow \text{4113}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left(\int \tan^{m+n}(c+dx)(A-C+B\tan(c+dx))dx+\frac{C\tan^{m+n+1}(c+dx)}{d(m+n+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left(\int \tan(c+dx)^{m+n}(A-C+B\tan(c+dx))dx+\frac{C\tan^{m+n+1}(c+dx)}{d(m+n+1)}\right)$$

$$\Big\downarrow \text{4021}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left((A-C)\int \tan^{m+n}(c+dx)dx+B\int \tan^{m+n+1}(c+dx)dx+\frac{C\tan^{m+n+1}(c+dx)}{d(m+n+1)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left((A-C)\int \tan(c+dx)^{m+n}dx+B\int \tan(c+dx)^{m+n+1}dx+\frac{C\tan^{m+n+1}(c+dx)}{d(m+n+1)}\right)$$

$$\Big\downarrow \text{3957}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left(\frac{(A-C)\int \frac{\tan^{m+n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{B\int \frac{\tan^{m+n+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{C\tan^{m+n+1}(c+dx)}{d(m+n+1)}\right)$$

$$\Big\downarrow \text{278}$$

$$\tan^{-n}(c+dx)(b\tan(c+dx))^n\left(\frac{(A-C)\tan^{m+n+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(c+dx)\right)}{d(m+n+1)}+\frac{B\tan\right.$$

input
```
Int[Tan[c + d*x]^m*(b*Tan[c + d*x])^n*(A + B*Tan[c + d*x] + C*Tan[c + d*x]
^2),x]
```

output
```
((b*Tan[c + d*x])^n*((C*Tan[c + d*x]^(1 + m + n))/(d*(1 + m + n)) + ((A -
C)*Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[c + d*x]^2]*Tan
[c + d*x]^(1 + m + n))/(d*(1 + m + n)) + (B*Hypergeometric2F1[1, (2 + m +
n)/2, (4 + m + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m + n))/(d*(2 + m
+ n))))/Tan[c + d*x]^n
```

3.46. $\int \tan^m(c+dx)(b\tan(c+dx))^n\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.46.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] && !IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

3.46.4 Maple [F]

$$\int \tan(dx+c)^m (b\tan(dx+c))^n \left(A+B\tan(dx+c)+C\tan(dx+c)^2\right) dx$$

input
```
int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

output
```
int(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

3.46.5 Fricas [F]

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int (C\tan(dx+c)^2+B\tan(dx+c)+A)(b\tan(dx+c))^n \tan(dx+c)^m \, dx$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="fricas")
```

output
```
integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d*
x + c)^m, x)
```

3.46.6 Sympy [F]

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$= \int (b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) \tan^m(c+dx)\, dx$$

input
```
integrate(tan(d*x+c)**m*(b*tan(d*x+c))**n*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)
,x)
```

output
```
Integral((b*tan(c + d*x))**n*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(
c + d*x)**m, x)
```

3.46.7 Maxima [F]

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$=\int \left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\left(b\tan\left(dx+c\right)\right)^n\tan\left(dx+c\right)^m dx$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="maxima")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d
*x + c)^m, x)
```

3.46.8 Giac [F]

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$=\int \left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\left(b\tan\left(dx+c\right)\right)^n\tan\left(dx+c\right)^m dx$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^n*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x,
 algorithm="giac")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*(b*tan(d*x + c))^n*tan(d
*x + c)^m, x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$$
$$=\int \tan(c+d\,x)^m\,\left(b\tan(c+d\,x)\right)^n\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right) dx$$

input
```
int(tan(c + d*x)^m*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)
^2),x)
```

output
```
int(tan(c + d*x)^m*(b*tan(c + d*x))^n*(A + B*tan(c + d*x) + C*tan(c + d*x)
^2), x)
```

3.46. $\int \tan^m(c+dx)(b\tan(c+dx))^n \left(A+B\tan(c+dx)+C\tan^2(c+dx)\right) dx$

3.47 $\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}(A+B\tan(c+dx)+C$

3.47.1 Optimal result

Integrand size = 41, antiderivative size = 170

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2C\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)}$$
$$+\frac{2(A-C)\operatorname{Hypergeometric2F1}\left(1,\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(3+2m)}$$
$$+\frac{2B\operatorname{Hypergeometric2F1}\left(1,\frac{1}{4}(5+2m),\frac{1}{4}(9+2m),-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)\sqrt{b\tan(c+dx)}}{d(5+2m)}$$

output
```
2*C*(b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m)/d/(3+2*m)+2*(A-C)*hypergeom([1,
3/4+1/2*m],[7/4+1/2*m],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1/2)*tan(d*x+c)^(1+m
)/d/(3+2*m)+2*B*hypergeom([1, 5/4+1/2*m],[9/4+1/2*m],-tan(d*x+c)^2)*(b*tan
(d*x+c))^(1/2)*tan(d*x+c)^(2+m)/d/(5+2*m)
```

3.47.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\frac{2\tan^{1+m}(c+dx)\sqrt{b\tan(c+dx)}\left(C(5+2m)+(A-C)(5+2m)\,\text{Hypergeometric2F1}\left(1,\tfrac{1}{4}(3+2m),\tfrac{1}{4}(7\right.\right.}{d(3+}$$

input
```
Integrate[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[
c + d*x]^2),x]
```

output
```
(2*Tan[c + d*x]^(1 + m)*Sqrt[b*Tan[c + d*x]]*(C*(5 + 2*m) + (A - C)*(5 + 2
*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2] + B*(3
 + 2*m)*Hypergeometric2F1[1, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[c + d*x]^2]*Ta
n[c + d*x]))/(d*(3 + 2*m)*(5 + 2*m))
```

3.47.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(c+dx)}\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt{b\tan(c+dx)}\int \tan^{m+\frac{1}{2}}(c+dx)\left(C\tan^2(c+dx)+B\tan(c+dx)+A\right)dx}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(c+dx)}\int \tan(c+dx)^{m+\frac{1}{2}}\left(C\tan(c+dx)^2+B\tan(c+dx)+A\right)dx}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4113}$$

$$\frac{\sqrt{b\tan(c+dx)}\left(\int \tan^{m+\frac{1}{2}}(c+dx)(A-C+B\tan(c+dx))dx+\frac{2C\tan^{m+\frac{3}{2}}(c+dx)}{d(2m+3)}\right)}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(c+dx)}\left(\int \tan(c+dx)^{m+\frac{1}{2}}(A-C+B\tan(c+dx))dx+\frac{2C\tan^{m+\frac{3}{2}}(c+dx)}{d(2m+3)}\right)}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4021}$$

$$\frac{\sqrt{b\tan(c+dx)}\left((A-C)\int \tan^{m+\frac{1}{2}}(c+dx)dx+B\int \tan^{m+\frac{3}{2}}(c+dx)dx+\frac{2C\tan^{m+\frac{3}{2}}(c+dx)}{d(2m+3)}\right)}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(c+dx)}\left((A-C)\int \tan(c+dx)^{m+\frac{1}{2}}dx+B\int \tan(c+dx)^{m+\frac{3}{2}}dx+\frac{2C\tan^{m+\frac{3}{2}}(c+dx)}{d(2m+3)}\right)}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{b\tan(c+dx)}\left(\frac{(A-C)\int \frac{\tan^{m+\frac{1}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{B\int \frac{\tan^{m+\frac{3}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{2C\tan^{m+\frac{3}{2}}(c+dx)}{d(2m+3)}\right)}{\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{b\tan(c+dx)}\left(\frac{2(A-C)\tan^{m+\frac{3}{2}}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),-\tan^2(c+dx)\right)}{d(2m+3)}+\frac{2B\tan^{m+\frac{5}{2}}(c+dx)\text{Hypergeometric}}{d}\right.}{\sqrt{\tan(c+dx)}}$$

input
```
Int[Tan[c + d*x]^m*Sqrt[b*Tan[c + d*x]]*(A + B*Tan[c + d*x] + C*Tan[c + d*
x]^2),x]
```

output
```
(Sqrt[b*Tan[c + d*x]]*((2*C*Tan[c + d*x]^(3/2 + m))/(d*(3 + 2*m)) + (2*(A
- C)*Hypergeometric2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c
 + d*x]^(3/2 + m))/(d*(3 + 2*m)) + (2*B*Hypergeometric2F1[1, (5 + 2*m)/4,
(9 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(5/2 + m))/(d*(5 + 2*m))))/Sqrt
[Tan[c + d*x]]
```

3.47. $\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$

3.47.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.47.4 Maple [F]

$$\int \tan(dx+c)^{m} \sqrt{b\tan(dx+c)}\left(A+B\tan(dx+c)+C\tan(dx+c)^{2}\right)dx$$

input
```
int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

output
```
int(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2),x)
```

3.47.5 Fricas [F]

$$\int \tan^{m}(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^{2}(c+dx)\right)\,dx$$
$$=\int \left(C\tan(dx+c)^{2}+B\tan(dx+c)+A\right)\sqrt{b\tan(dx+c)}\tan(dx+c)^{m}\,dx$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2
),x, algorithm="fricas")
```

output
```
integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(
d*x + c)^m, x)
```

3.47.6 Sympy [F]

$$\int \tan^{m}(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^{2}(c+dx)\right)\,dx$$
$$=\int \sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^{2}(c+dx)\right)\tan^{m}(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)
**2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x))*(A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan
(c + d*x)**m, x)
```

3.47.7 Maxima [F(-1)]

Timed out.

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2
),x, algorithm="maxima")
```

output
```
Timed out
```

3.47.8 Giac [F]

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\int \left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\sqrt{b\tan\left(dx+c\right)}\tan\left(dx+c\right)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)+C*tan(d*x+c)^2
),x, algorithm="giac")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan
(d*x + c)^m, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)\sqrt{b\tan(c+dx)}\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)\,dx$$
$$=\int \tan(c+d\,x)^m\,\sqrt{b\tan\left(c+d\,x\right)}\,\left(C\tan(c+d\,x)^2+B\tan(c+d\,x)+A\right)\,dx$$

input
```
int(tan(c + d*x)^m*(b*tan(c + d*x))^(1/2)*(A + B*tan(c + d*x) + C*tan(c +
d*x)^2),x)
```

output
```
int(tan(c + d*x)^m*(b*tan(c + d*x))^(1/2)*(A + B*tan(c + d*x) + C*tan(c +
d*x)^2), x)
```

3.48 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx)+C\tan^2(c+dx))}{\sqrt{b\tan(c+dx)}}\,dx$

3.48.1 Optimal result

Integrand size = 41, antiderivative size = 170

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \frac{2C\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{2(A-C)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+2m)\sqrt{b\tan(c+dx)}} + \frac{2B\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{d(3+2m)\sqrt{b\tan(c+dx)}}$$

output
```
2*C*tan(d*x+c)^(1+m)/d/(1+2*m)/(b*tan(d*x+c))^(1/2)+2*(A-C)*hypergeom([1,
1/4+1/2*m],[5/4+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1+2*m)/(b*tan(d*
x+c))^(1/2)+2*B*hypergeom([1, 3/4+1/2*m],[7/4+1/2*m],-tan(d*x+c)^2)*tan(d*
x+c)^(2+m)/d/(3+2*m)/(b*tan(d*x+c))^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \frac{2\tan^{1+m}(c+dx)\left(C(3+2m)+(A-C)(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),-\tan^2(\right.\right.}{d(1+2m)(3+2m)}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*
Tan[c + d*x]],x]
```

output
```
(2*Tan[c + d*x]^(1 + m)*(C*(3 + 2*m) + (A - C)*(3 + 2*m)*Hypergeometric2F1
[1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2] + B*(1 + 2*m)*Hypergeometri
c2F1[1, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]))/(d*(1 +
2*m)*(3 + 2*m)*Sqrt[b*Tan[c + d*x]])
```

3.48.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2034, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt{\tan(c+dx)}\int \tan^{m-\frac{1}{2}}(c+dx)\left(C\tan^2(c+dx)+B\tan(c+dx)+A\right)dx}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\tan(c+dx)}\int \tan(c+dx)^{m-\frac{1}{2}}\left(C\tan(c+dx)^2+B\tan(c+dx)+A\right)dx}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{4113}$$

$$\frac{\sqrt{\tan(c+dx)}\left(\int \tan^{m-\frac{1}{2}}(c+dx)(A-C+B\tan(c+dx))dx+\frac{2C\tan^{m+\frac{1}{2}}(c+dx)}{d(2m+1)}\right)}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\tan(c+dx)}\left(\int \tan(c+dx)^{m-\frac{1}{2}}(A-C+B\tan(c+dx))dx+\frac{2C\tan^{m+\frac{1}{2}}(c+dx)}{d(2m+1)}\right)}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{4021}$$

$$\frac{\sqrt{\tan(c+dx)}\left((A-C)\int \tan^{m-\frac{1}{2}}(c+dx)dx+B\int \tan^{m+\frac{1}{2}}(c+dx)dx+\frac{2C\tan^{m+\frac{1}{2}}(c+dx)}{d(2m+1)}\right)}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\tan(c+dx)}\left((A-C)\int \tan(c+dx)^{m-\frac{1}{2}}dx+B\int \tan(c+dx)^{m+\frac{1}{2}}dx+\frac{2C\tan^{m+\frac{1}{2}}(c+dx)}{d(2m+1)}\right)}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\sqrt{\tan(c+dx)}\left(\frac{(A-C)\int \frac{\tan^{m-\frac{1}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{B\int \frac{\tan^{m+\frac{1}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{2C\tan^{m+\frac{1}{2}}(c+dx)}{d(2m+1)}\right)}{\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{\tan(c+dx)}\left(\frac{2(A-C)\tan^{m+\frac{1}{2}}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),-\tan^2(c+dx)\right)}{d(2m+1)}+\frac{2B\tan^{m+\frac{3}{2}}(c+dx)\,\text{Hypergeometric2}}{d(}\right.}{\sqrt{b\tan(c+dx)}}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[b*Tan[c
 + d*x]],x]
```

output
```
(Sqrt[Tan[c + d*x]]*((2*C*Tan[c + d*x]^(1/2 + m))/(d*(1 + 2*m)) + (2*(A -
C)*Hypergeometric2F1[1, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c +
 d*x]^(1/2 + m))/(d*(1 + 2*m)) + (2*B*Hypergeometric2F1[1, (3 + 2*m)/4, (7
 + 2*m)/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(3/2 + m))/(d*(3 + 2*m))))/Sqrt[b
*Tan[c + d*x]]
```

3.48. $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx)+C\tan^2(c+dx))}{\sqrt{b\tan(c+dx)}}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] && !IntegerQ[2*m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && !LeQ[m, -1]
```

3.48. $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx)+C\tan^2(c+dx))}{\sqrt{b\tan(c+dx)}}\,dx$

3.48.4 Maple [F]

$$\int \frac{\tan (dx+c)^m \left(A+B\tan (dx+c)+C\tan (dx+c)^2\right)}{\sqrt{b\tan (dx+c)}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x)
```

3.48.5 Fricas [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{\left(C\tan (dx+c)^2+B\tan (dx+c)+A\right)\tan (dx+c)^m}{\sqrt{b\tan (dx+c)}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2
),x, algorithm="fricas")
```

output
```
integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c))*tan(
d*x + c)^m/(b*tan(d*x + c)), x)
```

3.48.6 Sympy [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{\left(A+B\tan (c+dx)+C\tan^2 (c+dx)\right)\tan^m (c+dx)}{\sqrt{b\tan (c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(b*tan(d*x+c))**(
1/2),x)
```

output
```
Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(b*t
an(c + d*x)), x)
```

3.48.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2
),x, algorithm="maxima")
```

output
```
Timed out
```

3.48.8 Giac [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(b*tan(d*x+c))^(1/2
),x, algorithm="giac")
```

output
```
Timed out
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^m\,\left(C\,\tan(c+d\,x)^2+B\,\tan(c+d\,x)+A\right)}{\sqrt{b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(b*tan(c + d*
x))^(1/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(b*tan(c + d*
x))^(1/2), x)
```

3.49 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx)+C\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

3.49.1 Optimal result

Integrand size = 43, antiderivative size = 328

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx =$$

$$-\frac{\left(bB+\sqrt{-b^2}(A-C)\right)\text{AppellF1}\left(\frac{1}{2},1,-m,\frac{3}{2},\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},1+\frac{b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-}}{b\left(a-\sqrt{-b^2}\right)d}$$

$$-\frac{\left(bB-\sqrt{-b^2}(A-C)\right)\text{AppellF1}\left(\frac{1}{2},1,-m,\frac{3}{2},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}},1+\frac{b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-}}{b\left(a+\sqrt{-b^2}\right)d}$$

$$+\frac{2C\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+\frac{b\tan(c+dx)}{a}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}\sqrt{a+b\tan(c+dx)}}{bd}$$

output
```
2*C*hypergeom([1/2, -m],[3/2],1+b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^(1/2)*tan
(d*x+c)^m/b/d/((-b*tan(d*x+c)/a)^m)-AppellF1(1/2,1,-m,3/2,(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)),1+b*tan(d*x+c)/a)*(B*b-(A-C)*(-b^2)^(1/2))*(a+b*tan(d*x+
c))^(1/2)*tan(d*x+c)^m/b/d/(a+(-b^2)^(1/2))/((-b*tan(d*x+c)/a)^m)-AppellF1
(1/2,1,-m,3/2,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),1+b*tan(d*x+c)/a)*(B*b+(A-
C)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1/2)*tan(d*x+c)^m/b/d/(a-(-b^2)^(1/2))/
((-b*tan(d*x+c)/a)^m)
```

3.49.2 Mathematica [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a
+ b*Tan[c + d*x]],x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a
+ b*Tan[c + d*x]], x]
```

3.49.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {3042, 4138, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^m\left(A+B\tan(c+dx)+C\tan(c+dx)^2\right)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 4138$$

$$\frac{\int \frac{\tan^m(c+dx)\left(C\tan^2(c+dx)+B\tan(c+dx)+A\right)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 2353$$

$$\frac{\int \left(\frac{C\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}+\frac{(i(A-C)-B)\tan^m(c+dx)}{2(i-\tan(c+dx))\sqrt{a+b\tan(c+dx)}}+\frac{(B+i(A-C))\tan^m(c+dx)}{2(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(A+iB-C)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,\text{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{(A-iB-C)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+}}{2(m+}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x] + C*Tan[c + d*x]^2))/Sqrt[a + b*Ta
n[c + d*x]],x]
```

output
```
((2*C*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*Tan[c + d*x])/a]*Tan[c + d*x]
^m*Sqrt[a + b*Tan[c + d*x]])/(b*(-((b*Tan[c + d*x])/a))^m) + ((A + I*B - C
)*AppellF1[1 + m, 1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]
*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*(1 + m)*Sqrt[a + b*
Tan[c + d*x]]) + ((A - I*B - C)*AppellF1[1 + m, 1/2, 1, 2 + m, -((b*Tan[c
+ d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])
/a])/(2*(1 + m)*Sqrt[a + b*Tan[c + d*x]]))/d
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.49.4 Maple [F]

$$\int \frac{\tan (dx+c)^m \left(A+B\tan (dx+c)+C\tan (dx+c)^2\right)}{\sqrt{a+b\tan (dx+c)}}\,dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1/2),x)
```

3.49.5 Fricas [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\left(C\tan (dx+c)^2+B\tan (dx+c)+A\right)\tan (dx+c)^m}{\sqrt{b\tan (dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1
/2),x, algorithm="fricas")
```

output
```
integral((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan
(d*x + c) + a), x)
```

3.49.6 Sympy [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\left(A+B\tan (c+dx)+C\tan^2 (c+dx)\right)\tan^m (c+dx)}{\sqrt{a+b\tan (c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c)+C*tan(d*x+c)**2)/(a+b*tan(d*x+c))*
*(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x) + C*tan(c + d*x)**2)*tan(c + d*x)**m/sqrt(a +
 b*tan(c + d*x)), x)
```

3.49.7 Maxima [F]

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\left(C\tan\left(dx+c\right)^2+B\tan\left(dx+c\right)+A\right)\tan\left(dx+c\right)^m}{\sqrt{b\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1
/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(d*x + c)^2 + B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*ta
n(d*x + c) + a), x)
```

3.49.8 Giac [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c)+C*tan(d*x+c)^2)/(a+b*tan(d*x+c))^(1
/2),x, algorithm="giac")
```

output
```
Timed out
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)\left(A+B\tan(c+dx)+C\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{{\mathrm{tan}\left(c+d\,x\right)}^m\,\left(C\,{\mathrm{tan}\left(c+d\,x\right)}^2+B\,\mathrm{tan}\left(c+d\,x\right)+A\right)}{\sqrt{a+b\,\mathrm{tan}\left(c+d\,x\right)}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c
+ d*x))^(1/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x) + C*tan(c + d*x)^2))/(a + b*tan(c
+ d*x))^(1/2), x)
```

3.49. $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx)+C\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

3.50 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$

3.50.1 Optimal result

Integrand size = 43, antiderivative size = 353

$$\begin{aligned}
&\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx \\
&= \left(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)-3a^2b(Bc+(A-C)d)+b^3(Bc+(A-C)d)\right)x \\
&\quad -\frac{(3a^2b(Ac-cC-Bd)-b^3(Ac-cC-Bd)+a^3(Bc+(A-C)d)-3ab^2(Bc+(A-C)d))\log(\cos(e+fx))}{f} \\
&\quad +\frac{b(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\tan(e+fx)}{f} \\
&\quad +\frac{(Abc+aBc-bcC+aAd-bBd-aCd)(a+b\tan(e+fx))^2}{2f} \\
&\quad +\frac{(Bc+(A-C)d)(a+b\tan(e+fx))^3}{3f} \\
&\quad -\frac{(aCd-5b(cC+Bd))(a+b\tan(e+fx))^4}{20b^2f}+\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}
\end{aligned}$$

output
```
(a^3*(A*c-B*d-C*c)-3*a*b^2*(A*c-B*d-C*c)-3*a^2*b*(B*c+(A-C)*d)+b^3*(B*c+(A
-C)*d))*x-(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C)*d)-3*a*b
^2*(B*c+(A-C)*d))*ln(cos(f*x+e))/f+b*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d
)-b^2*(B*c+(A-C)*d))*tan(f*x+e)/f+1/2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c
)*(a+b*tan(f*x+e))^2/f+1/3*(B*c+(A-C)*d)*(a+b*tan(f*x+e))^3/f-1/20*(C*a*d-
5*b*(B*d+C*c))*(a+b*tan(f*x+e))^4/b^2/f+1/5*C*d*tan(f*x+e)*(a+b*tan(f*x+e)
)^4/b/f
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.85

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$
$$-\frac{\frac{(aCd-5b(cC+Bd))(a+b\tan(e+fx))^4}{4bf}-\frac{5(3(Abc-aBc-bcC-aAd-bBd+aCd)((ia-b)^3\log(i-\tan(e+fx))-(ia+b)^3\log(i+\tan(e+fx))+6a}{}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f) - (((a*C*d - 5*b*(c*C +
B*d))*(a + b*Tan[e + f*x])^4)/(4*b*f) - (5*(3*(A*b*c - a*B*c - b*c*C - a*A
*d - b*B*d + a*C*d)*((I*a - b)^3*Log[I - Tan[e + f*x]] - (I*a + b)^3*Log[I
 + Tan[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^3*Tan[e + f*x]^2) - (B*c + (A
- C)*d)*((3*I)*(a + I*b)^4*Log[I - Tan[e + f*x]] - (3*I)*(a - I*b)^4*Log[I
 + Tan[e + f*x]] - 6*b^2*(6*a^2 - b^2)*Tan[e + f*x] - 12*a*b^3*Tan[e + f*x
]^2 - 2*b^4*Tan[e + f*x]^3)))/(6*f))/(5*b)
```

3.50.3 Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {3042, 4120, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4120$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}-\frac{\int -(a+b\tan(e+fx))^3\left(-((aCd-5b(cC+Bd))\tan^2(e+fx))+5b(Bc+(A-C)d)\tan(e+fx)+5Abc-aC\right.}{5b}$$

$\big\downarrow$ 25

$$\frac{\int (a+b\tan(e+fx))^3\left(-((aCd-5b(cC+Bd))\tan^2(e+fx))+5b(Bc+(A-C)d)\tan(e+fx)+5Abc-aCd\right)}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$\big\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^3\left(-((aCd-5b(cC+Bd))\tan(e+fx)^2)+5b(Bc+(A-C)d)\tan(e+fx)+5Abc-aCd\right)}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$\big\downarrow$ 4113

$$\frac{\int (a+b\tan(e+fx))^3(5b(Ac-Cc-Bd)+5b(Bc+(A-C)d)\tan(e+fx))dx-\frac{(aCd-5b(Bd+cC))(a+b\tan(e+fx))^4}{4bf}}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$\big\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^3(5b(Ac-Cc-Bd)+5b(Bc+(A-C)d)\tan(e+fx))dx-\frac{(aCd-5b(Bd+cC))(a+b\tan(e+fx))^4}{4bf}}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$\big\downarrow$ 4011

$$\frac{\int (a+b\tan(e+fx))^2(5b(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)-5b(bBc+b(A-C)d-a(Ac-C}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$\big\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^2(5b(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)-5b(bBc+b(A-C)d-a(Ac-C}{5b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

3.50. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\downarrow \text{4011}$$

$$\frac{\int (a+b\tan(e+fx))\left(5b\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)+5b\left((Bc+(A-C)d)\right.\right.}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$$\downarrow \text{3042}$$

$$\frac{\int (a+b\tan(e+fx))\left(5b\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)+5b\left((Bc+(A-C)d)\right.\right.}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$$\downarrow \text{4008}$$

$$\frac{5b\left(a^3(d(A-C)+Bc)+3a^2b(Ac-Bd-cC)-3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)\int \tan(e+fx)dx+}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$$\downarrow \text{3042}$$

$$\frac{5b\left(a^3(d(A-C)+Bc)+3a^2b(Ac-Bd-cC)-3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)\int \tan(e+fx)dx+}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

$$\downarrow \text{3956}$$

$$\frac{\frac{5b^2\tan(e+fx)\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}-\frac{5b\log(\cos(e+fx))\left(a^3(d(A-C)+Bc)+3a^2b(Ac-Bd-cC)-3ab^2(d(A\cdot\right.}{f}}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^4}{5bf}$$

input
```
Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Ta
n[e + f*x]^2),x]
```

3.50. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
(C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^4)/(5*b*f) + (5*b*(a^3*(A*c - c*C -
 B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*c +
 (A - C)*d))*x - (5*b*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) +
 a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Log[Cos[e + f*x]])/f +
 (5*b^2*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A -
 C)*d))*Tan[e + f*x])/f + (5*b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*
C*d)*(a + b*Tan[e + f*x])^2)/(2*f) + (5*b*(B*c + (A - C)*d)*(a + b*Tan[e +
 f*x])^3)/(3*f) - ((a*C*d - 5*b*(c*C + B*d))*(a + b*Tan[e + f*x])^4)/(4*b*
f))/(5*b)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.98

method	result
parts	$\frac{\left(A\,a^{3} d+3A\,a^{2} b c+B\,a^{3} c\right) \ln\left(1+\tan(fx+e)^{2}\right)}{2f}+\frac{\left(B\,b^{3} d+3C a\,b^{2} d+C\,b^{3} c\right)\left(\frac{\tan(fx+e)^{4}}{4}-\frac{\tan(fx+e)^{2}}{2}+\frac{\ln\left(1+\tan(fx+e)^{2}\right.}{2}\right.}{f}$
norman	$(A\,a^{3} c-3A\,a^{2} b d-3A a\,b^{2} c+A\,b^{3} d-B\,a^{3} d-3B\,a^{2} b c+3B a\,b^{2} d+B\,b^{3} c-C\,a^{3} c+3$
derivativedivides	$\frac{\frac{\left(A\,a^{3} d+3A\,a^{2} b c-3A a\,b^{2} d-A\,b^{3} c+B\,a^{3} c-3B\,a^{2} b d-3B a\,b^{2} c+B\,b^{3} d-a^{3} C d-3C\,a^{2} b c+3C a\,b^{2} d+C\,b^{3} c\right) \ln\left(1+\tan(fx+e)^{2}\right)}{2}+(A\,a^{3}}{}$
default	$\frac{\frac{\left(A\,a^{3} d+3A\,a^{2} b c-3A a\,b^{2} d-A\,b^{3} c+B\,a^{3} c-3B\,a^{2} b d-3B a\,b^{2} c+B\,b^{3} d-a^{3} C d-3C\,a^{2} b c+3C a\,b^{2} d+C\,b^{3} c\right) \ln\left(1+\tan(fx+e)^{2}\right)}{2}+(A\,a^{3}}{}$
parallelrisch	$\frac{30C\,a^{3} d\tan(fx+e)^{2}-30C\,b^{3} c\tan(fx+e)^{2}+20B\,b^{3} c\tan(fx+e)^{3}-20C\,b^{3} d\tan(fx+e)^{3}+15B\,b^{3} d\tan(fx+e)^{4}+15C\,b^{3} c}{}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*(A*a^3*d+3*A*a^2*b*c+B*a^3*c)/f*ln(1+tan(f*x+e)^2)+(B*b^3*d+3*C*a*b^2*
d+C*b^3*c)/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+(A
*b^3*d+3*B*a*b^2*d+B*b^3*c+3*C*a^2*b*d+3*C*a*b^2*c)/f*(1/3*tan(f*x+e)^3-ta
n(f*x+e)+arctan(tan(f*x+e)))+(3*A*a^2*b*d+3*A*a*b^2*c+B*a^3*d+3*B*a^2*b*c+
C*a^3*c)/f*(tan(f*x+e)-arctan(tan(f*x+e)))+(3*A*a*b^2*d+A*b^3*c+3*B*a^2*b*
d+3*B*a*b^2*c+C*a^3*d+3*C*a^2*b*c)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)
^2))+A*a^3*c*x+C*b^3*d/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arc
tan(tan(f*x+e)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.18

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$
$$= \frac{12\, Cb^3 d \tan\left(fx+e\right)^5 + 15\left(Cb^3c + (3\, Cab^2 + Bb^3)d\right)\tan\left(fx+e\right)^4 + 20\left((3\, Cab^2 + Bb^3)c + (3\, Ca^2b + 3\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="fricas")
```

output
```
1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan
(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)
*b^3)*d)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 +
 B*b^3)*c - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*f*x + 3
0*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C*a^3 + 3*B*a^2*b + 3*(A - C
)*a*b^2 - B*b^3)*d)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)
*d)*log(1/(tan(f*x + e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2
 - B*b^3)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan(f
*x + e))/f
```

3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(316) = 632.

Time = 0.29 (sec) , antiderivative size = 1001, normalized size of antiderivative = 2.84

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e
)**2),x)
```

output
```
Piecewise((A*a**3*c*x + A*a**3*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2
*b*c*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*a**2*b*d*x + 3*A*a**2*b*d*tan(e
+ f*x)/f - 3*A*a*b**2*c*x + 3*A*a*b**2*c*tan(e + f*x)/f - 3*A*a*b**2*d*log
(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a*b**2*d*tan(e + f*x)**2/(2*f) - A*b**3*
c*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c*tan(e + f*x)**2/(2*f) + A*b**3
*d*x + A*b**3*d*tan(e + f*x)**3/(3*f) - A*b**3*d*tan(e + f*x)/f + B*a**3*c
*log(tan(e + f*x)**2 + 1)/(2*f) - B*a**3*d*x + B*a**3*d*tan(e + f*x)/f - 3
*B*a**2*b*c*x + 3*B*a**2*b*c*tan(e + f*x)/f - 3*B*a**2*b*d*log(tan(e + f*x
)**2 + 1)/(2*f) + 3*B*a**2*b*d*tan(e + f*x)**2/(2*f) - 3*B*a*b**2*c*log(ta
n(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*c*tan(e + f*x)**2/(2*f) + 3*B*a*b**2
*d*x + B*a*b**2*d*tan(e + f*x)**3/f - 3*B*a*b**2*d*tan(e + f*x)/f + B*b**3
*c*x + B*b**3*c*tan(e + f*x)**3/(3*f) - B*b**3*c*tan(e + f*x)/f + B*b**3*d
*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**3*d*tan(e + f*x)**4/(4*f) - B*b**3*
d*tan(e + f*x)**2/(2*f) - C*a**3*c*x + C*a**3*c*tan(e + f*x)/f - C*a**3*d*
log(tan(e + f*x)**2 + 1)/(2*f) + C*a**3*d*tan(e + f*x)**2/(2*f) - 3*C*a**2
*b*c*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*b*c*tan(e + f*x)**2/(2*f) +
 3*C*a**2*b*d*x + C*a**2*b*d*tan(e + f*x)**3/f - 3*C*a**2*b*d*tan(e + f*x)
/f + 3*C*a*b**2*c*x + C*a*b**2*c*tan(e + f*x)**3/f - 3*C*a*b**2*c*tan(e +
f*x)/f + 3*C*a*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a*b**2*d*tan(e
+ f*x)**4/(4*f) - 3*C*a*b**2*d*tan(e + f*x)**2/(2*f) + C*b**3*c*log(tan...
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.18

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{12\,Cb^3d\tan{(fx+e)}^5+15\,(Cb^3c+(3\,Cab^2+Bb^3)d)\tan{(fx+e)}^4+20\,((3\,Cab^2+Bb^3)c+(3\,Ca^2b+3}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="maxima")
```

output
```
1/60*(12*C*b^3*d*tan(f*x + e)^5 + 15*(C*b^3*c + (3*C*a*b^2 + B*b^3)*d)*tan
(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3)*c + (3*C*a^2*b + 3*B*a*b^2 + (A - C)
*b^3)*d)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c + (C
*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*d)*tan(f*x + e)^2 + 60*(((A -
C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c - (B*a^3 + 3*(A - C)*a^2*b
 - 3*B*a*b^2 - (A - C)*b^3)*d)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b -
3*B*a*b^2 - (A - C)*b^3)*c + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 +
B*b^3)*d)*log(tan(f*x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b
^2 - B*b^3)*c + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*tan
(f*x + e))/f
```

3.50.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10353 vs. $2(345) = 690$.

Time = 9.66 (sec) , antiderivative size = 10353, normalized size of antiderivative = 29.33

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="giac")
```

output
```
1/60*(60*A*a^3*c*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a^3*c*f*x*tan(f*x)^5*tan(e
)^5 - 180*B*a^2*b*c*f*x*tan(f*x)^5*tan(e)^5 - 180*A*a*b^2*c*f*x*tan(f*x)^5
*tan(e)^5 + 180*C*a*b^2*c*f*x*tan(f*x)^5*tan(e)^5 + 60*B*b^3*c*f*x*tan(f*x
)^5*tan(e)^5 - 60*B*a^3*d*f*x*tan(f*x)^5*tan(e)^5 - 180*A*a^2*b*d*f*x*tan(
f*x)^5*tan(e)^5 + 180*C*a^2*b*d*f*x*tan(f*x)^5*tan(e)^5 + 180*B*a*b^2*d*f*
x*tan(f*x)^5*tan(e)^5 + 60*A*b^3*d*f*x*tan(f*x)^5*tan(e)^5 - 60*C*b^3*d*f*
x*tan(f*x)^5*tan(e)^5 - 30*B*a^3*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)
*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5
*tan(e)^5 - 90*A*a^2*b*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) +
1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5
+ 90*C*a^2*b*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*a*b
^2*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e
)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 30*A*b^3*c*log(4*(
tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*C*b^3*c*log(4*(tan(f*x)^2*t
an(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e
)^2 + 1))*tan(f*x)^5*tan(e)^5 - 30*A*a^3*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*ta
n(f*x)^5*tan(e)^5 + 30*C*a^3*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*...
```

3.50. $\int (a + b\tan(e + fx))^3 (c + d\tan(e + fx)) \left(A + B\tan(e + fx) + C\tan^2(e + fx)\right) dx$

3.50.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.35

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \,dx$$
$$= x \left(A a^3 c + A b^3 d - B a^3 d + B b^3 c - C a^3 c - C b^3 d - 3 A a b^2 c - 3 A a^2 b d - 3 B a^2 b c + 3 B a b^2 d + 3 C a b^2 c + 3 C a^2 b d\right) + \frac{\tan(e+f x)^4 \left(\frac{B b^3 d}{4} + \frac{C b^3 c}{4} + \frac{3 C a b^2 d}{4}\right)}{f}$$
$$+ \frac{\tan(e+f x)^3 \left(\frac{A b^3 d}{3} + \frac{B b^3 c}{3} - \frac{C b^3 d}{3} + B a b^2 d + C a b^2 c + C a^2 b d\right)}{f}$$
$$+ \frac{\tan(e+f x)^2 \left(\frac{A b^3 c}{2} - \frac{B b^3 d}{2} + \frac{C a^3 d}{2} - \frac{C b^3 c}{2} + \frac{3 A a b^2 d}{2} + \frac{3 B a b^2 c}{2} + \frac{3 B a^2 b d}{2} + \frac{3 C a^2 b c}{2} - \frac{3 C a b^2 d}{2}\right)}{f}$$
$$+ \frac{\ln\left(\tan(e+f x)^2+1\right) \left(\frac{A a^3 d}{2} - \frac{A b^3 c}{2} + \frac{B a^3 c}{2} + \frac{B b^3 d}{2} - \frac{C a^3 d}{2} + \frac{C b^3 c}{2} + \frac{3 A a^2 b c}{2} - \frac{3 A a b^2 d}{2} - \frac{3 B a b^2 c}{2} - 3 I\right.}{f}$$
$$+ \frac{\tan(e+f x) \left(B a^3 d - A b^3 d - B b^3 c + C a^3 c + C b^3 d + 3 A a b^2 c + 3 A a^2 b d + 3 B a^2 b c - 3 B a b^2 d\right.}{f}$$
$$+ \frac{C b^3 d \tan(e+f x)^5}{5 f}$$

input
```
int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2),x)
```

output
```
x*(A*a^3*c + A*b^3*d - B*a^3*d + B*b^3*c - C*a^3*c - C*b^3*d - 3*A*a*b^2*c
 - 3*A*a^2*b*d - 3*B*a^2*b*c + 3*B*a*b^2*d + 3*C*a*b^2*c + 3*C*a^2*b*d) +
(tan(e + f*x)^4*((B*b^3*d)/4 + (C*b^3*c)/4 + (3*C*a*b^2*d)/4))/f + (tan(e
+ f*x)^3*((A*b^3*d)/3 + (B*b^3*c)/3 - (C*b^3*d)/3 + B*a*b^2*d + C*a*b^2*c
+ C*a^2*b*d))/f + (tan(e + f*x)^2*((A*b^3*c)/2 - (B*b^3*d)/2 + (C*a^3*d)/2
 - (C*b^3*c)/2 + (3*A*a*b^2*d)/2 + (3*B*a*b^2*c)/2 + (3*B*a^2*b*d)/2 + (3*
C*a^2*b*c)/2 - (3*C*a*b^2*d)/2))/f + (log(tan(e + f*x)^2 + 1)*((A*a^3*d)/2
 - (A*b^3*c)/2 + (B*a^3*c)/2 + (B*b^3*d)/2 - (C*a^3*d)/2 + (C*b^3*c)/2 + (
3*A*a^2*b*c)/2 - (3*A*a*b^2*d)/2 - (3*B*a*b^2*c)/2 - (3*B*a^2*b*d)/2 - (3*
C*a^2*b*c)/2 + (3*C*a*b^2*d)/2))/f + (tan(e + f*x)*(B*a^3*d - A*b^3*d - B*
b^3*c + C*a^3*c + C*b^3*d + 3*A*a*b^2*c + 3*A*a^2*b*d + 3*B*a^2*b*c - 3*B*
a*b^2*d - 3*C*a*b^2*c - 3*C*a^2*b*d))/f + (C*b^3*d*tan(e + f*x)^5)/(5*f)
```

3.51 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$

3.51.1 Optimal result

Integrand size = 43, antiderivative size = 248

$$\begin{aligned}
&\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx \\
&= \left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)-2ab(Bc+(A-C)d)\right)x \\
&\quad -\frac{(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\log(\cos(e+fx))}{f} \\
&\quad +\frac{b(Abc+aBc-bcC+aAd-bBd-aCd)\tan(e+fx)}{f} \\
&\quad +\frac{(Bc+(A-C)d)(a+b\tan(e+fx))^2}{2f} \\
&\quad -\frac{(aCd-4b(cC+Bd))(a+b\tan(e+fx))^3}{12b^2f}+\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}
\end{aligned}$$

output
```
(a^2*(A*c-B*d-C*c)-b^2*(A*c-B*d-C*c)-2*a*b*(B*c+(A-C)*d))*x-(2*a*b*(A*c-B*
d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*ln(cos(f*x+e))/f+b*(A*a*d+A*b*
c+B*a*c-B*b*d-C*a*d-C*b*c)*tan(f*x+e)/f+1/2*(B*c+(A-C)*d)*(a+b*tan(f*x+e))
^2/f-1/12*(C*a*d-4*b*(B*d+C*c))*(a+b*tan(f*x+e))^3/b^2/f+1/4*C*d*tan(f*x+e
)*(a+b*tan(f*x+e))^3/b/f
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$
$$= \frac{\frac{(-aCd+4b(cC+Bd))(a+b\tan(e+fx))^3}{b}+3Cd\tan(e+fx)(a+b\tan(e+fx))^3-6(Abc-aBc-bcC-aAd-bB}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(((-(a*C*d) + 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/b + 3*C*d*Tan[e + f
*x]*(a + b*Tan[e + f*x])^3 - 6*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*
C*d)*(I*((a + I*b)^2*Log[I - Tan[e + f*x]] - (a - I*b)^2*Log[I + Tan[e + f
*x]]) - 2*b^2*Tan[e + f*x]) + 6*(B*c + (A - C)*d)*((I*a - b)^3*Log[I - Tan
[e + f*x]] - (I*a + b)^3*Log[I + Tan[e + f*x]] + 6*a*b^2*Tan[e + f*x] + b^
3*Tan[e + f*x]^2))/(12*b*f)
```

3.51.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {3042, 4120, 25, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}-$$
$$\frac{\int -(a+b\tan(e+fx))^2\left(-((aCd-4b(cC+Bd))\tan^2(e+fx))+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-aC\right.}{4b}$$

$\downarrow$ 25

$$\frac{\int (a+b\tan(e+fx))^2\left(-\left((aCd-4b(cC+Bd))\tan^2(e+fx)\right)+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-aCd\right)}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^2\left(-\left((aCd-4b(cC+Bd))\tan(e+fx)^2\right)+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-aCd\right)}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 4113

$$\frac{\int (a+b\tan(e+fx))^2(4b(Ac-Cc-Bd)+4b(Bc+(A-C)d)\tan(e+fx))dx-\frac{(aCd-4b(Bd+cC))(a+b\tan(e+fx))^3}{3bf}}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^2(4b(Ac-Cc-Bd)+4b(Bc+(A-C)d)\tan(e+fx))dx-\frac{(aCd-4b(Bd+cC))(a+b\tan(e+fx))^3}{3bf}}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 4011

$$\frac{\int (a+b\tan(e+fx))(4b(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)-4b(bBc+b(A-C)d-a(Ac-Cc}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))(4b(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)-4b(bBc+b(A-C)d-a(Ac-Cc}{4b}$$
$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

$\downarrow$ 4008

3.51. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{4b\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)\int\tan(e+fx)dx+4bx\left(a^2(Ac-Bd-cC)-\right.}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

↓ 3042

$$\frac{4b\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)\int\tan(e+fx)dx+4bx\left(a^2(Ac-Bd-cC)-\right.}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

↓ 3956

$$\frac{-\frac{4b\log(\cos(e+fx))\left(a^2(d(A-C)+Bc)+2ab(Ac-Bd-cC)-b^2(d(A-C)+Bc)\right)}{f}+4bx\left(a^2(Ac-Bd-cC)-2ab(d(A-C)+Bc)-\right.}{}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^3}{4bf}$$

input
```
Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Ta
n[e + f*x]^2),x]
```

output
```
(C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^3)/(4*b*f) + (4*b*(a^2*(A*c - c*C -
 B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d))*x - (4*b*(2*a*b*(
A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Log[Cos[
e + f*x]])/f + (4*b^2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*Tan[
e + f*x])/f + (2*b*(B*c + (A - C)*d)*(a + b*Tan[e + f*x])^2)/f - ((a*C*d -
 4*b*(c*C + B*d))*(a + b*Tan[e + f*x])^3)/(3*b*f))/(4*b)
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.51.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.99

method	result
parts	$\frac{(A a^2 d+2Aabc+B a^2 c)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(B b^2 d+2Cabd+C b^2 c)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right.}{f}$
norman	$(A a^2 c-2Aabd-A b^2 c-B a^2 d-2Babc+B b^2 d-C a^2 c+2Cabd+C b^2 c)\,x+\frac{(2Aab}{}$
derivativedivides	$\frac{dC b^2\tan(fx+e)^4}{4}+\frac{B b^2 d\tan(fx+e)^3}{3}+\frac{2Cabd\tan(fx+e)^3}{3}+\frac{C b^2 c\tan(fx+e)^3}{3}+\frac{A b^2 d\tan(fx+e)^2}{2}+Babd\tan(fx+e)^2+\frac{B b^2 c\,t}{}$
default	$\frac{dC b^2\tan(fx+e)^4}{4}+\frac{B b^2 d\tan(fx+e)^3}{3}+\frac{2Cabd\tan(fx+e)^3}{3}+\frac{C b^2 c\tan(fx+e)^3}{3}+\frac{A b^2 d\tan(fx+e)^2}{2}+Babd\tan(fx+e)^2+\frac{B b^2 c\,t}{}$
parallelrisch	$\frac{-12\tan(fx+e)C b^2 c+3dC b^2\tan(fx+e)^4+4B b^2 d\tan(fx+e)^3+4C b^2 c\tan(fx+e)^3+6A b^2 d\tan(fx+e)^2+6B b^2 c\tan(f}{}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*(A*a^2*d+2*A*a*b*c+B*a^2*c)/f*ln(1+tan(f*x+e)^2)+(B*b^2*d+2*C*a*b*d+C*
b^2*c)/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(A*b^2*d+2*B*a*b
*d+B*b^2*c+C*a^2*d+2*C*a*b*c)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+
(2*A*a*b*d+A*b^2*c+B*a^2*d+2*B*a*b*c+C*a^2*c)/f*(tan(f*x+e)-arctan(tan(f*x
+e)))+A*a^2*c*x+d*C*b^2/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(
f*x+e)^2))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.10

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,Cb^2d\tan(fx+e)^4+4\,(Cb^2c+(2\,Cab+Bb^2)d)\tan(fx+e)^3+12\,(((A-C)a^2-2\,Bab-(A-C)b^2)}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="fricas")
```

output
```
1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
 + e)^3 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A - C)
*a*b - B*b^2)*d)*f*x + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)
*b^2)*d)*tan(f*x + e)^2 - 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A - C)*
a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a^2 +
 2*B*a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x + e
))/f
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(218) = 436$.

Time = 0.21 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.49

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \begin{cases} Aa^2cx + \frac{Aa^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{Aabc\log\left(\tan^2\left(e+fx\right)+1\right)}{f} - 2Aabdx + \frac{2Aabd\tan\left(e+fx\right)}{f} - Ab^2cx + \frac{Ab^2c\tan\left(e+fx\right)}{f} \\ x(a+b\tan\left(e\right))^2\left(c+d\tan\left(e\right)\right)\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

input
```
integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e
)**2),x)
```

output
```
Piecewise((A*a**2*c*x + A*a**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*a*b*c*
log(tan(e + f*x)**2 + 1)/f - 2*A*a*b*d*x + 2*A*a*b*d*tan(e + f*x)/f - A*b*
*2*c*x + A*b**2*c*tan(e + f*x)/f - A*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f)
 + A*b**2*d*tan(e + f*x)**2/(2*f) + B*a**2*c*log(tan(e + f*x)**2 + 1)/(2*f
) - B*a**2*d*x + B*a**2*d*tan(e + f*x)/f - 2*B*a*b*c*x + 2*B*a*b*c*tan(e +
 f*x)/f - B*a*b*d*log(tan(e + f*x)**2 + 1)/f + B*a*b*d*tan(e + f*x)**2/f -
 B*b**2*c*log(tan(e + f*x)**2 + 1)/(2*f) + B*b**2*c*tan(e + f*x)**2/(2*f)
+ B*b**2*d*x + B*b**2*d*tan(e + f*x)**3/(3*f) - B*b**2*d*tan(e + f*x)/f -
C*a**2*c*x + C*a**2*c*tan(e + f*x)/f - C*a**2*d*log(tan(e + f*x)**2 + 1)/(
2*f) + C*a**2*d*tan(e + f*x)**2/(2*f) - C*a*b*c*log(tan(e + f*x)**2 + 1)/f
 + C*a*b*c*tan(e + f*x)**2/f + 2*C*a*b*d*x + 2*C*a*b*d*tan(e + f*x)**3/(3*
f) - 2*C*a*b*d*tan(e + f*x)/f + C*b**2*c*x + C*b**2*c*tan(e + f*x)**3/(3*f
) - C*b**2*c*tan(e + f*x)/f + C*b**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*
b**2*d*tan(e + f*x)**4/(4*f) - C*b**2*d*tan(e + f*x)**2/(2*f), Ne(f, 0)),
(x*(a + b*tan(e))**2*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.10

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,Cb^2d\tan{(fx+e)}^4+4\,(Cb^2c+(2\,Cab+Bb^2)d)\tan{(fx+e)}^3+6\,((2\,Cab+Bb^2)c+(Ca^2+2\,Bab+(}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="maxima")
```

output
```
1/12*(3*C*b^2*d*tan(f*x + e)^4 + 4*(C*b^2*c + (2*C*a*b + B*b^2)*d)*tan(f*x
 + e)^3 + 6*((2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + (A - C)*b^2)*d)*tan(
f*x + e)^2 + 12*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c - (B*a^2 + 2*(A -
 C)*a*b - B*b^2)*d)*(f*x + e) + 6*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c + ((A
 - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1) + 12*((C*a^2
 + 2*B*a*b + (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d)*tan(f*x +
 e))/f
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5631 vs. 2(242) = 484.

Time = 4.13 (sec) , antiderivative size = 5631, normalized size of antiderivative = 22.71

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="giac")
```

output
```
1/12*(12*A*a^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a^2*c*f*x*tan(f*x)^4*tan(e
)^4 - 24*B*a*b*c*f*x*tan(f*x)^4*tan(e)^4 - 12*A*b^2*c*f*x*tan(f*x)^4*tan(e
)^4 + 12*C*b^2*c*f*x*tan(f*x)^4*tan(e)^4 - 12*B*a^2*d*f*x*tan(f*x)^4*tan(e
)^4 - 24*A*a*b*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*a*b*d*f*x*tan(f*x)^4*tan(e
)^4 + 12*B*b^2*d*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a^2*c*log(4*(tan(f*x)^2*tan
(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^
2 + 1))*tan(f*x)^4*tan(e)^4 - 12*A*a*b*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*ta
n(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(
f*x)^4*tan(e)^4 + 12*C*a*b*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e
) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e
)^4 + 6*B*b^2*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 6*A*a^2
*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 6*C*a^2*d*log(4*(tan
(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^
2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 12*B*a*b*d*log(4*(tan(f*x)^2*tan(
e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2
 + 1))*tan(f*x)^4*tan(e)^4 + 6*A*b^2*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(
f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*
x)^4*tan(e)^4 - 6*C*b^2*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e...
```

3.51.9 Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.21

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \frac{\tan(e+f\,x)^2\left(\frac{A\,b^2\,d}{2}+\frac{B\,b^2\,c}{2}+\frac{C\,a^2\,d}{2}-\frac{C\,b^2\,d}{2}+B\,a\,b\,d+C\,a\,b\,c\right)}{f}$$
$$-x\left(A\,b^2\,c-A\,a^2\,c+B\,a^2\,d+C\,a^2\,c-B\,b^2\,d-C\,b^2\,c+2\,A\,a\,b\,d+2\,B\,a\,b\,c-2\,C\,a\,b\,d\right)$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{A\,b^2\,d}{2}-\frac{B\,a^2\,c}{2}-\frac{A\,a^2\,d}{2}+\frac{B\,b^2\,c}{2}+\frac{C\,a^2\,d}{2}-\frac{C\,b^2\,d}{2}-A\,a\,b\,c+B\,a\,b\,d+C\,a\,b\,c\right)}{f}$$
$$+\frac{\tan(e+f\,x)\left(A\,b^2\,c+B\,a^2\,d+C\,a^2\,c-B\,b^2\,d-C\,b^2\,c+2\,A\,a\,b\,d+2\,B\,a\,b\,c-2\,C\,a\,b\,d\right)}{f}$$
$$+\frac{\tan(e+f\,x)^3\left(\frac{B\,b^2\,d}{3}+\frac{C\,b^2\,c}{3}+\frac{2\,C\,a\,b\,d}{3}\right)}{f}+\frac{C\,b^2\,d\tan(e+f\,x)^4}{4\,f}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^2*((A*b^2*d)/2 + (B*b^2*c)/2 + (C*a^2*d)/2 - (C*b^2*d)/2 + B
*a*b*d + C*a*b*c))/f - x*(A*b^2*c - A*a^2*c + B*a^2*d + C*a^2*c - B*b^2*d
- C*b^2*c + 2*A*a*b*d + 2*B*a*b*c - 2*C*a*b*d) - (log(tan(e + f*x)^2 + 1)*
((A*b^2*d)/2 - (B*a^2*c)/2 - (A*a^2*d)/2 + (B*b^2*c)/2 + (C*a^2*d)/2 - (C*
b^2*d)/2 - A*a*b*c + B*a*b*d + C*a*b*c))/f + (tan(e + f*x)*(A*b^2*c + B*a^
2*d + C*a^2*c - B*b^2*d - C*b^2*c + 2*A*a*b*d + 2*B*a*b*c - 2*C*a*b*d))/f
+ (tan(e + f*x)^3*((B*b^2*d)/3 + (C*b^2*c)/3 + (2*C*a*b*d)/3))/f + (C*b^2*
d*tan(e + f*x)^4)/(4*f)
```

3.51. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.52 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+f$

3.52.1 Optimal result

Integrand size = 41, antiderivative size = 161

$$\begin{aligned}&\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx\\&=(a(Ac-cC-Bd)-b(Bc+(A-C)d))x\\&\quad-\frac{(Abc+aBc-bcC+aAd-bBd-aCd)\log(\cos(e+fx))}{f}\\&\quad+\frac{(Ab+aB-bC)d\tan(e+fx)}{f}-\frac{(bcC-3bBd-3aCd)(c+d\tan(e+fx))^2}{6d^2f}\\&\quad+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}\end{aligned}$$

output
```
(a*(A*c-B*d-C*c)-b*(B*c+(A-C)*d))*x-(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*
ln(cos(f*x+e))/f+(A*b+B*a-C*b)*d*tan(f*x+e)/f-1/6*(-3*B*b*d-3*C*a*d+C*b*c)
*(c+d*tan(f*x+e))^2/d^2/f+1/3*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^2/d/f
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx\\&=\frac{3(a+ib)(A+iB-C)(-ic+d)\log(i-\tan(e+fx))+3(a-ib)(A-iB-C)(ic+d)\log(i+\tan(e+f}{6f}\end{aligned}$$

input
```
Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] +
C*Tan[e + f*x]^2),x]
```

output
```
(3*(a + I*b)*(A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]] + 3*(a - I*b
)*(A - I*B - C)*(I*c + d)*Log[I + Tan[e + f*x]] + 6*(A*b + a*B - b*C)*d*Ta
n[e + f*x] + ((-(b*c*C) + 3*b*B*d + 3*a*C*d)*(c + d*Tan[e + f*x])^2)/d^2 +
 (2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/d)/(6*f)
```

3.52.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3042, 4120, 3042, 4113, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{\int (c+d\tan(e+fx))\left((bcC-3adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx)+bcC-3aAd\right)dx}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{\int (c+d\tan(e+fx))\left((bcC-3adC-3bBd)\tan(e+fx)^2-3(Ab-Cb+aB)d\tan(e+fx)+bcC-3aAd\right)dx}{3d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{\int (c+d\tan(e+fx))(3(bB-a(A-C))d-3(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-3aCd-3bBd+bcC)(c+d\tan(e+fx))^2}{2df}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{\int(c+d\tan(e+fx))(3(bB-a(A-C))d-3(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-3aCd-3bBd+bcC)(c+d\tan(e+fx))^2}{2df}}{3d}$$

$$\Big\downarrow \text{4008}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{-3d(aAd+aBc-aCd+Abc-bBd-bcC)\int\tan(e+fx)dx+3dx(-a(Ac-Bd-cC)+bd(A-C)+bBc)-}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{-3d(aAd+aBc-aCd+Abc-bBd-bcC)\int\tan(e+fx)dx+3dx(-a(Ac-Bd-cC)+bd(A-C)+bBc)-}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^2}{3df}-\frac{\frac{3d\log(\cos(e+fx))(aAd+aBc-aCd+Abc-bBd-bcC)}{f}+3dx(-a(Ac-Bd-cC)+bd(A-C)+bBc)-\frac{3d^2\tan(e+fx)(aB+Ab-}{f}}{3d}$$

input
```
Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[
e + f*x]^2),x]
```

output
```
(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^2)/(3*d*f) - (3*d*(b*B*c + b*(A - C
)*d - a*(A*c - c*C - B*d))*x + (3*d*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d
 - a*C*d)*Log[Cos[e + f*x]])/f - (3*(A*b + a*B - b*C)*d^2*Tan[e + f*x])/f
+ ((b*c*C - 3*b*B*d - 3*a*C*d)*(c + d*Tan[e + f*x])^2)/(2*d*f))/(3*d)
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

method	result
parts	$\frac{(Aad+Abc+Bac)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(bdB+Cad+Cbc)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{(Abd+Bad+Bbc+C\cdots}{}$
norman	$(Aac-Abd-Bad-Bbc-Cac+Cbd)\,x+\frac{(Abd+Bad+Bbc+Cac-Cbd)\tan(fx+e)}{f}+\frac{(bdB+Ca\cdots}{}$
derivativedivides	$\frac{\frac{C\tan(fx+e)^3bd}{3}+\frac{B\tan(fx+e)^2bd}{2}+\frac{C\tan(fx+e)^2ad}{2}+\frac{C\tan(fx+e)^2bc}{2}+A\tan(fx+e)bd+B\tan(fx+e)ad+B\tan(fx+e)bc+\cdots}{}$
default	$\frac{\frac{C\tan(fx+e)^3bd}{3}+\frac{B\tan(fx+e)^2bd}{2}+\frac{C\tan(fx+e)^2ad}{2}+\frac{C\tan(fx+e)^2bc}{2}+A\tan(fx+e)bd+B\tan(fx+e)ad+B\tan(fx+e)bc+\cdots}{}$
parallelrisch	$\frac{2C\tan(fx+e)^3bd+6Aacfx-6Abdfx-6Badfx-6Bbcfx+3B\tan(fx+e)^2bd-6Cacfx+6Cbdfx+3C\tan(fx+e)^2ad+3C\cdots}{}$
risch	$-Badx+Cbdx+\frac{2iBace}{f}-\frac{2iBbde}{f}-\frac{2iCade}{f}-\frac{2iCbce}{f}+\frac{2iAade}{f}+\frac{2iAbce}{f}+\frac{2i\left(-3iCad\,e^{2i(fx+e)}\cdots\right.}{}$

input
```
int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,me
thod=_RETURNVERBOSE)
```

output
```
1/2*(A*a*d+A*b*c+B*a*c)/f*ln(1+tan(f*x+e)^2)+(B*b*d+C*a*d+C*b*c)/f*(1/2*ta
n(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+(A*b*d+B*a*d+B*b*c+C*a*c)/f*(tan(f*x+e)
-arctan(tan(f*x+e)))+A*a*c*x+C*b*d/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(t
an(f*x+e)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{2\,Cbd\tan{(fx+e)}^3+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)fx+3\,(Cbc+(Ca+Bb)d)\tan{(fx+e}}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x, algorithm="fricas")
```

output
```
1/6*(2*C*b*d*tan(f*x + e)^3 + 6*(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d
)*f*x + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 - 3*((B*a + (A - C)*b)*c
+ ((A - C)*a - B*b)*d)*log(1/(tan(f*x + e)^2 + 1)) + 6*((C*a + B*b)*c + (B
*a + (A - C)*b)*d)*tan(f*x + e))/f
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(148) = 296.

Time = 0.15 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.02

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} Aacx+\frac{Aad\log{(\tan^2{(e+fx)}+1)}}{2f}+\frac{Abc\log{(\tan^2{(e+fx)}+1)}}{2f}-Abdx+\frac{Abd\tan{(e+fx)}}{f}+\frac{Bac\log{(\tan^2{(e+fx)}+1)}}{2f}-Badx \\ x(a+b\tan{(e)})\,(c+d\tan{(e)})\,(A+B\tan{(e)}+C\tan^2{(e)}) \end{cases}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**
2),x)
```

output
```
Piecewise((A*a*c*x + A*a*d*log(tan(e + f*x)**2 + 1)/(2*f) + A*b*c*log(tan(
e + f*x)**2 + 1)/(2*f) - A*b*d*x + A*b*d*tan(e + f*x)/f + B*a*c*log(tan(e
+ f*x)**2 + 1)/(2*f) - B*a*d*x + B*a*d*tan(e + f*x)/f - B*b*c*x + B*b*c*ta
n(e + f*x)/f - B*b*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*b*d*tan(e + f*x)**
2/(2*f) - C*a*c*x + C*a*c*tan(e + f*x)/f - C*a*d*log(tan(e + f*x)**2 + 1)/
(2*f) + C*a*d*tan(e + f*x)**2/(2*f) - C*b*c*log(tan(e + f*x)**2 + 1)/(2*f)
 + C*b*c*tan(e + f*x)**2/(2*f) + C*b*d*x + C*b*d*tan(e + f*x)**3/(3*f) - C
*b*d*tan(e + f*x)/f, Ne(f, 0)), (x*(a + b*tan(e))*(c + d*tan(e))*(A + B*ta
n(e) + C*tan(e)**2), True))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \frac{2\,Cbd\tan\left(fx+e\right)^3+3\,(Cbc+(Ca+Bb)d)\tan\left(fx+e\right)^2+6\,(((A-C)a-Bb)c-(Ba+(A-C)b)d)(}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x, algorithm="maxima")
```

output
```
1/6*(2*C*b*d*tan(f*x + e)^3 + 3*(C*b*c + (C*a + B*b)*d)*tan(f*x + e)^2 + 6
*(((A - C)*a - B*b)*c - (B*a + (A - C)*b)*d)*(f*x + e) + 3*((B*a + (A - C)
*b)*c + ((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1) + 6*((C*a + B*b)*c +
(B*a + (A - C)*b)*d)*tan(f*x + e))/f
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2475 vs. 2(157) = 314.

Time = 1.79 (sec) , antiderivative size = 2475, normalized size of antiderivative = 15.37

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x, algorithm="giac")
```

output
```
1/6*(6*A*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6*C*a*c*f*x*tan(f*x)^3*tan(e)^3 - 6
*B*b*c*f*x*tan(f*x)^3*tan(e)^3 - 6*B*a*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*b*d
*f*x*tan(f*x)^3*tan(e)^3 + 6*C*b*d*f*x*tan(f*x)^3*tan(e)^3 - 3*B*a*c*log(4
*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(
f*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 3*A*b*c*log(4*(tan(f*x)^2*ta
n(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)
^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*C*b*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(
f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*
x)^3*tan(e)^3 - 3*A*a*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1
)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 +
 3*C*a*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*t
an(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*B*b*d*log(4*
(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f
*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 18*A*a*c*f*x*tan(f*x)^2*tan(e
)^2 + 18*C*a*c*f*x*tan(f*x)^2*tan(e)^2 + 18*B*b*c*f*x*tan(f*x)^2*tan(e)^2
+ 18*B*a*d*f*x*tan(f*x)^2*tan(e)^2 + 18*A*b*d*f*x*tan(f*x)^2*tan(e)^2 - 18
*C*b*d*f*x*tan(f*x)^2*tan(e)^2 + 3*C*b*c*tan(f*x)^3*tan(e)^3 + 3*C*a*d*tan
(f*x)^3*tan(e)^3 + 3*B*b*d*tan(f*x)^3*tan(e)^3 + 9*B*a*c*log(4*(tan(f*x)^2
*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan
(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 9*A*b*c*log(4*(tan(f*x)^2*tan(e)^2 - ...
```

3.52.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,a\,d}{2}+\frac{A\,b\,c}{2}+\frac{B\,a\,c}{2}-\frac{B\,b\,d}{2}-\frac{C\,a\,d}{2}-\frac{C\,b\,c}{2}\right)}{f}$$
$$-x\,(A\,b\,d-A\,a\,c+B\,a\,d+B\,b\,c+C\,a\,c-C\,b\,d)+\frac{\tan(e+f\,x)^2\,\left(\frac{B\,b\,d}{2}+\frac{C\,a\,d}{2}+\frac{C\,b\,c}{2}\right)}{f}$$
$$+\frac{\tan(e+f\,x)\,\left(A\,b\,d+B\,a\,d+B\,b\,c+C\,a\,c-C\,b\,d\right)}{f}+\frac{C\,b\,d\tan(e+f\,x)^3}{3\,f}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(
e + f*x)^2),x)
```

output
```
(log(tan(e + f*x)^2 + 1)*((A*a*d)/2 + (A*b*c)/2 + (B*a*c)/2 - (B*b*d)/2 -
(C*a*d)/2 - (C*b*c)/2))/f - x*(A*b*d - A*a*c + B*a*d + B*b*c + C*a*c - C*b
*d) + (tan(e + f*x)^2*((B*b*d)/2 + (C*a*d)/2 + (C*b*c)/2))/f + (tan(e + f*
x)*(A*b*d + B*a*d + B*b*c + C*a*c - C*b*d))/f + (C*b*d*tan(e + f*x)^3)/(3*
f)
```

3.52. $\int (a+b\tan(e+fx))(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.53 $\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

3.53.1 Optimal result

Integrand size = 31, antiderivative size = 73

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=(Ac-cC-Bd)x-\frac{(Bc+(A-C)d)\log(\cos(e+fx))}{f}$$
$$+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

output
```
(A*c-B*d-C*c)*x-(B*c+(A-C)*d)*ln(cos(f*x+e))/f+B*d*tan(f*x+e)/f+1/2*C*(c+d
*tan(f*x+e))^2/d/f
```

3.53.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{2Acfx-2(cC+Bd)\arctan(\tan(e+fx))-2(Bc+(A-C)d)\log(\cos(e+fx))+2(cC+Bd)\tan(e+fx)+Cd\tan^2(e+fx)}{2f}$$

input
```
Integrate[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*A*c*f*x - 2*(c*C + B*d)*ArcTan[Tan[e + f*x]] - 2*(B*c + (A - C)*d)*Log[
Cos[e + f*x]] + 2*(c*C + B*d)*Tan[e + f*x] + C*d*Tan[e + f*x]^2)/(2*f)
```

3.53.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 4113, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \; 4113$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))dx+\frac{C(c+d\tan(e+fx))^2}{2df}$$

$$\Big\downarrow \; 3042$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))dx+\frac{C(c+d\tan(e+fx))^2}{2df}$$

$$\Big\downarrow \; 4008$$

$$(d(A-C)+Bc)\int \tan(e+fx)dx+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

$$\Big\downarrow \; 3042$$

$$(d(A-C)+Bc)\int \tan(e+fx)dx+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

$$\Big\downarrow \; 3956$$

$$-\frac{(d(A-C)+Bc)\log(\cos(e+fx))}{f}+x(Ac-Bd-cC)+\frac{Bd\tan(e+fx)}{f}+\frac{C(c+d\tan(e+fx))^2}{2df}$$

input
```
Int[(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(A*c - c*C - B*d)*x - ((B*c + (A - C)*d)*Log[Cos[e + f*x]])/f + (B*d*Tan[e
 + f*x])/f + (C*(c + d*Tan[e + f*x])^2)/(2*d*f)
```

3.53.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

method	result
norman	$(Ac-Bd-Cc)\,x+\frac{(Bd+Cc)\tan(fx+e)}{f}+\frac{Cd\tan(fx+e)^2}{2f}+\frac{(Ad+Bc-Cd)\ln\left(1+\tan(fx+e)^2\right)}{2f}$
derivativedivides	$\frac{\frac{C\tan(fx+e)^2 d}{2}+B\tan(fx+e)d+C\tan(fx+e)c+\frac{(Ad+Bc-Cd)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Ac-Bd-Cc)\arctan(\tan(fx+e))}{f}$
default	$\frac{\frac{C\tan(fx+e)^2 d}{2}+B\tan(fx+e)d+C\tan(fx+e)c+\frac{(Ad+Bc-Cd)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Ac-Bd-Cc)\arctan(\tan(fx+e))}{f}$
parts	$Acx+\frac{(Ad+Bc)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(Bd+Cc)(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{Cd\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan\right.}{}\right.}{f}$
parallelrisch	$\frac{2Acfx-2Bdfx-2Ccfx+C\tan(fx+e)^2 d+A\ln\left(1+\tan(fx+e)^2\right)d+B\ln\left(1+\tan(fx+e)^2\right)c+2B\tan(fx+e)d-C\ln\left(1+\right.}{2f}$
risch	$\frac{2iAde}{f}-iCdx+iAdx+Acx-Bdx-Ccx-\frac{2iCde}{f}+iBcx+\frac{2iBce}{f}+\frac{2i\left(-iCd\,e^{2i(fx+e)}+Bd\right.}{f(}$

input
```
int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVERBO
SE)
```

output
```
(A*c-B*d-C*c)*x+(B*d+C*c)/f*tan(f*x+e)+1/2*C*d/f*tan(f*x+e)^2+1/2*(A*d+B*c
-C*d)/f*ln(1+tan(f*x+e)^2)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{Cd\tan\left(fx+e\right)^2+2\left((A-C)c-Bd\right)fx-\left(Bc+(A-C)d\right)\log\left(\frac{1}{\tan(fx+e)^2+1}\right)+2\left(Cc+Bd\right)\tan\left(fx+\right.}{2\,f}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="f
ricas")
```

output
```
1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*f*x - (B*c + (A - C)*d)*log(
1/(tan(f*x + e)^2 + 1)) + 2*(C*c + B*d)*tan(f*x + e))/f
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(60) = 120.

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\begin{cases} Acx+\frac{Ad\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Bc\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-Bdx+\frac{Bd\tan\left(e+fx\right)}{f}-Ccx+\frac{Cc\tan\left(e+fx\right)}{f}-\frac{Cd\log\left(\tan^2\left(e\right.\right.}{2f} \\ x(c+d\tan\left(e\right))\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Piecewise((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) + B*c*log(tan(e + f*
x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x + C*c*tan(e + f*x)/f
 - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x)**2/(2*f), Ne(f, 0
)), (x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2), True))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int (c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{Cd\tan(fx+e)^2+2\,((A-C)c-Bd)(fx+e)+(Bc+(A-C)d)\log\left(\tan(fx+e)^2+1\right)+2\,(Cc+Bd)\tan(fx+e)}{2\,f}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="m
axima")
```

output
```
1/2*(C*d*tan(f*x + e)^2 + 2*((A - C)*c - B*d)*(f*x + e) + (B*c + (A - C)*d
)*log(tan(f*x + e)^2 + 1) + 2*(C*c + B*d)*tan(f*x + e))/f
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. 2(71) = 142.

Time = 0.78 (sec) , antiderivative size = 761, normalized size of antiderivative = 10.42

$$\int (c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="g
iac")
```

output
```
1/2*(2*A*c*f*x*tan(f*x)^2*tan(e)^2 - 2*C*c*f*x*tan(f*x)^2*tan(e)^2 - 2*B*d
*f*x*tan(f*x)^2*tan(e)^2 - B*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan
(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^2*tan
(e)^2 - A*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + C*d*log(4*(
tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*
x)^2 + tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - 4*A*c*f*x*tan(f*x)*tan(e) + 4*
C*c*f*x*tan(f*x)*tan(e) + 4*B*d*f*x*tan(f*x)*tan(e) + C*d*tan(f*x)^2*tan(e
)^2 + 2*B*c*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^
2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)*tan(e) + 2*A*d*log(4*(ta
n(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 + tan(e)^2 + 1))*tan(f*x)*tan(e) - 2*C*d*log(4*(tan(f*x)^2*tan(e)^2 - 2
*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*t
an(f*x)*tan(e) - 2*C*c*tan(f*x)^2*tan(e) - 2*B*d*tan(f*x)^2*tan(e) - 2*C*c
*tan(f*x)*tan(e)^2 - 2*B*d*tan(f*x)*tan(e)^2 + 2*A*c*f*x - 2*C*c*f*x - 2*B
*d*f*x + C*d*tan(f*x)^2 + C*d*tan(e)^2 - B*c*log(4*(tan(f*x)^2*tan(e)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))
- A*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 + tan(e)^2 + 1)) + C*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*ta
n(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1)) +...
```

3.53.9 Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int (c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{\tan(e+f\,x)\,(B\,d+C\,c)}{f}-x\,(B\,d-A\,c+C\,c) + \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,d}{2}+\frac{B\,c}{2}-\frac{C\,d}{2}\right)}{f}+\frac{C\,d\tan(e+f\,x)^2}{2\,f}$$

input
```
int((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)*(B*d + C*c))/f - x*(B*d - A*c + C*c) + (log(tan(e + f*x)^2 +
 1)*((A*d)/2 + (B*c)/2 - (C*d)/2))/f + (C*d*tan(e + f*x)^2)/(2*f)
```

3.54 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

3.54.1 Optimal result

Integrand size = 43, antiderivative size = 156

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{(a(Ac-cC-Bd)+b(Bc+(A-C)d))x}{a^2+b^2} + \frac{(Abc-aBc-bcC-aAd-bBd+aCd)\log(\cos(e+fx))}{(a^2+b^2)\,f} + \frac{\left(Ab^2-a(bB-aC)\right)(bc-ad)\log(a+b\tan(e+fx))}{b^2\left(a^2+b^2\right)f} + \frac{Cd\tan(e+fx)}{bf}$$

output
```
(a*(A*c-B*d-C*c)+b*(B*c+(A-C)*d))*x/(a^2+b^2)+(-A*a*d+A*b*c-B*a*c-B*b*d+C*
a*d-C*b*c)*ln(cos(f*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)*ln(a+
b*tan(f*x+e))/b^2/(a^2+b^2)/f+C*d*tan(f*x+e)/b/f
```

3.54.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{\frac{(A+iB-C)(-ic+d)\log(i-\tan(e+fx))}{a+ib} + \frac{(A-iB-C)(ic+d)\log(i+\tan(e+fx))}{a-ib} + \frac{2\left(Ab^2+a(-bB+aC)\right)(bc-ad)\log(a+b\tan(e+fx))}{b^2(a^2+b^2)} + 2C}{2f}$$

input
```
Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x]),x]
```

output
```
(((A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]])/(a + I*b) + ((A - I*B
- C)*(I*c + d)*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 + a*(-(b*B) +
a*C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)) + (2*C*d*Tan[
e + f*x])/b)/(2*f)
```

3.54.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4120, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{Cd\tan(e+fx)}{bf}-\frac{\int -\frac{(bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{a+b\tan(e+fx)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{a+b\tan(e+fx)}dx}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(bcC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{a+b\tan(e+fx)}dx}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$$\Big\downarrow \text{4109}$$

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{b(-aAd-aBc+aCd+Abc-bBd-bcC)\int \tan(e+fx)dx}{a^2+b^2}+\frac{bx(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$\downarrow$ 3042

$$\frac{-\frac{b(-aAd-aBc+aCd+Abc-bBd-bcC)\int \tan(e+fx)dx}{a^2+b^2}+\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{bx(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$\downarrow$ 3956

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{b\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f(a^2+b^2)}+\frac{bx(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$\downarrow$ 4100

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf(a^2+b^2)}+\frac{b\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f(a^2+b^2)}+\frac{bx(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}}{b}+\frac{Cd\tan(e+fx)}{bf}$$

$\downarrow$ 16

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{bf(a^2+b^2)}+\frac{b\log(\cos(e+fx))(-aAd-aBc+aCd+Abc-bBd-bcC)}{f(a^2+b^2)}+\frac{bx(a(Ac-Bd-cC)+bd(A-C)+bBc)}{a^2+b^2}}{b}+\frac{Cd\tan(e+fx)}{bf}$$

input
```
Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*
Tan[e + f*x]),x]
```

output
```
((b*(b*B*c + b*(A - C)*d + a*(A*c - c*C - B*d))*x)/(a^2 + b^2) + (b*(A*b*c
 - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d)*Log[Cos[e + f*x]])/((a^2 + b^2)*
f) + ((A*b^2 - a*(b*B - a*C))*(b*c - a*d)*Log[a + b*Tan[e + f*x]])/(b*(a^2
 + b^2)*f))/b + (C*d*Tan[e + f*x])/(b*f)
```

3.54. $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

3.54.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.54. $\int \frac{(c+d\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.54.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)Cd}{b}+\frac{\left(-Aa\,b^2d+A\,b^3c+B\,a^2bd-Ba\,b^2c-a^3Cd+C\,a^2bc\right)\ln(a+b\tan(fx+e))}{b^2\left(a^2+b^2\right)}+\frac{(Aad-Abc+Bac+bdB-Cad+Cbc)\ln\left(1+\tan\right.}{2}}{f}$
default	$\frac{\frac{\tan(fx+e)Cd}{b}+\frac{\left(-Aa\,b^2d+A\,b^3c+B\,a^2bd-Ba\,b^2c-a^3Cd+C\,a^2bc\right)\ln(a+b\tan(fx+e))}{b^2\left(a^2+b^2\right)}+\frac{(Aad-Abc+Bac+bdB-Cad+Cbc)\ln\left(1+\mathrm{ta}\right.}{2}}{f}$
norman	$\frac{(Aac+Abd-Bad+Bbc-Cac-Cbd)x}{a^2+b^2}+\frac{Cd\tan(fx+e)}{bf}+\frac{(Aad-Abc+Bac+bdB-Cad+Cbc)\ln\left(1+\tan(fx+e)^2\right)}{2(a^2+b^2)f}-$
parallelrisch	$\frac{2Aa\,b^2cfx+2A\,b^3dfx-2Ba\,b^2dfx+2B\,b^3cfx-2Ca\,b^2cfx-2C\,b^3dfx+A\ln\left(1+\tan(fx+e)^2\right)a\,b^2d-A\ln\left(1+\tan(fx+e)^2\right.}{}$
risch	$-\frac{2iB\,a^2de}{bf(a^2+b^2)}+\frac{2ia^3Cde}{b^2f(a^2+b^2)}-\frac{2iC\,a^2ce}{bf(a^2+b^2)}-\frac{xAc}{ib-a}+\frac{xBd}{ib-a}+\frac{xCc}{ib-a}-\frac{2iCade}{b^2f}+\frac{2iAade}{f(a^2+b^2)}-\frac{2ibAce}{f(a^2+b^2)}-$

input
```
int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,me
thod=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*d/b+(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^
2*b*c)/b^2/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(a^2+b^2)*(1/2*(A*a*d-A*b*c+B*a*
c+B*b*d-C*a*d+C*b*c)*ln(1+tan(f*x+e)^2)+(A*a*c+A*b*d-B*a*d+B*b*c-C*a*c-C*b
*d)*arctan(tan(f*x+e))))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.45

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$= \frac{2\,(((A-C)ab^2+Bb^3)c-(Bab^2-(A-C)b^3)d)fx+2\,(Ca^2b+Cb^3)d\tan(fx+e)+((Ca^2b-Bab^2+}{}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a*b^2 + B*b^3)*c - (B*a*b^2 - (A - C)*b^3)*d)*f*x + 2*(C*
a^2*b + C*b^3)*d*tan(f*x + e) + ((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 -
B*a^2*b + A*a*b^2)*d)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/
(tan(f*x + e)^2 + 1)) - ((C*a^2*b + C*b^3)*c - (C*a^3 - B*a^2*b + C*a*b^2
- B*b^3)*d)*log(1/(tan(f*x + e)^2 + 1)))/((a^2*b^2 + b^4)*f)
```

3.54.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 2387, normalized size of antiderivative = 15.30

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e
)),x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(a,
 0) & Eq(b, 0) & Eq(f, 0)), ((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f) +
 B*c*log(tan(e + f*x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*x +
 C*c*tan(e + f*x)/f - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e + f*x
)**2/(2*f))/a, Eq(b, 0)), (I*A*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*
I*b*f) + A*c*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*A*c/(2*b*f*tan(e + f*x
) - 2*I*b*f) + A*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*A*d
*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) - A*d/(2*b*f*tan(e + f*x) - 2*I*b*f) +
 B*c*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*B*c*f*x/(2*b*f*ta
n(e + f*x) - 2*I*b*f) - B*c/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*B*d*f*x*tan
(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + B*d*f*x/(2*b*f*tan(e + f*x) - 2
*I*b*f) + B*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) -
2*I*b*f) - I*B*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) -
 I*B*d/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*C*c*f*x*tan(e + f*x)/(2*b*f*tan(
e + f*x) - 2*I*b*f) + C*c*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + C*c*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c*log(
tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*C*c/(2*b*f*tan(e +
 f*x) - 2*I*b*f) - 3*C*d*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) +
 3*I*C*d*f*x/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*C*d*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + C*d*log(tan(e + f*x)**...
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.17

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\frac{2\,Cd\tan(fx+e)}{b}+\frac{2\,(((A-C)a+Bb)c-(Ba-(A-C)b)d)(fx+e)}{a^2+b^2}+\frac{2\,((Ca^2b-Bab^2+Ab^3)c-(Ca^3-Ba^2b+Aab^2)d)\log(b\tan(fx+e)+a)}{a^2b^2+b^4}+\cdots}{2\,f}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x, algorithm="maxima")
```

output
```
1/2*(2*C*d*tan(f*x + e)/b + 2*(((A - C)*a + B*b)*c - (B*a - (A - C)*b)*d)*
(f*x + e)/(a^2 + b^2) + 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B*a^2*
b + A*a*b^2)*d)*log(b*tan(f*x + e) + a)/(a^2*b^2 + b^4) + ((B*a - (A - C)*
b)*c + ((A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2))/f
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\frac{2\,Cd\tan(fx+e)}{b}+\frac{2\,(Aac-Cac+Bbc-Bad+Abd-Cbd)(fx+e)}{a^2+b^2}+\frac{(Bac-Abc+Cbc+Aad-Cad+Bbd)\log\left(\tan(fx+e)^2+1\right)}{a^2+b^2}+\frac{2\,(Ca^2bc-Ba\cdots}{\cdots}}{2\,f}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x, algorithm="giac")
```

output
```
1/2*(2*C*d*tan(f*x + e)/b + 2*(A*a*c - C*a*c + B*b*c - B*a*d + A*b*d - C*b
*d)*(f*x + e)/(a^2 + b^2) + (B*a*c - A*b*c + C*b*c + A*a*d - C*a*d + B*b*d
)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b*c - B*a*b^2*c + A*b^3*c
 - C*a^3*d + B*a^2*b*d - A*a*b^2*d)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2
+ b^4))/f
```

3.54.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,d+B\,c-C\,d-A\,c\,1\mathrm{i}+B\,d\,1\mathrm{i}+C\,c\,1\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,d+A\,d\,1\mathrm{i}+B\,c\,1\mathrm{i}-A\,c+C\,c-C\,d\,1\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^2\left(A\,a\,d+B\,a\,c\right)-b\left(B\,a^2\,d+C\,a^2\,c\right)-A\,b^3\,c+C\,a^3\,d\right)}{f\,\left(a^2\,b^2+b^4\right)}$$
$$+\frac{C\,d\tan(e+f\,x)}{b\,f}$$

input
```
int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*
tan(e + f*x)),x)
```

output
```
(log(tan(e + f*x) - 1i)*(A*d - A*c*1i + B*c + B*d*1i + C*c*1i - C*d))/(2*f
*(a + b*1i)) + (log(tan(e + f*x) + 1i)*(A*d*1i - A*c + B*c*1i + B*d + C*c
- C*d*1i))/(2*f*(a*1i + b)) - (log(a + b*tan(e + f*x))*(b^2*(A*a*d + B*a*c
) - b*(B*a^2*d + C*a^2*c) - A*b^3*c + C*a^3*d))/(f*(b^4 + a^2*b^2)) + (C*d
*tan(e + f*x))/(b*f)
```

3.55 $\int \frac{(c+d\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.55.1 Optimal result

Integrand size = 43, antiderivative size = 265

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A-C)d))\,x}{(a^2+b^2)^2}$$
$$+ \frac{(2ab(Ac-cC-Bd)-a^2(Bc+(A-C)d)+b^2(Bc+(A-C)d))\log(\cos(e+fx))}{(a^2+b^2)^2 f}$$
$$+ \frac{(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3C)d))\log(a+b\tan(e+fx))}{b^2(a^2+b^2)^2 f}$$
$$- \frac{(Ab^2-a(bB-aC))\,(bc-ad)}{b^2(a^2+b^2)\,f(a+b\tan(e+fx))}$$

output
```
(a^2*(A*c-B*d-C*c)-b^2*(A*c-B*d-C*c)+2*a*b*(B*c+(A-C)*d))*x/(a^2+b^2)^2+(2
*a*b*(A*c-B*d-C*c)-a^2*(B*c+(A-C)*d)+b^2*(B*c+(A-C)*d))*ln(cos(f*x+e))/(a^
2+b^2)^2/f+(a^4*C*d+b^4*(A*d+B*c)+2*a*b^3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*
C)*d))*ln(a+b*tan(f*x+e))/b^2/(a^2+b^2)^2/f-(A*b^2-a*(B*b-C*a))*(-a*d+b*c)
/b^2/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.55.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.82

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \frac{\frac{(A+iB-C)(-ic+d)\log(i-\tan(e+fx))}{(a+ib)^2}+\frac{(A-iB-C)(ic+d)\log(i+\tan(e+fx))}{(a-ib)^2}+\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3}{b^2(a^2+b^2)^2}}{2f}$$

input
```
Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^2,x]
```

output
```
(((A + I*B - C)*((-I)*c + d)*Log[I - Tan[e + f*x]])/(a + I*b)^2 + ((A - I*
B - C)*(I*c + d)*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (2*(a^4*C*d + b^4*(B
*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Log[a
 + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2) - (2*(A*b^2 + a*(-(b*B) + a*C))*(b
*c - a*d))/(b^2*(a^2 + b^2)*(a + b*Tan[e + f*x])))/(2*f)
```

3.55.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4118}$$

$$\frac{\int \frac{Cda^2+b(Ac-Cc-Bd)a+(a^2+b^2)Cd\tan^2(e+fx)+b^2(Bc+Ad)-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{a+b\tan(e+fx)}dx}{b\left(a^2+b^2\right)} - \frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{Cda^2+b(Ac-Cc-Bd)a+(a^2+b^2)Cd\tan(e+fx)^2+b^2(Bc+Ad)-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{a+b\tan(e+fx)}dx}{b(a^2+b^2)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 4109$$

$$\frac{-\frac{b(-(a^2(d(A-C)+Bc))+2ab(Ac-Bd-cC)+b^2(d(A-C)+Bc))\int\tan(e+fx)dx}{a^2+b^2}+\frac{(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+I}{a^2+b^2}}{b(a^2+b^2)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{b(-(a^2(d(A-C)+Bc))+2ab(Ac-Bd-cC)+b^2(d(A-C)+Bc))\int\tan(e+fx)dx}{a^2+b^2}+\frac{(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+I}{a^2+b^2}}{b(a^2+b^2)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc))\int\frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{b\log(\cos(e+fx))(-(a^2(d(A-C)+Bc))+2ab(Ac-Bd-cC}{f(a^2+b^2)}}{b(a^2+b^2)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 4100$$

$$\frac{\frac{(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc))\int\frac{1}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf(a^2+b^2)}+\frac{b\log(\cos(e+fx))(-(a^2(d(A-C)+Bc))+2ab(}{f(a^2+b^2)}}{b(a^2+b^2)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{b\log(\cos(e+fx))(-(a^2(d(A-C)+Bc))+2ab(Ac-Bd-cC)+b^2(d(A-C)+Bc))}{f(a^2+b^2)}+\frac{bx(a^2(Ac-Bd-cC)+2ab(d(A-C)+Bc)-b^2(Ac-Bd-cC))}{a^2+b^2}+}{b(a^2+b^2)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{b^2f(a^2+b^2)(a+b\tan(e+fx))}$$

3.55. $\int \frac{(c+d\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

input
```
Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*
Tan[e + f*x])^2,x]
```

output
```
((b*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)*
d))*x)/(a^2 + b^2) + (b*(2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C)*d) +
 b^2*(B*c + (A - C)*d))*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((a^4*C*d + b
^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*
Log[a + b*Tan[e + f*x]])/(b*(a^2 + b^2)*f))/(b*(a^2 + b^2)) - ((A*b^2 - a*
(b*B - a*C))*(b*c - a*d))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

3.55.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.55.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.21

method	result
derivativedivides	$-\frac{-A a b^2 d+A b^3 c+B a^2 b d-B a b^2 c-a^3 C d+C a^2 b c}{b^2(a^2+b^2)(a+b\tan(fx+e))}+\frac{(-A a^2 b^2 d+2A a b^3 c+A b^4 d-B a^2 b^2 c-2B a b^3 d+B b^4 c+a^4 C d+3C a^2 b^2 d-2C\cdots}{(a^2+b^2)^2 b^2}\cdots$
default	$-\frac{-A a b^2 d+A b^3 c+B a^2 b d-B a b^2 c-a^3 C d+C a^2 b c}{b^2(a^2+b^2)(a+b\tan(fx+e))}+\frac{(-A a^2 b^2 d+2A a b^3 c+A b^4 d-B a^2 b^2 c-2B a b^3 d+B b^4 c+a^4 C d+3C a^2 b^2 d-2C\cdots}{(a^2+b^2)^2 b^2}\cdots$
norman	$\frac{\frac{a(A a^2 c+2A a b d-A b^2 c-B a^2 d+2B a b c+B b^2 d-C a^2 c-2C a b d+C b^2 c)x}{a^4+2a^2b^2+b^4}+\frac{A a b^2 d-A b^3 c-B a^2 b d+B a b^2 c+a^3 C d-C a^2 b c}{b^2 f(a^2+b^2)}+\frac{b(A a^2\cdots}{\cdots}}{a+b\tan(fx+e)}\cdots$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,x,
method=_RETURNVERBOSE)
```

output
```
1/f*(-(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^2*b*c)/b^2/(a^2+
b^2)/(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(-A*a^2*b^2*d+2*A*a*b^3*c+A*b^4*d-B*a^
2*b^2*c-2*B*a*b^3*d+B*b^4*c+C*a^4*d+3*C*a^2*b^2*d-2*C*a*b^3*c)/b^2*ln(a+b*
tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(A*a^2*d-2*A*a*b*c-A*b^2*d+B*a^2*c+2*B*a*b*
d-B*b^2*c-C*a^2*d+2*C*a*b*c+C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c+2*A*a*b*d
-A*b^2*c-B*a^2*d+2*B*a*b*c+B*b^2*d-C*a^2*c-2*C*a*b*d+C*b^2*c)*arctan(tan(f
*x+e))))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(265) = 530.

Time = 0.39 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.10

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{2\,(((A-C)a^3b^2+2\,Ba^2b^3-(A-C)ab^4)c-(Ba^3b^2-2\,(A-C)a^2b^3-Bab^4)d)fx-2\,(Ca^2b^3-Bab^4+}{}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c - (B*a^3*b^2 - 2
*(A - C)*a^2*b^3 - B*a*b^4)*d)*f*x - 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c + 2
*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*d - ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B
*a*b^4)*c - (C*a^5 - (A - 3*C)*a^3*b^2 - 2*B*a^2*b^3 + A*a*b^4)*d + ((B*a^
2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*
b^4 + A*b^5)*d)*tan(f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e)
 + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^4*b + 2*C*a^2*b^3 + C*b^5)*d*tan(f*x
 + e) + (C*a^5 + 2*C*a^3*b^2 + C*a*b^4)*d)*log(1/(tan(f*x + e)^2 + 1)) + 2
*((((A - C)*a^2*b^3 + 2*B*a*b^4 - (A - C)*b^5)*c - (B*a^2*b^3 - 2*(A - C)*
a*b^4 - B*b^5)*d)*f*x + (C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c - (C*a^4*b - B
*a^3*b^2 + A*a^2*b^3)*d)*tan(f*x + e))/((a^4*b^3 + 2*a^2*b^5 + b^7)*f*tan(
f*x + e) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*f)
```

3.55.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 9721, normalized size of antiderivative = 36.68

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e
))**2,x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq
(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c*x + A*d*log(tan(e + f*x)**2 + 1)/(2*f
) + B*c*log(tan(e + f*x)**2 + 1)/(2*f) - B*d*x + B*d*tan(e + f*x)/f - C*c*
x + C*c*tan(e + f*x)/f - C*d*log(tan(e + f*x)**2 + 1)/(2*f) + C*d*tan(e +
f*x)**2/(2*f))/a**2, Eq(b, 0)), (-A*c*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e
+ f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c*f*x*tan(e + f*x)
/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c*f*x
/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*c*tan
(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f)
+ 2*I*A*c/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f)
+ I*A*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
 f*x) - 4*b**2*f) + 2*A*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I
*b**2*f*tan(e + f*x) - 4*b**2*f) - I*A*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8
*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*A*d*tan(e + f*x)/(4*b**2*f*tan(e +
f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + I*B*c*f*x*tan(e + f*x)**2/
(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*B*c*f*
x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**
2*f) - I*B*c*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b
**2*f) + I*B*c*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
 f*x) - 4*b**2*f) + B*d*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 -...
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.28

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2\left(\left(\left((A-C)a^2+2\,Bab-(A-C)b^2\right)c-\left(Ba^2-2\,(A-C)ab-Bb^2\right)d\right)(fx+e)}{a^4+2\,a^2b^2+b^4} - \frac{2\left(\left(Ba^2b^2-2\,(A-C)ab^3-Bb^4\right)c-\left(Ca^4-(A-3\,C)a^2b^2-2\,Bab^3+Ab^4\right.\right.}{a^4b^2+2\,a^2b^4+b^6}}{}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c - (B*a^2 - 2*(A - C)*a*b -
 B*b^2)*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^2 - 2*(A - C)*a
*b^3 - B*b^4)*c - (C*a^4 - (A - 3*C)*a^2*b^2 - 2*B*a*b^3 + A*b^4)*d)*log(b
*tan(f*x + e) + a)/(a^4*b^2 + 2*a^2*b^4 + b^6) + ((B*a^2 - 2*(A - C)*a*b -
 B*b^2)*c + ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 +
1)/(a^4 + 2*a^2*b^2 + b^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c - (C*a^3 - B
*a^2*b + A*a*b^2)*d)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(f*x + e)))/f
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.95

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2\,(Aa^2c-Ca^2c+2\,Babc-Ab^2c+Cb^2c-Ba^2d+2\,Aabd-2\,Cabd+Bb^2d)(fx+e)}{a^4+2\,a^2b^2+b^4}+\frac{(Ba^2c-2\,Aabc+2\,Cabc-Bb^2c+Aa^2d-Ca^2d+2\,Babd-Ab^2d+}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c - B*a^2*d + 2*A*
a*b*d - 2*C*a*b*d + B*b^2*d)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c
- 2*A*a*b*c + 2*C*a*b*c - B*b^2*c + A*a^2*d - C*a^2*d + 2*B*a*b*d - A*b^2*
d + C*b^2*d)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^
2*c - 2*A*a*b^3*c + 2*C*a*b^3*c - B*b^4*c - C*a^4*d + A*a^2*b^2*d - 3*C*a^
2*b^2*d + 2*B*a*b^3*d - A*b^4*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^2 + 2
*a^2*b^4 + b^6) + 2*(B*a^2*b^2*c*tan(f*x + e) - 2*A*a*b^3*c*tan(f*x + e) +
 2*C*a*b^3*c*tan(f*x + e) - B*b^4*c*tan(f*x + e) - C*a^4*d*tan(f*x + e) +
A*a^2*b^2*d*tan(f*x + e) - 3*C*a^2*b^2*d*tan(f*x + e) + 2*B*a*b^3*d*tan(f*
x + e) - A*b^4*d*tan(f*x + e) - C*a^4*c + 2*B*a^3*b*c - 3*A*a^2*b^2*c + C*
a^2*b^2*c - A*b^4*c - B*a^4*d + 2*A*a^3*b*d - 2*C*a^3*b*d + B*a^2*b^2*d)/(
(a^4*b + 2*a^2*b^3 + b^5)*(b*tan(f*x + e) + a)))/f
```

3.55.9 Mupad [B] (verification not implemented)

Time = 21.30 (sec) , antiderivative size = 1875, normalized size of antiderivative = 7.08

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Too large to display}$$

input
```
int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*
tan(e + f*x))^2,x)
```

output
```
(log(a + b*tan(e + f*x))*(b^4*(A*d + B*c) - b^3*(2*B*a*d - 2*A*a*c + 2*C*a
*c) - b^2*(A*a^2*d + B*a^2*c - 3*C*a^2*d) + C*a^4*d))/(f*(b^6 + 2*a^2*b^4
+ a^4*b^2)) - (log((A*B*b^4*d^2 - A*B*b^4*c^2 + B*C*a^4*d^2 + B*C*b^4*c^2
- A^2*b^4*c*d + B^2*b^4*c*d + C^2*a^4*c*d - A^2*a*b^3*c^2 + A^2*a*b^3*d^2
+ B^2*a*b^3*c^2 - B^2*a*b^3*d^2 - C^2*a*b^3*c^2 + C^2*a*b^3*d^2 + A*B*a^2*
b^2*c^2 - A*B*a^2*b^2*d^2 - B*C*a^2*b^2*c^2 + 3*B*C*a^2*b^2*d^2 + A^2*a^2*
b^2*c*d - B^2*a^2*b^2*c*d + 3*C^2*a^2*b^2*c*d - A*C*a^4*c*d + A*C*b^4*c*d
+ 2*A*C*a*b^3*c^2 - 2*A*C*a*b^3*d^2 - 4*A*C*a^2*b^2*c*d + 4*A*B*a*b^3*c*d
- 4*B*C*a*b^3*c*d)/(b*(a^2 + b^2)^2) + (tan(e + f*x)*(A^2*b^4*c^2 + B^2*b^
4*d^2 + C^2*a^4*d^2 + C^2*b^4*c^2 + C^2*b^4*d^2 + A^2*a^2*b^2*d^2 + B^2*a^
2*b^2*c^2 + 3*C^2*a^2*b^2*d^2 - A*C*a^4*d^2 - 2*A*C*b^4*c^2 - A*C*b^4*d^2
- 4*A*C*a^2*b^2*d^2 - 2*A*B*b^4*c*d - B*C*a^4*c*d + B*C*b^4*c*d - 2*A*B*a*
b^3*c^2 + 2*A*B*a*b^3*d^2 + 2*B*C*a*b^3*c^2 - 2*B*C*a*b^3*d^2 - 2*A^2*a*b^
3*c*d + 2*B^2*a*b^3*c*d - 2*C^2*a*b^3*c*d + 2*A*B*a^2*b^2*c*d - 4*B*C*a^2*
b^2*c*d + 4*A*C*a*b^3*c*d))/(b*(a^2 + b^2)^2) + ((c + d*1i)*(A + B*1i - C)
*(A*b*c - B*b*d - 4*C*a*d - C*b*c + (tan(e + f*x)*(3*A*b^4*d + 3*B*b^4*c +
 2*C*a^4*d - 5*C*b^4*d + 4*A*a*b^3*c - 4*B*a*b^3*d - 4*C*a*b^3*c - A*a^2*b
^2*d - B*a^2*b^2*c + C*a^2*b^2*d))/(b*(a^2 + b^2)) + (b*(c + d*1i)*(4*a*b
- a^2*tan(e + f*x) + 3*b^2*tan(e + f*x))*(A + B*1i - C)*1i)/(a*1i - b)^2)*
1i)/(2*(a*1i - b)^2))*(A*c + A*d*1i + B*c*1i - B*d - C*c - C*d*1i))/(2*...
```

3.56 $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

3.56.1 Optimal result

Integrand size = 43, antiderivative size = 320

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{(a^3(Ac-cC-Bd)-3ab^2(Ac-cC-Bd)+3a^2b(Bc+(A-C)d)-b^3(Bc+(A-C)d))\,x}{(a^2+b^2)^3}$$
$$+\frac{(3a^2b(Ac-cC-Bd)-b^3(Ac-cC-Bd)-a^3(Bc+(A-C)d)+3ab^2(Bc+(A-C)d))\log(a\cos(e}{(a^2+b^2)^3 f}$$
$$-\frac{(Ab^2-a(bB-aC))\,(bc-ad)}{2b^2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$
$$-\frac{a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3C)d)}{b^2\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$

output
```
(a^3*(A*c-B*d-C*c)-3*a*b^2*(A*c-B*d-C*c)+3*a^2*b*(B*c+(A-C)*d)-b^3*(B*c+(A
-C)*d))*x/(a^2+b^2)^3+(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)-a^3*(B*c+(A
-C)*d)+3*a*b^2*(B*c+(A-C)*d))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/f-
1/2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)/b^2/(a^2+b^2)/f/(a+b*tan(f*x+e))^2+(-a^
4*C*d-b^4*(A*d+B*c)-2*a*b^3*(A*c-B*d-C*c)+a^2*b^2*(B*c+(A-3*C)*d))/b^2/(a^
2+b^2)^2/f/(a+b*tan(f*x+e))
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = -\frac{C(c+d\tan(e+fx))}{bf(a+b\tan(e+fx))^2}$$

$$-\frac{-\frac{bcC-bBd-aCd}{2bf(a+b\tan(e+fx))^2}+\frac{\frac{\left(-2b^3(Ac-cC-Bd)+2ab^2(Bc+(A-C)d)\right)\left(-\frac{\log(i-\tan(e+fx))}{2(ia-b)^3}+\frac{\log(i+\tan(e+fx))}{2(ia+b)^3}+\frac{b\left(3a^2-b^2\right)\log(a+b\tan(e+fx))}{\left(a^2+b^2\right)^3}-\frac{}{2(a\cdots}\right.}{b}}{}}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^3,x]
```

output
```
-((C*(c + d*Tan[e + f*x]))/(b*f*(a + b*Tan[e + f*x])^2)) - (-1/2*(b*c*C -
b*B*d - a*C*d)/(b*f*(a + b*Tan[e + f*x])^2) + (((-2*b^3*(A*c - c*C - B*d)
+ 2*a*b^2*(B*c + (A - C)*d))*(-1/2*Log[I - Tan[e + f*x]]/(I*a - b)^3 + Log
[I + Tan[e + f*x]]/(2*(I*a + b)^3) + (b*(3*a^2 - b^2)*Log[a + b*Tan[e + f*
x]])/(a^2 + b^2)^3 - b/(2*(a^2 + b^2)*(a + b*Tan[e + f*x])^2) - (2*a*b)/((
a^2 + b^2)^2*(a + b*Tan[e + f*x]))))/b - 2*b*(B*c + (A - C)*d)*(((-1/2*I)*
Log[I - Tan[e + f*x]])/(a + I*b)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(a - I*
b)^2 + (2*a*b*Log[a + b*Tan[e + f*x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a +
 b*Tan[e + f*x]))))/(2*b*f))/b
```

3.56.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4118, 3042, 4111, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$\Big\downarrow$ 4118

$$\frac{\int \frac{Cda^2+b(Ac-Cc-Bd)a+\left(a^2+b^2\right)Cd\tan^2(e+fx)+b^2(Bc+Ad)-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{(a+b\tan(e+fx))^2}dx}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{Cda^2+b(Ac-Cc-Bd)a+\left(a^2+b^2\right)Cd\tan(e+fx)^2+b^2(Bc+Ad)-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{(a+b\tan(e+fx))^2}dx}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 4111

$$\frac{\frac{\int \frac{b\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{a^4Cd-a^2b^2(d(A\cdots}{\cdots}}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{b\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{a^4Cd-a^2b^2(d(A\cdots}{\cdots}}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{b\left(-\left(a^3(d(A-C)+Bc)\right)+3a^2b(Ac-Bd-cC)+3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)\int\frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{bx\left(a^3(Ac-Bd-cC)+3a^2b(d(A-C)+Bc)-3ab^2(Ac-B\cdots\right.}{a^2+b^2}}{a^2+b^2}\cdots}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{b\left(-\left(a^3(d(A-C)+Bc)\right)+3a^2b(Ac-Bd-cC)+3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)\int\frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{bx\left(a^3(Ac-Bd-cC)+3a^2b(d(A-C)+Bc)-3ab^2(Ac-B\cdots\right.}{a^2+b^2}}{a^2+b^2}\cdots}{b\left(a^2+b^2\right)}-\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

3.56. $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$\Big\downarrow \text{4013}$$

$$\frac{\frac{\frac{b\left(-\left(a^3(d(A-C)+Bc)\right)+3a^2b(Ac-Bd-cC)+3ab^2(d(A-C)+Bc)-b^3(Ac-Bd-cC)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)}+\frac{bx\left(a^3(Ac-Bd-cC)+3a^2b(d(A-C)+Bc)-3a\right.}{a^2+b^2}}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)}{2b^2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

input
```
Int[((c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + b*
Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(b*c - a*d))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e
 + f*x])^2) + (((b*(a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) + 3*
a^2*b*(B*c + (A - C)*d) - b^3*(B*c + (A - C)*d))*x)/(a^2 + b^2) + (b*(3*a^
2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) - a^3*(B*c + (A - C)*d) + 3*
a*b^2*(B*c + (A - C)*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2
)*f))/(a^2 + b^2) - (a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d)
 - a^2*b^2*(B*c + (A - 3*C)*d))/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])))/(b
*(a^2 + b^2))
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.56.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{-Aa\,b^2d+A\,b^3c+B\,a^2bd-Ba\,b^2c-a^3Cd+C\,a^2bc}{2b^2\left(a^2+b^2\right)(a+b\tan(fx+e))^2}-\frac{-A\,a^2b^2d+2Aa\,b^3c+A\,b^4d-B\,a^2b^2c-2Ba\,b^3d+B\,b^4c+a^4Cd+3C\,a^2b^2d-2Ca}{\left(a^2+b^2\right)^2b^2(a+b\tan(fx+e))}$
default	$-\frac{-Aa\,b^2d+A\,b^3c+B\,a^2bd-Ba\,b^2c-a^3Cd+C\,a^2bc}{2b^2\left(a^2+b^2\right)(a+b\tan(fx+e))^2}-\frac{-A\,a^2b^2d+2Aa\,b^3c+A\,b^4d-B\,a^2b^2c-2Ba\,b^3d+B\,b^4c+a^4Cd+3C\,a^2b^2d-2Ca}{\left(a^2+b^2\right)^2b^2(a+b\tan(fx+e))}$
norman	$\frac{\left(A\,a^2b^2d-2Aa\,b^3c-A\,b^4d+B\,a^2b^2c+2Ba\,b^3d-B\,b^4c-a^4Cd-3C\,a^2b^2d+2Ca\,b^3c\right)\tan(fx+e)}{fb\left(a^4+2a^2b^2+b^4\right)}+\left(A\,a^3c+3A\,a^2bd-3Aa\,b^2c-A\,b^3d-\right.$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,x,
method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*(-A*a*b^2*d+A*b^3*c+B*a^2*b*d-B*a*b^2*c-C*a^3*d+C*a^2*b*c)/b^2/(
a^2+b^2)/(a+b*tan(f*x+e))^2-(-A*a^2*b^2*d+2*A*a*b^3*c+A*b^4*d-B*a^2*b^2*c-
2*B*a*b^3*d+B*b^4*c+C*a^4*d+3*C*a^2*b^2*d-2*C*a*b^3*c)/(a^2+b^2)^2/b^2/(a+
b*tan(f*x+e))-(A*a^3*d-3*A*a^2*b*c-3*A*a*b^2*d+A*b^3*c+B*a^3*c+3*B*a^2*b*d
-3*B*a*b^2*c-B*b^3*d-C*a^3*d+3*C*a^2*b*c+3*C*a*b^2*d-C*b^3*c)/(a^2+b^2)^3*
ln(a+b*tan(f*x+e))+1/(a^2+b^2)^3*(1/2*(A*a^3*d-3*A*a^2*b*c-3*A*a*b^2*d+A*b
^3*c+B*a^3*c+3*B*a^2*b*d-3*B*a*b^2*c-B*b^3*d-C*a^3*d+3*C*a^2*b*c+3*C*a*b^2
*d-C*b^3*c)*ln(1+tan(f*x+e)^2)+(A*a^3*c+3*A*a^2*b*d-3*A*a*b^2*c-A*b^3*d-B*
a^3*d+3*B*a^2*b*c+3*B*a*b^2*d-B*b^3*c-C*a^3*c-3*C*a^2*b*d+3*C*a*b^2*c+C*b^
3*d)*arctan(tan(f*x+e))))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(316) = 632$.

Time = 0.30 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.08

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{2\,(((A-C)a^5+3\,Ba^4b-3\,(A-C)a^3b^2-Ba^2b^3)c-(Ba^5-3\,(A-C)a^4b-3\,Ba^3b^2+(A-C)a^2b^3)d)}{}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B*a^2*b^3)*c - (B*a
^5 - 3*(A - C)*a^4*b - 3*B*a^3*b^2 + (A - C)*a^2*b^3)*d)*f*x + (2*(((A - C
)*a^3*b^2 + 3*B*a^2*b^3 - 3*(A - C)*a*b^4 - B*b^5)*c - (B*a^3*b^2 - 3*(A -
 C)*a^2*b^3 - 3*B*a*b^4 + (A - C)*b^5)*d)*f*x + (C*a^4*b - 3*B*a^3*b^2 + 5
*(A - C)*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c + (C*a^5 + B*a^4*b - (3*A - 7*C)*a
^3*b^2 - 5*B*a^2*b^3 + 3*A*a*b^4)*d)*tan(f*x + e)^2 - (3*C*a^4*b - 5*B*a^3
*b^2 + (7*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5 - 3*B*a^4*b + 5*(
A - C)*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*d - (((B*a^3*b^2 - 3*(A - C)*a^2*b
^3 - 3*B*a*b^4 + (A - C)*b^5)*c + ((A - C)*a^3*b^2 + 3*B*a^2*b^3 - 3*(A -
C)*a*b^4 - B*b^5)*d)*tan(f*x + e)^2 + (B*a^5 - 3*(A - C)*a^4*b - 3*B*a^3*b
^2 + (A - C)*a^2*b^3)*c + ((A - C)*a^5 + 3*B*a^4*b - 3*(A - C)*a^3*b^2 - B
*a^2*b^3)*d + 2*((B*a^4*b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^
4)*c + ((A - C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b^4)*d)*tan(
f*x + e))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e
)^2 + 1)) + 2*(2*(((A - C)*a^4*b + 3*B*a^3*b^2 - 3*(A - C)*a^2*b^3 - B*a*b
^4)*c - (B*a^4*b - 3*(A - C)*a^3*b^2 - 3*B*a^2*b^3 + (A - C)*a*b^4)*d)*f*x
 + (C*a^5 - 2*B*a^4*b + 3*(A - C)*a^3*b^2 + 3*B*a^2*b^3 - (3*A - 2*C)*a*b^
4 - B*b^5)*c + (B*a^5 - (2*A - 3*C)*a^4*b - 3*B*a^3*b^2 + 3*(A - C)*a^2*b^
3 + 2*B*a*b^4 - A*b^5)*d)*tan(f*x + e))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6
+ b^8)*f*tan(f*x + e)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*f*t...
```

3.56.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e
))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.79

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{\frac{2\left(\left((A-C)a^3+3\,Ba^2b-3\,(A-C)ab^2-Bb^3\right)c-\left(Ba^3-3\,(A-C)a^2b-3\,Bab^2+(A-C)b^3\right)d\right)(fx+e)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\left(\left(Ba^3-3\,(A-C)a^2b-3\,Bab^2+(A-C)b^3\right)\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c - (B*a^3 - 3
*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/(a^6 + 3*a^4*b^2 +
3*a^2*b^4 + b^6) - 2*((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*
c + ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d)*log(b*tan(f*x +
 e) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((B*a^3 - 3*(A - C)*a^2*b -
 3*B*a*b^2 + (A - C)*b^3)*c + ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 -
 B*b^3)*d)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
(C*a^4*b - 3*B*a^3*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c + (C*a^5
 + B*a^4*b - (3*A - 5*C)*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*d - 2*((B*a^2*b^
3 - 2*(A - C)*a*b^4 - B*b^5)*c - (C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4
+ A*b^5)*d)*tan(f*x + e))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 + 2*a^
2*b^6 + b^8)*tan(f*x + e)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(f*x + e)
))/f
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. 2(316) = 632.

Time = 0.81 (sec) , antiderivative size = 1006, normalized size of antiderivative = 3.14

$$\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{\frac{2\left(Aa^3c-Ca^3c+3\,Ba^2bc-3\,Aab^2c+3\,Cab^2c-Bb^3c-Ba^3d+3\,Aa^2bd-3\,Ca^2bd+3\,Bab^2d-Ab^3d+Cb^3d\right)(fx+e)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(Ba^3c-3\,Aa^2bc+3\,Ca^2bc-3\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^
3*c - B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d + 3*B*a*b^2*d - A*b^3*d + C*b^3*
d)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3*c - 3*A*a^2*b*c
+ 3*C*a^2*b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c + A*a^3*d - C*a^3*d + 3*B*
a^2*b*d - 3*A*a*b^2*d + 3*C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b*c - 3*A*a^2*b^2*c + 3*C*a^2*
b^2*c - 3*B*a*b^3*c + A*b^4*c - C*b^4*c + A*a^3*b*d - C*a^3*b*d + 3*B*a^2*
b^2*d - 3*A*a*b^3*d + 3*C*a*b^3*d - B*b^4*d)*log(abs(b*tan(f*x + e) + a))/
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + (3*B*a^3*b^4*c*tan(f*x + e)^2 - 9*
A*a^2*b^5*c*tan(f*x + e)^2 + 9*C*a^2*b^5*c*tan(f*x + e)^2 - 9*B*a*b^6*c*ta
n(f*x + e)^2 + 3*A*b^7*c*tan(f*x + e)^2 - 3*C*b^7*c*tan(f*x + e)^2 + 3*A*a
^3*b^4*d*tan(f*x + e)^2 - 3*C*a^3*b^4*d*tan(f*x + e)^2 + 9*B*a^2*b^5*d*tan
(f*x + e)^2 - 9*A*a*b^6*d*tan(f*x + e)^2 + 9*C*a*b^6*d*tan(f*x + e)^2 - 3*
B*b^7*d*tan(f*x + e)^2 + 8*B*a^4*b^3*c*tan(f*x + e) - 22*A*a^3*b^4*c*tan(f
*x + e) + 22*C*a^3*b^4*c*tan(f*x + e) - 18*B*a^2*b^5*c*tan(f*x + e) + 2*A*
a*b^6*c*tan(f*x + e) - 2*C*a*b^6*c*tan(f*x + e) - 2*B*b^7*c*tan(f*x + e) -
 2*C*a^6*b*d*tan(f*x + e) + 8*A*a^4*b^3*d*tan(f*x + e) - 14*C*a^4*b^3*d*ta
n(f*x + e) + 22*B*a^3*b^4*d*tan(f*x + e) - 18*A*a^2*b^5*d*tan(f*x + e) + 1
2*C*a^2*b^5*d*tan(f*x + e) - 2*B*a*b^6*d*tan(f*x + e) - 2*A*b^7*d*tan(f*x
+ e) - C*a^6*b*c + 6*B*a^5*b^2*c - 14*A*a^4*b^3*c + 11*C*a^4*b^3*c - 7*...
```

3.56.9 Mupad [B] (verification not implemented)

Time = 15.53 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.57

$$\int \frac{(c+d\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{\frac{A\,b^5\,c+C\,a^5\,d+A\,a\,b^4\,d+B\,a\,b^4\,c+B\,a^4\,b\,d+C\,a^4\,b\,c+5\,A\,a^2\,b^3\,c-3\,A\,a^3\,b^2\,d-3\,B\,a^3\,b^2\,c-3\,B\,a^2\,b^3\,d-3\,C\,a^2\,b^3\,c+5\,C\,a^3\,b^2\,d}{2\,b^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(e+f\,x)\,(}{}}{f\,\left(a^2+2\,a\,b\tan\left(e+f\,x\right)+b^2\tan\left(e+f\right.\right.}$$

$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,d+A\,d\,1\mathrm{i}+B\,c\,1\mathrm{i}-A\,c+C\,c-C\,d\,1\mathrm{i}\right)}{2\,f\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,d+B\,c-C\,d-A\,c\,1\mathrm{i}+B\,d\,1\mathrm{i}+C\,c\,1\mathrm{i}\right)}{2\,f\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$

$$-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(\left(A\,d+B\,c-C\,d\right)\,a^3+\left(3\,B\,d-3\,A\,c+3\,C\,c\right)\,a^2\,b+\left(3\,C\,d-3\,B\,c-3\,A\,d\right)\,a\right.}{f\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}$$

input
```
int(((c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a + b*
tan(e + f*x))^3,x)
```

output
```
- ((A*b^5*c + C*a^5*d + A*a*b^4*d + B*a*b^4*c + B*a^4*b*d + C*a^4*b*c + 5*
A*a^2*b^3*c - 3*A*a^3*b^2*d - 3*B*a^3*b^2*c - 3*B*a^2*b^3*d - 3*C*a^2*b^3*
c + 5*C*a^3*b^2*d)/(2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(e + f*x)*(A*b^4*
d + B*b^4*c + C*a^4*d + 2*A*a*b^3*c - 2*B*a*b^3*d - 2*C*a*b^3*c - A*a^2*b^
2*d - B*a^2*b^2*c + 3*C*a^2*b^2*d))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 +
 b^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))) - (log(tan(e + f*x) + 1i)*(A*d*
1i - A*c + B*c*1i + B*d + C*c - C*d*1i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i
 + b^3)) - (log(tan(e + f*x) - 1i)*(A*d - A*c*1i + B*c + B*d*1i + C*c*1i -
 C*d))/(2*f*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(a + b*tan(e + f*x)
)*(a^3*(A*d + B*c - C*d) - b^3*(B*d - A*c + C*c) + a^2*b*(3*B*d - 3*A*c +
3*C*c) - a*b^2*(3*A*d + 3*B*c - 3*C*d)))/(f*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2))
```

3.56. $\int \frac{(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

3.57 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2 (A+B\tan(e+$

3.57.1 Optimal result

Integrand size = 45, antiderivative size = 661

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= -\Big(\big(a^3\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-3ab^2\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)$$
$$+3a^2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)-b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\big)x\Big)$$
$$+\frac{(3a^2b(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b^3(c^2C+2Bcd-Cd^2-A(c^2-d^2))-a^3(2c(A-C)d+B(}{f}$$
$$+\frac{d(3a^2b(Ac-cC-Bd)-b^3(Ac-cC-Bd)+a^3(Bc+(A-C)d)-3ab^2(Bc+(A-C)d))\tan(e+f}{f}$$
$$+\frac{(a^3B-3ab^2B+3a^2b(A-C)-b^3(A-C))\,(c+d\tan(e+fx))^2}{2f}$$
$$+\frac{(4a^3Cd^3-3a^2bd^2(3cC-16Bd)+3ab^2d(2c^2C-5Bcd+20(A-C)d^2)-b^3(c^3C-2Bc^2d+5c(A-C}{60d^4f}$$
$$+\frac{b(5b(Ab+aB-bC)d^2+(bc-ad)(bcC-2bBd-aCd))\tan(e+fx)(c+d\tan(e+fx))^3}{20d^3f}$$
$$-\frac{(bcC-2bBd-aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{10d^2f}$$
$$+\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}$$

output
```
-(a^3*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^
2-d^2))+3*a^2*b*(2*c*(A-C)*d+B*(c^2-d^2))-b^3*(2*c*(A-C)*d+B*(c^2-d^2)))*x
+(3*a^2*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C+2*B*c*d-C*d^2-A*(c^
2-d^2))-a^3*(2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*l
n(cos(f*x+e))/f+d*(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C)*
d)-3*a*b^2*(B*c+(A-C)*d))*tan(f*x+e)/f+1/2*(B*a^3-3*B*a*b^2+3*a^2*b*(A-C)-
b^3*(A-C))*(c+d*tan(f*x+e))^2/f+1/60*(4*a^3*C*d^3-3*a^2*b*d^2*(-16*B*d+3*C
*c)+3*a*b^2*d*(2*c^2*C-5*B*c*d+20*(A-C)*d^2)-b^3*(c^3*C-2*B*c^2*d+5*c*(A-C
)*d^2+20*B*d^3))*(c+d*tan(f*x+e))^3/d^4/f+1/20*b*(5*b*(A*b+B*a-C*b)*d^2+(-
a*d+b*c)*(-2*B*b*d-C*a*d+C*b*c))*tan(f*x+e)*(c+d*tan(f*x+e))^3/d^3/f-1/10*
(-2*B*b*d-C*a*d+C*b*c)*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3/d^2/f+1/6*C*(
a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^3/d/f
```

3.57.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 573, normalized size of antiderivative = 0.87

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$= \frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} + \frac{-\frac{3(bcC-2bBd-aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} + \frac{\frac{3b\left(5b(Ab+aB-bC)d^2+(bc-ad)(bcC-2bBd-aCd)\right)\tan(e+fx)(c+d\tan(e+fx))^3}{2df} - \frac{(-}{}}{}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d*f) + ((-3*(b*c*C -
2*b*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) +
((3*b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d))*
Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(2*d*f) - (((-24*a*d*(5*b*(A*b + a*B
- b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d)) + b*(-120*(a^2*B - b^2
*B + 2*a*b*(A - C))*d^3 + 6*c*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*
c*C - 2*b*B*d - a*C*d))))*(c + d*Tan[e + f*x])^3)/(3*d*f) - (60*(d^2*(3*a^
2*b*(A*c - c*C + B*d) - b^3*(A*c - c*C + B*d) + a^3*(B*c - (A - C)*d) - 3*
a*b^2*(B*c - (A - C)*d))*(I*(c + I*d)^2*Log[I - Tan[e + f*x]] - I*(c - I*d
)^2*Log[I + Tan[e + f*x]] - 2*d^2*Tan[e + f*x]) + (a^3*B - 3*a*b^2*B + 3*a
^2*b*(A - C) - b^3*(A - C))*d^2*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c
+ d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2))
)/f)/(4*d))/(5*d))/(6*d)
```

3.57.3 Rubi [A] (verified)

Time = 3.41 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 25, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4130$$

$$\frac{\int -3(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left((bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+f\right.}{6d}$$
$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}$$

$$\Big\downarrow \; 27$$

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}-$$
$$\frac{\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left((bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx)\right.}{2d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left((bcC-adC-2bBd)\tan(e+fx)^2-2(Ab-Cb+aB)d\tan(e+fx)\right.}{2d}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{\int -2(a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(c(cC-2Bd)b^2-ad(2cC+3Bd)b+a^2(5A-4C)d^2+\left(5b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-2bBd)\right)\tan^2\right.}{5d}}{2d}$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(c(cC-2Bd)b^2-ad(2cC+3Bd)b+a^2(5A-4C\right.}{}}{2d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(c(cC-2Bd)b^2-ad(2cC+3Bd)b+a^2(5A-4C\right.}{}}{2d}$$

$\downarrow$ 4120

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{b\tan(e+fx)(c+d\tan(e+fx))^3\left(5bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC)\right)}{4df} - \frac{\int -(c-}{}\right.}{}$$

$\downarrow$ 25

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int (c+d\tan(e+fx))^2\left(-c\left(Cc^2-2Bdc+5(A-C)d^2\right)b^3+3acd(2cC-5Bd)b^2-3a^2d^2(3cC+4Bc\right.}{}\right.}{}$$

$\downarrow$ 3042

3.57. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int(c+d\tan(e+fx))^2\left(-c\left(Cc^2-2Bdc+5(A-C)d^2\right)b^3+3acd(2cC-5Bd)b^2-3a^2d^2(3cC+4Bc}{}\right.}{}}{}$$

$\downarrow$ 4113

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int(c+d\tan(e+fx))^2\left(20\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-20\left(-\left((A-C\right.\right.}{}\right.}{}}{}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int(c+d\tan(e+fx))^2\left(20\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-20\left(-\left((A-C\right.\right.}{}\right.}{}}{}$$

$\downarrow$ 4011

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int(c+d\tan(e+fx))\left(20\left((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3(Bc+\right.\right.}{}\right.}{}}{}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{\int(c+d\tan(e+fx))\left(20\left((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3(Bc+\right.\right.}{}\right.}{}}{}$$

$\downarrow$ 4008

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df} - \frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - \frac{2\left(\frac{-20d^3\left(-\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+3ab^2\left(2\right.\right.}{}\right.}{}}{}$$

3.57. $\int(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}-$$

$$\frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}-\frac{2\left(\frac{-20d^3\left(-\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+3ab^2\left(2\right.\right.}{}\right.}{}}{}$$

$\Big\downarrow$ 3956

$$\frac{C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^3}{6df}-$$

$$\frac{\frac{(-aCd-2bBd+bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}-\frac{2\left(\frac{\frac{20d^3\log(\cos(e+fx))\left(-\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-C\right.\right.}{f}}{}\right.}{}}{}$$

input

```
Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

output

```
(C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^3)/(6*d*f) - (((b*c*C - 2*b
*B*d - a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) - (2*
((b*(5*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d))*Ta
n[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f) + (-20*d^3*(a^3*(c^2*C + 2*B*c*
d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d
^2)) + 3*a^2*b*(2*c*(A - C)*d + B*(c^2 - d^2)) - b^3*(2*c*(A - C)*d + B*(c
^2 - d^2)))*x + (20*d^3*(3*a^2*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))
 - b^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^3*(2*c*(A - C)*d + B*
(c^2 - d^2)) + 3*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])
/f + (20*d^4*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3*(B*c
 + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))*Tan[e + f*x])/f + (10*(a^3*B -
3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*d^3*(c + d*Tan[e + f*x])^2)/f +
 ((4*a^3*C*d^3 - 3*a^2*b*d^2*(3*c*C - 16*B*d) + 3*a*b^2*d*(2*c^2*C - 5*B*c
*d + 20*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 5*c*(A - C)*d^2 + 20*B*d^3
))*(c + d*Tan[e + f*x])^3)/(3*d*f))/(4*d)))/(5*d))/(2*d)
```

3.57. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.57.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.83

method	result
parts	$\frac{(2A\,a^3cd+3A\,a^2b\,c^2+B\,a^3c^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(B\,b^3d^2+3Ca\,b^2d^2+2C\,b^3cd)\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx\right.}{f}$
norman	$(A\,a^3c^2-A\,a^3d^2-6A\,a^2bcd-3Aa\,b^2c^2+3Aa\,b^2d^2+2A\,b^3cd-2B\,a^3cd-3B\,a^2b\,c^2+$
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*A*a^3*c*d+3*A*a^2*b*c^2+B*a^3*c^2)/f*ln(1+tan(f*x+e)^2)+(B*b^3*d^2+
3*C*a*b^2*d^2+2*C*b^3*c*d)/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)
-arctan(tan(f*x+e)))+(A*b^3*d^2+3*B*a*b^2*d^2+2*B*b^3*c*d+3*C*a^2*b*d^2+6*
C*a*b^2*c*d+C*b^3*c^2)/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f
*x+e)^2))+(A*a^3*d^2+6*A*a^2*b*c*d+3*A*a*b^2*c^2+2*B*a^3*c*d+3*B*a^2*b*c^2
+C*a^3*c^2)/f*(tan(f*x+e)-arctan(tan(f*x+e)))+(3*A*a*b^2*d^2+2*A*b^3*c*d+3
*B*a^2*b*d^2+6*B*a*b^2*c*d+B*b^3*c^2+C*a^3*d^2+6*C*a^2*b*c*d+3*C*a*b^2*c^2
)/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(3*A*a^2*b*d^2+6*A*a*
b^2*c*d+A*b^3*c^2+B*a^3*d^2+6*B*a^2*b*c*d+3*B*a*b^2*c^2+2*C*a^3*c*d+3*C*a^
2*b*c^2)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+A*a^3*c^2*x+C*b^3*d^2
/f*(1/6*tan(f*x+e)^6-1/4*tan(f*x+e)^4+1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)
^2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.04

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$= \frac{10\,Cb^3d^2\tan\left(fx+e\right)^6+12\left(2\,Cb^3cd+\left(3\,Cab^2+Bb^3\right)d^2\right)\tan\left(fx+e\right)^5+15\left(Cb^3c^2+2\left(3\,Cab^2+Bb^3\right)c\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*
d^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2
*b + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3
)*c^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b +
 3*(A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^3 - 3*B*a^2
*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2
 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*
d^2)*f*x + 30*((3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*
a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c*d + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^
2 - (A - C)*b^3)*d^2)*tan(f*x + e)^2 - 30*((B*a^3 + 3*(A - C)*a^2*b - 3*B*
a*b^2 - (A - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 +
B*b^3)*c*d - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(
1/(tan(f*x + e)^2 + 1)) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3
)*c^2 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A -
C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(604) = 1208$.

Time = 0.39 (sec) , antiderivative size = 1819, normalized size of antiderivative = 2.75

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Piecewise((A*a**3*c**2*x + A*a**3*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**3*
d**2*x + A*a**3*d**2*tan(e + f*x)/f + 3*A*a**2*b*c**2*log(tan(e + f*x)**2
+ 1)/(2*f) - 6*A*a**2*b*c*d*x + 6*A*a**2*b*c*d*tan(e + f*x)/f - 3*A*a**2*b
*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*a**2*b*d**2*tan(e + f*x)**2/(2*
f) - 3*A*a*b**2*c**2*x + 3*A*a*b**2*c**2*tan(e + f*x)/f - 3*A*a*b**2*c*d*l
og(tan(e + f*x)**2 + 1)/f + 3*A*a*b**2*c*d*tan(e + f*x)**2/f + 3*A*a*b**2*
d**2*x + A*a*b**2*d**2*tan(e + f*x)**3/f - 3*A*a*b**2*d**2*tan(e + f*x)/f
- A*b**3*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*c**2*tan(e + f*x)**2
/(2*f) + 2*A*b**3*c*d*x + 2*A*b**3*c*d*tan(e + f*x)**3/(3*f) - 2*A*b**3*c*
d*tan(e + f*x)/f + A*b**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**3*d**
2*tan(e + f*x)**4/(4*f) - A*b**3*d**2*tan(e + f*x)**2/(2*f) + B*a**3*c**2*
log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*a**3*c*d*x + 2*B*a**3*c*d*tan(e + f*x
)/f - B*a**3*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*a**3*d**2*tan(e + f*x
)**2/(2*f) - 3*B*a**2*b*c**2*x + 3*B*a**2*b*c**2*tan(e + f*x)/f - 3*B*a**2
*b*c*d*log(tan(e + f*x)**2 + 1)/f + 3*B*a**2*b*c*d*tan(e + f*x)**2/f + 3*B
*a**2*b*d**2*x + B*a**2*b*d**2*tan(e + f*x)**3/f - 3*B*a**2*b*d**2*tan(e +
 f*x)/f - 3*B*a*b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*c**2
*tan(e + f*x)**2/(2*f) + 6*B*a*b**2*c*d*x + 2*B*a*b**2*c*d*tan(e + f*x)**3
/f - 6*B*a*b**2*c*d*tan(e + f*x)/f + 3*B*a*b**2*d**2*log(tan(e + f*x)**2 +
 1)/(2*f) + 3*B*a*b**2*d**2*tan(e + f*x)**4/(4*f) - 3*B*a*b**2*d**2*tan...
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.05

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{10\,Cb^3d^2\tan\left(fx+e\right)^6+12\left(2\,Cb^3cd+\left(3\,Cab^2+Bb^3\right)d^2\right)\tan\left(fx+e\right)^5+15\left(Cb^3c^2+2\left(3\,Cab^2+Bb^3\right)c\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
1/60*(10*C*b^3*d^2*tan(f*x + e)^6 + 12*(2*C*b^3*c*d + (3*C*a*b^2 + B*b^3)*
d^2)*tan(f*x + e)^5 + 15*(C*b^3*c^2 + 2*(3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2
*b + 3*B*a*b^2 + (A - C)*b^3)*d^2)*tan(f*x + e)^4 + 20*((3*C*a*b^2 + B*b^3
)*c^2 + 2*(3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*c*d + (C*a^3 + 3*B*a^2*b +
 3*(A - C)*a*b^2 - B*b^3)*d^2)*tan(f*x + e)^3 + 30*((3*C*a^2*b + 3*B*a*b^2
 + (A - C)*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c*d
+ (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*tan(f*x + e)^2
+ 60*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 - 2*(B*a^3 +
 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2*b
 - 3*(A - C)*a*b^2 + B*b^3)*d^2)*(f*x + e) + 30*((B*a^3 + 3*(A - C)*a^2*b
- 3*B*a*b^2 - (A - C)*b^3)*c^2 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*
b^2 + B*b^3)*c*d - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2
)*log(tan(f*x + e)^2 + 1) + 60*((C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b
^3)*c^2 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d + ((A
- C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*tan(f*x + e))/f
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21368 vs. 2(650) = 1300.

Time = 23.58 (sec) , antiderivative size = 21368, normalized size of antiderivative = 32.33

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
1/60*(60*A*a^3*c^2*f*x*tan(f*x)^6*tan(e)^6 - 60*C*a^3*c^2*f*x*tan(f*x)^6*t
an(e)^6 - 180*B*a^2*b*c^2*f*x*tan(f*x)^6*tan(e)^6 - 180*A*a*b^2*c^2*f*x*ta
n(f*x)^6*tan(e)^6 + 180*C*a*b^2*c^2*f*x*tan(f*x)^6*tan(e)^6 + 60*B*b^3*c^2
*f*x*tan(f*x)^6*tan(e)^6 - 120*B*a^3*c*d*f*x*tan(f*x)^6*tan(e)^6 - 360*A*a
^2*b*c*d*f*x*tan(f*x)^6*tan(e)^6 + 360*C*a^2*b*c*d*f*x*tan(f*x)^6*tan(e)^6
 + 360*B*a*b^2*c*d*f*x*tan(f*x)^6*tan(e)^6 + 120*A*b^3*c*d*f*x*tan(f*x)^6*
tan(e)^6 - 120*C*b^3*c*d*f*x*tan(f*x)^6*tan(e)^6 - 60*A*a^3*d^2*f*x*tan(f*
x)^6*tan(e)^6 + 60*C*a^3*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*B*a^2*b*d^2*f*x
*tan(f*x)^6*tan(e)^6 + 180*A*a*b^2*d^2*f*x*tan(f*x)^6*tan(e)^6 - 180*C*a*b
^2*d^2*f*x*tan(f*x)^6*tan(e)^6 - 60*B*b^3*d^2*f*x*tan(f*x)^6*tan(e)^6 - 30
*B*a^3*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90*A*a^2*b*c
^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 90*C*a^2*b*c^2*log(4
*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(
f*x)^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 90*B*a*b^2*c^2*log(4*(tan(f*
x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 +
 tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*A*b^3*c^2*log(4*(tan(f*x)^2*tan(e
)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2
+ 1))*tan(f*x)^6*tan(e)^6 - 30*C*b^3*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2...
```

3.57. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$

3.57.9 Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.35

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= x\left(A\,a^3\,c^2-A\,a^3\,d^2+B\,b^3\,c^2-C\,a^3\,c^2-B\,b^3\,d^2+C\,a^3\,d^2+2\,A\,b^3\,c\,d-2\,B\,a^3\,c\,d-2\,C\,b^3\,c\,d-3\,A\,a\,b^2\,c^2+3\,A\,a\,b^2\,d^2-3\,B\,a^2\,b\,c^2+3\,B\,a^2\,b\,d^2+3\,C\,a\,b^2\,c^2-3\,C\,a\,b^2\,d^2-6\,A\,a^2\,b\,c\,d+6\,B\,a\,b^2\,c\,d+6\,C\,a^2\,b\,c\,d\right)$$
$$-\frac{\tan(e+f\,x)\,\left(B\,b^3\,c^2-A\,a^3\,d^2-b^2\,d\,(B\,b\,d+3\,C\,a\,d+2\,C\,b\,c)-C\,a^3\,c^2+C\,a^3\,d^2+2\,A\,b^3\,c\,d-2\,B\,a$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,b^3\,c^2}{2}-\frac{B\,a^3\,c^2}{2}-\frac{A\,b^3\,d^2}{2}+\frac{B\,a^3\,d^2}{2}-\frac{C\,b^3\,c^2}{2}+\frac{C\,b^3\,d^2}{2}-A\,a^3\,c\,d-B\,b^3\,c\,d+C\,a^3\,c$$
$$+\frac{\tan(e+f\,x)^4\,\left(\frac{A\,b^3\,d^2}{4}+\frac{C\,b^3\,c^2}{4}-\frac{C\,b^3\,d^2}{4}+\frac{B\,b^3\,c\,d}{2}+\frac{3\,B\,a\,b^2\,d^2}{4}+\frac{3\,C\,a^2\,b\,d^2}{4}+\frac{3\,C\,a\,b^2\,c\,d}{2}\right)}{f}$$
$$+\frac{\tan(e+f\,x)^3\,\left(\frac{B\,b^3\,c^2}{3}-\frac{b^2\,d\,(B\,b\,d+3\,C\,a\,d+2\,C\,b\,c)}{3}+\frac{C\,a^3\,d^2}{3}+\frac{2\,A\,b^3\,c\,d}{3}+A\,a\,b^2\,d^2+B\,a^2\,b\,d^2+C\,a\,b^2\,c^2+2\,B$$
$$+\frac{\tan(e+f\,x)^2\,\left(\frac{A\,b^3\,c^2}{2}-\frac{A\,b^3\,d^2}{2}+\frac{B\,a^3\,d^2}{2}-\frac{C\,b^3\,c^2}{2}+\frac{C\,b^3\,d^2}{2}-B\,b^3\,c\,d+C\,a^3\,c\,d+\frac{3\,A\,a^2\,b\,d^2}{2}+\frac{3\,B\,a\,b^2\,c^2}{2}-\frac{3\,E$$
$$+\frac{b^2\,d\tan(e+f\,x)^5\,(B\,b\,d+3\,C\,a\,d+2\,C\,b\,c)}{5\,f}+\frac{C\,b^3\,d^2\tan(e+f\,x)^6}{6\,f}$$

input
```
int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
x*(A*a^3*c^2 - A*a^3*d^2 + B*b^3*c^2 - C*a^3*c^2 - B*b^3*d^2 + C*a^3*d^2 +
 2*A*b^3*c*d - 2*B*a^3*c*d - 2*C*b^3*c*d - 3*A*a*b^2*c^2 + 3*A*a*b^2*d^2 -
 3*B*a^2*b*c^2 + 3*B*a^2*b*d^2 + 3*C*a*b^2*c^2 - 3*C*a*b^2*d^2 - 6*A*a^2*b
*c*d + 6*B*a*b^2*c*d + 6*C*a^2*b*c*d) - (tan(e + f*x)*(B*b^3*c^2 - A*a^3*d
^2 - b^2*d*(B*b*d + 3*C*a*d + 2*C*b*c) - C*a^3*c^2 + C*a^3*d^2 + 2*A*b^3*c
*d - 2*B*a^3*c*d - 3*A*a*b^2*c^2 + 3*A*a*b^2*d^2 - 3*B*a^2*b*c^2 + 3*B*a^2
*b*d^2 + 3*C*a*b^2*c^2 - 6*A*a^2*b*c*d + 6*B*a*b^2*c*d + 6*C*a^2*b*c*d))/f
 - (log(tan(e + f*x)^2 + 1)*((A*b^3*c^2)/2 - (B*a^3*c^2)/2 - (A*b^3*d^2)/2
 + (B*a^3*d^2)/2 - (C*b^3*c^2)/2 + (C*b^3*d^2)/2 - A*a^3*c*d - B*b^3*c*d +
 C*a^3*c*d - (3*A*a^2*b*c^2)/2 + (3*A*a^2*b*d^2)/2 + (3*B*a*b^2*c^2)/2 - (
3*B*a*b^2*d^2)/2 + (3*C*a^2*b*c^2)/2 - (3*C*a^2*b*d^2)/2 + 3*A*a*b^2*c*d +
 3*B*a^2*b*c*d - 3*C*a*b^2*c*d))/f + (tan(e + f*x)^4*((A*b^3*d^2)/4 + (C*b
^3*c^2)/4 - (C*b^3*d^2)/4 + (B*b^3*c*d)/2 + (3*B*a*b^2*d^2)/4 + (3*C*a^2*b
*d^2)/4 + (3*C*a*b^2*c*d)/2))/f + (tan(e + f*x)^3*((B*b^3*c^2)/3 - (b^2*d*
(B*b*d + 3*C*a*d + 2*C*b*c))/3 + (C*a^3*d^2)/3 + (2*A*b^3*c*d)/3 + A*a*b^2
*d^2 + B*a^2*b*d^2 + C*a*b^2*c^2 + 2*B*a*b^2*c*d + 2*C*a^2*b*c*d))/f + (ta
n(e + f*x)^2*((A*b^3*c^2)/2 - (A*b^3*d^2)/2 + (B*a^3*d^2)/2 - (C*b^3*c^2)/
2 + (C*b^3*d^2)/2 - B*b^3*c*d + C*a^3*c*d + (3*A*a^2*b*d^2)/2 + (3*B*a*b^2
*c^2)/2 - (3*B*a*b^2*d^2)/2 + (3*C*a^2*b*c^2)/2 - (3*C*a^2*b*d^2)/2 + 3*A*
a*b^2*c*d + 3*B*a^2*b*c*d - 3*C*a*b^2*c*d))/f + (b^2*d*tan(e + f*x)^5*(...
```

3.57. $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

3.58 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+\right.$

3.58.1 Optimal result

Integrand size = 45, antiderivative size = 443

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= -\Big(\big(a^2\big(c^2C+2Bcd-Cd^2-A\big(c^2-d^2\big)\big)-b^2\big(c^2C+2Bcd-Cd^2-A\big(c^2-d^2\big)\big) + 2ab\big(2c(A-C)d+B\big(c^2-d^2\big)\big)\big)\,x\Big)$$
$$+\frac{(2ab(c^2C+2Bcd-Cd^2-A(c^2-d^2))-a^2(2c(A-C)d+B(c^2-d^2))+b^2(2c(A-C)d+B(c^2-d^2)}{f}$$
$$+\frac{d(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\tan(e+fx)}{f}$$
$$+\frac{(a^2B-b^2B+2ab(A-C))\,(c+d\tan(e+fx))^2}{2f}$$
$$+\frac{(8a^2Cd^2-10abd(cC-4Bd)+b^2(2c^2C-5Bcd+20(A-C)d^2))\,(c+d\tan(e+fx))^3}{60d^3f}$$
$$-\frac{b(2bcC-5bBd-2aCd)\tan(e+fx)(c+d\tan(e+fx))^3}{20d^2f}$$
$$+\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}$$

output
```
-(a^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^
2))+2*a*b*(2*c*(A-C)*d+B*(c^2-d^2)))*x+(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-
d^2))-a^2*(2*c*(A-C)*d+B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*ln(cos(
f*x+e))/f+d*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*tan(
f*x+e)/f+1/2*(B*a^2-B*b^2+2*a*b*(A-C))*(c+d*tan(f*x+e))^2/f+1/60*(8*a^2*C*
d^2-10*a*b*d*(-4*B*d+C*c)+b^2*(2*c^2*C-5*B*c*d+20*(A-C)*d^2))*(c+d*tan(f*x
+e))^3/d^3/f-1/20*b*(-5*B*b*d-2*C*a*d+2*C*b*c)*tan(f*x+e)*(c+d*tan(f*x+e))
^3/d^2/f+1/5*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3/d/f
```

3.58.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.55 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.86

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}$$
$$+ \frac{\frac{b(-2bcC+5bBd+2aCd)\tan(e+fx)(c+d\tan(e+fx))^3}{4df} - \frac{\frac{\left(-8a^2Cd^2+10abd(cC-4Bd)-b^2\left(2c^2C-5Bcd+20(A-C)d^2\right)\right)(c+d\tan(e+fx))^3}{3df} - \frac{10\left(d\left(2a\right.\right.}{}}{}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) + ((b*(-2*b*c*C
+ 5*b*B*d + 2*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f) - (((-8*
a^2*C*d^2 + 10*a*b*d*(c*C - 4*B*d) - b^2*(2*c^2*C - 5*B*c*d + 20*(A - C)*d
^2))*(c + d*Tan[e + f*x])^3)/(3*d*f) - (10*(d*(2*a*b*(A*c - c*C + B*d) + a
^2*(B*c - (A - C)*d) - b^2*(B*c - (A - C)*d))*(I*(c + I*d)^2*Log[I - Tan[e
 + f*x]] - I*(c - I*d)^2*Log[I + Tan[e + f*x]] - 2*d^2*Tan[e + f*x]) + (a^
2*B - b^2*B + 2*a*b*(A - C))*d*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c +
 d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2)))
/f)/(4*d))/(5*d)
```

3.58.3 Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.311$, Rules used = {3042, 4130, 25, 3042, 4120, 25, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -((a+b\tan(e+fx))(c+d\tan(e+fx))^2\left((2bcC-2adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+\right.}{5d}$$
$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}$$

$$\Big\downarrow \text{25}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left((2bcC-2adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)\right.}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left((2bcC-2adC-5bBd)\tan(e+fx)^2-5(Ab-Cb+aB)d\tan(e+fx)\right.}{5d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df}-$$
$$\frac{\frac{b\tan(e+fx)(-2aCd-5bBd+2bcC)(c+d\tan(e+fx))^3}{4df}-\frac{\int -(c+d\tan(e+fx))^2\left(-c(2cC-5Bd)b^2+10acCdb-4a^2(5A-3C)d^2-((2Cc^2-5Bdc+2\right.}{}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\frac{\int (c+d\tan(e+fx))^2\left(-c(2cC-5Bd)b^2+10acCdb-4a^2(5A-3C)d^2-\left((2Cc^2-5Bdc+20(A-C)d^2)b^2-10ad(cC-4Bd)b+8a^2Cd^2\right)\tan^2(e+fx)-20(}{4d}}{5d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\frac{\int (c+d\tan(e+fx))^2\left(-c(2cC-5Bd)b^2+10acCdb-4a^2(5A-3C)d^2-\left((2Cc^2-5Bdc+20(A-C)d^2)b^2-10ad(cC-4Bd)b+8a^2Cd^2\right)\tan(e+fx)^2-20(}{4d}}{5d}$$

$\downarrow$ 4113

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\frac{\int (c+d\tan(e+fx))^2\left(20\left(-((A-C)a^2)+2bBa+b^2(A-C)\right)d^2-20\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4\right.}{3df}}{4d}}{5d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\frac{\int (c+d\tan(e+fx))^2\left(20\left(-((A-C)a^2)+2bBa+b^2(A-C)\right)d^2-20\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4\right.}{3df}}{4d}}{5d}$$

$\downarrow$ 4011

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\int (c+d\tan(e+fx))\left(-20\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-20\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)\right)\right.}{4d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$\frac{\int (c+d\tan(e+fx))\left(-20\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-20\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)\right)\right.}{4d}$$

$\downarrow$ 4008

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} -$$

$$20d^2\left(-\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)\int\tan(e+fx)dx-\frac{(c+d\tan(e+fx))^3\left(8a^2C\right.}{}$$

3.58. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

$$\Big\downarrow \text{3042}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - $$

$$20d^2\left(-\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)+b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)\int\tan(e+fx)dx-\frac{(c+d\tan(e+fx))^3\left(8a^2C\right.}{}$$

$$\Big\downarrow \text{3956}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}{5df} - $$

$$-\frac{(c+d\tan(e+fx))^3\left(8a^2Cd^2-10abd(cC-4Bd)+b^2\left(20d^2(A-C)-5Bcd+2c^2C\right)\right)}{3df}-\frac{20d^2\log(\cos(e+fx))\left(-\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bc\right.}{f}$$

input
```
Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3)/(5*d*f) - ((b*(2*b*c*C -
 5*b*B*d - 2*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f) + (20*d^2
*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2*B*c*d - C
*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x - (20*d^2
*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B
*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])/f
- (20*d^3*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A
 - C)*d))*Tan[e + f*x])/f - (10*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(c + d
*Tan[e + f*x])^2)/f - ((8*a^2*C*d^2 - 10*a*b*d*(c*C - 4*B*d) + b^2*(2*c^2*
C - 5*B*c*d + 20*(A - C)*d^2))*(c + d*Tan[e + f*x])^3)/(3*d*f))/(4*d))/(5*
d)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.58. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.58.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.88

method	result
parts	$\frac{(2Aa^2cd+2Aabc^2+Ba^2c^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(Bb^2d^2+2Cabd^2+2Cb^2cd)\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(\right.}{2}\right.}{f}$
norman	$(Aa^2c^2-Aa^2d^2-4Aabcd-Ab^2c^2+Ab^2d^2-2Ba^2cd-2Babc^2+2Babd^2+2Bb^2c$
derivativedivides	$\frac{Ca^2d^2\tan(fx+e)^3}{3}+\frac{Cb^2c^2\tan(fx+e)^3}{3}-\frac{Cb^2d^2\tan(fx+e)^3}{3}+\frac{Ba^2d^2\tan(fx+e)^2}{2}+\frac{Bb^2c^2\tan(fx+e)^2}{2}-\frac{Bb^2d^2\tan(fx+e)^2}{2}-t$
default	$\frac{Ca^2d^2\tan(fx+e)^3}{3}+\frac{Cb^2c^2\tan(fx+e)^3}{3}-\frac{Cb^2d^2\tan(fx+e)^3}{3}+\frac{Ba^2d^2\tan(fx+e)^2}{2}+\frac{Bb^2c^2\tan(fx+e)^2}{2}-\frac{Bb^2d^2\tan(fx+e)^2}{2}-t$
parallelrisch	$\frac{20Ca^2d^2\tan(fx+e)^3+20Cb^2c^2\tan(fx+e)^3-20Cb^2d^2\tan(fx+e)^3+30Ba^2d^2\tan(fx+e)^2+30Bb^2c^2\tan(fx+e)^2-30}{}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*A*a^2*c*d+2*A*a*b*c^2+B*a^2*c^2)/f*ln(1+tan(f*x+e)^2)+(B*b^2*d^2+2*
C*a*b*d^2+2*C*b^2*c*d)/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f
*x+e)^2))+(A*b^2*d^2+2*B*a*b*d^2+2*B*b^2*c*d+C*a^2*d^2+4*C*a*b*c*d+C*b^2*c
^2)/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(A*a^2*d^2+4*A*a*b*
c*d+A*b^2*c^2+2*B*a^2*c*d+2*B*a*b*c^2+C*a^2*c^2)/f*(tan(f*x+e)-arctan(tan(
f*x+e)))+(2*A*a*b*d^2+2*A*b^2*c*d+B*a^2*d^2+4*B*a*b*c*d+B*b^2*c^2+2*C*a^2*
c*d+2*C*a*b*c^2)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+A*a^2*c^2*x+C
*b^2*d^2/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arctan(tan(f*x+e)
))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.04

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$=\frac{12\,Cb^2d^2\tan{(fx+e)}^5+15\,(2\,Cb^2cd+(2\,Cab+Bb^2)d^2)\tan{(fx+e)}^4+20\,(Cb^2c^2+2\,(2\,Cab+Bb^2)cd}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^
2)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B
*a*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A
- C)*b^2)*c^2 - 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B
*a*b - (A - C)*b^2)*d^2)*f*x + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*
a*b + (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*tan(f*x + e)
^2 - 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*a*b -
(A - C)*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(1/(tan(f*x + e
)^2 + 1)) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A - C)
*a*b - B*b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x + e
))/f
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(396) = 792.

Time = 0.30 (sec) , antiderivative size = 1134, normalized size of antiderivative = 2.56

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Piecewise((A*a**2*c**2*x + A*a**2*c*d*log(tan(e + f*x)**2 + 1)/f - A*a**2*
d**2*x + A*a**2*d**2*tan(e + f*x)/f + A*a*b*c**2*log(tan(e + f*x)**2 + 1)/
f - 4*A*a*b*c*d*x + 4*A*a*b*c*d*tan(e + f*x)/f - A*a*b*d**2*log(tan(e + f*
x)**2 + 1)/f + A*a*b*d**2*tan(e + f*x)**2/f - A*b**2*c**2*x + A*b**2*c**2*
tan(e + f*x)/f - A*b**2*c*d*log(tan(e + f*x)**2 + 1)/f + A*b**2*c*d*tan(e
+ f*x)**2/f + A*b**2*d**2*x + A*b**2*d**2*tan(e + f*x)**3/(3*f) - A*b**2*d
**2*tan(e + f*x)/f + B*a**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*a**2
*c*d*x + 2*B*a**2*c*d*tan(e + f*x)/f - B*a**2*d**2*log(tan(e + f*x)**2 + 1
)/(2*f) + B*a**2*d**2*tan(e + f*x)**2/(2*f) - 2*B*a*b*c**2*x + 2*B*a*b*c**
2*tan(e + f*x)/f - 2*B*a*b*c*d*log(tan(e + f*x)**2 + 1)/f + 2*B*a*b*c*d*ta
n(e + f*x)**2/f + 2*B*a*b*d**2*x + 2*B*a*b*d**2*tan(e + f*x)**3/(3*f) - 2*
B*a*b*d**2*tan(e + f*x)/f - B*b**2*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + B
*b**2*c**2*tan(e + f*x)**2/(2*f) + 2*B*b**2*c*d*x + 2*B*b**2*c*d*tan(e + f
*x)**3/(3*f) - 2*B*b**2*c*d*tan(e + f*x)/f + B*b**2*d**2*log(tan(e + f*x)*
*2 + 1)/(2*f) + B*b**2*d**2*tan(e + f*x)**4/(4*f) - B*b**2*d**2*tan(e + f*
x)**2/(2*f) - C*a**2*c**2*x + C*a**2*c**2*tan(e + f*x)/f - C*a**2*c*d*log(
tan(e + f*x)**2 + 1)/f + C*a**2*c*d*tan(e + f*x)**2/f + C*a**2*d**2*x + C*
a**2*d**2*tan(e + f*x)**3/(3*f) - C*a**2*d**2*tan(e + f*x)/f - C*a*b*c**2*
log(tan(e + f*x)**2 + 1)/f + C*a*b*c**2*tan(e + f*x)**2/f + 4*C*a*b*c*d*x
+ 4*C*a*b*c*d*tan(e + f*x)**3/(3*f) - 4*C*a*b*c*d*tan(e + f*x)/f + C*a*...
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.05

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{12\,Cb^2d^2\tan\left(fx+e\right)^5 + 15\left(2\,Cb^2cd + (2\,Cab + Bb^2)d^2\right)\tan\left(fx+e\right)^4 + 20\left(Cb^2c^2 + 2\left(2\,Cab + Bb^2\right)cd\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
1/60*(12*C*b^2*d^2*tan(f*x + e)^5 + 15*(2*C*b^2*c*d + (2*C*a*b + B*b^2)*d^
2)*tan(f*x + e)^4 + 20*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B
*a*b + (A - C)*b^2)*d^2)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^2 + 2*(C
*a^2 + 2*B*a*b + (A - C)*b^2)*c*d + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*t
an(f*x + e)^2 + 60*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2 - 2*(B*a^2 +
 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*(
f*x + e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2 + 2*((A - C)*a^2 - 2*B*
a*b - (A - C)*b^2)*c*d - (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x
+ e)^2 + 1) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2 + 2*(B*a^2 + 2*(A -
C)*a*b - B*b^2)*c*d + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^2)*tan(f*x +
 e))/f
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11957 vs. $2(436) = 872$.

Time = 10.52 (sec) , antiderivative size = 11957, normalized size of antiderivative = 26.99

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
1/60*(60*A*a^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a^2*c^2*f*x*tan(f*x)^5*t
an(e)^5 - 120*B*a*b*c^2*f*x*tan(f*x)^5*tan(e)^5 - 60*A*b^2*c^2*f*x*tan(f*x
)^5*tan(e)^5 + 60*C*b^2*c^2*f*x*tan(f*x)^5*tan(e)^5 - 120*B*a^2*c*d*f*x*ta
n(f*x)^5*tan(e)^5 - 240*A*a*b*c*d*f*x*tan(f*x)^5*tan(e)^5 + 240*C*a*b*c*d*
f*x*tan(f*x)^5*tan(e)^5 + 120*B*b^2*c*d*f*x*tan(f*x)^5*tan(e)^5 - 60*A*a^2
*d^2*f*x*tan(f*x)^5*tan(e)^5 + 60*C*a^2*d^2*f*x*tan(f*x)^5*tan(e)^5 + 120*
B*a*b*d^2*f*x*tan(f*x)^5*tan(e)^5 + 60*A*b^2*d^2*f*x*tan(f*x)^5*tan(e)^5 -
 60*C*b^2*d^2*f*x*tan(f*x)^5*tan(e)^5 - 30*B*a^2*c^2*log(4*(tan(f*x)^2*tan
(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^
2 + 1))*tan(f*x)^5*tan(e)^5 - 60*A*a*b*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*
tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*ta
n(f*x)^5*tan(e)^5 + 60*C*a*b*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*t
an(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*t
an(e)^5 + 30*B*b^2*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 -
60*A*a^2*c*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)
^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 60*C*a^2*c
*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 120*B*a*b*c*d*log(4*
(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + ta...
```

3.58.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.27

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= x\left(A\,a^2\,c^2-A\,a^2\,d^2-A\,b^2\,c^2+A\,b^2\,d^2-C\,a^2\,c^2+C\,a^2\,d^2+C\,b^2\,c^2-C\,b^2\,d^2-2\,B\,a\,b\,c^2+2\,B\,a\,b\,d^2-2\,B\,a^2\,c\,d+2\,B\,b^2\,c\,d-4\,A\,a\,b\,c\,d+4\,C\,a\,b\,c\,d\right)$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{B\,a^2\,d^2}{2}-\frac{B\,a^2\,c^2}{2}+\frac{B\,b^2\,c^2}{2}-\frac{B\,b^2\,d^2}{2}-A\,a\,b\,c^2+A\,a\,b\,d^2-A\,a^2\,c\,d+C\,a\,b\,c^2+A\,b\right.}{f}$$
$$+\frac{\tan(e+f\,x)^2\left(\frac{B\,a^2\,d^2}{2}+\frac{B\,b^2\,c^2}{2}-\frac{b\,d\,(B\,b\,d+2\,C\,a\,d+2\,C\,b\,c)}{2}+A\,a\,b\,d^2+C\,a\,b\,c^2+A\,b^2\,c\,d+C\,a^2\,c\,d+2\,B\,a\,b\right.}{f}$$
$$+\frac{\tan(e+f\,x)^3\left(\frac{A\,b^2\,d^2}{3}+\frac{C\,a^2\,d^2}{3}+\frac{C\,b^2\,c^2}{3}-\frac{C\,b^2\,d^2}{3}+\frac{2\,B\,a\,b\,d^2}{3}+\frac{2\,B\,b^2\,c\,d}{3}+\frac{4\,C\,a\,b\,c\,d}{3}\right)}{f}$$
$$+\frac{\tan(e+f\,x)\,\left(A\,a^2\,d^2+A\,b^2\,c^2-A\,b^2\,d^2+C\,a^2\,c^2-C\,a^2\,d^2-C\,b^2\,c^2+C\,b^2\,d^2+2\,B\,a\,b\,c^2-2\,B\,a\,b\,d^2\right.}{f}$$
$$+\frac{b\,d\tan(e+f\,x)^4\,(B\,b\,d+2\,C\,a\,d+2\,C\,b\,c)}{4\,f}+\frac{C\,b^2\,d^2\tan(e+f\,x)^5}{5\,f}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
x*(A*a^2*c^2 - A*a^2*d^2 - A*b^2*c^2 + A*b^2*d^2 - C*a^2*c^2 + C*a^2*d^2 +
 C*b^2*c^2 - C*b^2*d^2 - 2*B*a*b*c^2 + 2*B*a*b*d^2 - 2*B*a^2*c*d + 2*B*b^2
*c*d - 4*A*a*b*c*d + 4*C*a*b*c*d) - (log(tan(e + f*x)^2 + 1)*((B*a^2*d^2)/
2 - (B*a^2*c^2)/2 + (B*b^2*c^2)/2 - (B*b^2*d^2)/2 - A*a*b*c^2 + A*a*b*d^2
- A*a^2*c*d + C*a*b*c^2 + A*b^2*c*d - C*a*b*d^2 + C*a^2*c*d - C*b^2*c*d +
2*B*a*b*c*d))/f + (tan(e + f*x)^2*((B*a^2*d^2)/2 + (B*b^2*c^2)/2 - (b*d*(B
*b*d + 2*C*a*d + 2*C*b*c))/2 + A*a*b*d^2 + C*a*b*c^2 + A*b^2*c*d + C*a^2*c
*d + 2*B*a*b*c*d))/f + (tan(e + f*x)^3*((A*b^2*d^2)/3 + (C*a^2*d^2)/3 + (C
*b^2*c^2)/3 - (C*b^2*d^2)/3 + (2*B*a*b*d^2)/3 + (2*B*b^2*c*d)/3 + (4*C*a*b
*c*d)/3))/f + (tan(e + f*x)*(A*a^2*d^2 + A*b^2*c^2 - A*b^2*d^2 + C*a^2*c^2
 - C*a^2*d^2 - C*b^2*c^2 + C*b^2*d^2 + 2*B*a*b*c^2 - 2*B*a*b*d^2 + 2*B*a^2
*c*d - 2*B*b^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d))/f + (b*d*tan(e + f*x)^4*(
B*b*d + 2*C*a*d + 2*C*b*c))/(4*f) + (C*b^2*d^2*tan(e + f*x)^5)/(5*f)
```

3.58. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.59 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2 (A+B\tan(e+$

3.59.1 Optimal result

Integrand size = 43, antiderivative size = 266

$$\begin{aligned}
&\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx \\
&= -\left(\left(a\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)x\right) \\
&\quad -\frac{(a(Bc^2-2cCd-Bd^2)-b(c^2C+2Bcd-Cd^2)+A(2acd+b(c^2-d^2)))\log(\cos(e+fx))}{f} \\
&\quad +\frac{d(Abc+aBc-bcC+aAd-bBd-aCd)\tan(e+fx)}{f} \\
&\quad +\frac{(Ab+aB-bC)(c+d\tan(e+fx))^2}{2f} \\
&\quad -\frac{(bcC-4bBd-4aCd)(c+d\tan(e+fx))^3}{12d^2f}+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}
\end{aligned}$$

output
```
-(a*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))+b*(2*c*(A-C)*d+B*(c^2-d^2)))*x-(a*(B
*c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d+b*(c^2-d^2)))*ln(co
s(f*x+e))/f+d*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*tan(f*x+e)/f+1/2*(A*b+
B*a-C*b)*(c+d*tan(f*x+e))^2/f-1/12*(-4*B*b*d-4*C*a*d+C*b*c)*(c+d*tan(f*x+e
))^3/d^2/f+1/4*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^3/d/f
```

3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.91

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{\frac{(-bcC+4bBd+4aCd)(c+d\tan(e+fx))^3}{d}+3bC\tan(e+fx)(c+d\tan(e+fx))^3+6(Abc+aBc-bcC-aAd+bB}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(((-(b*c*C) + 4*b*B*d + 4*a*C*d)*(c + d*Tan[e + f*x])^3)/d + 3*b*C*Tan[e +
 f*x]*(c + d*Tan[e + f*x])^3 + 6*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d +
a*C*d)*(I*((c + I*d)^2*Log[I - Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e +
 f*x]]) - 2*d^2*Tan[e + f*x]) + 6*(A*b + a*B - b*C)*((I*c - d)^3*Log[I - T
an[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] +
d^3*Tan[e + f*x]^2))/(12*d*f)
```

3.59.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4120, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$
$$\Big\downarrow \ 4120$$
$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-$$
$$\frac{\int (c+d\tan(e+fx))^2\left((bcC-4adC-4bBd)\tan^2(e+fx)-4(Ab-Cb+aB)d\tan(e+fx)+bcC-4aAd\right)dx}{4d}$$

$\Big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{\int(c+d\tan(e+fx))^2\left((bcC-4adC-4bBd)\tan(e+fx)^2-4(Ab-Cb+aB)d\tan(e+fx)+bcC-4aAd\right)dx}{4d}$$

$\Big\downarrow$ 4113

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{\int(c+d\tan(e+fx))^2(4(bB-a(A-C))d-4(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-4aCd-4bBd+bcC)(c+d\tan(e+fx))}{3df}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{\int(c+d\tan(e+fx))^2(4(bB-a(A-C))d-4(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-4aCd-4bBd+bcC)(c+d\tan(e+fx))}{3df}}{4d}$$

$\Big\downarrow$ 4011

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{\int(c+d\tan(e+fx))(4d(bBc+b(A-C)d-a(Ac-Cc-Bd))-4d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{4d}$$

$\Big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{\int(c+d\tan(e+fx))(4d(bBc+b(A-C)d-a(Ac-Cc-Bd))-4d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{4d}$$

$\Big\downarrow$ 4008

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{-4d\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)\int\tan(e+fx)dx+4dx\left(a\left(-A\left(c^2-\right.\right.\right.}{}$$

$\Big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df}-\frac{-4d\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)\int\tan(e+fx)dx+4dx\left(a\left(-A\left(c^2-\right.\right.\right.}{}$$

$\Big\downarrow$ 3956

3.59. $\int(a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

$$\frac{\frac{bC\tan(e+fx)(c+d\tan(e+fx))^3}{4df} - \frac{\frac{4d\log(\cos(e+fx))(2aAcd+aB(c^2-d^2)-2acCd+Ab(c^2-d^2)-b(2Bcd+c^2C-Cd^2))}{f} + 4dx(a(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)+}{}}{}$$

input
```
Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Ta
n[e + f*x]^2),x]
```

output
```
(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^3)/(4*d*f) - (4*d*(a*(c^2*C + 2*B*c
*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x + (4*d*
(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*
B*c*d - C*d^2))*Log[Cos[e + f*x]])/f - (4*d^2*(A*b*c + a*B*c - b*c*C + a*A
*d - b*B*d - a*C*d)*Tan[e + f*x])/f - (2*(A*b + a*B - b*C)*d*(c + d*Tan[e
 + f*x])^2)/f + ((b*c*C - 4*b*B*d - 4*a*C*d)*(c + d*Tan[e + f*x])^3)/(3*d*f
))/(4*d)
```

3.59.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.59.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.92

method	result
parts	$\frac{(2Aacd+Ab\,c^2+Ba\,c^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(Bb\,d^2+Ca\,d^2+2Cbcd)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right.}{f}$
norman	$(Aa\,c^2-Aa\,d^2-2Abcd-2Bacd-Bb\,c^2+Bb\,d^2-Ca\,c^2+Ca\,d^2+2Cbcd)\,x+\frac{(Aa\,d}{}$
derivativedivides	$\frac{\frac{Cb\,d^2\tan(fx+e)^4}{4}+\frac{Bb\,d^2\tan(fx+e)^3}{3}+\frac{Ca\,d^2\tan(fx+e)^3}{3}+\frac{2Cbcd\tan(fx+e)^3}{3}+\frac{Ab\,d^2\tan(fx+e)^2}{2}+\frac{Ba\,d^2\tan(fx+e)^2}{2}+Bbcd\,\mathrm{ta}}{}$
default	$\frac{\frac{Cb\,d^2\tan(fx+e)^4}{4}+\frac{Bb\,d^2\tan(fx+e)^3}{3}+\frac{Ca\,d^2\tan(fx+e)^3}{3}+\frac{2Cbcd\tan(fx+e)^3}{3}+\frac{Ab\,d^2\tan(fx+e)^2}{2}+\frac{Ba\,d^2\tan(fx+e)^2}{2}+Bbcd\,\mathrm{ta}}{}$
parallelrisch	$\frac{12A\ln\left(1+\tan(fx+e)^2\right)acd-12B\ln\left(1+\tan(fx+e)^2\right)bcd-12C\ln\left(1+\tan(fx+e)^2\right)acd+3Cb\,d^2\tan(fx+e)^4+4Bb\,d^2\,\mathrm{ta}}{}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*(2*A*a*c*d+A*b*c^2+B*a*c^2)/f*ln(1+tan(f*x+e)^2)+(B*b*d^2+C*a*d^2+2*C*
b*c*d)/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(A*b*d^2+B*a*d^2
+2*B*b*c*d+2*C*a*c*d+C*b*c^2)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+
(A*a*d^2+2*A*b*c*d+2*B*a*c*d+B*b*c^2+C*a*c^2)/f*(tan(f*x+e)-arctan(tan(f*x
+e)))+A*a*c^2*x+C*b*d^2/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(
f*x+e)^2))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,Cbd^2\tan\left(fx+e\right)^4+4\left(2\,Cbcd+(Ca+Bb)d^2\right)\tan\left(fx+e\right)^3+12\left(\left((A-C)a-Bb\right)c^2-2\left(Ba+(A-\right.\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="fricas")
```

output
```
1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
 e)^3 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a -
 B*b)*d^2)*f*x + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*t
an(f*x + e)^2 - 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d - (B*a
+ (A - C)*b)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 12*((C*a + B*b)*c^2 + 2*(B
*a + (A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

3.59.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(246) = 492.

Time = 0.20 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.32

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\begin{cases}Aac^2x+\frac{Aacd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-Aad^2x+\frac{Aad^2\tan\left(e+fx\right)}{f}+\frac{Abc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Abcdx+\frac{2Abcd\tan\left(e+fx\right)}{f} \\ x(a+b\tan\left(e\right))\left(c+d\tan\left(e\right)\right)^2\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right)\end{cases}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e
)**2),x)
```

output
```
Piecewise((A*a*c**2*x + A*a*c*d*log(tan(e + f*x)**2 + 1)/f - A*a*d**2*x +
A*a*d**2*tan(e + f*x)/f + A*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*A*b*
c*d*x + 2*A*b*c*d*tan(e + f*x)/f - A*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f)
 + A*b*d**2*tan(e + f*x)**2/(2*f) + B*a*c**2*log(tan(e + f*x)**2 + 1)/(2*f
) - 2*B*a*c*d*x + 2*B*a*c*d*tan(e + f*x)/f - B*a*d**2*log(tan(e + f*x)**2
+ 1)/(2*f) + B*a*d**2*tan(e + f*x)**2/(2*f) - B*b*c**2*x + B*b*c**2*tan(e
+ f*x)/f - B*b*c*d*log(tan(e + f*x)**2 + 1)/f + B*b*c*d*tan(e + f*x)**2/f
+ B*b*d**2*x + B*b*d**2*tan(e + f*x)**3/(3*f) - B*b*d**2*tan(e + f*x)/f -
C*a*c**2*x + C*a*c**2*tan(e + f*x)/f - C*a*c*d*log(tan(e + f*x)**2 + 1)/f
+ C*a*c*d*tan(e + f*x)**2/f + C*a*d**2*x + C*a*d**2*tan(e + f*x)**3/(3*f)
- C*a*d**2*tan(e + f*x)/f - C*b*c**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*
c**2*tan(e + f*x)**2/(2*f) + 2*C*b*c*d*x + 2*C*b*c*d*tan(e + f*x)**3/(3*f)
 - 2*C*b*c*d*tan(e + f*x)/f + C*b*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + C*
b*d**2*tan(e + f*x)**4/(4*f) - C*b*d**2*tan(e + f*x)**2/(2*f), Ne(f, 0)),
(x*(a + b*tan(e))*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2), True))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{3\,Cbd^2\tan\left(fx+e\right)^4+4\left(2\,Cbcd+(Ca+Bb)d^2\right)\tan\left(fx+e\right)^3+6\left(Cbc^2+2\left(Ca+Bb\right)cd+(Ba+(A-\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="maxima")
```

output
```
1/12*(3*C*b*d^2*tan(f*x + e)^4 + 4*(2*C*b*c*d + (C*a + B*b)*d^2)*tan(f*x +
 e)^3 + 6*(C*b*c^2 + 2*(C*a + B*b)*c*d + (B*a + (A - C)*b)*d^2)*tan(f*x +
e)^2 + 12*(((A - C)*a - B*b)*c^2 - 2*(B*a + (A - C)*b)*c*d - ((A - C)*a -
B*b)*d^2)*(f*x + e) + 6*((B*a + (A - C)*b)*c^2 + 2*((A - C)*a - B*b)*c*d -
 (B*a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1) + 12*((C*a + B*b)*c^2 + 2*
(B*a + (A - C)*b)*c*d + ((A - C)*a - B*b)*d^2)*tan(f*x + e))/f
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5631 vs. $2(260) = 520$.

Time = 4.09 (sec) , antiderivative size = 5631, normalized size of antiderivative = 21.17

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="giac")
```

output
```
1/12*(12*A*a*c^2*f*x*tan(f*x)^4*tan(e)^4 - 12*C*a*c^2*f*x*tan(f*x)^4*tan(e
)^4 - 12*B*b*c^2*f*x*tan(f*x)^4*tan(e)^4 - 24*B*a*c*d*f*x*tan(f*x)^4*tan(e
)^4 - 24*A*b*c*d*f*x*tan(f*x)^4*tan(e)^4 + 24*C*b*c*d*f*x*tan(f*x)^4*tan(e
)^4 - 12*A*a*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*C*a*d^2*f*x*tan(f*x)^4*tan(e
)^4 + 12*B*b*d^2*f*x*tan(f*x)^4*tan(e)^4 - 6*B*a*c^2*log(4*(tan(f*x)^2*tan
(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^
2 + 1))*tan(f*x)^4*tan(e)^4 - 6*A*b*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan
(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f
*x)^4*tan(e)^4 + 6*C*b*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e)
+ 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^
4 - 12*A*a*c*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*
x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 12*C*a*c
*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^
2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 12*B*b*c*d*log(4*(ta
n(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 6*B*a*d^2*log(4*(tan(f*x)^2*tan(
e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2
 + 1))*tan(f*x)^4*tan(e)^4 + 6*A*b*d^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(
f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*
x)^4*tan(e)^4 - 6*C*b*d^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e...
```

3.59.9 Mupad [B] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.13

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{\tan(e+f\,x)^2\left(\frac{A\,b\,d^2}{2}+\frac{B\,a\,d^2}{2}+\frac{C\,b\,c^2}{2}-\frac{C\,b\,d^2}{2}+B\,b\,c\,d+C\,a\,c\,d\right)}{f}$$
$$-x\left(A\,a\,d^2-A\,a\,c^2+B\,b\,c^2+C\,a\,c^2-B\,b\,d^2-C\,a\,d^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d-2\,C\,b\,c\,d\right)$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{A\,b\,d^2}{2}-\frac{B\,a\,c^2}{2}-\frac{A\,b\,c^2}{2}+\frac{B\,a\,d^2}{2}+\frac{C\,b\,c^2}{2}-\frac{C\,b\,d^2}{2}-A\,a\,c\,d+B\,b\,c\,d+C\,a\,c\,d\right)}{f}$$
$$+\frac{\tan(e+f\,x)\left(A\,a\,d^2+B\,b\,c^2+C\,a\,c^2-B\,b\,d^2-C\,a\,d^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d-2\,C\,b\,c\,d\right)}{f}$$
$$+\frac{\tan(e+f\,x)^3\left(\frac{B\,b\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,C\,b\,c\,d}{3}\right)}{f}+\frac{C\,b\,d^2\tan(e+f\,x)^4}{4\,f}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^2*((A*b*d^2)/2 + (B*a*d^2)/2 + (C*b*c^2)/2 - (C*b*d^2)/2 + B
*b*c*d + C*a*c*d))/f - x*(A*a*d^2 - A*a*c^2 + B*b*c^2 + C*a*c^2 - B*b*d^2
- C*a*d^2 + 2*A*b*c*d + 2*B*a*c*d - 2*C*b*c*d) - (log(tan(e + f*x)^2 + 1)*
((A*b*d^2)/2 - (B*a*c^2)/2 - (A*b*c^2)/2 + (B*a*d^2)/2 + (C*b*c^2)/2 - (C*
b*d^2)/2 - A*a*c*d + B*b*c*d + C*a*c*d))/f + (tan(e + f*x)*(A*a*d^2 + B*b*
c^2 + C*a*c^2 - B*b*d^2 - C*a*d^2 + 2*A*b*c*d + 2*B*a*c*d - 2*C*b*c*d))/f
+ (tan(e + f*x)^3*((B*b*d^2)/3 + (C*a*d^2)/3 + (2*C*b*c*d)/3))/f + (C*b*d^
2*tan(e + f*x)^4)/(4*f)
```

3.60 $\int (c+d\tan(e+fx))^2 (A+B\tan(e+fx)+C\tan^2(e+$

3.60.1 Optimal result

Integrand size = 33, antiderivative size = 131

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=-\left(\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)x\right)-\frac{(2c(A-C)d+B(c^2-d^2))\log(\cos(e+fx))}{f}$$
$$+\frac{d(Bc+(A-C)d)\tan(e+fx)}{f}+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

output
```
-(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))*x-(2*c*(A-C)*d+B*(c^2-d^2))*ln(cos(f*x+
e))/f+d*(B*c+(A-C)*d)*tan(f*x+e)/f+1/2*B*(c+d*tan(f*x+e))^2/f+1/3*C*(c+d*t
an(f*x+e))^3/d/f
```

3.60.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.34

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{2C(c+d\tan(e+fx))^3+3(Bc+(-A+C)d)\left(i((c+id)^2\log(i-\tan(e+fx))-(c-id)^2\log(i+\tan(e\right.}{}$$

input
```
Integrate[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x
]
```

output
```
(2*C*(c + d*Tan[e + f*x])^3 + 3*(B*c + (-A + C)*d)*(I*((c + I*d)^2*Log[I -
 Tan[e + f*x]] - (c - I*d)^2*Log[I + Tan[e + f*x]]) - 2*d^2*Tan[e + f*x])
+ 3*B*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x
]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2))/(6*d*f)
```

3.60.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4113}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^2dx+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^2dx+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{4011}$$

$$\int (c+d\tan(e+fx))(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{4008}$$

$$(2cd(A-C)+B(c^2-d^2))\int \tan(e+fx)dx-x(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{3042}$$

$$(2cd(A-C)+B(c^2-d^2))\int \tan(e+fx)dx-x(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(2cd(A-C)+B(c^2-d^2))\log(\cos(e+fx))}{f}-x(-A(c^2-d^2)+2Bcd+c^2C-Cd^2)+\frac{d\tan(e+fx)(d(A-C)+Bc)}{f}+\frac{B(c+d\tan(e+fx))^2}{2f}+\frac{C(c+d\tan(e+fx))^3}{3df}$$

input
```
Int[(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
-((c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x) - ((2*c*(A - C)*d + B*(c^2
- d^2))*Log[Cos[e + f*x]])/f + (d*(B*c + (A - C)*d)*Tan[e + f*x])/f + (B*(
c + d*Tan[e + f*x])^2)/(2*f) + (C*(c + d*Tan[e + f*x])^3)/(3*d*f)
```

3.60.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

method	result
norman	$(A c^2 - A d^2 - 2Bcd - c^2C + C d^2) x + \frac{(A d^2+2Bcd+c^2C-C d^2)\tan(fx+e)}{f} + \frac{C d^2 \tan(fx+e)^3}{3f} +$
parts	$A c^2 x + \frac{(2Acd+B c^2)\ln\left(1+\tan(fx+e)^2\right)}{2f} + \frac{(B d^2+2Ccd)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f} + \frac{(A d^2+2Bcd+c}{}$
derivativedivides	$\frac{\frac{C d^2 \tan(fx+e)^3}{3}+\frac{B d^2 \tan(fx+e)^2}{2}+Ccd\tan(fx+e)^2+\tan(fx+e)A d^2+2\tan(fx+e)Bcd+\tan(fx+e)c^2C-\tan(fx+e)C}{f}$
default	$\frac{\frac{C d^2 \tan(fx+e)^3}{3}+\frac{B d^2 \tan(fx+e)^2}{2}+Ccd\tan(fx+e)^2+\tan(fx+e)A d^2+2\tan(fx+e)Bcd+\tan(fx+e)c^2C-\tan(fx+e)C}{f}$
parallelrisch	$\frac{2C d^2\tan(fx+e)^3+6A c^2 fx-6A d^2 fx-12Bcdfx+3B d^2\tan(fx+e)^2-6C c^2 fx+6C d^2 fx+6Ccd\tan(fx+e)^2+6A\ln\left(1\right.}{}$
risch	$-\frac{4iCcde}{f} + \frac{4iAcde}{f} + 2iAcdx - \frac{2iB d^2 e}{f} + A c^2 x - A d^2 x - 2Bcdx - C c^2 x + C d^2 x + \frac{2i(-6}{}$

input
```
int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVER
BOSE)
```

output
```
(A*c^2-A*d^2-2*B*c*d-C*c^2+C*d^2)*x+(A*d^2+2*B*c*d+C*c^2-C*d^2)/f*tan(f*x+
e)+1/3*C*d^2/f*tan(f*x+e)^3+1/2*d*(B*d+2*C*c)/f*tan(f*x+e)^2+1/2*(2*A*c*d+
B*c^2-B*d^2-2*C*c*d)/f*ln(1+tan(f*x+e)^2)
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{2\,Cd^2\tan\left(fx+e\right)^3+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)fx+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2-3\left(Bc^2+\right.}{6\,f}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"fricas")
```

output
```
1/6*(2*C*d^2*tan(f*x + e)^3 + 6*((A - C)*c^2 - 2*B*c*d - (A - C)*d^2)*f*x
+ 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 - 3*(B*c^2 + 2*(A - C)*c*d - B*d^2)*l
og(1/(tan(f*x + e)^2 + 1)) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x + e
))/f
```

3.60.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.84

$$\int (c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} Ac^2x+\frac{Acd\log\left(\tan^2\left(e+fx\right)+1\right)}{f}-Ad^2x+\frac{Ad^2\tan\left(e+fx\right)}{f}+\frac{Bc^2\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-2Bcdx+\frac{2Bcd\tan\left(e+fx\right)}{f}-\frac{Bd^2}{} \\ x(c+d\tan\left(e\right))^2\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

input
```
integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Piecewise((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)/f - A*d**2*x + A*d**2
*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2*f) - 2*B*c*d*x + 2*B*
c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + B*d**2*tan(e
+ f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f - C*c*d*log(tan(e + f*x
)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C*d**2*tan(e + f*x)**3/
(3*f) - C*d**2*tan(e + f*x)/f, Ne(f, 0)), (x*(c + d*tan(e))**2*(A + B*tan(
e) + C*tan(e)**2), True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\int (c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{2\,Cd^2\tan\left(fx+e\right)^3+3\left(2\,Ccd+Bd^2\right)\tan\left(fx+e\right)^2+6\left((A-C)c^2-2\,Bcd-(A-C)d^2\right)(fx+e)+3\,(}{6\,f}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"maxima")
```

output
```
1/6*(2*C*d^2*tan(f*x + e)^3 + 3*(2*C*c*d + B*d^2)*tan(f*x + e)^2 + 6*((A -
 C)*c^2 - 2*B*c*d - (A - C)*d^2)*(f*x + e) + 3*(B*c^2 + 2*(A - C)*c*d - B*
d^2)*log(tan(f*x + e)^2 + 1) + 6*(C*c^2 + 2*B*c*d + (A - C)*d^2)*tan(f*x +
 e))/f
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1825 vs. 2(127) = 254.

Time = 1.36 (sec) , antiderivative size = 1825, normalized size of antiderivative = 13.93

$$\int (c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"giac")
```

output
```
1/6*(6*A*c^2*f*x*tan(f*x)^3*tan(e)^3 - 6*C*c^2*f*x*tan(f*x)^3*tan(e)^3 - 1
2*B*c*d*f*x*tan(f*x)^3*tan(e)^3 - 6*A*d^2*f*x*tan(f*x)^3*tan(e)^3 + 6*C*d^
2*f*x*tan(f*x)^3*tan(e)^3 - 3*B*c^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x
)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^
3*tan(e)^3 - 6*A*c*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(
tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 6*
C*c*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(
e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 + 3*B*d^2*log(4*(ta
n(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 + tan(e)^2 + 1))*tan(f*x)^3*tan(e)^3 - 18*A*c^2*f*x*tan(f*x)^2*tan(e)^2
 + 18*C*c^2*f*x*tan(f*x)^2*tan(e)^2 + 36*B*c*d*f*x*tan(f*x)^2*tan(e)^2 + 1
8*A*d^2*f*x*tan(f*x)^2*tan(e)^2 - 18*C*d^2*f*x*tan(f*x)^2*tan(e)^2 + 6*C*c
*d*tan(f*x)^3*tan(e)^3 + 3*B*d^2*tan(f*x)^3*tan(e)^3 + 9*B*c^2*log(4*(tan(
f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2
 + tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 + 18*A*c*d*log(4*(tan(f*x)^2*tan(e)^
2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 +
1))*tan(f*x)^2*tan(e)^2 - 18*C*c*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)
*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^2
*tan(e)^2 - 9*B*d^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(t
an(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^2*tan(e)^2 - ...
```

3.60.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int (c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{\tan(e+f\,x)^2\left(\frac{B\,d^2}{2}+C\,c\,d\right)}{f}-x\left(A\,d^2-A\,c^2+C\,c^2-C\,d^2+2\,B\,c\,d\right)$$
$$+\frac{\tan(e+f\,x)\,\left(A\,d^2+C\,c^2-C\,d^2+2\,B\,c\,d\right)}{f}$$
$$+\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{B\,c^2}{2}-\frac{B\,d^2}{2}+A\,c\,d-C\,c\,d\right)}{f}+\frac{C\,d^2\tan(e+f\,x)^3}{3\,f}$$

input
```
int((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
(tan(e + f*x)^2*((B*d^2)/2 + C*c*d))/f - x*(A*d^2 - A*c^2 + C*c^2 - C*d^2
+ 2*B*c*d) + (tan(e + f*x)*(A*d^2 + C*c^2 - C*d^2 + 2*B*c*d))/f + (log(tan
(e + f*x)^2 + 1)*((B*c^2)/2 - (B*d^2)/2 + A*c*d - C*c*d))/f + (C*d^2*tan(e
 + f*x)^3)/(3*f)
```

3.60. $\int (c + d\tan(e+fx))^2 \left(A + B\tan(e+fx) + C\tan^2(e+fx)\right) dx$

3.61 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.61.1 Optimal result

Integrand size = 45, antiderivative size = 254

$$\begin{aligned}
&\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx \\
&= -\frac{(a(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b(2c(A-C)d+B(c^2-d^2)))\,x}{a^2+b^2} \\
&\quad -\frac{(a(Bc^2-2cCd-Bd^2)+b(c^2C+2Bcd-Cd^2)+A(2acd-b(c^2-d^2)))\log(\cos(e+fx))}{(a^2+b^2)\,f} \\
&\quad +\frac{(Ab^2-a(bB-aC))\,(bc-ad)^2\log(a+b\tan(e+fx))}{b^3\,(a^2+b^2)\,f} \\
&\quad +\frac{d(bcC+bBd-aCd)\tan(e+fx)}{b^2f}+\frac{C(c+d\tan(e+fx))^2}{2bf}
\end{aligned}$$

output
```
-(a*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b*(2*c*(A-C)*d+B*(c^2-d^2)))*x/(a^2+
b^2)-(a*(B*c^2-B*d^2-2*C*c*d)+b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d-b*(c^2-d^
2)))*ln(cos(f*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^2*ln(a+b*ta
n(f*x+e))/b^3/(a^2+b^2)/f+d*(B*b*d-C*a*d+C*b*c)*tan(f*x+e)/b^2/f+1/2*C*(c+
d*tan(f*x+e))^2/b/f
```

3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.75

$$\int \frac{(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\frac{b(-iA+B+iC)(c+id)^2\log(i-\tan(e+fx))}{a+ib}+\frac{b(iA+B-iC)(c-id)^2\log(i+\tan(e+fx))}{a-ib}+\frac{2\left(Ab^2+a(-bB+aC)\right)(bc-ad)^2\log(a+b\tan(e+fx))}{b^2(a^2+b^2)}}{2bf}$$

input
```
Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x]),x]
```

output
```
((b*((-I)*A + B + I*C)*(c + I*d)^2*Log[I - Tan[e + f*x]])/(a + I*b) + (b*(
I*A + B - I*C)*(c - I*d)^2*Log[I + Tan[e + f*x]])/(a - I*b) + (2*(A*b^2 +
a*(-(b*B) + a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2))
 + (2*d*(b*c*C + b*B*d - a*C*d)*Tan[e + f*x])/b + C*(c + d*Tan[e + f*x])^2
)/(2*b*f)
```

3.61.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4130, 27, 3042, 4120, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{a+b\tan(e+fx)}dx$$
$$\Big\downarrow \text{4130}$$
$$\frac{\int \frac{2(c+d\tan(e+fx))\left((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{2b}+\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 27

$$\frac{\int \frac{(c+d\tan(e+fx))\left((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))\left((bcC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 4120

$$\frac{\frac{d\tan(e+fx)(-aCd+bBd+bcC)}{bf}-\frac{\int -\frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(\left(Cc^2+2Bdc+(A-C)d^2\right)b^2-ad(2cC+Bd)b+a^2Cd^2\right)\tan^2(e+fx)-a}{a+b\tan(e+fx)}}{b}}{b}\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(\left(Cc^2+2Bdc+(A-C)d^2\right)b^2-ad(2cC+Bd)b+a^2Cd^2\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))}{a+b\tan(e+fx)}dx}{b}+\frac{d\tan(e+fx)}{}}{b}\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(\left(Cc^2+2Bdc+(A-C)d^2\right)b^2-ad(2cC+Bd)b+a^2Cd^2\right)\tan(e+fx)^2+ad(aCd-b(2cC+Bd))}{a+b\tan(e+fx)}dx}{b}+\frac{d\tan(e+fx)}{}}{b}\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 4109

$$\frac{\frac{\frac{b^2\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)\int\tan(e+fx)dx}{a^2+b^2}+\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan^2(e+fx)+1}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{b^2x\left(a\left(-A\left(c^2-d^2\right)+2\right.\right.}{}}{b}}{b}\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 3042

3.61. $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$

$$\frac{\frac{\frac{b^2\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)\int \tan(e+fx)dx}{a^2+b^2}+\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{b^2x\left(a\left(-A\left(c^2-d^2\right)+2\right.\right.}{}}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 3956

$$\frac{\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{b^2\log(\cos(e+fx))\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}-\frac{b^2x\left(a\left(-A\left(c^2-d^2\right)+2\right.\right.}{}}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 4100

$$\frac{\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{1}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\left(a^2+b^2\right)}-\frac{b^2\log(\cos(e+fx))\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}-\frac{b^2x\left(a\left(-A\right.\right.}{}}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^2}{2bf}$$

$\downarrow$ 16

$$\frac{\frac{-\frac{b^2\log(\cos(e+fx))\left(2aAcd+aB\left(c^2-d^2\right)-2acCd-Ab\left(c^2-d^2\right)+b\left(2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}-\frac{b^2x\left(a\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)}{a^2+b^2}+}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^2}{2bf}$$

input
```
Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
b*Tan[e + f*x]),x]
```

output
```
(C*(c + d*Tan[e + f*x])^2)/(2*b*f) + ((-((b^2*(a*(c^2*C + 2*B*c*d - C*d^2
- A*(c^2 - d^2)) - b*(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 + b^2)) - (b
^2*(2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2) + a*B*(c^2 - d^2) + b*(c^2*C +
 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((A*b^2 - a*(b*B -
 a*C))*(b*c - a*d)^2*Log[a + b*Tan[e + f*x]])/(b*(a^2 + b^2)*f))/b + (d*(b
*c*C + b*B*d - a*C*d)*Tan[e + f*x])/(b*f))/b
```

3.61. $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

3.61.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.61.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{d\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB-\tan(fx+e)Cad+2\tan(fx+e)Cbc\right)}{b^2}+\frac{\left(A a^2 d^2 b^2-2Aa b^3 cd+A b^4 c^2-B a^3 d^2 b+2B a^2 cd b^2-Ba \cdots\right.}{b^3\left(a^2+\cdots\right.}}{}$
default	$\frac{\frac{d\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB-\tan(fx+e)Cad+2\tan(fx+e)Cbc\right)}{b^2}+\frac{\left(A a^2 d^2 b^2-2Aa b^3 cd+A b^4 c^2-B a^3 d^2 b+2B a^2 cd b^2-Ba \cdots\right.}{b^3\left(a^2+\cdots\right.}}{}$
norman	$\frac{(Aa c^2-Aa d^2+2Abcd-2Bacd+Bb c^2-Bb d^2-Ca c^2+Ca d^2-2Cbcd)x}{a^2+b^2}+\frac{d(bdB-Cad+2Cbc)\tan(fx+e)}{b^2 f}+\frac{C d^2 \tan\cdots}{2\cdots}$
parallelrisch	$\frac{2A\ln(a+b\tan(fx+e))a^2b^2d^2+B\ln\left(1+\tan(fx+e)^2\right)a b^3 c^2-B\ln\left(1+\tan(fx+e)^2\right)a b^3 d^2+2B\ln\left(1+\tan(fx+e)^2\right)b^4 cd\cdots}{}$
risch	Expression too large to display

input
```
int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
1/f*(d/b^2*(1/2*tan(f*x+e)^2*C*b*d+tan(f*x+e)*b*d*B-tan(f*x+e)*C*a*d+2*tan
(f*x+e)*C*b*c)+1/b^3*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a^3*b*d^2+2*
B*a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2)/(a^2+b^2)
*ln(a+b*tan(f*x+e))+1/(a^2+b^2)*(1/2*(2*A*a*c*d-A*b*c^2+A*b*d^2+B*a*c^2-B*
a*d^2+2*B*b*c*d-2*C*a*c*d+C*b*c^2-C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a
*d^2+2*A*b*c*d-2*B*a*c*d+B*b*c^2-B*b*d^2-C*a*c^2+C*a*d^2-2*C*b*c*d)*arctan
(tan(f*x+e))))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.56

$$\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$= \frac{(Ca^2b^2+Cb^4)d^2\tan(fx+e)^2+2(((A-C)ab^3+Bb^4)c^2-2(Bab^3-(A-C)b^4)cd-((A-C)ab^3+B}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
1/2*((C*a^2*b^2 + C*b^4)*d^2*tan(f*x + e)^2 + 2*(((A - C)*a*b^3 + B*b^4)*c
^2 - 2*(B*a*b^3 - (A - C)*b^4)*c*d - ((A - C)*a*b^3 + B*b^4)*d^2)*f*x + ((
C*a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d +
 (C*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*
x + e) + a^2)/(tan(f*x + e)^2 + 1)) - ((C*a^2*b^2 + C*b^4)*c^2 - 2*(C*a^3*
b - B*a^2*b^2 + C*a*b^3 - B*b^4)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2 - B*a*
b^3 + (A - C)*b^4)*d^2)*log(1/(tan(f*x + e)^2 + 1)) + 2*(2*(C*a^2*b^2 + C*
b^4)*c*d - (C*a^3*b - B*a^2*b^2 + C*a*b^3 - B*b^4)*d^2)*tan(f*x + e))/((a^
2*b^3 + b^5)*f)
```

3.61.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 4444, normalized size of antiderivative = 17.50

$$\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e)),x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq
(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**2*x + A*c*d*log(tan(e + f*x)**2 + 1)
/f - A*d**2*x + A*d**2*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)/(2
*f) - 2*B*c*d*x + 2*B*c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 + 1)
/(2*f) + B*d**2*tan(e + f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/f -
 C*c*d*log(tan(e + f*x)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x + C
*d**2*tan(e + f*x)**3/(3*f) - C*d**2*tan(e + f*x)/f)/a, Eq(b, 0)), (I*A*c*
*2*f*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + A*c**2*f*x/(2*b*f*tan
(e + f*x) - 2*I*b*f) + I*A*c**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*A*c*d*f
*x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - 2*I*A*c*d*f*x/(2*b*f*tan(
e + f*x) - 2*I*b*f) - 2*A*c*d/(2*b*f*tan(e + f*x) - 2*I*b*f) + I*A*d**2*f*
x*tan(e + f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + A*d**2*f*x/(2*b*f*tan(e +
f*x) - 2*I*b*f) + A*d**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(
e + f*x) - 2*I*b*f) - I*A*d**2*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x
) - 2*I*b*f) - I*A*d**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + B*c**2*f*x*tan(e
+ f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) - I*B*c**2*f*x/(2*b*f*tan(e + f*x) -
 2*I*b*f) - B*c**2/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*I*B*c*d*f*x*tan(e +
f*x)/(2*b*f*tan(e + f*x) - 2*I*b*f) + 2*B*c*d*f*x/(2*b*f*tan(e + f*x) - 2*
I*b*f) + 2*B*c*d*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*b*f*tan(e + f*x)
 - 2*I*b*f) - 2*I*B*c*d*log(tan(e + f*x)**2 + 1)/(2*b*f*tan(e + f*x) - ...
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$

$$= \frac{\frac{2\left(\left(\left(A-C\right)a+Bb\right)c^2-2\left(Ba-\left(A-C\right)b\right)cd-\left(\left(A-C\right)a+Bb\right)d^2\right)\left(fx+e\right)}{a^2+b^2}+\frac{2\left(\left(Ca^2b^2-Bab^3+Ab^4\right)c^2-2\left(Ca^3b-Ba^2b^2+Aab^3\right)cd+\left(Ca^4-Ba^3b+\right.\right.}{a^2b^3+b^5}}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a + B*b)*c^2 - 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b
)*d^2)*(f*x + e)/(a^2 + b^2) + 2*((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C
*a^3*b - B*a^2*b^2 + A*a*b^3)*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2)*d^2)*log
(b*tan(f*x + e) + a)/(a^2*b^3 + b^5) + ((B*a - (A - C)*b)*c^2 + 2*((A - C)
*a + B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2)
 + (C*b*d^2*tan(f*x + e)^2 + 2*(2*C*b*c*d - (C*a - B*b)*d^2)*tan(f*x + e))
/b^2)/f
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.30

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\frac{2\left(Aac^2-Cac^2+Bbc^2-2\,Bacd+2\,Abcd-2\,Cbcd-Aad^2+Cad^2-Bbd^2\right)(fx+e)}{a^2+b^2}+\frac{\left(Bac^2-Abc^2+Cbc^2+2\,Aacd-2\,Cacd+2\,Bbcd-Bad^2+Abd^2-\right.}{a^2+b^2}}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 - 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d -
A*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2 + b^2) + (B*a*c^2 - A*b*c^2 +
C*b*c^2 + 2*A*a*c*d - 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)
*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 2*(C*a^2*b^2*c^2 - B*a*b^3*c^2 + A*
b^4*c^2 - 2*C*a^3*b*c*d + 2*B*a^2*b^2*c*d - 2*A*a*b^3*c*d + C*a^4*d^2 - B*
a^3*b*d^2 + A*a^2*b^2*d^2)*log(abs(b*tan(f*x + e) + a))/(a^2*b^3 + b^5) +
(C*b*d^2*tan(f*x + e)^2 + 4*C*b*c*d*tan(f*x + e) - 2*C*a*d^2*tan(f*x + e)
+ 2*B*b*d^2*tan(f*x + e))/b^2)/f
```

3.61.9 Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.28

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\tan(e+f\,x)\,\left(\frac{B\,d^2+2\,C\,c\,d}{b}-\frac{C\,a\,d^2}{b^2}\right)}{f}$$
$$+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^2\,\left(C\,a^2\,c^2+2\,B\,a^2\,c\,d+A\,a^2\,d^2\right)-b\,\left(B\,a^3\,d^2+2\,C\,c\,a^3\,d\right)-b^3\,\left(B\,a\,c^2+2\,A\,a\right.\right.}{f\,\left(a^2\,b^3+b^5\right)}$$
$$+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,d^2-A\,c^2+B\,c^2\,1\mathrm{i}-B\,d^2\,1\mathrm{i}+C\,c^2-C\,d^2+A\,c\,d\,2\mathrm{i}+2\,B\,c\,d-C\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(b+a\,1\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,c^2-B\,d^2+2\,A\,c\,d-2\,C\,c\,d-A\,c^2\,1\mathrm{i}+A\,d^2\,1\mathrm{i}+C\,c^2\,1\mathrm{i}-C\,d^2\,1\mathrm{i}+B\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(a+b\,1\mathrm{i}\right)}$$
$$+\frac{C\,d^2\tan(e+f\,x)^2}{2\,b\,f}$$

input
```
int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x)),x)
```

output
```
(tan(e + f*x)*((B*d^2 + 2*C*c*d)/b - (C*a*d^2)/b^2))/f + (log(a + b*tan(e
+ f*x))*(b^2*(A*a^2*d^2 + C*a^2*c^2 + 2*B*a^2*c*d) - b*(B*a^3*d^2 + 2*C*a^
3*c*d) - b^3*(B*a*c^2 + 2*A*a*c*d) + A*b^4*c^2 + C*a^4*d^2))/(f*(b^5 + a^2
*b^3)) + (log(tan(e + f*x) + 1i)*(A*d^2 - A*c^2 + B*c^2*1i - B*d^2*1i + C*
c^2 - C*d^2 + A*c*d*2i + 2*B*c*d - C*c*d*2i))/(2*f*(a*1i + b)) + (log(tan(
e + f*x) - 1i)*(A*d^2*1i - A*c^2*1i + B*c^2 - B*d^2 + C*c^2*1i - C*d^2*1i
+ 2*A*c*d + B*c*d*2i - 2*C*c*d))/(2*f*(a + b*1i)) + (C*d^2*tan(e + f*x)^2)
/(2*b*f)
```

3.62 $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

3.62.1 Optimal result

Integrand size = 45, antiderivative size = 415

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx =$$
$$-\frac{(a^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-2ab(2c(A-C)d+B(c$$
$$(a^2+b^2)^2$$
$$-\frac{(2ab(c^2C+2Bcd-Cd^2-A(c^2-d^2))+a^2(2c(A-C)d+B(c^2-d^2))-b^2(2c(A-C)d+B(c^2-d^2)}{(a^2+b^2)^2 f}$$
$$-\frac{(bc-ad)\left(a^3bBd-2a^4Cd-b^4(Bc+2Ad)-ab^3(2Ac-2cC-3Bd)+a^2b^2(Bc-4Cd)\right)\log(a+b\tan}{b^3(a^2+b^2)^2 f}$$
$$+\frac{(Ab^2-abB+2a^2C+b^2C)\,d^2\tan(e+fx)}{b^2(a^2+b^2)f}-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}$$

output
```
-(a^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^
2))-2*a*b*(2*c*(A-C)*d+B*(c^2-d^2)))*x/(a^2+b^2)^2-(2*a*b*(c^2*C+2*B*c*d-C
*d^2-A*(c^2-d^2))+a^2*(2*c*(A-C)*d+B*(c^2-d^2))-b^2*(2*c*(A-C)*d+B*(c^2-d^
2)))*ln(cos(f*x+e))/(a^2+b^2)^2/f-(-a*d+b*c)*(a^3*b*B*d-2*a^4*C*d-b^4*(2*A
*d+B*c)-a*b^3*(2*A*c-3*B*d-2*C*c)+a^2*b^2*(B*c-4*C*d))*ln(a+b*tan(f*x+e))/
b^3/(a^2+b^2)^2/f+(A*b^2-B*a*b+2*C*a^2+C*b^2)*d^2*tan(f*x+e)/b^2/(a^2+b^2)
/f-(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^2/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.67

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$=\frac{\frac{(-iA+B+iC)(c+id)^2\log(i-\tan(e+fx))}{(a+ib)^2}+\frac{(iA+B-iC)(c-id)^2\log(i+\tan(e+fx))}{(a-ib)^2}+\frac{2(bc-ad)(-a^3bBd+2a^4Cd+b^4(Bc+2Ad)+ab^3(2Ac-2}{b^3(a^2+b}}{2f}$$

input
```
Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x])^2,x]
```

output
```
((((-I)*A + B + I*C)*(c + I*d)^2*Log[I - Tan[e + f*x]])/(a + I*b)^2 + ((I*
A + B - I*C)*(c - I*d)^2*Log[I + Tan[e + f*x]])/(a - I*b)^2 + (2*(b*c - a*
d)*(-(a^3*b*B*d) + 2*a^4*C*d + b^4*(B*c + 2*A*d) + a*b^3*(2*A*c - 2*c*C -
3*B*d) + a^2*b^2*(-(B*c) + 4*C*d))*Log[a + b*Tan[e + f*x]])/(b^3*(a^2 + b^
2)^2) - (2*(A*b^2 - a*b*B + 2*a^2*C + b^2*C)*(b*c - a*d)^2)/(b^3*(a^2 + b^
2)*(a + b*Tan[e + f*x])) + (2*C*(c + d*Tan[e + f*x])^2)/(b*(a + b*Tan[e +
f*x])))/(2*f)
```

3.62.3 Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4128, 3042, 4120, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{(a+b\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{(c+d\tan(e+fx))\left((2Ca^2-bBa+Ab^2+b^2C)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-2ad)+Ab(ac+2bd)\right)}{a+b\tan(e+fx)}dx}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))\left((2Ca^2-bBa+Ab^2+b^2C)d\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-2ad)+Ab(ac+2bd)\right)}{a+b\tan(e+fx)}dx}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4120

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{\int \frac{-\left(\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\right)-c((bB-aC)(bc-2ad)+Ab(}{a+b\tan(e+fx)}}{b}}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{\int \frac{-\left(\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\right)-c((bB-aC)(bc-2ad)+Ab(}{a+b\tan(e+fx)}}{b}}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4109

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{-\frac{b^2\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)\int \tan(e+f}{a^2+b^2}}{}}{}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{-\frac{b^2\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)\int \tan(e+f}{a^2+b^2}}{}}{}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

3.62. $\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$\Big\downarrow$ 3956

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{\frac{(bc-ad)\left(-2a^4Cd+a^3bBd+a^2b^2(Bc-4Cd)-ab^3(2Ac-3Bd-2cC)-b^4(2Ad+Bc)\right)\int\frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{b^2\log(\cos}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 4100

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{\frac{(bc-ad)\left(-2a^4Cd+a^3bBd+a^2b^2(Bc-4Cd)-ab^3(2Ac-3Bd-2cC)-b^4(2Ad+Bc)\right)\int\frac{1}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\left(a^2+b^2\right)}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 16

$$\frac{\frac{d^2\tan(e+fx)(2a^2C-abB+Ab^2+b^2C)}{bf}-\frac{\frac{b^2\log(\cos(e+fx))\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right.}{f\left(a^2+b^2\right)}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

input
```
Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
b*Tan[e + f*x])^2,x]
```

output
```
-(((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^2)/(b*(a^2 + b^2)*f*(a + b
*Tan[e + f*x]))) + (-(((b^2*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))
 - b^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d +
B*(c^2 - d^2)))*x)/(a^2 + b^2) + (b^2*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*
(c^2 - d^2)) + a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d +
B*(c^2 - d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((b*c - a*d)*(a^3*b*B
*d - 2*a^4*C*d - b^4*(B*c + 2*A*d) - a*b^3*(2*A*c - 2*c*C - 3*B*d) + a^2*b
^2*(B*c - 4*C*d))*Log[a + b*Tan[e + f*x]])/(b*(a^2 + b^2)*f))/b) + ((A*b^2
 - a*b*B + 2*a^2*C + b^2*C)*d^2*Tan[e + f*x])/(b*f))/(b*(a^2 + b^2))
```

3.62. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.62.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.62. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)C\,d^2}{b^2}-\frac{A\,a^2d^2b^2-2Aa\,b^3cd+A\,b^4c^2-B\,a^3d^2b+2B\,a^2cd\,b^2-Ba\,b^3c^2+a^4C\,d^2-2C\,a^3cdb+C\,a^2c^2b^2}{b^3\left(a^2+b^2\right)(a+b\tan(fx+e))}+\frac{\left(-2A\,a^2b^3cd+2Aa\right.}{}}{}$
default	$\frac{\frac{\tan(fx+e)C\,d^2}{b^2}-\frac{A\,a^2d^2b^2-2Aa\,b^3cd+A\,b^4c^2-B\,a^3d^2b+2B\,a^2cd\,b^2-Ba\,b^3c^2+a^4C\,d^2-2C\,a^3cdb+C\,a^2c^2b^2}{b^3\left(a^2+b^2\right)(a+b\tan(fx+e))}+\frac{\left(-2A\,a^2b^3cd+2Aa\right.}{}}{}$
norman	$\frac{\frac{a\left(A\,a^2c^2-A\,a^2d^2+4Aabcd-A\,b^2c^2+A\,b^2d^2-2B\,a^2cd+2Bab\,c^2-2Bab\,d^2+2B\,b^2cd-C\,a^2c^2+a^2C\,d^2-4Cabcd+C\,b^2c^2-C\,b^2d^2\right)x}{a^4+2a^2b^2+b^4}+}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*d^2/b^2-1/b^3*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a
^3*b*d^2+2*B*a^2*b^2*c*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2
)/(a^2+b^2)/(a+b*tan(f*x+e))+(-2*A*a^2*b^3*c*d+2*A*a*b^4*c^2-2*A*a*b^4*d^2
+2*A*b^5*c*d+B*a^4*b*d^2-B*a^2*b^3*c^2+3*B*a^2*b^3*d^2-4*B*a*b^4*c*d+B*b^5
*c^2-2*C*a^5*d^2+2*C*a^4*b*c*d-4*C*a^3*b^2*d^2+6*C*a^2*b^3*c*d-2*C*a*b^4*c
^2)/b^3/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2*(1/2*(2*A*a^2*c*d-2*A
*a*b*c^2+2*A*a*b*d^2-2*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2+4*B*a*b*c*d-B*b^2*c^2
+B*b^2*d^2-2*C*a^2*c*d+2*C*a*b*c^2-2*C*a*b*d^2+2*C*b^2*c*d)*ln(1+tan(f*x+e
)^2)+(A*a^2*c^2-A*a^2*d^2+4*A*a*b*c*d-A*b^2*c^2+A*b^2*d^2-2*B*a^2*c*d+2*B*
a*b*c^2-2*B*a*b*d^2+2*B*b^2*c*d-C*a^2*c^2+C*a^2*d^2-4*C*a*b*c*d+C*b^2*c^2-
C*b^2*d^2)*arctan(tan(f*x+e))))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(413) = 826$.

Time = 0.59 (sec) , antiderivative size = 964, normalized size of antiderivative = 2.32

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{2\,(Ca^4b^2+2\,Ca^2b^4+Cb^6)d^2\tan\left(fx+e\right)^2-2\,(Ca^2b^4-Bab^5+Ab^6)c^2+4\,(Ca^3b^3-Ba^2b^4+Aab^5)cd-}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
1/2*(2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*d^2*tan(f*x + e)^2 - 2*(C*a^2*b^4
 - B*a*b^5 + A*b^6)*c^2 + 4*(C*a^3*b^3 - B*a^2*b^4 + A*a*b^5)*c*d - 2*(C*a
^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*d^2 + 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 -
(A - C)*a*b^5)*c^2 - 2*(B*a^3*b^3 - 2*(A - C)*a^2*b^4 - B*a*b^5)*c*d - ((A
 - C)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*d^2)*f*x - ((B*a^3*b^3 - 2*(A
 - C)*a^2*b^4 - B*a*b^5)*c^2 - 2*(C*a^5*b - (A - 3*C)*a^3*b^3 - 2*B*a^2*b^
4 + A*a*b^5)*c*d + (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 3*B*a^3*b^3 + 2*A*a^
2*b^4)*d^2 + ((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (
A - 3*C)*a^2*b^4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a
^3*b^3 - 3*B*a^2*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))*log((b^2*tan(f*x + e)
^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (2*(C*a^5*b + 2*C*a
^3*b^3 + C*a*b^5)*c*d - (2*C*a^6 - B*a^5*b + 4*C*a^4*b^2 - 2*B*a^3*b^3 + 2
*C*a^2*b^4 - B*a*b^5)*d^2 + (2*(C*a^4*b^2 + 2*C*a^2*b^4 + C*b^6)*c*d - (2*
C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 2*B*a^2*b^4 + 2*C*a*b^5 - B*b^6)*d^2)*
tan(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) + 2*((C*a^3*b^3 - B*a^2*b^4 + A*
a*b^5)*c^2 - 2*(C*a^4*b^2 - B*a^3*b^3 + A*a^2*b^4)*c*d + (2*C*a^5*b - B*a^
4*b^2 + (A + 2*C)*a^3*b^3 + C*a*b^5)*d^2 + (((A - C)*a^2*b^4 + 2*B*a*b^5 -
 (A - C)*b^6)*c^2 - 2*(B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c*d - ((A - C)
*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*d^2)*f*x)*tan(f*x + e))/((a^4*b^4 + 2*
a^2*b^6 + b^8)*f*tan(f*x + e) + (a^5*b^3 + 2*a^3*b^5 + a*b^7)*f)
```

3.62.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.97 (sec) , antiderivative size = 16225, normalized size of antiderivative = 39.10

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e))**2,x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
 Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**2*x + A*c*d*log(tan(e + f*x)**2 +
 1)/f - A*d**2*x + A*d**2*tan(e + f*x)/f + B*c**2*log(tan(e + f*x)**2 + 1)
/(2*f) - 2*B*c*d*x + 2*B*c*d*tan(e + f*x)/f - B*d**2*log(tan(e + f*x)**2 +
 1)/(2*f) + B*d**2*tan(e + f*x)**2/(2*f) - C*c**2*x + C*c**2*tan(e + f*x)/
f - C*c*d*log(tan(e + f*x)**2 + 1)/f + C*c*d*tan(e + f*x)**2/f + C*d**2*x
+ C*d**2*tan(e + f*x)**3/(3*f) - C*d**2*tan(e + f*x)/f)/a**2, Eq(b, 0)), (
-A*c**2*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e +
 f*x) - 4*b**2*f) + 2*I*A*c**2*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2
- 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c**2*f*x/(4*b**2*f*tan(e + f*x)*
*2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*c**2*tan(e + f*x)/(4*b**2*f*t
an(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c**2/(4*b**2*
f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c*d*f*x*ta
n(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2
*f) + 4*A*c*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(
e + f*x) - 4*b**2*f) - 2*I*A*c*d*f*x/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*
f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c*d*tan(e + f*x)/(4*b**2*f*tan(e + f*x)
**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*d**2*f*x*tan(e + f*x)**2/(4*
b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - 2*I*A*d**2*
f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - ...
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.20

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2Cd^2\tan(fx+e)}{b^2}+\frac{2\left(\left(((A-C)a^2+2Bab-(A-C)b^2)c^2-2\left(Ba^2-2(A-C)ab-Bb^2\right)cd-((A-C)a^2+2Bab-(A-C)b^2)d^2\right)(fx+e)}{a^4+2a^2b^2+b^4}-\frac{2\left((Ba^2\right.}{}}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c
^2 - 2*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A -
 C)*b^2)*d^2)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a^2*b^3 - 2*(A - C
)*a*b^4 - B*b^5)*c^2 - 2*(C*a^4*b - (A - 3*C)*a^2*b^3 - 2*B*a*b^4 + A*b^5)
*c*d + (2*C*a^5 - B*a^4*b + 4*C*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*d^2)*lo
g(b*tan(f*x + e) + a)/(a^4*b^3 + 2*a^2*b^5 + b^7) + ((B*a^2 - 2*(A - C)*a*
b - B*b^2)*c^2 + 2*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 - 2*
(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4)
- 2*((C*a^2*b^2 - B*a*b^3 + A*b^4)*c^2 - 2*(C*a^3*b - B*a^2*b^2 + A*a*b^3)
*c*d + (C*a^4 - B*a^3*b + A*a^2*b^2)*d^2)/(a^3*b^3 + a*b^5 + (a^2*b^4 + b^
6)*tan(f*x + e)))/f
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(413) = 826$.

Time = 0.84 (sec) , antiderivative size = 893, normalized size of antiderivative = 2.15

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2\,Cd^2\tan(fx+e)}{b^2} + \frac{2\,(Aa^2c^2-Ca^2c^2+2\,Babc^2-Ab^2c^2+Cb^2c^2-2\,Ba^2cd+4\,Aabcd-4\,Cabcd+2\,Bb^2cd-Aa^2d^2+Ca^2d^2-2\,Babd^2+Ab^2d^2-Cb^2}{a^4+2\,a^2b^2+b^4}}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*C*d^2*tan(f*x + e)/b^2 + 2*(A*a^2*c^2 - C*a^2*c^2 + 2*B*a*b*c^2 - A
*b^2*c^2 + C*b^2*c^2 - 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d + 2*B*b^2*c
*d - A*a^2*d^2 + C*a^2*d^2 - 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e
)/(a^4 + 2*a^2*b^2 + b^4) + (B*a^2*c^2 - 2*A*a*b*c^2 + 2*C*a*b*c^2 - B*b^2
*c^2 + 2*A*a^2*c*d - 2*C*a^2*c*d + 4*B*a*b*c*d - 2*A*b^2*c*d + 2*C*b^2*c*d
 - B*a^2*d^2 + 2*A*a*b*d^2 - 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 +
 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^2*b^3*c^2 - 2*A*a*b^4*c^2 + 2*C*a*b^4
*c^2 - B*b^5*c^2 - 2*C*a^4*b*c*d + 2*A*a^2*b^3*c*d - 6*C*a^2*b^3*c*d + 4*B
*a*b^4*c*d - 2*A*b^5*c*d + 2*C*a^5*d^2 - B*a^4*b*d^2 + 4*C*a^3*b^2*d^2 - 3
*B*a^2*b^3*d^2 + 2*A*a*b^4*d^2)*log(abs(b*tan(f*x + e) + a))/(a^4*b^3 + 2*
a^2*b^5 + b^7) + 2*(B*a^2*b^4*c^2*tan(f*x + e) - 2*A*a*b^5*c^2*tan(f*x + e
) + 2*C*a*b^5*c^2*tan(f*x + e) - B*b^6*c^2*tan(f*x + e) - 2*C*a^4*b^2*c*d*
tan(f*x + e) + 2*A*a^2*b^4*c*d*tan(f*x + e) - 6*C*a^2*b^4*c*d*tan(f*x + e)
 + 4*B*a*b^5*c*d*tan(f*x + e) - 2*A*b^6*c*d*tan(f*x + e) + 2*C*a^5*b*d^2*t
an(f*x + e) - B*a^4*b^2*d^2*tan(f*x + e) + 4*C*a^3*b^3*d^2*tan(f*x + e) -
3*B*a^2*b^4*d^2*tan(f*x + e) + 2*A*a*b^5*d^2*tan(f*x + e) - C*a^4*b^2*c^2
+ 2*B*a^3*b^3*c^2 - 3*A*a^2*b^4*c^2 + C*a^2*b^4*c^2 - A*b^6*c^2 - 2*B*a^4*
b^2*c*d + 4*A*a^3*b^3*c*d - 4*C*a^3*b^3*c*d + 2*B*a^2*b^4*c*d + C*a^6*d^2
- A*a^4*b^2*d^2 + 3*C*a^4*b^2*d^2 - 2*B*a^3*b^3*d^2 + A*a^2*b^4*d^2)/((a^4
*b^3 + 2*a^2*b^5 + b^7)*(b*tan(f*x + e) + a)))/f
```

3.62.9 Mupad [B] (verification not implemented)

Time = 32.73 (sec) , antiderivative size = 3958, normalized size of antiderivative = 9.54

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x))^2,x)
```

output
```
(log((2*C^2*a^5*d^4 + 4*C^2*a^3*b^2*d^4 - 2*C^2*a^5*c^2*d^2 - A*B*b^5*c^4
- 2*A*C*a^5*d^4 + B*C*b^5*c^4 - A^2*a*b^4*c^4 - A^2*a*b^4*d^4 + B^2*a*b^4*
c^4 + B^2*a*b^4*d^4 - C^2*a*b^4*c^4 + 2*A^2*b^5*c*d^3 - 2*A^2*b^5*c^3*d +
C^2*a*b^4*d^4 + 2*B^2*b^5*c^3*d - 4*C^2*a^3*b^2*c^2*d^2 + A*B*a^2*b^3*c^4
+ 3*A*B*a^2*b^3*d^4 - 4*A*C*a^3*b^2*d^4 - B*C*a^2*b^3*c^4 + 5*A*B*b^5*c^2*
d^2 + 2*A*C*a^5*c^2*d^2 - 3*B*C*a^2*b^3*d^4 - B*C*b^5*c^2*d^2 + 2*B^2*a^4*
b*c*d^3 - 2*C^2*a^4*b*c*d^3 + 2*C^2*a^4*b*c^3*d + 6*A^2*a*b^4*c^2*d^2 - 2*
A^2*a^2*b^3*c*d^3 + 2*A^2*a^2*b^3*c^3*d - 6*B^2*a*b^4*c^2*d^2 + 6*B^2*a^2*
b^3*c*d^3 - 2*B^2*a^2*b^3*c^3*d + 4*C^2*a*b^4*c^2*d^2 - 6*C^2*a^2*b^3*c*d^
3 + 6*C^2*a^2*b^3*c^3*d + A*B*a^4*b*d^4 + 2*A*C*a*b^4*c^4 - B*C*a^4*b*d^4
- 2*A*C*b^5*c*d^3 + 2*A*C*b^5*c^3*d - 4*B*C*a^5*c*d^3 - 8*A*B*a*b^4*c*d^3
+ 8*A*B*a*b^4*c^3*d + 2*A*C*a^4*b*c*d^3 - 2*A*C*a^4*b*c^3*d + 4*B*C*a*b^4*
c*d^3 - 8*B*C*a*b^4*c^3*d - A*B*a^4*b*c^2*d^2 - 10*A*C*a*b^4*c^2*d^2 + 8*A
*C*a^2*b^3*c*d^3 - 8*A*C*a^2*b^3*c^3*d - 8*B*C*a^3*b^2*c*d^3 + 5*B*C*a^4*b
*c^2*d^2 - 8*A*B*a^2*b^3*c^2*d^2 + 4*A*C*a^3*b^2*c^2*d^2 + 16*B*C*a^2*b^3*
c^2*d^2)/(b^2*(a^2 + b^2)^2) + ((c*1i + d)^2*((tan(e + f*x)*(3*B*b^5*c^2 -
 5*B*b^5*d^2 - 4*C*a^5*d^2 + 6*A*b^5*c*d - 10*C*b^5*c*d + 4*A*a*b^4*c^2 -
4*A*a*b^4*d^2 + 2*B*a^4*b*d^2 - 4*C*a*b^4*c^2 + 8*C*a*b^4*d^2 - B*a^2*b^3*
c^2 + B*a^2*b^3*d^2 - 8*B*a*b^4*c*d + 4*C*a^4*b*c*d - 2*A*a^2*b^3*c*d + 2*
C*a^2*b^3*c*d))/(b^2*(a^2 + b^2)) - (A*b^2*d^2 - A*b^2*c^2 - 8*C*a^2*d^...
```

3.62. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.63 $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.63.1 Optimal result

Integrand size = 45, antiderivative size = 597

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{(a^3(c^2C+2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C+2Bcd-Cd^2-A(c^2-d^2))-3a^2b(2c(A-C)d+}{(a^2+b^2)^3}$$

$$-\frac{(3a^2b(c^2C+2Bcd-Cd^2-A(c^2-d^2))-b^3(c^2C+2Bcd-Cd^2-A(c^2-d^2))+a^3(2c(A-C)d+B(}{(a^2+b^2)^3\,f}$$

$$+\frac{(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C+2Bcd-2Cd^2-A(c^2-d^2))+b^6(c(cC+2Bd)-A(c^2-d^2))-a^3b}{b^3\,(a^2+b^2)^3\,f}$$

$$-\frac{(bc-ad)\,(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3C)d))}{b^3\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$

$$-\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^2}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

output
```
-(a^3*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C+2*B*c*d-C*d^2-A*(c^
2-d^2))-3*a^2*b*(2*c*(A-C)*d+B*(c^2-d^2))+b^3*(2*c*(A-C)*d+B*(c^2-d^2)))*x
/(a^2+b^2)^3-(3*a^2*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C+2*B*c*d
-C*d^2-A*(c^2-d^2))+a^3*(2*c*(A-C)*d+B*(c^2-d^2))-3*a*b^2*(2*c*(A-C)*d+B*(
c^2-d^2)))*ln(cos(f*x+e))/(a^2+b^2)^3/f+(a^6*C*d^2+3*a^4*b^2*C*d^2-3*a^2*b
^4*(c^2*C+2*B*c*d-2*C*d^2-A*(c^2-d^2))+b^6*(c*(2*B*d+C*c)-A*(c^2-d^2))-a^3
*b^3*(2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^5*(2*c*(A-C)*d+B*(c^2-d^2)))*ln(a+b*t
an(f*x+e))/b^3/(a^2+b^2)^3/f-(-a*d+b*c)*(a^4*C*d+b^4*(A*d+B*c)+2*a*b^3*(A*
c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*d))/b^3/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2
*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^2/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.04 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.74

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{(3a^2Abc^2-Ab^3c^2-a^3Bc^2+3ab^2Bc^2-3a^2bc^2C+b^3c^2C-2a^3Acd+6aAb^2cd-6a^2bBcd+2b^3Bcd+$$

$$+\frac{(-3a^2Abc^2+Ab^3c^2+a^3Bc^2-3ab^2Bc^2+3a^2bc^2C-b^3c^2C+2a^3Acd-6aAb^2cd+6a^2bBcd-2b^3Bcd$$

$$+\frac{(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C+2Bcd-2Cd^2-A(c^2-d^2))+b^6(c(cC+2Bd)-A(c^2-d^2))-a^3b}{b^3\,(a^2+b^2)^3\,f}$$

$$-\frac{(Ab^2-a(bB-aC))\,(bc-ad)^2}{2b^3\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

$$+\frac{(bc-ad)\,(a^3bBd-2a^4Cd-b^4(Bc+2Ad)-ab^3(2Ac-2cC-3Bd)+a^2b^2(Bc-4Cd))}{b^3\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$

input
```
Integrate[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x])^3,x]
```

output
```
-1/2*((3*a^2*A*b*c^2 - A*b^3*c^2 - a^3*B*c^2 + 3*a*b^2*B*c^2 - 3*a^2*b*c^2
*C + b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d
 + 2*a^3*c*C*d - 6*a*b^2*c*C*d - 3*a^2*A*b*d^2 + A*b^3*d^2 + a^3*B*d^2 - 3
*a*b^2*B*d^2 + 3*a^2*b*C*d^2 - b^3*C*d^2 + I*(a^3*A*c^2 - 3*a*A*b^2*c^2 +
3*a^2*b*B*c^2 - b^3*B*c^2 - a^3*c^2*C + 3*a*b^2*c^2*C + 6*a^2*A*b*c*d - 2*
A*b^3*c*d - 2*a^3*B*c*d + 6*a*b^2*B*c*d - 6*a^2*b*c*C*d + 2*b^3*c*C*d - a^
3*A*d^2 + 3*a*A*b^2*d^2 - 3*a^2*b*B*d^2 + b^3*B*d^2 + a^3*C*d^2 - 3*a*b^2*
C*d^2))*Log[I - Tan[e + f*x]])/((a^2 + b^2)^3*f) + ((-3*a^2*A*b*c^2 + A*b^
3*c^2 + a^3*B*c^2 - 3*a*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C + 2*a^3*A*c*
d - 6*a*A*b^2*c*d + 6*a^2*b*B*c*d - 2*b^3*B*c*d - 2*a^3*c*C*d + 6*a*b^2*c*
C*d + 3*a^2*A*b*d^2 - A*b^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d^
2 + b^3*C*d^2 + I*(a^3*A*c^2 - 3*a*A*b^2*c^2 + 3*a^2*b*B*c^2 - b^3*B*c^2 -
 a^3*c^2*C + 3*a*b^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d - 2*a^3*B*c*d + 6
*a*b^2*B*c*d - 6*a^2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2*d^2 - 3
*a^2*b*B*d^2 + b^3*B*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2))*Log[I + Tan[e + f*x
]])/(2*(a^2 + b^2)^3*f) + ((a^6*C*d^2 + 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c^2*C
 + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2)) + b^6*(c*(c*C + 2*B*d) - A*(c^2 - d^
2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5*(2*c*(A - C)*d + B
*(c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b^3*(a^2 + b^2)^3*f) - ((A*b^2 -
a*(b*B - a*C))*(b*c - a*d)^2)/(2*b^3*(a^2 + b^2)*f*(a + b*Tan[e + f*x])...
```

3.63.3 Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {3042, 4128, 27, 3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{2(c+d\tan(e+fx))\left((a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^2}dx}{2b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\tan(e+fx))\left((a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^2}dx}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\tan(e+fx))\left((a^2+b^2)Cd\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^2}dx}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{4118}$$

$$\frac{\int \frac{Cd^2a^4-b^2\left(Cc^2+2Bdc-3Cd^2-A\left(c^2-d^2\right)\right)a^2+2b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a+\left(a^2+b^2\right)^2Cd^2\tan^2(e+fx)+b^4\left(c(cC+2Bd)-A\left(c^2-d^2\right)\right)+b^2\left(\left(2c(A-C)d+B\right.\right.}{a+b\tan(e+fx)}}{b(a^2+b^2)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{Cd^2a^4-b^2\left(Cc^2+2Bdc-3Cd^2-A\left(c^2-d^2\right)\right)a^2+2b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a+\left(a^2+b^2\right)^2Cd^2\tan(e+fx)^2+b^4\left(c(cC+2Bd)-A\left(c^2-d^2\right)\right)+b^2\left(\left(2c(A-C)d+B\right.\right.}{a+b\tan(e+fx)}}{b(a^2+b^2)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 4109

$$\frac{\frac{b^2\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-b^3\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)\right)\int \tan(e+fx)dx}{a^2+b^2}+}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{b^2\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-b^3\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)\right)\int \tan(e+fx)dx}{a^2+b^2}+}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3956

$$\frac{\frac{\left(a^6Cd^2+3a^4b^2Cd^2-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3ab^5\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+b^6\left(c(2Bd+cC)-A\left(c^2-d^2\right)\right)\right)\int}{a^2+b^2}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 4100

3.63. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\left(a^6Cd^2+3a^4b^2Cd^2-a^3b^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-2Cd^2\right)+3ab^5\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+b^6\left(c(2Bd+cC)-A\left(c^2-d^2\right)\right)\right)\int}{bf\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{ 16}$$

$$-\frac{b^2\log(\cos(e+fx))\left(a^3\left(2cd(A-C)+B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)-b^3\left(-A\left(c^2-d^2\right)+2Bcd+c^2C-Cd^2\right)\right)}{f\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^2}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

input
```
Int[((c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
b*Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^2)/(b*(a^2 + b^2)*f*(a
+ b*Tan[e + f*x])^2) + ((-((b^2*(a^3*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d
^2)) - 3*a*b^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A
 - C)*d + B*(c^2 - d^2)) + b^3*(2*c*(A - C)*d + B*(c^2 - d^2)))*x)/(a^2 +
b^2)) - (b^2*(3*a^2*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2
*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^3*(2*c*(A - C)*d + B*(c^2 - d^2)
) - 3*a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log[Cos[e + f*x]])/((a^2 + b^
2)*f) + ((a^6*C*d^2 + 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c^2*C + 2*B*c*d - 2*C*d
^2 - A*(c^2 - d^2)) + b^6*(c*(c*C + 2*B*d) - A*(c^2 - d^2)) - a^3*b^3*(2*c
*(A - C)*d + B*(c^2 - d^2)) + 3*a*b^5*(2*c*(A - C)*d + B*(c^2 - d^2)))*Log
[a + b*Tan[e + f*x]])/(b*(a^2 + b^2)*f))/(b*(a^2 + b^2)) - ((b*c - a*d)*(a
^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A -
 3*C)*d)))/(b^2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])))/(b*(a^2 + b^2))
```

3.63.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.63. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.63.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{A\,a^2 d^2 b^2-2 A a\,b^3 c d+A\,b^4 c^2-B\,a^3 d^2 b+2 B\,a^2 c d\,b^2-B a\,b^3 c^2+a^4 C\,d^2-2 C\,a^3 c d b+C\,a^2 c^2 b^2}{2 b^3 \left(a^2+b^2\right) (a+b\tan(f x+e))^2}-\frac{-2 A\,a^2 b^3 c d+2 A a\,b^4 c^2-2 A a\,b^4 d^2+}{}$
default	$-\frac{A\,a^2 d^2 b^2-2 A a\,b^3 c d+A\,b^4 c^2-B\,a^3 d^2 b+2 B\,a^2 c d\,b^2-B a\,b^3 c^2+a^4 C\,d^2-2 C\,a^3 c d b+C\,a^2 c^2 b^2}{2 b^3 \left(a^2+b^2\right) (a+b\tan(f x+e))^2}-\frac{-2 A\,a^2 b^3 c d+2 A a\,b^4 c^2-2 A a\,b^4 d^2+}{}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/2*(A*a^2*b^2*d^2-2*A*a*b^3*c*d+A*b^4*c^2-B*a^3*b*d^2+2*B*a^2*b^2*c
*d-B*a*b^3*c^2+C*a^4*d^2-2*C*a^3*b*c*d+C*a^2*b^2*c^2)/b^3/(a^2+b^2)/(a+b*t
an(f*x+e))^2-(-2*A*a^2*b^3*c*d+2*A*a*b^4*c^2-2*A*a*b^4*d^2+2*A*b^5*c*d+B*a
^4*b*d^2-B*a^2*b^3*c^2+3*B*a^2*b^3*d^2-4*B*a*b^4*c*d+B*b^5*c^2-2*C*a^5*d^2
+2*C*a^4*b*c*d-4*C*a^3*b^2*d^2+6*C*a^2*b^3*c*d-2*C*a*b^4*c^2)/b^3/(a^2+b^2
)^2/(a+b*tan(f*x+e))+1/(a^2+b^2)^3*(-2*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2-3*A*a
^2*b^4*d^2+6*A*a*b^5*c*d-A*b^6*c^2+A*b^6*d^2-B*a^3*b^3*c^2+B*a^3*b^3*d^2-6
*B*a^2*b^4*c*d+3*B*a*b^5*c^2-3*B*a*b^5*d^2+2*B*b^6*c*d+C*a^6*d^2+3*C*a^4*b
^2*d^2+2*C*a^3*b^3*c*d-3*C*a^2*b^4*c^2+6*C*a^2*b^4*d^2-6*C*a*b^5*c*d+C*b^6
*c^2)/b^3*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^3*(1/2*(2*A*a^3*c*d-3*A*a^2*b*c^2
+3*A*a^2*b*d^2-6*A*a*b^2*c*d+A*b^3*c^2-A*b^3*d^2+B*a^3*c^2-B*a^3*d^2+6*B*a
^2*b*c*d-3*B*a*b^2*c^2+3*B*a*b^2*d^2-2*B*b^3*c*d-2*C*a^3*c*d+3*C*a^2*b*c^2
-3*C*a^2*b*d^2+6*C*a*b^2*c*d-C*b^3*c^2+C*b^3*d^2)*ln(1+tan(f*x+e)^2)+(A*a^
3*c^2-A*a^3*d^2+6*A*a^2*b*c*d-3*A*a*b^2*c^2+3*A*a*b^2*d^2-2*A*b^3*c*d-2*B*
a^3*c*d+3*B*a^2*b*c^2-3*B*a^2*b*d^2+6*B*a*b^2*c*d-B*b^3*c^2+B*b^3*d^2-C*a^
3*c^2+C*a^3*d^2-6*C*a^2*b*c*d+3*C*a*b^2*c^2-3*C*a*b^2*d^2+2*C*b^3*c*d)*arc
tan(tan(f*x+e))))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1699 vs. $2(595) = 1190$.

Time = 0.74 (sec) , antiderivative size = 1699, normalized size of antiderivative = 2.85

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
-1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*
c^2 - 2*(C*a^5*b^3 - 3*B*a^4*b^4 + 5*(A - C)*a^3*b^5 + 3*B*a^2*b^6 - A*a*b
^7)*c*d - (C*a^6*b^2 + B*a^5*b^3 - (3*A - 7*C)*a^4*b^4 - 5*B*a^3*b^5 + 3*A
*a^2*b^6)*d^2 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*
a^2*b^6)*c^2 - 2*(B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A - C)*a^
2*b^6)*c*d - ((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B*a^2*b^
6)*d^2)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(A - C)*a^2*b^6 + 3*B*a*b^7 -
A*b^8)*c^2 + 2*(C*a^5*b^3 + B*a^4*b^4 - (3*A - 7*C)*a^3*b^5 - 5*B*a^2*b^6
+ 3*A*a*b^7)*c*d - (3*C*a^6*b^2 - B*a^5*b^3 - (A - 9*C)*a^4*b^4 - 7*B*a^3*
b^5 + 5*A*a^2*b^6)*d^2 + 2*(((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b
^7 - B*b^8)*c^2 - 2*(B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7 + (A - C)*b
^8)*c*d - ((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*d^2)*f
*x)*tan(f*x + e)^2 + ((B*a^5*b^3 - 3*(A - C)*a^4*b^4 - 3*B*a^3*b^5 + (A -
C)*a^2*b^6)*c^2 + 2*((A - C)*a^5*b^3 + 3*B*a^4*b^4 - 3*(A - C)*a^3*b^5 - B
*a^2*b^6)*c*d - (C*a^8 + 3*C*a^6*b^2 + B*a^5*b^3 - 3*(A - 2*C)*a^4*b^4 - 3
*B*a^3*b^5 + A*a^2*b^6)*d^2 + ((B*a^3*b^5 - 3*(A - C)*a^2*b^6 - 3*B*a*b^7
+ (A - C)*b^8)*c^2 + 2*((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 -
B*b^8)*c*d - (C*a^6*b^2 + 3*C*a^4*b^4 + B*a^3*b^5 - 3*(A - 2*C)*a^2*b^6 -
3*B*a*b^7 + A*b^8)*d^2)*tan(f*x + e)^2 + 2*((B*a^4*b^4 - 3*(A - C)*a^3*b^5
 - 3*B*a^2*b^6 + (A - C)*a*b^7)*c^2 + 2*((A - C)*a^4*b^4 + 3*B*a^3*b^5 ...
```

3.63.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 839, normalized size of antiderivative = 1.41

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{\frac{2\left(\left(\left((A-C)a^3+3\,Ba^2b-3\,(A-C)ab^2-Bb^3\right)c^2-2\left(Ba^3-3\,(A-C)a^2b-3\,Bab^2+(A-C)b^3\right)cd-\left((A-C)a^3+3\,Ba^2b-3\,(A-C)ab^2-Bb^3\right)d^2\right)(fx+e)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 - 2*(B*a^3
 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2
*b - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6) - 2*((B*a^3*b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2
+ 2*((A - C)*a^3*b^3 + 3*B*a^2*b^4 - 3*(A - C)*a*b^5 - B*b^6)*c*d - (C*a^6
 + 3*C*a^4*b^2 + B*a^3*b^3 - 3*(A - 2*C)*a^2*b^4 - 3*B*a*b^5 + A*b^6)*d^2)
*log(b*tan(f*x + e) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) + ((B*a^3
 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 + 2*((A - C)*a^3 + 3*B*a
^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2
 + (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6) - ((C*a^4*b^2 - 3*B*a^3*b^3 + (5*A - 3*C)*a^2*b^4 + B*a*b^5 + A*b^6
)*c^2 + 2*(C*a^5*b + B*a^4*b^2 - (3*A - 5*C)*a^3*b^3 - 3*B*a^2*b^4 + A*a*b
^5)*c*d - (3*C*a^6 - B*a^5*b - (A - 7*C)*a^4*b^2 - 5*B*a^3*b^3 + 3*A*a^2*b
^4)*d^2 - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^2 - 2*(C*a^4*b^2 - (A
 - 3*C)*a^2*b^4 - 2*B*a*b^5 + A*b^6)*c*d + (2*C*a^5*b - B*a^4*b^2 + 4*C*a^
3*b^3 - 3*B*a^2*b^4 + 2*A*a*b^5)*d^2)*tan(f*x + e))/(a^6*b^3 + 2*a^4*b^5 +
 a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(f*x + e)^2 + 2*(a^5*b^4 + 2*a^3
*b^6 + a*b^8)*tan(f*x + e)))/f
```

3.63.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. 2(595) = 1190.

Time = 1.01 (sec) , antiderivative size = 1668, normalized size of antiderivative = 2.79

$$\int \frac{(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*
c^2 - B*b^3*c^2 - 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d + 6*B*a*b^2*
c*d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 +
3*A*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6) + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^
2 + A*b^3*c^2 - C*b^3*c^2 + 2*A*a^3*c*d - 2*C*a^3*c*d + 6*B*a^2*b*c*d - 6*
A*a*b^2*c*d + 6*C*a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*
C*a^2*b*d^2 + 3*B*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 +
1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 2*(B*a^3*b^3*c^2 - 3*A*a^2*b^4*c^
2 + 3*C*a^2*b^4*c^2 - 3*B*a*b^5*c^2 + A*b^6*c^2 - C*b^6*c^2 + 2*A*a^3*b^3*
c*d - 2*C*a^3*b^3*c*d + 6*B*a^2*b^4*c*d - 6*A*a*b^5*c*d + 6*C*a*b^5*c*d -
2*B*b^6*c*d - C*a^6*d^2 - 3*C*a^4*b^2*d^2 - B*a^3*b^3*d^2 + 3*A*a^2*b^4*d^
2 - 6*C*a^2*b^4*d^2 + 3*B*a*b^5*d^2 - A*b^6*d^2)*log(abs(b*tan(f*x + e) +
a))/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) + (3*B*a^3*b^4*c^2*tan(f*x + e
)^2 - 9*A*a^2*b^5*c^2*tan(f*x + e)^2 + 9*C*a^2*b^5*c^2*tan(f*x + e)^2 - 9*
B*a*b^6*c^2*tan(f*x + e)^2 + 3*A*b^7*c^2*tan(f*x + e)^2 - 3*C*b^7*c^2*tan(
f*x + e)^2 + 6*A*a^3*b^4*c*d*tan(f*x + e)^2 - 6*C*a^3*b^4*c*d*tan(f*x + e)
^2 + 18*B*a^2*b^5*c*d*tan(f*x + e)^2 - 18*A*a*b^6*c*d*tan(f*x + e)^2 + 18*
C*a*b^6*c*d*tan(f*x + e)^2 - 6*B*b^7*c*d*tan(f*x + e)^2 - 3*C*a^6*b*d^2*ta
n(f*x + e)^2 - 9*C*a^4*b^3*d^2*tan(f*x + e)^2 - 3*B*a^3*b^4*d^2*tan(f*x...
```

3.63.9 Mupad [B] (verification not implemented)

Time = 27.62 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.35

$$\int \frac{(c+d\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(\frac{a^2\,\left(b^4\,\left(3\,A\,d^2-3\,A\,c^2+3\,C\,c^2-6\,C\,d^2+6\,B\,c\,d\right)+3\,C\,b^4\,d^2\right)-b^6\,\left(A\,d^2-A\,c^2+C\,c^2+2\,B\,c\,d\right)+C\,b^6\,d^2-a\,b^5\,(3\cdots}{a^6\,b^3+3\,a^4\,b^5+3\,a^2\,b^7+b^9}\cdots\right.}{f}$$

$$-\frac{\frac{A\,b^6\,c^2-3\,C\,a^6\,d^2+B\,a\,b^5\,c^2+B\,a^5\,b\,d^2+5\,A\,a^2\,b^4\,c^2-3\,A\,a^2\,b^4\,d^2+A\,a^4\,b^2\,d^2-3\,B\,a^3\,b^3\,c^2+5\,B\,a^3\,b^3\,d^2-3\,C\,a^2\,b^4\,c^2+C\,a^4\,b^2\,c^2-7\,C\,a^4\,b^2\,d^2\cdots}{2\,b^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}\cdots}{\cdots}$$

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,c^2-B\,d^2+2\,A\,c\,d-2\,C\,c\,d-A\,c^2\,1\mathrm{i}+A\,d^2\,1\mathrm{i}+C\,c^2\,1\mathrm{i}-C\,d^2\,1\mathrm{i}+B\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,d^2-A\,c^2+B\,c^2\,1\mathrm{i}-B\,d^2\,1\mathrm{i}+C\,c^2-C\,d^2+A\,c\,d\,2\mathrm{i}+2\,B\,c\,d-C\,c\,d\,2\mathrm{i}\right)}{2\,f\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

input
```
int(((c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x))^3,x)
```

output
```
- (log(a + b*tan(e + f*x))*((a^2*(b^4*(3*A*d^2 - 3*A*c^2 + 3*C*c^2 - 6*C*d
^2 + 6*B*c*d) + 3*C*b^4*d^2) - b^6*(A*d^2 - A*c^2 + C*c^2 + 2*B*c*d) + C*b
^6*d^2 - a*b^5*(3*B*c^2 - 3*B*d^2 + 6*A*c*d - 6*C*c*d) + a^3*b^3*(B*c^2 -
B*d^2 + 2*A*c*d - 2*C*c*d))/(b^9 + 3*a^2*b^7 + 3*a^4*b^5 + a^6*b^3) - (C*d
^2)/b^3))/f - ((A*b^6*c^2 - 3*C*a^6*d^2 + B*a*b^5*c^2 + B*a^5*b*d^2 + 5*A*
a^2*b^4*c^2 - 3*A*a^2*b^4*d^2 + A*a^4*b^2*d^2 - 3*B*a^3*b^3*c^2 + 5*B*a^3*
b^3*d^2 - 3*C*a^2*b^4*c^2 + C*a^4*b^2*c^2 - 7*C*a^4*b^2*d^2 + 2*A*a*b^5*c*
d + 2*C*a^5*b*c*d - 6*A*a^3*b^3*c*d - 6*B*a^2*b^4*c*d + 2*B*a^4*b^2*c*d +
10*C*a^3*b^3*c*d)/(2*b^3*(a^4 + b^4 + 2*a^2*b^2)) + (tan(e + f*x)*(B*b^5*c
^2 - 2*C*a^5*d^2 + 2*A*b^5*c*d + 2*A*a*b^4*c^2 - 2*A*a*b^4*d^2 + B*a^4*b*d
^2 - 2*C*a*b^4*c^2 - B*a^2*b^3*c^2 + 3*B*a^2*b^3*d^2 - 4*C*a^3*b^2*d^2 - 4
*B*a*b^4*c*d + 2*C*a^4*b*c*d - 2*A*a^2*b^3*c*d + 6*C*a^2*b^3*c*d))/(b^2*(a
^4 + b^4 + 2*a^2*b^2)))/(f*(a^2 + b^2*tan(e + f*x)^2 + 2*a*b*tan(e + f*x))
) - (log(tan(e + f*x) - 1i)*(A*d^2*1i - A*c^2*1i + B*c^2 - B*d^2 + C*c^2*1
i - C*d^2*1i + 2*A*c*d + B*c*d*2i - 2*C*c*d))/(2*f*(3*a*b^2 - a^2*b*3i - a
^3 + b^3*1i)) - (log(tan(e + f*x) + 1i)*(A*d^2 - A*c^2 + B*c^2*1i - B*d^2*
1i + C*c^2 - C*d^2 + A*c*d*2i + 2*B*c*d - C*c*d*2i))/(2*f*(a*b^2*3i - 3*a^
2*b - a^3*1i + b^3))
```

3.63. $\int \frac{(c+d\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.64 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3 (A+B\tan(e+$

3.64.1 Optimal result

Integrand size = 45, antiderivative size = 603

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \left(a^2\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right) + b^2\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right) - 2ab\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)x$$
$$+ \frac{(2ab(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))-a^2((A-C)d(3c^2-d^2)+B(c^3-3cd^2))+b^2((A}{f}$$
$$- \frac{d(2ab(c^2C+2Bcd-Cd^2-A(c^2-d^2))-a^2(2c(A-C)d+B(c^2-d^2))+b^2(2c(A-C)d+B(c^2-d^2}{f}$$
$$+ \frac{(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\,(c+d\tan(e+fx))^2}{2f}$$
$$+ \frac{(a^2B-b^2B+2ab(A-C))\,(c+d\tan(e+fx))^3}{3f}$$
$$+ \frac{(5a^2Cd^2-6abd(cC-5Bd)+b^2(c^2C-3Bcd+15(A-C)d^2))\,(c+d\tan(e+fx))^4}{60d^3f}$$
$$- \frac{b(bcC-3bBd-aCd)\tan(e+fx)(c+d\tan(e+fx))^4}{15d^2f}$$
$$+ \frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}$$

output
```
(a^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+b^2*(c^3*C+3*B*c^2*
d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d
^2)))*x+(2*a*b*(c^3*C+3*B*c^2*d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))-a^2*((A-C
)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))
)*ln(cos(f*x+e))/f-d*(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-a^2*(2*c*(A-
C)*d+B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*tan(f*x+e)/f+1/2*(2*a*b*(
A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*(c+d*tan(f*x+e))^2/f+1/3
*(B*a^2-B*b^2+2*a*b*(A-C))*(c+d*tan(f*x+e))^3/f+1/60*(5*a^2*C*d^2-6*a*b*d*
(-5*B*d+C*c)+b^2*(c^2*C-3*B*c*d+15*(A-C)*d^2))*(c+d*tan(f*x+e))^4/d^3/f-1/
15*b*(-3*B*b*d-C*a*d+C*b*c)*tan(f*x+e)*(c+d*tan(f*x+e))^4/d^2/f+1/6*C*(a+b
*tan(f*x+e))^2*(c+d*tan(f*x+e))^4/d/f
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.66 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.69

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}$$
$$+\frac{-\frac{2b(bcC-3bBd-aCd)\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{-\frac{\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\left(c^2C-3Bcd+15(A-C)d^2\right)\right)(c+d\tan(e+fx))^4}{2df}+\frac{5\left(3d\left(2ab(\right.\right.}{}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f) + ((-2*b*(b*c*C
- 3*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (-1/2*((
5*a^2*C*d^2 - 6*a*b*d*(c*C - 5*B*d) + b^2*(c^2*C - 3*B*c*d + 15*(A - C)*d^
2))*(c + d*Tan[e + f*x])^4)/(d*f) + (5*(3*d*(2*a*b*(A*c - c*C + B*d) + a^2
*(B*c - (A - C)*d) - b^2*(B*c - (A - C)*d))*((I*c - d)^3*Log[I - Tan[e + f
*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[
e + f*x]^2) + (a^2*B - b^2*B + 2*a*b*(A - C))*d*((3*I)*(c + I*d)^4*Log[I -
 Tan[e + f*x]] - (3*I)*(c - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(6*c^2 -
d^2)*Tan[e + f*x] - 12*c*d^3*Tan[e + f*x]^2 - 2*d^4*Tan[e + f*x]^3)))/f)/(
5*d))/(6*d)
```

3.64.3 Rubi [A] (verified)

Time = 2.85 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.356, Rules used = {3042, 4130, 27, 3042, 4120, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -2(a+b\tan(e+fx))(c+d\tan(e+fx))^3\left((bcC-adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx\right.}{6d}$$
$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left((bcC-adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx)+\right.}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left((bcC-adC-3bBd)\tan(e+fx)^2-3(Ab-Cb+aB)d\tan(e+fx)+\right.}{3d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-$$
$$\frac{\frac{b\tan(e+fx)(-aCd-3bBd+bcC)(c+d\tan(e+fx))^4}{5df}-\int -(c+d\tan(e+fx))^3\left(-c(cC-3Bd)b^2+6acCdb-5a^2(3A-2C)d^2-\left((Cc^2-3Bdc+15(A-\right.\right.}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^3\left(-c(cC-3Bd)b^2+6acCdb-5a^2(3A-2C)d^2-\left(\left(Cc^2-3Bdc+15(A-C)d^2\right)b^2-6ad(cC-5Bd)b+5a^2Cd^2\right)\tan^2(e+fx)-15\left(Ba^2\right.\right.}{5d}}{3d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^3\left(-c(cC-3Bd)b^2+6acCdb-5a^2(3A-2C)d^2-\left(\left(Cc^2-3Bdc+15(A-C)d^2\right)b^2-6ad(cC-5Bd)b+5a^2Cd^2\right)\tan(e+fx)^2-15\left(Ba^2\right.\right.}{5d}}{3d}$$

$\downarrow$ 4113

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^3\left(15\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5B\right.}{4df}}{5d}}{3d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^3\left(15\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5B\right.}{4df}}{5d}}{3d}$$

$\downarrow$ 4011

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^2\left(-15\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-15\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d\right.\right.}{5d}}{}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\frac{\int(c+d\tan(e+fx))^2\left(-15\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-15\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d\right.\right.}{5d}}{}$$

$\downarrow$ 4011

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df}-\frac{\int(c+d\tan(e+fx))\left(15\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+15\left(-\left((2c(A-C)a\right.\right.\right.}{}$$

3.64. $\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

$$\Big\downarrow \; 3042$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df} -$$

$$\frac{\frac{\int (c+d\tan(e+fx))\left(15\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+15\left(-\left(\left(2c(A-C)c\right.\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 4008$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df} -$$

$$\frac{\frac{15d^2\left(-\left(a^2\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)+2ab\left(-A\left(c^3-3cd^2\right)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+b^2\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)\int \tan(e+}{}}{}$$

$$\Big\downarrow \; 3042$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df} -$$

$$\frac{\frac{15d^2\left(-\left(a^2\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)+2ab\left(-A\left(c^3-3cd^2\right)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+b^2\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)\int \tan(e+}{}}{}$$

$$\Big\downarrow \; 3956$$

$$\frac{C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^4}{6df} -$$

$$\frac{-\frac{(c+d\tan(e+fx))^4\left(5a^2Cd^2-6abd(cC-5Bd)+b^2\left(15d^2(A-C)-3Bcd+c^2C\right)\right)}{4df}+\frac{15d^3\tan(e+fx)\left(-\left(a^2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\right)+2ab\left(-A\left(c^2-d^2\right)+2Bcd+c^2C\right.\right.}{f}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

3.64. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
(C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^4)/(6*d*f) - ((b*(b*c*C - 3
*b*B*d - a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) + (-15*d^2*(a
^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + b^2*(c^3*
C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*
(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x - (15*d^2*(2*a*b*(c^3*C + 3*B*c^2*d
- 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) - a^2*((A - C)*d*(3*c^2 - d^2) +
B*(c^3 - 3*c*d^2)) + b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Lo
g[Cos[e + f*x]])/f + (15*d^3*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^
2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 -
d^2)))*Tan[e + f*x])/f - (15*d^2*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A
- C)*d) - b^2*(B*c + (A - C)*d))*(c + d*Tan[e + f*x])^2)/(2*f) - (5*(a^2*B
 - b^2*B + 2*a*b*(A - C))*d^2*(c + d*Tan[e + f*x])^3)/f - ((5*a^2*C*d^2 -
6*a*b*d*(c*C - 5*B*d) + b^2*(c^2*C - 3*B*c*d + 15*(A - C)*d^2))*(c + d*Tan
[e + f*x])^4)/(4*d*f))/(5*d))/(3*d)
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.64. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.64.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.91

method	result
parts	$\frac{(3A\,a^2c^2d+2Aab\,c^3+B\,a^2c^3)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(B\,b^2d^3+2Cab\,d^3+3C\,b^2c\,d^2)\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx$
norman	$(A\,a^2c^3-3A\,a^2c\,d^2-6Aab\,c^2d+2Aab\,d^3-A\,b^2c^3+3A\,b^2c\,d^2-3B\,a^2c^2d+B\,a^2d^3-$
derivativedivides	Expression too large to display
default	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x,method=_RETURNVERBOSE)
```

output
```
1/2*(3*A*a^2*c^2*d+2*A*a*b*c^3+B*a^2*c^3)/f*ln(1+tan(f*x+e)^2)+(B*b^2*d^3+
2*C*a*b*d^3+3*C*b^2*c*d^2)/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)
-arctan(tan(f*x+e)))+(A*b^2*d^3+2*B*a*b*d^3+3*B*b^2*c*d^2+C*a^2*d^3+6*C*a*
b*c*d^2+3*C*b^2*c^2*d)/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f
*x+e)^2))+(3*A*a^2*c*d^2+6*A*a*b*c^2*d+A*b^2*c^3+3*B*a^2*c^2*d+2*B*a*b*c^3
+C*a^2*c^3)/f*(tan(f*x+e)-arctan(tan(f*x+e)))+(2*A*a*b*d^3+3*A*b^2*c*d^2+B
*a^2*d^3+6*B*a*b*c*d^2+3*B*b^2*c^2*d+3*C*a^2*c*d^2+6*C*a*b*c^2*d+C*b^2*c^3
)/f*(1/3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(A*a^2*d^3+6*A*a*b*c*
d^2+3*A*b^2*c^2*d+3*B*a^2*c*d^2+6*B*a*b*c^2*d+B*b^2*c^3+3*C*a^2*c^2*d+2*C*
a*b*c^3)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+A*a^2*c^3*x+C*b^2*d^3
/f*(1/6*tan(f*x+e)^6-1/4*tan(f*x+e)^4+1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)
^2))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.13

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$=\frac{10\,Cb^2d^3\tan\left(fx+e\right)^6+12\left(3\,Cb^2cd^2+\left(2\,Cab+Bb^2\right)d^3\right)\tan\left(fx+e\right)^5+15\left(3\,Cb^2c^2d+3\left(2\,Cab+Bb^2\right.\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*
d^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a
^2 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a
*b + B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(
A - C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 60*(((A - C)*a^2 - 2*B*a*b - (A
- C)*b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d - 3*((A - C)*a^2 -
 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)*a*b - B*b^2)*d^3)*f*x +
 30*((2*C*a*b + B*b^2)*c^3 + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(
B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^
2)*d^3)*tan(f*x + e)^2 - 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A -
 C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)
*c*d^2 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(1/(tan(f*x + e)^2
+ 1)) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a*b
 - B*b^2)*c^2*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2 +
 2*(A - C)*a*b - B*b^2)*d^3)*tan(f*x + e))/f
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(547) = 1094$.

Time = 0.39 (sec) , antiderivative size = 1819, normalized size of antiderivative = 3.02

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Piecewise((A*a**2*c**3*x + 3*A*a**2*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f)
- 3*A*a**2*c*d**2*x + 3*A*a**2*c*d**2*tan(e + f*x)/f - A*a**2*d**3*log(tan
(e + f*x)**2 + 1)/(2*f) + A*a**2*d**3*tan(e + f*x)**2/(2*f) + A*a*b*c**3*l
og(tan(e + f*x)**2 + 1)/f - 6*A*a*b*c**2*d*x + 6*A*a*b*c**2*d*tan(e + f*x)
/f - 3*A*a*b*c*d**2*log(tan(e + f*x)**2 + 1)/f + 3*A*a*b*c*d**2*tan(e + f*
x)**2/f + 2*A*a*b*d**3*x + 2*A*a*b*d**3*tan(e + f*x)**3/(3*f) - 2*A*a*b*d*
*3*tan(e + f*x)/f - A*b**2*c**3*x + A*b**2*c**3*tan(e + f*x)/f - 3*A*b**2*
c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*A*b**2*c**2*d*tan(e + f*x)**2/(2
*f) + 3*A*b**2*c*d**2*x + A*b**2*c*d**2*tan(e + f*x)**3/f - 3*A*b**2*c*d**
2*tan(e + f*x)/f + A*b**2*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + A*b**2*d**
3*tan(e + f*x)**4/(4*f) - A*b**2*d**3*tan(e + f*x)**2/(2*f) + B*a**2*c**3*
log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*a**2*c**2*d*x + 3*B*a**2*c**2*d*tan(e
 + f*x)/f - 3*B*a**2*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a**2*c*d*
*2*tan(e + f*x)**2/(2*f) + B*a**2*d**3*x + B*a**2*d**3*tan(e + f*x)**3/(3*
f) - B*a**2*d**3*tan(e + f*x)/f - 2*B*a*b*c**3*x + 2*B*a*b*c**3*tan(e + f*
x)/f - 3*B*a*b*c**2*d*log(tan(e + f*x)**2 + 1)/f + 3*B*a*b*c**2*d*tan(e +
f*x)**2/f + 6*B*a*b*c*d**2*x + 2*B*a*b*c*d**2*tan(e + f*x)**3/f - 6*B*a*b*
c*d**2*tan(e + f*x)/f + B*a*b*d**3*log(tan(e + f*x)**2 + 1)/f + B*a*b*d**3
*tan(e + f*x)**4/(2*f) - B*a*b*d**3*tan(e + f*x)**2/f - B*b**2*c**3*log(ta
n(e + f*x)**2 + 1)/(2*f) + B*b**2*c**3*tan(e + f*x)**2/(2*f) + 3*B*b**2...
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.13

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{10\,Cb^2d^3\tan\left(fx+e\right)^6+12\left(3\,Cb^2cd^2+(2\,Cab+Bb^2)d^3\right)\tan\left(fx+e\right)^5+15\left(3\,Cb^2c^2d+3\left(2\,Cab+Bb^2\right)\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
1/60*(10*C*b^2*d^3*tan(f*x + e)^6 + 12*(3*C*b^2*c*d^2 + (2*C*a*b + B*b^2)*
d^3)*tan(f*x + e)^5 + 15*(3*C*b^2*c^2*d + 3*(2*C*a*b + B*b^2)*c*d^2 + (C*a
^2 + 2*B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e)^4 + 20*(C*b^2*c^3 + 3*(2*C*a
*b + B*b^2)*c^2*d + 3*(C*a^2 + 2*B*a*b + (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(
A - C)*a*b - B*b^2)*d^3)*tan(f*x + e)^3 + 30*((2*C*a*b + B*b^2)*c^3 + 3*(C
*a^2 + 2*B*a*b + (A - C)*b^2)*c^2*d + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*
d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*tan(f*x + e)^2 + 60*(((A
- C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*
c^2*d - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2 + 2*(A - C)
*a*b - B*b^2)*d^3)*(f*x + e) + 30*((B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3 + 3
*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 + 2*(A - C)*a*b -
B*b^2)*c*d^2 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)
^2 + 1) + 60*((C*a^2 + 2*B*a*b + (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a
*b - B*b^2)*c^2*d + 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2
 + 2*(A - C)*a*b - B*b^2)*d^3)*tan(f*x + e))/f
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21368 vs. $2(593) = 1186$.

Time = 23.09 (sec) , antiderivative size = 21368, normalized size of antiderivative = 35.44

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
1/60*(60*A*a^2*c^3*f*x*tan(f*x)^6*tan(e)^6 - 60*C*a^2*c^3*f*x*tan(f*x)^6*t
an(e)^6 - 120*B*a*b*c^3*f*x*tan(f*x)^6*tan(e)^6 - 60*A*b^2*c^3*f*x*tan(f*x
)^6*tan(e)^6 + 60*C*b^2*c^3*f*x*tan(f*x)^6*tan(e)^6 - 180*B*a^2*c^2*d*f*x*
tan(f*x)^6*tan(e)^6 - 360*A*a*b*c^2*d*f*x*tan(f*x)^6*tan(e)^6 + 360*C*a*b*
c^2*d*f*x*tan(f*x)^6*tan(e)^6 + 180*B*b^2*c^2*d*f*x*tan(f*x)^6*tan(e)^6 -
180*A*a^2*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*C*a^2*c*d^2*f*x*tan(f*x)^6*t
an(e)^6 + 360*B*a*b*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 180*A*b^2*c*d^2*f*x*ta
n(f*x)^6*tan(e)^6 - 180*C*b^2*c*d^2*f*x*tan(f*x)^6*tan(e)^6 + 60*B*a^2*d^3
*f*x*tan(f*x)^6*tan(e)^6 + 120*A*a*b*d^3*f*x*tan(f*x)^6*tan(e)^6 - 120*C*a
*b*d^3*f*x*tan(f*x)^6*tan(e)^6 - 60*B*b^2*d^3*f*x*tan(f*x)^6*tan(e)^6 - 30
*B*a^2*c^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 - 60*A*a*b*c^3
*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 60*C*a*b*c^3*log(4*(ta
n(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 + tan(e)^2 + 1))*tan(f*x)^6*tan(e)^6 + 30*B*b^2*c^3*log(4*(tan(f*x)^2*t
an(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e
)^2 + 1))*tan(f*x)^6*tan(e)^6 - 90*A*a^2*c^2*d*log(4*(tan(f*x)^2*tan(e)^2
- 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1)
)*tan(f*x)^6*tan(e)^6 + 90*C*a^2*c^2*d*log(4*(tan(f*x)^2*tan(e)^2 - 2*t...
```

3.64. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.64.9 Mupad [B] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.48

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= x\left(A\,a^2\,c^3-A\,b^2\,c^3+B\,a^2\,d^3-C\,a^2\,c^3-B\,b^2\,d^3+C\,b^2\,c^3+2\,A\,a\,b\,d^3-2\,B\,a\,b\,c^3-2\,C\,a\,b\,d^3-3\,A\,a^2\,c\,d^2+3\,A\,b^2\,c\,d^2-3\,B\,a^2\,c^2\,d+3\,B\,b^2\,c^2\,d+3\,C\,a^2\,c\,d^2-3\,C\,b^2\,c\,d^2-6\,A\,a\,b\,c^2\,d+6\,B\,a\,b\,c\,d^2+6\,C\,a\,b\,c^2\,d\right)$$
$$-\frac{\tan(e+f\,x)\,\left(B\,a^2\,d^3-A\,b^2\,c^3-b\,d^2\,(B\,b\,d+2\,C\,a\,d+3\,C\,b\,c)-C\,a^2\,c^3+C\,b^2\,c^3+2\,A\,a\,b\,d^3-2\,B\,a\cdots\right.}{\cdots}$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,a^2\,d^3}{2}-\frac{B\,a^2\,c^3}{2}-\frac{A\,b^2\,d^3}{2}+\frac{B\,b^2\,c^3}{2}-\frac{C\,a^2\,d^3}{2}+\frac{C\,b^2\,d^3}{2}-A\,a\,b\,c^3-B\,a\,b\,d^3+C\,a\,b\,c\cdots\right.}{\cdots}$$
$$+\frac{\tan(e+f\,x)^4\,\left(\frac{A\,b^2\,d^3}{4}+\frac{C\,a^2\,d^3}{4}-\frac{C\,b^2\,d^3}{4}+\frac{B\,a\,b\,d^3}{2}+\frac{3\,B\,b^2\,c\,d^2}{4}+\frac{3\,C\,b^2\,c^2\,d}{4}+\frac{3\,C\,a\,b\,c\,d^2}{2}\right)}{f}$$
$$+\frac{\tan(e+f\,x)^3\,\left(\frac{B\,a^2\,d^3}{3}-\frac{b\,d^2\,(B\,b\,d+2\,C\,a\,d+3\,C\,b\,c)}{3}+\frac{C\,b^2\,c^3}{3}+\frac{2\,A\,a\,b\,d^3}{3}+A\,b^2\,c\,d^2+B\,b^2\,c^2\,d+C\,a^2\,c\,d^2+2\,B\cdots\right.}{f}$$
$$+\frac{\tan(e+f\,x)^2\,\left(\frac{A\,a^2\,d^3}{2}-\frac{A\,b^2\,d^3}{2}+\frac{B\,b^2\,c^3}{2}-\frac{C\,a^2\,d^3}{2}+\frac{C\,b^2\,d^3}{2}-B\,a\,b\,d^3+C\,a\,b\,c^3+\frac{3\,A\,b^2\,c^2\,d}{2}+\frac{3\,B\,a^2\,c\,d^2}{2}-\frac{3\,B\cdots}{}\right.}{f}$$
$$+\frac{b\,d^2\tan(e+f\,x)^5\,(B\,b\,d+2\,C\,a\,d+3\,C\,b\,c)}{5\,f}+\frac{C\,b^2\,d^3\tan(e+f\,x)^6}{6\,f}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
x*(A*a^2*c^3 - A*b^2*c^3 + B*a^2*d^3 - C*a^2*c^3 - B*b^2*d^3 + C*b^2*c^3 +
 2*A*a*b*d^3 - 2*B*a*b*c^3 - 2*C*a*b*d^3 - 3*A*a^2*c*d^2 + 3*A*b^2*c*d^2 -
 3*B*a^2*c^2*d + 3*B*b^2*c^2*d + 3*C*a^2*c*d^2 - 3*C*b^2*c*d^2 - 6*A*a*b*c
^2*d + 6*B*a*b*c*d^2 + 6*C*a*b*c^2*d) - (tan(e + f*x)*(B*a^2*d^3 - A*b^2*c
^3 - b*d^2*(B*b*d + 2*C*a*d + 3*C*b*c) - C*a^2*c^3 + C*b^2*c^3 + 2*A*a*b*d
^3 - 2*B*a*b*c^3 - 3*A*a^2*c*d^2 + 3*A*b^2*c*d^2 - 3*B*a^2*c^2*d + 3*B*b^2
*c^2*d + 3*C*a^2*c*d^2 - 6*A*a*b*c^2*d + 6*B*a*b*c*d^2 + 6*C*a*b*c^2*d))/f
 - (log(tan(e + f*x)^2 + 1)*((A*a^2*d^3)/2 - (B*a^2*c^3)/2 - (A*b^2*d^3)/2
 + (B*b^2*c^3)/2 - (C*a^2*d^3)/2 + (C*b^2*d^3)/2 - A*a*b*c^3 - B*a*b*d^3 +
 C*a*b*c^3 - (3*A*a^2*c^2*d)/2 + (3*A*b^2*c^2*d)/2 + (3*B*a^2*c*d^2)/2 - (
3*B*b^2*c*d^2)/2 + (3*C*a^2*c^2*d)/2 - (3*C*b^2*c^2*d)/2 + 3*A*a*b*c*d^2 +
 3*B*a*b*c^2*d - 3*C*a*b*c*d^2))/f + (tan(e + f*x)^4*((A*b^2*d^3)/4 + (C*a
^2*d^3)/4 - (C*b^2*d^3)/4 + (B*a*b*d^3)/2 + (3*B*b^2*c*d^2)/4 + (3*C*b^2*c
^2*d)/4 + (3*C*a*b*c*d^2)/2))/f + (tan(e + f*x)^3*((B*a^2*d^3)/3 - (b*d^2*
(B*b*d + 2*C*a*d + 3*C*b*c))/3 + (C*b^2*c^3)/3 + (2*A*a*b*d^3)/3 + A*b^2*c
*d^2 + B*b^2*c^2*d + C*a^2*c*d^2 + 2*B*a*b*c*d^2 + 2*C*a*b*c^2*d))/f + (ta
n(e + f*x)^2*((A*a^2*d^3)/2 - (A*b^2*d^3)/2 + (B*b^2*c^3)/2 - (C*a^2*d^3)/
2 + (C*b^2*d^3)/2 - B*a*b*d^3 + C*a*b*c^3 + (3*A*b^2*c^2*d)/2 + (3*B*a^2*c
*d^2)/2 - (3*B*b^2*c*d^2)/2 + (3*C*a^2*c^2*d)/2 - (3*C*b^2*c^2*d)/2 + 3*A*
a*b*c*d^2 + 3*B*a*b*c^2*d - 3*C*a*b*c*d^2))/f + (b*d^2*tan(e + f*x)^5*(...
```

3.64. $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.65 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\,(A+B\tan(e+$

3.65.1 Optimal result

Integrand size = 43, antiderivative size = 389

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\left(a\left(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3\right)-b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)x$$
$$-\frac{(A(bc^3+3ac^2d-3bcd^2-ad^3)-b(c^3C+3Bc^2d-3cCd^2-Bd^3)+a(Bc^3-3c^2Cd-3Bcd^2+Cd^3))}{f}$$
$$+\frac{d(a(Bc^2-2cCd-Bd^2)-b(c^2C+2Bcd-Cd^2)+A(2acd+b(c^2-d^2)))\tan(e+fx)}{f}$$
$$+\frac{(Abc+aBc-bcC+aAd-bBd-aCd)(c+d\tan(e+fx))^2}{2f}$$
$$+\frac{(Ab+aB-bC)(c+d\tan(e+fx))^3}{3f}$$
$$-\frac{(bcC-5bBd-5aCd)(c+d\tan(e+fx))^4}{20d^2f}+\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}$$

output
```
(a*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)-b*((A-C)*d*(3*c^2-d^2
)+B*(c^3-3*c*d^2)))*x-(A*(3*a*c^2*d-a*d^3+b*c^3-3*b*c*d^2)-b*(3*B*c^2*d-B*
d^3+C*c^3-3*C*c*d^2)+a*(B*c^3-3*B*c*d^2-3*C*c^2*d+C*d^3))*ln(cos(f*x+e))/f
+d*(a*(B*c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d+b*(c^2-d^2)
))*tan(f*x+e)/f+1/2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*(c+d*tan(f*x+e))
^2/f+1/3*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^3/f-1/20*(-5*B*b*d-5*C*a*d+C*b*c)*
(c+d*tan(f*x+e))^4/d^2/f+1/5*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^4/d/f
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.76

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}$$
$$-\frac{\frac{(bcC-5bBd-5aCd)(c+d\tan(e+fx))^4}{4df}+\frac{5\left(3(Abc+aBc-bcC-aAd+bBd+aCd)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+fx))+6\right.}{}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (((b*c*C - 5*b*B*d - 5
*a*C*d)*(c + d*Tan[e + f*x])^4)/(4*d*f) + (5*(3*(A*b*c + a*B*c - b*c*C - a
*A*d + b*B*d + a*C*d)*((I*c - d)^3*Log[I - Tan[e + f*x]] - (I*c + d)^3*Log
[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d^3*Tan[e + f*x]^2) + (A*b + a
*B - b*C)*((3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] - (3*I)*(c - I*d)^4*Log
[I + Tan[e + f*x]] - 6*d^2*(6*c^2 - d^2)*Tan[e + f*x] - 12*c*d^3*Tan[e + f
*x]^2 - 2*d^4*Tan[e + f*x]^3)))/(6*f))/(5*d)
```

3.65.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.279, Rules used = {3042, 4120, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$
$$\Big\downarrow \text{4120}$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^3\left((bcC-5adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)+bcC-5aAd\right)dx}{5d}$$

$\big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^3\left((bcC-5adC-5bBd)\tan(e+fx)^2-5(Ab-Cb+aB)d\tan(e+fx)+bcC-5aAd\right)dx}{5d}$$

$\big\downarrow$ 4113

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^3(5(bB-a(A-C))d-5(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-5aCd-5bBd+bcC)(c+d\tan(e+fx))}{4df}}{5d}$$

$\big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^3(5(bB-a(A-C))d-5(Ab-Cb+aB)d\tan(e+fx))dx+\frac{(-5aCd-5bBd+bcC)(c+d\tan(e+fx))}{4df}}{5d}$$

$\big\downarrow$ 4011

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^2(5d(bBc+b(A-C)d-a(Ac-Cc-Bd))-5d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{5d}$$

$\big\downarrow$ 3042

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))^2(5d(bBc+b(A-C)d-a(Ac-Cc-Bd))-5d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{5d}$$

$\big\downarrow$ 4011

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df}-\frac{\int(c+d\tan(e+fx))\left(5d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-5d\left(2aAcd-2a\right.\right.}{}$$

$\big\downarrow$ 3042

3.65. $\int(a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df} - \frac{\int (c+d\tan(e+fx))\left(5d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-5d\left(2aAcd-2a\right.\right.}{}$$

$$\Big\downarrow \; 4008$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df} - \frac{-5d\left(A\left(3ac^2d-ad^3+bc^3-3bcd^2\right)+a\left(Bc^3-3Bcd^2-3c^2Cd+Cd^3\right)-b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)\int \tan}{}$$

$$\Big\downarrow \; 3042$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df} - \frac{-5d\left(A\left(3ac^2d-ad^3+bc^3-3bcd^2\right)+a\left(Bc^3-3Bcd^2-3c^2Cd+Cd^3\right)-b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)\int \tan}{}$$

$$\Big\downarrow \; 3956$$

$$\frac{bC\tan(e+fx)(c+d\tan(e+fx))^4}{5df} - \frac{-\frac{5d^2\tan(e+fx)\left(2aAcd+aB\left(c^2-d^2\right)-2acCd+Ab\left(c^2-d^2\right)-b\left(2Bcd+c^2C-Cd^2\right)\right)}{f}+\frac{5d\log(\cos(e+fx))\left(A\left(3ac^2d-ad^3+bc^3-3bcd^2\right)+a\left(Bc^3-3\right.\right.}{f}}{}$$

input
```
Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Ta
n[e + f*x]^2),x]
```

output
```
(b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^4)/(5*d*f) - (5*d*(a*(c^3*C + 3*B*c
^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A - C)*d*(3*c^2 - d^2)
 + B*(c^3 - 3*c*d^2)))*x + (5*d*(A*(b*c^3 + 3*a*c^2*d - 3*b*c*d^2 - a*d^3)
 - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) + a*(B*c^3 - 3*c^2*C*d - 3*B*
c*d^2 + C*d^3))*Log[Cos[e + f*x]])/f - (5*d^2*(2*a*A*c*d - 2*a*c*C*d + A*b
*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*C + 2*B*c*d - C*d^2))*Tan[e + f*x]
)/f - (5*d*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan[e +
f*x])^2)/(2*f) - (5*(A*b + a*B - b*C)*d*(c + d*Tan[e + f*x])^3)/(3*f) + ((
b*c*C - 5*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^4)/(4*d*f))/(5*d)
```

3.65. $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.65.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.89

method	result
parts	$\frac{(3Aac^2d+Abc^3+Bac^3)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(Bbd^3+Cad^3+3Cbcd^2)\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right.}{2}\right.}{f}$
norman	$(Aac^3-3Aacd^2-3Abc^2d+Abd^3-3Bac^2d+Bad^3-Bbc^3+3Bbcd^2-Cac^3+3$
derivativedivides	$\frac{\frac{Cbc^3\tan(fx+e)^2}{2}-\frac{Cbd^3\tan(fx+e)^3}{3}+\frac{Aad^3\tan(fx+e)^2}{2}-\frac{Bbd^3\tan(fx+e)^2}{2}-\frac{Cad^3\tan(fx+e)^2}{2}+\frac{Cbd^3\tan(fx+e)^5}{5}+\frac{Bbd^3\tan}{}}{}$
default	$\frac{\frac{Cbc^3\tan(fx+e)^2}{2}-\frac{Cbd^3\tan(fx+e)^3}{3}+\frac{Aad^3\tan(fx+e)^2}{2}-\frac{Bbd^3\tan(fx+e)^2}{2}-\frac{Cad^3\tan(fx+e)^2}{2}+\frac{Cbd^3\tan(fx+e)^5}{5}+\frac{Bbd^3\tan}{}}{}$
parallelrisch	$\frac{30Cbc^3\tan(fx+e)^2-20Cbd^3\tan(fx+e)^3+30Aad^3\tan(fx+e)^2-30Bbd^3\tan(fx+e)^2-30Cad^3\tan(fx+e)^2+12Cbd^3}{}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,
method=_RETURNVERBOSE)
```

output
```
1/2*(3*A*a*c^2*d+A*b*c^3+B*a*c^3)/f*ln(1+tan(f*x+e)^2)+(B*b*d^3+C*a*d^3+3*
C*b*c*d^2)/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2+1/2*ln(1+tan(f*x+e)^2))+(A
*b*d^3+B*a*d^3+3*B*b*c*d^2+3*C*a*c*d^2+3*C*b*c^2*d)/f*(1/3*tan(f*x+e)^3-ta
n(f*x+e)+arctan(tan(f*x+e)))+(3*A*a*c*d^2+3*A*b*c^2*d+3*B*a*c^2*d+B*b*c^3+
C*a*c^3)/f*(tan(f*x+e)-arctan(tan(f*x+e)))+(A*a*d^3+3*A*b*c*d^2+3*B*a*c*d^
2+3*B*b*c^2*d+3*C*a*c^2*d+C*b*c^3)/f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)
^2))+A*a*c^3*x+C*b*d^3/f*(1/5*tan(f*x+e)^5-1/3*tan(f*x+e)^3+tan(f*x+e)-arc
tan(tan(f*x+e)))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.99

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$=\frac{12\,Cbd^3\tan(fx+e)^5+15\,(3\,Cbcd^2+(Ca+Bb)d^3)\tan(fx+e)^4+20\,(3\,Cbc^2d+3\,(Ca+Bb)cd^2+(B}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="fricas")
```

output
```
1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f
*x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)
*tan(f*x + e)^3 + 60*(((A - C)*a - B*b)*c^3 - 3*(B*a + (A - C)*b)*c^2*d -
3*((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x + 30*(C*b*c^3 + 3*(
C*a + B*b)*c^2*d + 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*tan(
f*x + e)^2 - 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d - 3*(B*
a + (A - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1))
+ 60*((C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*c*
d^2 - (B*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

3.65.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(379) = 758$.

Time = 0.28 (sec) , antiderivative size = 1001, normalized size of antiderivative = 2.57

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e
)**2),x)
```

output
```
Piecewise((A*a*c**3*x + 3*A*a*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*
a*c*d**2*x + 3*A*a*c*d**2*tan(e + f*x)/f - A*a*d**3*log(tan(e + f*x)**2 +
1)/(2*f) + A*a*d**3*tan(e + f*x)**2/(2*f) + A*b*c**3*log(tan(e + f*x)**2 +
 1)/(2*f) - 3*A*b*c**2*d*x + 3*A*b*c**2*d*tan(e + f*x)/f - 3*A*b*c*d**2*lo
g(tan(e + f*x)**2 + 1)/(2*f) + 3*A*b*c*d**2*tan(e + f*x)**2/(2*f) + A*b*d*
*3*x + A*b*d**3*tan(e + f*x)**3/(3*f) - A*b*d**3*tan(e + f*x)/f + B*a*c**3
*log(tan(e + f*x)**2 + 1)/(2*f) - 3*B*a*c**2*d*x + 3*B*a*c**2*d*tan(e + f*
x)/f - 3*B*a*c*d**2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*c*d**2*tan(e +
f*x)**2/(2*f) + B*a*d**3*x + B*a*d**3*tan(e + f*x)**3/(3*f) - B*a*d**3*tan
(e + f*x)/f - B*b*c**3*x + B*b*c**3*tan(e + f*x)/f - 3*B*b*c**2*d*log(tan(
e + f*x)**2 + 1)/(2*f) + 3*B*b*c**2*d*tan(e + f*x)**2/(2*f) + 3*B*b*c*d**2
*x + B*b*c*d**2*tan(e + f*x)**3/f - 3*B*b*c*d**2*tan(e + f*x)/f + B*b*d**3
*log(tan(e + f*x)**2 + 1)/(2*f) + B*b*d**3*tan(e + f*x)**4/(4*f) - B*b*d**
3*tan(e + f*x)**2/(2*f) - C*a*c**3*x + C*a*c**3*tan(e + f*x)/f - 3*C*a*c**
2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a*c**2*d*tan(e + f*x)**2/(2*f) +
3*C*a*c*d**2*x + C*a*c*d**2*tan(e + f*x)**3/f - 3*C*a*c*d**2*tan(e + f*x)/
f + C*a*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*a*d**3*tan(e + f*x)**4/(4*
f) - C*a*d**3*tan(e + f*x)**2/(2*f) - C*b*c**3*log(tan(e + f*x)**2 + 1)/(2
*f) + C*b*c**3*tan(e + f*x)**2/(2*f) + 3*C*b*c**2*d*x + C*b*c**2*d*tan(e +
 f*x)**3/f - 3*C*b*c**2*d*tan(e + f*x)/f + 3*C*b*c*d**2*log(tan(e + f*x...
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.99

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{12\,Cbd^3\tan(fx+e)^5+15\,(3\,Cbcd^2+(Ca+Bb)d^3)\tan(fx+e)^4+20\,(3\,Cbc^2d+3\,(Ca+Bb)cd^2+(B\dots}{}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="maxima")
```

output
```
1/60*(12*C*b*d^3*tan(f*x + e)^5 + 15*(3*C*b*c*d^2 + (C*a + B*b)*d^3)*tan(f
*x + e)^4 + 20*(3*C*b*c^2*d + 3*(C*a + B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)
*tan(f*x + e)^3 + 30*(C*b*c^3 + 3*(C*a + B*b)*c^2*d + 3*(B*a + (A - C)*b)*
c*d^2 + ((A - C)*a - B*b)*d^3)*tan(f*x + e)^2 + 60*(((A - C)*a - B*b)*c^3
- 3*(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b
)*d^3)*(f*x + e) + 30*((B*a + (A - C)*b)*c^3 + 3*((A - C)*a - B*b)*c^2*d -
 3*(B*a + (A - C)*b)*c*d^2 - ((A - C)*a - B*b)*d^3)*log(tan(f*x + e)^2 + 1
) + 60*((C*a + B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d + 3*((A - C)*a - B*b)*
c*d^2 - (B*a + (A - C)*b)*d^3)*tan(f*x + e))/f
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10353 vs. $2(381) = 762$.

Time = 9.71 (sec) , antiderivative size = 10353, normalized size of antiderivative = 26.61

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="giac")
```

output
```
1/60*(60*A*a*c^3*f*x*tan(f*x)^5*tan(e)^5 - 60*C*a*c^3*f*x*tan(f*x)^5*tan(e
)^5 - 60*B*b*c^3*f*x*tan(f*x)^5*tan(e)^5 - 180*B*a*c^2*d*f*x*tan(f*x)^5*ta
n(e)^5 - 180*A*b*c^2*d*f*x*tan(f*x)^5*tan(e)^5 + 180*C*b*c^2*d*f*x*tan(f*x
)^5*tan(e)^5 - 180*A*a*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 180*C*a*c*d^2*f*x*t
an(f*x)^5*tan(e)^5 + 180*B*b*c*d^2*f*x*tan(f*x)^5*tan(e)^5 + 60*B*a*d^3*f*
x*tan(f*x)^5*tan(e)^5 + 60*A*b*d^3*f*x*tan(f*x)^5*tan(e)^5 - 60*C*b*d^3*f*
x*tan(f*x)^5*tan(e)^5 - 30*B*a*c^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)
*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5
*tan(e)^5 - 30*A*b*c^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)
/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 +
30*C*b*c^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2
*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 - 90*A*a*c^2*d
*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2
+ tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*C*a*c^2*d*log(4*(ta
n(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)
^2 + tan(e)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*b*c^2*d*log(4*(tan(f*x)^2*t
an(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e
)^2 + 1))*tan(f*x)^5*tan(e)^5 + 90*B*a*c*d^2*log(4*(tan(f*x)^2*tan(e)^2 -
2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*
tan(f*x)^5*tan(e)^5 + 90*A*b*c*d^2*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f...
```

3.65. $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.65.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.23

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= x\left(A\,a\,c^3+A\,b\,d^3+B\,a\,d^3-B\,b\,c^3-C\,a\,c^3-C\,b\,d^3-3\,A\,a\,c\,d^2-3\,A\,b\,c^2\,d-3\,B\,a\,c^2\,d+3\,B\,b\,c\,d^2+3\,C\,a\,c\,d^2+3\,C\,b\,c^2\,d\right)+\frac{\tan(e+f\,x)^4\left(\frac{B\,b\,d^3}{4}+\frac{C\,a\,d^3}{4}+\frac{3\,C\,b\,c\,d^2}{4}\right)}{f}$$
$$+\frac{\tan(e+f\,x)^3\left(\frac{A\,b\,d^3}{3}+\frac{B\,a\,d^3}{3}-\frac{C\,b\,d^3}{3}+B\,b\,c\,d^2+C\,a\,c\,d^2+C\,b\,c^2\,d\right)}{f}$$
$$+\frac{\tan(e+f\,x)^2\left(\frac{A\,a\,d^3}{2}-\frac{B\,b\,d^3}{2}-\frac{C\,a\,d^3}{2}+\frac{C\,b\,c^3}{2}+\frac{3\,A\,b\,c\,d^2}{2}+\frac{3\,B\,a\,c\,d^2}{2}+\frac{3\,B\,b\,c^2\,d}{2}+\frac{3\,C\,a\,c^2\,d}{2}-\frac{3\,C\,b\,c\,d^2}{2}\right)}{f}$$
$$-\frac{\ln\left(\tan(e+f\,x)^2+1\right)\left(\frac{A\,a\,d^3}{2}-\frac{A\,b\,c^3}{2}-\frac{B\,a\,c^3}{2}-\frac{B\,b\,d^3}{2}-\frac{C\,a\,d^3}{2}+\frac{C\,b\,c^3}{2}-\frac{3\,A\,a\,c^2\,d}{2}+\frac{3\,A\,b\,c\,d^2}{2}+\frac{3\,B\,a\,c\,d^2}{2}+\frac{3}{}\right.}{f}$$
$$+\frac{\tan(e+f\,x)\left(B\,b\,c^3-B\,a\,d^3-A\,b\,d^3+C\,a\,c^3+C\,b\,d^3+3\,A\,a\,c\,d^2+3\,A\,b\,c^2\,d+3\,B\,a\,c^2\,d-3\,B\,b\,c\,d^2\right.}{f}$$
$$+\frac{C\,b\,d^3\tan(e+f\,x)^5}{5\,f}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2),x)
```

output
```
x*(A*a*c^3 + A*b*d^3 + B*a*d^3 - B*b*c^3 - C*a*c^3 - C*b*d^3 - 3*A*a*c*d^2
 - 3*A*b*c^2*d - 3*B*a*c^2*d + 3*B*b*c*d^2 + 3*C*a*c*d^2 + 3*C*b*c^2*d) +
(tan(e + f*x)^4*((B*b*d^3)/4 + (C*a*d^3)/4 + (3*C*b*c*d^2)/4))/f + (tan(e
+ f*x)^3*((A*b*d^3)/3 + (B*a*d^3)/3 - (C*b*d^3)/3 + B*b*c*d^2 + C*a*c*d^2
+ C*b*c^2*d))/f + (tan(e + f*x)^2*((A*a*d^3)/2 - (B*b*d^3)/2 - (C*a*d^3)/2
 + (C*b*c^3)/2 + (3*A*b*c*d^2)/2 + (3*B*a*c*d^2)/2 + (3*B*b*c^2*d)/2 + (3*
C*a*c^2*d)/2 - (3*C*b*c*d^2)/2))/f - (log(tan(e + f*x)^2 + 1)*((A*a*d^3)/2
 - (A*b*c^3)/2 - (B*a*c^3)/2 - (B*b*d^3)/2 - (C*a*d^3)/2 + (C*b*c^3)/2 - (
3*A*a*c^2*d)/2 + (3*A*b*c*d^2)/2 + (3*B*a*c*d^2)/2 + (3*B*b*c^2*d)/2 + (3*
C*a*c^2*d)/2 - (3*C*b*c*d^2)/2))/f + (tan(e + f*x)*(B*b*c^3 - B*a*d^3 - A*
b*d^3 + C*a*c^3 + C*b*d^3 + 3*A*a*c*d^2 + 3*A*b*c^2*d + 3*B*a*c^2*d - 3*B*
b*c*d^2 - 3*C*a*c*d^2 - 3*C*b*c^2*d))/f + (C*b*d^3*tan(e + f*x)^5)/(5*f)
```

3.66 $\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+\right.$

3.66.1 Optimal result

Integrand size = 33, antiderivative size = 191

$$\begin{aligned}&\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx\\&=-\left(\left(c^3C+3Bc^2d-3cCd^2-Bd^3-A\left(c^3-3cd^2\right)\right)x\right)\\&\quad-\frac{((A-C)d(3c^2-d^2)+B(c^3-3cd^2))\log(\cos(e+fx))}{f}\\&\quad+\frac{d(2c(A-C)d+B(c^2-d^2))\tan(e+fx)}{f}+\frac{(Bc+(A-C)d)(c+d\tan(e+fx))^2}{2f}\\&\quad+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}\end{aligned}$$

output
```
-(c^3*C+3*B*c^2*d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))*x-((A-C)*d*(3*c^2-d^2)+
B*(c^3-3*c*d^2))*ln(cos(f*x+e))/f+d*(2*c*(A-C)*d+B*(c^2-d^2))*tan(f*x+e)/f
+1/2*(B*c+(A-C)*d)*(c+d*tan(f*x+e))^2/f+1/3*B*(c+d*tan(f*x+e))^3/f+1/4*C*(
c+d*tan(f*x+e))^4/d/f
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.11

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{3C(c+d\tan(e+fx))^4-6(Bc+(-A+C)d)\left((ic-d)^3\log(i-\tan(e+fx))-(ic+d)^3\log(i+\tan(e+\right.}{}$$

input
```
Integrate[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x
]
```

output
```
(3*C*(c + d*Tan[e + f*x])^4 - 6*(B*c + (-A + C)*d)*((I*c - d)^3*Log[I - Ta
n[e + f*x]] - (I*c + d)^3*Log[I + Tan[e + f*x]] + 6*c*d^2*Tan[e + f*x] + d
^3*Tan[e + f*x]^2) + 2*B*((-3*I)*(c + I*d)^4*Log[I - Tan[e + f*x]] + (3*I)
*(c - I*d)^4*Log[I + Tan[e + f*x]] - 6*d^2*(-6*c^2 + d^2)*Tan[e + f*x] + 1
2*c*d^3*Tan[e + f*x]^2 + 2*d^4*Tan[e + f*x]^3))/(12*d*f)
```

3.66.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4113}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^3dx+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{3042}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^3dx+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{4011}$$

$$\int (c+d\tan(e+fx))^2(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^2(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{4011}$$

$$\int (c+d\tan(e+fx))\left(-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B(c^2-d^2)\right)\tan(e+fx)\right)dx+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))\left(-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B(c^2-d^2)\right)\tan(e+fx)\right)dx+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{4008}$$

$$\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\int \tan(e+fx)dx+\frac{d\tan(e+fx)\left(2cd(A-C)+B\left(c^2-d^2\right)\right)}{f}-x\left(-A\left(c^3-3cd^2\right)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{3042}$$

$$\left(d(A-C)\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\int \tan(e+fx)dx+\frac{d\tan(e+fx)\left(2cd(A-C)+B\left(c^2-d^2\right)\right)}{f}-x\left(-A\left(c^3-3cd^2\right)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

$$\Big\downarrow \text{3956}$$

3.66. $\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

$$\frac{d\tan(e+fx)\left(2cd(A-C)+B(c^2-d^2)\right)}{f}-\frac{\left(d(A-C)\left(3c^2-d^2\right)+B(c^3-3cd^2)\right)\log(\cos(e+fx))}{f}-x\left(-A(c^3-3cd^2)+3Bc^2d-Bd^3+c^3C-3cCd^2\right)+\frac{(d(A-C)+Bc)(c+d\tan(e+fx))^2}{2f}+\frac{B(c+d\tan(e+fx))^3}{3f}+\frac{C(c+d\tan(e+fx))^4}{4df}$$

input
```
Int[(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
-((c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2))*x) - (((A -
C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2))*Log[Cos[e + f*x]])/f + (d*(2*c*(A
- C)*d + B*(c^2 - d^2))*Tan[e + f*x])/f + ((B*c + (A - C)*d)*(c + d*Tan[e
+ f*x])^2)/(2*f) + (B*(c + d*Tan[e + f*x])^3)/(3*f) + (C*(c + d*Tan[e + f*
x])^4)/(4*d*f)
```

3.66.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.66.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.10

method	result
parts	$A c^3 x+\frac{(3A c^2 d+B c^3)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(B d^3+3C c d^2)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+$
norman	$(A c^3-3Ac d^2-3B c^2 d+B d^3-c^3 C+3C c d^2)\,x+\frac{(3Ac d^2+3B c^2 d-B d^3+c^3 C-3C c d^2)\tan(f$
derivativedivides	$\frac{C d^3\tan(fx+e)^4}{4}+\frac{B d^3\tan(fx+e)^3}{3}+C c d^2\tan(fx+e)^3+\frac{A d^3\tan(fx+e)^2}{2}+\frac{3B c d^2\tan(fx+e)^2}{2}+\frac{3C c^2 d\tan(fx+e)^2}{2}-\frac{C d^3\tan}{}$
default	$\frac{C d^3\tan(fx+e)^4}{4}+\frac{B d^3\tan(fx+e)^3}{3}+C c d^2\tan(fx+e)^3+\frac{A d^3\tan(fx+e)^2}{2}+\frac{3B c d^2\tan(fx+e)^2}{2}+\frac{3C c^2 d\tan(fx+e)^2}{2}-\frac{C d^3\tan}{}$
parallelrisch	$\frac{3C d^3\tan(fx+e)^4+4B d^3\tan(fx+e)^3+6A d^3\tan(fx+e)^2-6C d^3\tan(fx+e)^2-12\tan(fx+e)B d^3+12\tan(fx+e)c^3C+}{}$
risch	$A c^3 x+B d^3 x-C c^3 x+\frac{2i\left(9Ac d^2-12Cc d^2+9B c^2 d-4B d^3+3c^3 C-36Cc d^2 e^{4i(fx+e)}+27Ac d^2 e^{2i(fx+e)}+\right.}{}$

input
```
int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETURNVER
BOSE)
```

output
```
A*c^3*x+1/2*(3*A*c^2*d+B*c^3)/f*ln(1+tan(f*x+e)^2)+(B*d^3+3*C*c*d^2)/f*(1/
3*tan(f*x+e)^3-tan(f*x+e)+arctan(tan(f*x+e)))+(A*d^3+3*B*c*d^2+3*C*c^2*d)/
f*(1/2*tan(f*x+e)^2-1/2*ln(1+tan(f*x+e)^2))+(3*A*c*d^2+3*B*c^2*d+C*c^3)/f*
(tan(f*x+e)-arctan(tan(f*x+e)))+C*d^3/f*(1/4*tan(f*x+e)^4-1/2*tan(f*x+e)^2
+1/2*ln(1+tan(f*x+e)^2))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.05

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{3\,Cd^3\tan\left(fx+e\right)^4+4\left(3\,Ccd^2+Bd^3\right)\tan\left(fx+e\right)^3+12\left((A-C)c^3-3\,Bc^2d-3\,(A-C)cd^2+Bd^3\right)f}{}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"fricas")
```

output
```
1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 12*(
(A - C)*c^3 - 3*B*c^2*d - 3*(A - C)*c*d^2 + B*d^3)*f*x + 6*(3*C*c^2*d + 3*
B*c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 - 6*(B*c^3 + 3*(A - C)*c^2*d - 3*B*c
*d^2 - (A - C)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 12*(C*c^3 + 3*B*c^2*d +
3*(A - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(163) = 326$.

Time = 0.17 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.15

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \begin{cases} Ac^3x+\frac{3Ac^2d\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}-3Acd^2x+\frac{3Acd^2\tan\left(e+fx\right)}{f}-\frac{Ad^3\log\left(\tan^2\left(e+fx\right)+1\right)}{2f}+\frac{Ad^3\tan^2\left(e+fx\right)}{2f}+\frac{Bc^3\log\left(\mathrm{t}\right.}{} \\ x(c+d\tan\left(e\right))^3\left(A+B\tan\left(e\right)+C\tan^2\left(e\right)\right) \end{cases}$$

input
```
integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Piecewise((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) - 3*A*c*d*
*2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e + f*x)**2 + 1)/(2*f) +
 A*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f*x)**2 + 1)/(2*f) - 3*
B*c**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*log(tan(e + f*x)**2 +
1)/(2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3*x + B*d**3*tan(e + f*
x)**3/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c**3*tan(e + f*x)/f - 3
*C*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*d*tan(e + f*x)**2/(2*f
) + 3*C*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*c*d**2*tan(e + f*x)/f
+ C*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan(e + f*x)**4/(4*f) - C
*d**3*tan(e + f*x)**2/(2*f), Ne(f, 0)), (x*(c + d*tan(e))**3*(A + B*tan(e)
 + C*tan(e)**2), True))
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.06

$$\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{3\,Cd^3\tan\left(fx+e\right)^4+4\left(3\,Ccd^2+Bd^3\right)\tan\left(fx+e\right)^3+6\left(3\,Cc^2d+3\,Bcd^2+(A-C)d^3\right)\tan\left(fx+e\right)^2+\cdots}{}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"maxima")
```

output
```
1/12*(3*C*d^3*tan(f*x + e)^4 + 4*(3*C*c*d^2 + B*d^3)*tan(f*x + e)^3 + 6*(3
*C*c^2*d + 3*B*c*d^2 + (A - C)*d^3)*tan(f*x + e)^2 + 12*((A - C)*c^3 - 3*B
*c^2*d - 3*(A - C)*c*d^2 + B*d^3)*(f*x + e) + 6*(B*c^3 + 3*(A - C)*c^2*d -
 3*B*c*d^2 - (A - C)*d^3)*log(tan(f*x + e)^2 + 1) + 12*(C*c^3 + 3*B*c^2*d
+ 3*(A - C)*c*d^2 - B*d^3)*tan(f*x + e))/f
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3720 vs. 2(185) = 370.

Time = 3.19 (sec) , antiderivative size = 3720, normalized size of antiderivative = 19.48

$$\int (c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"giac")
```

output
```
1/12*(12*A*c^3*f*x*tan(f*x)^4*tan(e)^4 - 12*C*c^3*f*x*tan(f*x)^4*tan(e)^4
- 36*B*c^2*d*f*x*tan(f*x)^4*tan(e)^4 - 36*A*c*d^2*f*x*tan(f*x)^4*tan(e)^4
+ 36*C*c*d^2*f*x*tan(f*x)^4*tan(e)^4 + 12*B*d^3*f*x*tan(f*x)^4*tan(e)^4 -
6*B*c^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*ta
n(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 18*A*c^2*d*log(
4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan
(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 18*C*c^2*d*log(4*(tan(f*x)^
2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + ta
n(e)^2 + 1))*tan(f*x)^4*tan(e)^4 + 18*B*c*d^2*log(4*(tan(f*x)^2*tan(e)^2 -
 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))
*tan(f*x)^4*tan(e)^4 + 6*A*d^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan
(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan
(e)^4 - 6*C*d^3*log(4*(tan(f*x)^2*tan(e)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f
*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)^2 + 1))*tan(f*x)^4*tan(e)^4 - 48*A*c^
3*f*x*tan(f*x)^3*tan(e)^3 + 48*C*c^3*f*x*tan(f*x)^3*tan(e)^3 + 144*B*c^2*d
*f*x*tan(f*x)^3*tan(e)^3 + 144*A*c*d^2*f*x*tan(f*x)^3*tan(e)^3 - 144*C*c*d
^2*f*x*tan(f*x)^3*tan(e)^3 - 48*B*d^3*f*x*tan(f*x)^3*tan(e)^3 + 18*C*c^2*d
*tan(f*x)^4*tan(e)^4 + 18*B*c*d^2*tan(f*x)^4*tan(e)^4 + 6*A*d^3*tan(f*x)^4
*tan(e)^4 - 9*C*d^3*tan(f*x)^4*tan(e)^4 + 24*B*c^3*log(4*(tan(f*x)^2*tan(e
)^2 - 2*tan(f*x)*tan(e) + 1)/(tan(f*x)^2*tan(e)^2 + tan(f*x)^2 + tan(e)...
```

3.66.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= x\left(A\,c^3+B\,d^3-C\,c^3-3\,A\,c\,d^2-3\,B\,c^2\,d+3\,C\,c\,d^2\right) + \frac{\tan(e+f\,x)\,\left(C\,c^3-B\,d^3+3\,A\,c\,d^2+3\,B\,c^2\,d-3\,C\,c\,d^2\right)}{f} + \frac{\tan(e+f\,x)^3\,\left(\frac{B\,d^3}{3}+C\,c\,d^2\right)}{f} - \frac{\ln\left(\tan(e+f\,x)^2+1\right)\,\left(\frac{A\,d^3}{2}-\frac{B\,c^3}{2}-\frac{C\,d^3}{2}-\frac{3\,A\,c^2\,d}{2}+\frac{3\,B\,c\,d^2}{2}+\frac{3\,C\,c^2\,d}{2}\right)}{f} + \frac{\tan(e+f\,x)^2\,\left(\frac{A\,d^3}{2}-\frac{C\,d^3}{2}+\frac{3\,B\,c\,d^2}{2}+\frac{3\,C\,c^2\,d}{2}\right)}{f} + \frac{C\,d^3\,\tan(e+f\,x)^4}{4\,f}$$

input
```
int((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
x*(A*c^3 + B*d^3 - C*c^3 - 3*A*c*d^2 - 3*B*c^2*d + 3*C*c*d^2) + (tan(e + f
*x)*(C*c^3 - B*d^3 + 3*A*c*d^2 + 3*B*c^2*d - 3*C*c*d^2))/f + (tan(e + f*x)
^3*((B*d^3)/3 + C*c*d^2))/f - (log(tan(e + f*x)^2 + 1)*((A*d^3)/2 - (B*c^3
)/2 - (C*d^3)/2 - (3*A*c^2*d)/2 + (3*B*c*d^2)/2 + (3*C*c^2*d)/2))/f + (tan
(e + f*x)^2*((A*d^3)/2 - (C*d^3)/2 + (3*B*c*d^2)/2 + (3*C*c^2*d)/2))/f + (
C*d^3*tan(e + f*x)^4)/(4*f)
```

3.66. $\int (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$

3.67 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.67.1 Optimal result

Integrand size = 45, antiderivative size = 363

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx =$$
$$-\frac{(a(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))-b((A-C)d(3c^2-d^2)+B(c^3-3cd^2)))\,x}{a^2+b^2}$$
$$-\frac{(b(c^3C+3Bc^2d-3cCd^2-Bd^3)+a(Bc^3-3c^2Cd-3Bcd^2+Cd^3)+A(ad(3c^2-d^2)-b(c^3-3cd^2))}{(a^2+b^2)\,f}$$
$$+\frac{(Ab^2-a(bB-aC))\,(bc-ad)^3\log(a+b\tan(e+fx))}{b^4\,(a^2+b^2)\,f}$$
$$+\frac{d(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aCd))\tan(e+fx)}{b^3f}$$
$$+\frac{(bcC+bBd-aCd)(c+d\tan(e+fx))^2}{2b^2f}+\frac{C(c+d\tan(e+fx))^3}{3bf}$$

output
```
-(a*(c^3*C+3*B*c^2*d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))-b*((A-C)*d*(3*c^2-d^
2)+B*(c^3-3*c*d^2)))*x/(a^2+b^2)-(b*(3*B*c^2*d-B*d^3+C*c^3-3*C*c*d^2)+a*(B
*c^3-3*B*c*d^2-3*C*c^2*d+C*d^3)+A*(a*d*(3*c^2-d^2)-b*(c^3-3*c*d^2)))*ln(co
s(f*x+e))/(a^2+b^2)/f+(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^3*ln(a+b*tan(f*x+e))/
b^4/(a^2+b^2)/f+d*(b^2*d*(B*c+(A-C)*d)+(-a*d+b*c)*(B*b*d-C*a*d+C*b*c))*tan
(f*x+e)/b^3/f+1/2*(B*b*d-C*a*d+C*b*c)*(c+d*tan(f*x+e))^2/b^2/f+1/3*C*(c+d*
tan(f*x+e))^3/b/f
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.70

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\frac{3b^2(-iA+B+iC)(c+id)^3\log(i-\tan(e+fx))}{a+ib} - \frac{3b^2(A-iB-C)(ic+d)^3\log(i+\tan(e+fx))}{a-ib} + \frac{6(Ab^2+a(-bB+aC))(bc-ad)^3\log(a+b\tan(e+f}{b^2(a^2+b^2)}}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x]),x]
```

output
```
((3*b^2*((-I)*A + B + I*C)*(c + I*d)^3*Log[I - Tan[e + f*x]])/(a + I*b) -
(3*b^2*(A - I*B - C)*(I*c + d)^3*Log[I + Tan[e + f*x]])/(a - I*b) + (6*(A*
b^2 + a*(-(b*B) + a*C))*(b*c - a*d)^3*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 +
 b^2)) + 6*b*d^2*(B*c + (A - C)*d)*Tan[e + f*x] + (6*d*(b*c - a*d)*(b*c*C
+ b*B*d - a*C*d)*Tan[e + f*x])/b + 3*(b*c*C + b*B*d - a*C*d)*(c + d*Tan[e
+ f*x])^2 + 2*b*C*(c + d*Tan[e + f*x])^3)/(6*b^2*f)
```

3.67.3 Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \; 4130$$

$$\frac{\int \frac{3(c+d\tan(e+fx))^2((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd)}{a+b\tan(e+fx)}dx}{3b}+\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+d\tan(e+fx))^2((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd)}{a+b\tan(e+fx)}dx}{b}+\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^2((bcC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd)}{a+b\tan(e+fx)}dx}{b}+\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 4130

$$\frac{\frac{\int \frac{2(c+d\tan(e+fx))\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))\right)}{a+b\tan(e+fx)}dx}{2b}+}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(c+d\tan(e+fx))\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))\right)}{a+b\tan(e+fx)}dx}{b}+(}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{(c+d\tan(e+fx))\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan(e+fx)^2+ad(aCd-b(2cC+Bd))\right)}{a+b\tan(e+fx)}dx}{b}+(}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\Big\downarrow$ 4120

3.67. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

$$\frac{\frac{d\tan(e+fx)\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}-\int -\frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2-\left(-\left(\left(Cc^3+3Bdc^2+3(A-C\cdots\right.\right.\right.}{}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2-\left(-\left(\left(Cc^3+3Bdc^2+3(A-C)d^2c-Bd^3\right)b^3\right)+ad\left(3Cc^2+3Bdc+(A-C)d^2\right)b^2-a^2d^2(3cC+Bd)\cdots\right.}{a+b\tan(e+fx)}}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2-\left(-\left(\left(Cc^3+3Bdc^2+3(A-C)d^2c-Bd^3\right)b^3\right)+ad\left(3Cc^2+3Bdc+(A-C)d^2\right)b^2-a^2d^2(3cC+Bd)\cdots\right.}{a+b\tan(e+fx)}}{b}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\downarrow$ 4109

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\tan^2(e+fx)+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{b^3\left(aAd\left(3c^2-d^2\right)-a\left(Cd\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-Ab\left(c^3-3cd^2\right)+b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)\int\tan(e+\cdots}{a^2+b^2}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\downarrow$ 3042

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}+\frac{b^3\left(aAd\left(3c^2-d^2\right)-a\left(Cd\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-Ab\left(c^3-3cd^2\right)+b\left(3Bc^2d-Bd^3+c^3C-3cCd^2\right)\right)\int\tan(e+\cdots}{a^2+b^2}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$\downarrow$ 3956

3.67. $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{a+b\tan(e+fx)}dx}{a^2+b^2}-\frac{b^3\log(\cos(e+fx))\left(aAd\left(3c^2-d^2\right)-a\left(Cd\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-Ab\left(c^3-3cd^2\right)+b\left(3Bc^2d-Bd^3+c^3C-3cC\right.\right.}{f\left(a^2+b^2\right)}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$$\Big\downarrow \; 4100$$

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\left(a^2+b^2\right)}-\frac{b^3\log(\cos(e+fx))\left(aAd\left(3c^2-d^2\right)-a\left(Cd\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-Ab\left(c^3-3cd^2\right)+b\left(3Bc^2d-Bd\right.\right.}{f\left(a^2+b^2\right)}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\log(a+b\tan(e+fx))}{bf\left(a^2+b^2\right)}-\frac{b^3\log(\cos(e+fx))\left(aAd\left(3c^2-d^2\right)-a\left(Cd\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-Ab\left(c^3-3cd^2\right)+b\left(3Bc^2d-Bd^3+c^3C-3cC\right.\right.}{f\left(a^2+b^2\right)}}{b}$$

$$\frac{C(c+d\tan(e+fx))^3}{3bf}$$

input
```
Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
b*Tan[e + f*x]),x]
```

output
```
(C*(c + d*Tan[e + f*x])^3)/(3*b*f) + (((b*c*C + b*B*d - a*C*d)*(c + d*Tan[
e + f*x])^2)/(2*b*f) + ((-((b^3*(a*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3
- A*(c^3 - 3*c*d^2)) - b*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)
/(a^2 + b^2)) - (b^3*(a*A*d*(3*c^2 - d^2) - A*b*(c^3 - 3*c*d^2) + b*(c^3*C
 + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) - a*(C*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^
2)))*Log[Cos[e + f*x]])/((a^2 + b^2)*f) + ((A*b^2 - a*(b*B - a*C))*(b*c -
a*d)^3*Log[a + b*Tan[e + f*x]])/(b*(a^2 + b^2)*f))/b + (d*(b^2*d*(B*c + (A
 - C)*d) + (b*c - a*d)*(b*c*C + b*B*d - a*C*d))*Tan[e + f*x])/(b*f))/b)/b
```

3.67. $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.67. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.67.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.38

method	result
norman	$\frac{(A a c^3-3Aac d^2+3Ab c^2d-Ab d^3-3Ba c^2d+Ba d^3+Bb c^3-3Bbc d^2-Ca c^3+3Cac d^2-3Cb c^2d+Cb d^3)x}{a^2+b^2}+\frac{(A b^2d^2-$
derivativedivides	$\frac{\frac{d\left(\frac{C b^2d^2\tan(fx+e)^3}{3}+\frac{B b^2d^2\tan(fx+e)^2}{2}-\frac{Cab d^2\tan(fx+e)^2}{2}+\frac{3C b^2cd\tan(fx+e)^2}{2}+\tan(fx+e)A b^2d^2-\tan(fx+e)Bab d^2+3\text{ta}\right.}{b^3}}{}$
default	$\frac{\frac{d\left(\frac{C b^2d^2\tan(fx+e)^3}{3}+\frac{B b^2d^2\tan(fx+e)^2}{2}-\frac{Cab d^2\tan(fx+e)^2}{2}+\frac{3C b^2cd\tan(fx+e)^2}{2}+\tan(fx+e)A b^2d^2-\tan(fx+e)Bab d^2+3\text{ta}\right.}{b^3}}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
(A*a*c^3-3*A*a*c*d^2+3*A*b*c^2*d-A*b*d^3-3*B*a*c^2*d+B*a*d^3+B*b*c^3-3*B*b
*c*d^2-C*a*c^3+3*C*a*c*d^2-3*C*b*c^2*d+C*b*d^3)/(a^2+b^2)*x+(A*b^2*d^2-B*a
*b*d^2+3*B*b^2*c*d+C*a^2*d^2-3*C*a*b*c*d+3*C*b^2*c^2-C*b^2*d^2)*d/f/b^3*ta
n(f*x+e)+1/3*C*d^3/b/f*tan(f*x+e)^3+1/2*d^2*(B*b*d-C*a*d+3*C*b*c)/b^2/f*ta
n(f*x+e)^2+1/2*(3*A*a*c^2*d-A*a*d^3-A*b*c^3+3*A*b*c*d^2+B*a*c^3-3*B*a*c*d^
2+3*B*b*c^2*d-B*b*d^3-3*C*a*c^2*d+C*a*d^3+C*b*c^3-3*C*b*c*d^2)/f/(a^2+b^2)
*ln(1+tan(f*x+e)^2)-(A*a^3*b^2*d^3-3*A*a^2*b^3*c*d^2+3*A*a*b^4*c^2*d-A*b^5
*c^3-B*a^4*b*d^3+3*B*a^3*b^2*c*d^2-3*B*a^2*b^3*c^2*d+B*a*b^4*c^3+C*a^5*d^3
-3*C*a^4*b*c*d^2+3*C*a^3*b^2*c^2*d-C*a^2*b^3*c^3)/(a^2+b^2)/b^4/f*ln(a+b*t
an(f*x+e))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.72

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$
$$= \frac{2\left(Ca^2b^3+Cb^5\right)d^3\tan\left(fx+e\right)^3+6\left(\left(\left(A-C\right)ab^4+Bb^5\right)c^3-3\left(Bab^4-\left(A-C\right)b^5\right)c^2d-3\left(\left(A-C\right)ab^4\right.\right.}{}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
1/6*(2*(C*a^2*b^3 + C*b^5)*d^3*tan(f*x + e)^3 + 6*(((A - C)*a*b^4 + B*b^5)
*c^3 - 3*(B*a*b^4 - (A - C)*b^5)*c^2*d - 3*((A - C)*a*b^4 + B*b^5)*c*d^2 +
 (B*a*b^4 - (A - C)*b^5)*d^3)*f*x + 3*(3*(C*a^2*b^3 + C*b^5)*c*d^2 - (C*a^
3*b^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*d^3)*tan(f*x + e)^2 + 3*((C*a^2*b^3 -
 B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C*a
^4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)*l
og((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) -
 3*((C*a^2*b^3 + C*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + C*a*b^4 - B*b^5)*
c^2*d + 3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (A - C)*b^5)*c*d^2
- (C*a^5 - B*a^4*b + A*a^3*b^2 + (A - C)*a*b^4 + B*b^5)*d^3)*log(1/(tan(f*
x + e)^2 + 1)) + 6*(3*(C*a^2*b^3 + C*b^5)*c^2*d - 3*(C*a^3*b^2 - B*a^2*b^3
 + C*a*b^4 - B*b^5)*c*d^2 + (C*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + (
A - C)*b^5)*d^3)*tan(f*x + e))/((a^2*b^4 + b^6)*f)
```

3.67.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.20 (sec) , antiderivative size = 7096, normalized size of antiderivative = 19.55

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e)),x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq
(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)**2
 + 1)/(2*f) - 3*A*c*d**2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan(e
+ f*x)**2 + 1)/(2*f) + A*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e + f
*x)**2 + 1)/(2*f) - 3*B*c**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d**2*
log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d**3
*x + B*d**3*tan(e + f*x)**3/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x + C*c
**3*tan(e + f*x)/f - 3*C*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c**2*
d*tan(e + f*x)**2/(2*f) + 3*C*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3*C*
c*d**2*tan(e + f*x)/f + C*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*tan
(e + f*x)**4/(4*f) - C*d**3*tan(e + f*x)**2/(2*f))/a, Eq(b, 0)), (3*I*A*c*
*3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 3*A*c**3*f*x/(6*b*f*t
an(e + f*x) - 6*I*b*f) + 3*I*A*c**3/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*A*c
**2*d*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*I*A*c**2*d*f*x/(
6*b*f*tan(e + f*x) - 6*I*b*f) - 9*A*c**2*d/(6*b*f*tan(e + f*x) - 6*I*b*f)
+ 9*I*A*c*d**2*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*A*c*d**
2*f*x/(6*b*f*tan(e + f*x) - 6*I*b*f) + 9*A*c*d**2*log(tan(e + f*x)**2 + 1)
*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*I*A*c*d**2*log(tan(e + f*
x)**2 + 1)/(6*b*f*tan(e + f*x) - 6*I*b*f) - 9*I*A*c*d**2/(6*b*f*tan(e + f*
x) - 6*I*b*f) - 9*A*d**3*f*x*tan(e + f*x)/(6*b*f*tan(e + f*x) - 6*I*b*f...
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.20

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$

$$= \frac{\frac{6\,(((A-C)a+Bb)c^3-3\,(Ba-(A-C)b)c^2d-3\,((A-C)a+Bb)cd^2+(Ba-(A-C)b)d^3)\,(fx+e)}{a^2+b^2} + \frac{6\,((Ca^2b^3-Bab^4+Ab^5)c^3-3\,(Ca^3b^2-Ba^2b^3+A}{}}{}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/6*(6*(((A - C)*a + B*b)*c^3 - 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
 B*b)*c*d^2 + (B*a - (A - C)*b)*d^3)*(f*x + e)/(a^2 + b^2) + 6*((C*a^2*b^3
 - B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c^2*d + 3*(C
*a^4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a^4*b + A*a^3*b^2)*d^3)
*log(b*tan(f*x + e) + a)/(a^2*b^4 + b^6) + 3*((B*a - (A - C)*b)*c^3 + 3*((
A - C)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 - ((A - C)*a + B*b)*d^3)
*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + (2*C*b^2*d^3*tan(f*x + e)^3 + 3*(3*
C*b^2*c*d^2 - (C*a*b - B*b^2)*d^3)*tan(f*x + e)^2 + 6*(3*C*b^2*c^2*d - 3*(
C*a*b - B*b^2)*c*d^2 + (C*a^2 - B*a*b + (A - C)*b^2)*d^3)*tan(f*x + e))/b^
3)/f
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.54

$$\int \frac{(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$$

$$= \frac{\frac{6 \left(Aac^3-Cac^3+Bbc^3-3\,Bac^2d+3\,Abc^2d-3\,Cbc^2d-3\,Aacd^2+3\,Cacd^2-3\,Bbcd^2+Bad^3-Abd^3+Cbd^3\right)(fx+e)}{a^2+b^2}+\frac{3 \left(Bac^3-Abc^3+Cbc^3+3\,Aac^2\right.}{}}{}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/6*(6*(A*a*c^3 - C*a*c^3 + B*b*c^3 - 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^
2*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 + B*a*d^3 - A*b*d^3 + C*b*d^
3)*(f*x + e)/(a^2 + b^2) + 3*(B*a*c^3 - A*b*c^3 + C*b*c^3 + 3*A*a*c^2*d -
3*C*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*C*b*c*d^2 - A*a*
d^3 + C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2 + b^2) + 6*(C*a^2*b^
3*c^3 - B*a*b^4*c^3 + A*b^5*c^3 - 3*C*a^3*b^2*c^2*d + 3*B*a^2*b^3*c^2*d -
3*A*a*b^4*c^2*d + 3*C*a^4*b*c*d^2 - 3*B*a^3*b^2*c*d^2 + 3*A*a^2*b^3*c*d^2
 - C*a^5*d^3 + B*a^4*b*d^3 - A*a^3*b^2*d^3)*log(abs(b*tan(f*x + e) + a))/(a
^2*b^4 + b^6) + (2*C*b^2*d^3*tan(f*x + e)^3 + 9*C*b^2*c*d^2*tan(f*x + e)^2
 - 3*C*a*b*d^3*tan(f*x + e)^2 + 3*B*b^2*d^3*tan(f*x + e)^2 + 18*C*b^2*c^2*
d*tan(f*x + e) - 18*C*a*b*c*d^2*tan(f*x + e) + 18*B*b^2*c*d^2*tan(f*x + e)
 + 6*C*a^2*d^3*tan(f*x + e) - 6*B*a*b*d^3*tan(f*x + e) + 6*A*b^2*d^3*tan(f
*x + e) - 6*C*b^2*d^3*tan(f*x + e))/b^3)/f
```

3.67.9 Mupad [B] (verification not implemented)

Time = 12.19 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.40

$$\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\tan(e+fx)^2\left(\frac{Bd^3+3Ccd^2}{2b}-\frac{Cad^3}{2b^2}\right)}{f} - \frac{\tan(e+fx)\left(\frac{a\left(\frac{Bd^3+3Ccd^2}{b}-\frac{Cad^3}{b^2}\right)}{b}-\frac{3Cc^2d+3Bcd^2+Ad^3}{b}+\frac{Cd^3}{b}\right)}{f} - \frac{\ln(a+b\tan(e+fx))\,(b^4(Bac^3+3Aadc^2)-b^3(Ca^2c^3+3Ba^2c^2d+3Aa^2cd^2)+b^2(3Ca^3c^2d}{f\,(a^2b^4+b^6)} - \frac{\ln(\tan(e+fx)+1\mathrm{i})\,(Ac^3+Ad^3\,1\mathrm{i}-Bc^3\,1\mathrm{i}+Bd^3-Cc^3-Cd^3\,1\mathrm{i}-3Acd^2-Ac^2d\,3\mathrm{i}+Bcd^2\,3\mathrm{i}}{2f\,(b+a\,1\mathrm{i})} - \frac{\ln(\tan(e+fx)-\mathrm{i})\,(Ad^3-Bc^3-Cd^3-3Ac^2d+3Bcd^2+3Cc^2d+Ac^3\,1\mathrm{i}+Bd^3\,1\mathrm{i}-Cc^3\,1\mathrm{i}-}{2f\,(a+b\,1\mathrm{i})} + \frac{Cd^3\tan(e+fx)^3}{3bf}$$

input
```
int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x)),x)
```

output
```
(tan(e + f*x)^2*((B*d^3 + 3*C*c*d^2)/(2*b) - (C*a*d^3)/(2*b^2)))/f - (tan(
e + f*x)*((a*((B*d^3 + 3*C*c*d^2)/b - (C*a*d^3)/b^2))/b - (A*d^3 + 3*B*c*d
^2 + 3*C*c^2*d)/b + (C*d^3)/b))/f - (log(a + b*tan(e + f*x))*(b^4*(B*a*c^3
 + 3*A*a*c^2*d) - b^3*(C*a^2*c^3 + 3*A*a^2*c*d^2 + 3*B*a^2*c^2*d) + b^2*(A
*a^3*d^3 + 3*B*a^3*c*d^2 + 3*C*a^3*c^2*d) - b*(B*a^4*d^3 + 3*C*a^4*c*d^2)
- A*b^5*c^3 + C*a^5*d^3))/(f*(b^6 + a^2*b^4)) - (log(tan(e + f*x) + 1i)*(A
*c^3 + A*d^3*1i - B*c^3*1i + B*d^3 - C*c^3 - C*d^3*1i - 3*A*c*d^2 - A*c^2*
d*3i + B*c*d^2*3i - 3*B*c^2*d + 3*C*c*d^2 + C*c^2*d*3i))/(2*f*(a*1i + b))
- (log(tan(e + f*x) - 1i)*(A*c^3*1i + A*d^3 - B*c^3 + B*d^3*1i - C*c^3*1i
- C*d^3 - A*c*d^2*3i - 3*A*c^2*d + 3*B*c*d^2 - B*c^2*d*3i + C*c*d^2*3i + 3
*C*c^2*d))/(2*f*(a + b*1i)) + (C*d^3*tan(e + f*x)^3)/(3*b*f)
```

3.68 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.68.1 Optimal result

Integrand size = 45, antiderivative size = 574

$$\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx =$$
$$-\frac{(b^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)+a^2(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2))}{(a^2+b^2)^2}$$
$$+\frac{(2ab(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)-a^2((A-C)d(3c^2-d^2)+B(c^3-3cd^2))+b^2((A}{(a^2+b^2)^2 f}$$
$$-\frac{(bc-ad)^2(2a^3bBd-3a^4Cd-b^4(Bc+3Ad)-2ab^3(Ac-cC-2Bd)+a^2b^2(Bc-(A+5C)d))\log(a}{b^4(a^2+b^2)^2 f}$$
$$-\frac{d^2(3a^3Cd-Ab^2(bc-ad)-b^3(2cC+Bd)-a^2b(3cC+2Bd)+ab^2(Bc+2Cd))\tan(e+fx)}{b^3(a^2+b^2)f}$$
$$+\frac{(2Ab^2-2abB+3a^2C+b^2C)d(c+d\tan(e+fx))^2}{2b^2(a^2+b^2)f}$$
$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^3}{b(a^2+b^2)f(a+b\tan(e+fx))}$$

output
```
-(b^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C+3*B*c^2
*d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*
d^2)))*x/(a^2+b^2)^2+(2*a*b*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d
^2)-a^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)+B*(
c^3-3*c*d^2)))*ln(cos(f*x+e))/(a^2+b^2)^2/f-(-a*d+b*c)^2*(2*a^3*b*B*d-3*a^
4*C*d-b^4*(3*A*d+B*c)-2*a*b^3*(A*c-2*B*d-C*c)+a^2*b^2*(B*c-(A+5*C)*d))*ln(
a+b*tan(f*x+e))/b^4/(a^2+b^2)^2/f-d^2*(3*a^3*C*d-A*b^2*(-a*d+b*c)-b^3*(B*d
+2*C*c)-a^2*b*(2*B*d+3*C*c)+a*b^2*(B*c+2*C*d))*tan(f*x+e)/b^3/(a^2+b^2)/f+
1/2*(2*A*b^2-2*B*a*b+3*C*a^2+C*b^2)*d*(c+d*tan(f*x+e))^2/b^2/(a^2+b^2)/f-(
A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^3/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.59 (sec) , antiderivative size = 1024, normalized size of antiderivative = 1.78

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \frac{C(c+d\tan(e+fx))^3}{2bf(a+b\tan(e+fx))}$$

$$+ \frac{\frac{(3bcC+2bBd-3aCd)(c+d\tan(e+fx))^2}{bf(a+b\tan(e+fx))} + \frac{2\left(-\frac{b^2\left(2aAbc^3-a^2Bc^3+b^2Bc^3-2abc^3C-3a^2Ac^2d+3Ab^2c^2d-6abBc^2d+3a^2c^2Cd-3b^2c^2Cd-6aAbcd^2+3\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x])^2,x]
```

output
```
(C*(c + d*Tan[e + f*x])^3)/(2*b*f*(a + b*Tan[e + f*x])) + (((3*b*c*C + 2*b
*B*d - 3*a*C*d)*(c + d*Tan[e + f*x])^2)/(b*f*(a + b*Tan[e + f*x])) + (2*(-
1/2*(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*a*b*c^3*C - 3*a^2*A*c^2*
d + 3*A*b^2*c^2*d - 6*a*b*B*c^2*d + 3*a^2*c^2*C*d - 3*b^2*c^2*C*d - 6*a*A*
b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*C*d^2 + a^2*A*d^3 - A*b^
2*d^3 + 2*a*b*B*d^3 - a^2*C*d^3 + b^2*C*d^3 + I*(a^2*A*c^3 - A*b^2*c^3 + 2
*a*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c^2*d - 3*a^2*B*c^2*d + 3*b^2
*B*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*b^2*c*d^2 - 6*a*b*B*c*d^2 +
 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 + a^2*B*d^3 - b^2*B*d^3 + 2*a
*b*C*d^3))*Log[I - Tan[e + f*x]])/((a^2 + b^2)^2*f) + (b^2*(-2*a*A*b*c^3 +
 a^2*B*c^3 - b^2*B*c^3 + 2*a*b*c^3*C + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a
*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^2*c^2*C*d + 6*a*A*b*c*d^2 - 3*a^2*B*c*d^2
 + 3*b^2*B*c*d^2 - 6*a*b*c*C*d^2 - a^2*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a
^2*C*d^3 - b^2*C*d^3 + I*(a^2*A*c^3 - A*b^2*c^3 + 2*a*b*B*c^3 - a^2*c^3*C
+ b^2*c^3*C + 6*a*A*b*c^2*d - 3*a^2*B*c^2*d + 3*b^2*B*c^2*d - 6*a*b*c^2*C*
d - 3*a^2*A*c*d^2 + 3*A*b^2*c*d^2 - 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*
c*C*d^2 - 2*a*A*b*d^3 + a^2*B*d^3 - b^2*B*d^3 + 2*a*b*C*d^3))*Log[I + Tan[
e + f*x]])/(2*(a^2 + b^2)^2*f) - ((b*c - a*d)^2*(2*a^3*b*B*d - 3*a^4*C*d -
 b^4*(B*c + 3*A*d) - 2*a*b^3*(A*c - c*C - 2*B*d) + a^2*b^2*(B*c - (A + 5*C
)*d))*Log[a + b*Tan[e + f*x]])/(b^2*(a^2 + b^2)^2*f) + ((b*c - a*d)^2*(...
```

3.68.3 Rubi [A] (verified)

Time = 3.33 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4128, 3042, 4130, 27, 3042, 4120, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^2\left((3Ca^2-2bBa+2Ab^2+b^2C)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-3ad)+Ab(ac+3bd)\right)}{a+b\tan(e+fx)}dx}{b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^2\left((3Ca^2-2bBa+2Ab^2+b^2C)d\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-3ad)+Ab(ac+3bd)\right)}{a+b\tan(e+fx)}dx}{b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 4130

$$\frac{\int -\frac{\frac{2(c+d\tan(e+fx))\left(-\left(\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\right)-c((bB-aC)(bc-3ad)+Ab(ac+3bd))b+a\left(3Ca^2-2bBa\right.}{a+b\tan(e+fx)}}{2b}}{b\left(a^2\right.}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)(c+d\tan(e+fx))^2}{2bf}-\int \frac{(c+d\tan(e+fx))\left(-\left(\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\right)\right.}{}}{b\left(a^2\right.}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)(c+d\tan(e+fx))^2}{2bf}-\int \frac{(c+d\tan(e+fx))\left(-\left(\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\right)\right.}{}}{b\left(a^2\right.}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\Big\downarrow$ 4120

$$\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)(c+d\tan(e+fx))^2}{2bf}-\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\int \frac{3Cd^3a^4-2bd^2(3}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

3.68. $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$

$\downarrow$ 3042

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\int\frac{3Cd^3a^4-2bd^2(3\cdots}{\cdots}}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4109

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\frac{b^3\left(a^2\left(d(A-C)\left(3\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\frac{b^3\left(a^2\left(d(A-C)\left(3\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3956

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{(bc-ad)^2\left(-3a^4\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4100

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{(bc-ad)^2\left(-3a^4\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 16

3.68. $\int\frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)(c+d\tan(e+fx))^2}{2bf}-\frac{d^2\tan(e+fx)\left(3a^3Cd-a^2b(2Bd+3cC)-Ab^2(bc-ad)+ab^2(Bc+2Cd)-b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{b^3\log(\cos(e+fx)}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{bf\,(a^2+b^2)\,(a+b\tan(e+fx))}$$

input
```
Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a + 
b*Tan[e + f*x])^2,x]
```

output
```
-(((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^3)/(b*(a^2 + b^2)*f*(a + b
*Tan[e + f*x]))) + ((((2*A*b^2 - 2*a*b*B + 3*a^2*C + b^2*C)*d*(c + d*Tan[e
 + f*x])^2)/(2*b*f) - (-((-((b^3*(b^2*(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^
2 + 3*c*C*d^2 + B*d^3) + a^2*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c
^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*x)/(
a^2 + b^2)) - (b^3*(2*a*b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3
- 3*c*d^2)) + a^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) - b^2*((A
- C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((a^2 + b^2)
*f) - ((b*c - a*d)^2*(2*a^3*b*B*d - 3*a^4*C*d - b^4*(B*c + 3*A*d) - 2*a*b^
3*(A*c - c*C - 2*B*d) + a^2*b^2*(B*c - (A + 5*C)*d))*Log[a + b*Tan[e + f*x
]])/(b*(a^2 + b^2)*f))/b) + (d^2*(3*a^3*C*d - A*b^2*(b*c - a*d) - b^3*(2*c
*C + B*d) - a^2*b*(3*c*C + 2*B*d) + a*b^2*(B*c + 2*C*d))*Tan[e + f*x])/(b*
f))/b)/(b*(a^2 + b^2))
```

3.68.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.68. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.68.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{d^2\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB-2\tan(fx+e)Cad+3\tan(fx+e)Cbc\right)}{b^3}-\frac{-A\,a^3d^3b^2+3A\,b^3c\,d^2a^2-3A\,b^4c^2da+A\,b^5c^3+B\,a^4d^3b}{}}{}$
default	$\frac{\frac{d^2\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB-2\tan(fx+e)Cad+3\tan(fx+e)Cbc\right)}{b^3}-\frac{-A\,a^3d^3b^2+3A\,b^3c\,d^2a^2-3A\,b^4c^2da+A\,b^5c^3+B\,a^4d^3b}{}}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(d^2/b^3*(1/2*tan(f*x+e)^2*C*b*d+tan(f*x+e)*b*d*B-2*tan(f*x+e)*C*a*d+3
*tan(f*x+e)*C*b*c)-(-A*a^3*b^2*d^3+3*A*a^2*b^3*c*d^2-3*A*a*b^4*c^2*d+A*b^5
*c^3+B*a^4*b*d^3-3*B*a^3*b^2*c*d^2+3*B*a^2*b^3*c^2*d-B*a*b^4*c^3-C*a^5*d^3
+3*C*a^4*b*c*d^2-3*C*a^3*b^2*c^2*d+C*a^2*b^3*c^3)/b^4/(a^2+b^2)/(a+b*tan(f
*x+e))+1/b^4*(A*a^4*b^2*d^3-3*A*a^2*b^4*c^2*d+3*A*a^2*b^4*d^3+2*A*a*b^5*c^
3-6*A*a*b^5*c*d^2+3*A*b^6*c^2*d-2*B*a^5*b*d^3+3*B*a^4*b^2*c*d^2-4*B*a^3*b^
3*d^3-B*a^2*b^4*c^3+9*B*a^2*b^4*c*d^2-6*B*a*b^5*c^2*d+B*b^6*c^3+3*C*a^6*d^
3-6*C*a^5*b*c*d^2+3*C*a^4*b^2*c^2*d+5*C*a^4*b^2*d^3-12*C*a^3*b^3*c*d^2+9*C
*a^2*b^4*c^2*d-2*C*a*b^5*c^3)/(a^2+b^2)^2*ln(a+b*tan(f*x+e))+1/(a^2+b^2)^2
*(1/2*(3*A*a^2*c^2*d-A*a^2*d^3-2*A*a*b*c^3+6*A*a*b*c*d^2-3*A*b^2*c^2*d+A*b
^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2+6*B*a*b*c^2*d-2*B*a*b*d^3-B*b^2*c^3+3*B*b^2
*c*d^2-3*C*a^2*c^2*d+C*a^2*d^3+2*C*a*b*c^3-6*C*a*b*c*d^2+3*C*b^2*c^2*d-C*b
^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2*c*d^2+6*A*a*b*c^2*d-2*A*a*b*
d^3-A*b^2*c^3+3*A*b^2*c*d^2-3*B*a^2*c^2*d+B*a^2*d^3+2*B*a*b*c^3-6*B*a*b*c*
d^2+3*B*b^2*c^2*d-B*b^2*d^3-C*a^2*c^3+3*C*a^2*c*d^2-6*C*a*b*c^2*d+2*C*a*b*
d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(tan(f*x+e))))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1512 vs. 2(571) = 1142.

Time = 1.16 (sec) , antiderivative size = 1512, normalized size of antiderivative = 2.63

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
1/2*((C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*d^3*tan(f*x + e)^3 - 2*(C*a^2*b^5 -
 B*a*b^6 + A*b^7)*c^3 + 6*(C*a^3*b^4 - B*a^2*b^5 + A*a*b^6)*c^2*d - 6*(C*a
^4*b^3 - B*a^3*b^4 + A*a^2*b^5)*c*d^2 + (3*C*a^5*b^2 - 2*B*a^4*b^3 + 2*(A
+ C)*a^3*b^4 + C*a*b^6)*d^3 + 2*(((A - C)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*
a*b^6)*c^3 - 3*(B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^2*d - 3*((A - C
)*a^3*b^4 + 2*B*a^2*b^5 - (A - C)*a*b^6)*c*d^2 + (B*a^3*b^4 - 2*(A - C)*a^
2*b^5 - B*a*b^6)*d^3)*f*x + (6*(C*a^4*b^3 + 2*C*a^2*b^5 + C*b^7)*c*d^2 - (
3*C*a^5*b^2 - 2*B*a^4*b^3 + 6*C*a^3*b^4 - 4*B*a^2*b^5 + 3*C*a*b^6 - 2*B*b^
7)*d^3)*tan(f*x + e)^2 - ((B*a^3*b^4 - 2*(A - C)*a^2*b^5 - B*a*b^6)*c^3 -
3*(C*a^5*b^2 - (A - 3*C)*a^3*b^4 - 2*B*a^2*b^5 + A*a*b^6)*c^2*d + 3*(2*C*a
^6*b - B*a^5*b^2 + 4*C*a^4*b^3 - 3*B*a^3*b^4 + 2*A*a^2*b^5)*c*d^2 - (3*C*a
^7 - 2*B*a^6*b + (A + 5*C)*a^5*b^2 - 4*B*a^4*b^3 + 3*A*a^3*b^4)*d^3 + ((B*
a^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5
- 2*B*a*b^6 + A*b^7)*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - 3*
B*a^2*b^5 + 2*A*a*b^6)*c*d^2 - (3*C*a^6*b - 2*B*a^5*b^2 + (A + 5*C)*a^4*b^
3 - 4*B*a^3*b^4 + 3*A*a^2*b^5)*d^3)*tan(f*x + e))*log((b^2*tan(f*x + e)^2
+ 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1)) - (3*(C*a^5*b^2 + 2*C*a^
3*b^4 + C*a*b^6)*c^2*d - 3*(2*C*a^6*b - B*a^5*b^2 + 4*C*a^4*b^3 - 2*B*a^3*
b^4 + 2*C*a^2*b^5 - B*a*b^6)*c*d^2 + (3*C*a^7 - 2*B*a^6*b + (A + 5*C)*a^5*
b^2 - 4*B*a^4*b^3 + (2*A + C)*a^3*b^4 - 2*B*a^2*b^5 + (A - C)*a*b^6)*d^...
```

3.68.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.69 (sec) , antiderivative size = 24300, normalized size of antiderivative = 42.33

$$\int \frac{(c + d\tan(e + fx))^3 (A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e))**2,x)
```

output
```
Piecewise((zoo*x*(c + d*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
 Eq(a, 0) & Eq(b, 0) & Eq(f, 0)), ((A*c**3*x + 3*A*c**2*d*log(tan(e + f*x)
**2 + 1)/(2*f) - 3*A*c*d**2*x + 3*A*c*d**2*tan(e + f*x)/f - A*d**3*log(tan
(e + f*x)**2 + 1)/(2*f) + A*d**3*tan(e + f*x)**2/(2*f) + B*c**3*log(tan(e
+ f*x)**2 + 1)/(2*f) - 3*B*c**2*d*x + 3*B*c**2*d*tan(e + f*x)/f - 3*B*c*d*
*2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*c*d**2*tan(e + f*x)**2/(2*f) + B*d
**3*x + B*d**3*tan(e + f*x)**3/(3*f) - B*d**3*tan(e + f*x)/f - C*c**3*x +
C*c**3*tan(e + f*x)/f - 3*C*c**2*d*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*c*
*2*d*tan(e + f*x)**2/(2*f) + 3*C*c*d**2*x + C*c*d**2*tan(e + f*x)**3/f - 3
*C*c*d**2*tan(e + f*x)/f + C*d**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*d**3*
tan(e + f*x)**4/(4*f) - C*d**3*tan(e + f*x)**2/(2*f))/a**2, Eq(b, 0)), (-A
*c**3*f*x*tan(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f
*x) - 4*b**2*f) + 2*I*A*c**3*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 -
8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + A*c**3*f*x/(4*b**2*f*tan(e + f*x)**2
 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) - A*c**3*tan(e + f*x)/(4*b**2*f*tan
(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 2*I*A*c**3/(4*b**2*f*
tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**2*f) + 3*I*A*c**2*d*f*x*t
an(e + f*x)**2/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*tan(e + f*x) - 4*b**
2*f) + 6*A*c**2*d*f*x*tan(e + f*x)/(4*b**2*f*tan(e + f*x)**2 - 8*I*b**2*f*
tan(e + f*x) - 4*b**2*f) - 3*I*A*c**2*d*f*x/(4*b**2*f*tan(e + f*x)**2 -...
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.19

$$\int \frac{(c+d\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{2\left(\left(((A-C)a^2+2\,Bab-(A-C)b^2\right)c^3-3\left(Ba^2-2\,(A-C)ab-Bb^2\right)c^2d-3\left((A-C)a^2+2\,Bab-(A-C)b^2\right)cd^2+\left(Ba^2-2\,(A-C)ab-Bb^2\right)d^3\right)(fx+e)}{a^4+2\,a^2b^2+b^4}\ \cdots}{\cdots}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 - 3*(B*a^2 - 2*(A - C)*a
*b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 + (B*a^2
 - 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^4) - 2*((B*a
^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c^3 - 3*(C*a^4*b^2 - (A - 3*C)*a^2*b^4 -
 2*B*a*b^5 + A*b^6)*c^2*d + 3*(2*C*a^5*b - B*a^4*b^2 + 4*C*a^3*b^3 - 3*B*a
^2*b^4 + 2*A*a*b^5)*c*d^2 - (3*C*a^6 - 2*B*a^5*b + (A + 5*C)*a^4*b^2 - 4*B
*a^3*b^3 + 3*A*a^2*b^4)*d^3)*log(b*tan(f*x + e) + a)/(a^4*b^4 + 2*a^2*b^6
+ b^8) + ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 + 3*((A - C)*a^2 + 2*B*a*b -
 (A - C)*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^2 - ((A - C)*a
^2 + 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) - 2*((C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 3*(C*a^3*b^2 - B*a^2*b^3 +
 A*a*b^4)*c^2*d + 3*(C*a^4*b - B*a^3*b^2 + A*a^2*b^3)*c*d^2 - (C*a^5 - B*a
^4*b + A*a^3*b^2)*d^3)/(a^3*b^4 + a*b^6 + (a^2*b^5 + b^7)*tan(f*x + e)) +
(C*b*d^3*tan(f*x + e)^2 + 2*(3*C*b*c*d^2 - (2*C*a - B*b)*d^3)*tan(f*x + e)
)/b^3)/f
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(571) = 1142$.

Time = 1.13 (sec) , antiderivative size = 1329, normalized size of antiderivative = 2.32

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 - 3*B*
a^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d + 3*B*b^2*c^2*d - 3*A*a^2*c*d^2
+ 3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 + B*a^2*d^
3 - 2*A*a*b*d^3 + 2*C*a*b*d^3 - B*b^2*d^3)*(f*x + e)/(a^4 + 2*a^2*b^2 + b^
4) + (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 + 3*A*a^2*c^2*d -
3*C*a^2*c^2*d + 6*B*a*b*c^2*d - 3*A*b^2*c^2*d + 3*C*b^2*c^2*d - 3*B*a^2*c*
d^2 + 6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 - A*a^2*d^3 + C*a^2*d^
3 - 2*B*a*b*d^3 + A*b^2*d^3 - C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4 + 2*
a^2*b^2 + b^4) - 2*(B*a^2*b^4*c^3 - 2*A*a*b^5*c^3 + 2*C*a*b^5*c^3 - B*b^6*
c^3 - 3*C*a^4*b^2*c^2*d + 3*A*a^2*b^4*c^2*d - 9*C*a^2*b^4*c^2*d + 6*B*a*b^
5*c^2*d - 3*A*b^6*c^2*d + 6*C*a^5*b*c*d^2 - 3*B*a^4*b^2*c*d^2 + 12*C*a^3*b
^3*c*d^2 - 9*B*a^2*b^4*c*d^2 + 6*A*a*b^5*c*d^2 - 3*C*a^6*d^3 + 2*B*a^5*b*d
^3 - A*a^4*b^2*d^3 - 5*C*a^4*b^2*d^3 + 4*B*a^3*b^3*d^3 - 3*A*a^2*b^4*d^3)*
log(abs(b*tan(f*x + e) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8) + 2*(B*a^2*b^5*c^
3*tan(f*x + e) - 2*A*a*b^6*c^3*tan(f*x + e) + 2*C*a*b^6*c^3*tan(f*x + e) -
 B*b^7*c^3*tan(f*x + e) - 3*C*a^4*b^3*c^2*d*tan(f*x + e) + 3*A*a^2*b^5*c^2
*d*tan(f*x + e) - 9*C*a^2*b^5*c^2*d*tan(f*x + e) + 6*B*a*b^6*c^2*d*tan(f*x
 + e) - 3*A*b^7*c^2*d*tan(f*x + e) + 6*C*a^5*b^2*c*d^2*tan(f*x + e) - 3*B*
a^4*b^3*c*d^2*tan(f*x + e) + 12*C*a^3*b^4*c*d^2*tan(f*x + e) - 9*B*a^2*b^5
*c*d^2*tan(f*x + e) + 6*A*a*b^6*c*d^2*tan(f*x + e) - 3*C*a^6*b*d^3*tan(...
```

3.68.9 Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.22

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\tan(e+f\,x)\,\left(\frac{B\,d^3+3\,C\,c\,d^2}{b^2}-\frac{2\,C\,a\,d^3}{b^3}\right)}{f}$$

$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,c^3-A\,d^3+C\,d^3+3\,A\,c^2\,d-3\,B\,c\,d^2-3\,C\,c^2\,d+A\,c^3\,1\mathrm{i}+B\,d^3\,1\mathrm{i}-C\,c^3\,1\mathrm{i}\right.}{2\,f\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}$$

$$+\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(b^4\,\left(3\,A\,a^2\,d^3-B\,a^2\,c^3-3\,A\,a^2\,c^2\,d+9\,B\,a^2\,c\,d^2+9\,C\,a^2\,c^2\,d\right)-b^5\,\left(2\,C\,a\,c^3-2\right.\right.}{}$$

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,c^3-A\,d^3\,1\mathrm{i}+B\,c^3\,1\mathrm{i}+B\,d^3-C\,c^3+C\,d^3\,1\mathrm{i}-3\,A\,c\,d^2+A\,c^2\,d\,3\mathrm{i}-B\,c\,d^2\,3\mathrm{i}\right.}{2\,f\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$

$$-\frac{-C\,a^5\,d^3+3\,C\,a^4\,b\,c\,d^2+B\,a^4\,b\,d^3-3\,C\,a^3\,b^2\,c^2\,d-3\,B\,a^3\,b^2\,c\,d^2-A\,a^3\,b^2\,d^3+C\,a^2\,b^3\,c^3+3\,B\,a^2\,b^3\,c}{b\,f\,\left(\tan\left(e+f\,x\right)\,b^4+a\,b^3\right)\,\left(a^2+b^2\right)}$$

$$+\frac{C\,d^3\,\tan(e+f\,x)^2}{2\,b^2\,f}$$

input
```
int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x))^2,x)
```

output
```
(tan(e + f*x)*((B*d^3 + 3*C*c*d^2)/b^2 - (2*C*a*d^3)/b^3))/f - (log(tan(e
+ f*x) + 1i)*(A*c^3*1i - A*d^3 + B*c^3 + B*d^3*1i - C*c^3*1i + C*d^3 - A*c
*d^2*3i + 3*A*c^2*d - 3*B*c*d^2 - B*c^2*d*3i + C*c*d^2*3i - 3*C*c^2*d))/(2
*f*(a*b*2i - a^2 + b^2)) + (log(a + b*tan(e + f*x))*(b^4*(3*A*a^2*d^3 - B*
a^2*c^3 - 3*A*a^2*c^2*d + 9*B*a^2*c*d^2 + 9*C*a^2*c^2*d) - b^5*(2*C*a*c^3
- 2*A*a*c^3 + 6*A*a*c*d^2 + 6*B*a*c^2*d) - b^3*(4*B*a^3*d^3 + 12*C*a^3*c*d
^2) + b^6*(B*c^3 + 3*A*c^2*d) - b*(2*B*a^5*d^3 + 6*C*a^5*c*d^2) + b^2*(A*a
^4*d^3 + 5*C*a^4*d^3 + 3*B*a^4*c*d^2 + 3*C*a^4*c^2*d) + 3*C*a^6*d^3))/(f*(
b^8 + 2*a^2*b^6 + a^4*b^4)) - (log(tan(e + f*x) - 1i)*(A*c^3 - A*d^3*1i +
B*c^3*1i + B*d^3 - C*c^3 + C*d^3*1i - 3*A*c*d^2 + A*c^2*d*3i - B*c*d^2*3i
- 3*B*c^2*d + 3*C*c*d^2 - C*c^2*d*3i))/(2*f*(2*a*b - a^2*1i + b^2*1i)) - (
A*b^5*c^3 - C*a^5*d^3 - B*a*b^4*c^3 + B*a^4*b*d^3 - A*a^3*b^2*d^3 + C*a^2*
b^3*c^3 + 3*A*a^2*b^3*c*d^2 + 3*B*a^2*b^3*c^2*d - 3*B*a^3*b^2*c*d^2 - 3*C*
a^3*b^2*c^2*d - 3*A*a*b^4*c^2*d + 3*C*a^4*b*c*d^2)/(b*f*(a*b^3 + b^4*tan(e
 + f*x))*(a^2 + b^2)) + (C*d^3*tan(e + f*x)^2)/(2*b^2*f)
```

3.68. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.69 $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.69.1 Optimal result

Integrand size = 45, antiderivative size = 798

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx =$$
$$-\frac{(3ab^2(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)+a^3(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2}{(a^2+b^2)^3}$$
$$-\frac{(b^3(Ac^3-c^3C-3Bc^2d-3Acd^2+3cCd^2+Bd^3)+3a^2b(c^3C+3Bc^2d-3cCd^2-Bd^3-A(c^3-3cd^2)}{(a^2}$$
$$-\frac{(bc-ad)\,(a^5bBd^2-3a^6Cd^2+a^4b^2d(Bc-9Cd)+a^3b^3B(c^2+3d^2)-b^6(c(cC+3Bd)-A(c^2-3d^2))}{b^4\,(a^2}$$
$$-\frac{d^2(a^3bBd-3a^4Cd-ab^3(2Ac-2cC-3Bd)+a^2b^2(Bc-6Cd)-b^4(Bc+(2A+C)d))\tan(e+fx)}{b^3\,(a^2+b^2)^2\,f}$$
$$+\frac{(a^3bBd-3a^4Cd-b^4(2Bc+3Ad)-ab^3(4Ac-4cC-5Bd)+a^2b^2(2Bc+(A-7C)d))\,(c+d\tan(e+}{2b^2\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$
$$-\frac{(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^3}{2b\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}$$

output
```
-(3*a*b^2*(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+a^3*(c^3*C+3*B
*c^2*d-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))-3*a^2*b*((A-C)*d*(3*c^2-d^2)+B*(c^
3-3*c*d^2))+b^3*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)))*x/(a^2+b^2)^3-(b^3*
(A*c^3-3*A*c*d^2-3*B*c^2*d+B*d^3-C*c^3+3*C*c*d^2)+3*a^2*b*(c^3*C+3*B*c^2*d
-3*C*c*d^2-B*d^3-A*(c^3-3*c*d^2))+a^3*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)
)-3*a*b^2*((A-C)*d*(3*c^2-d^2)+B*(c^3-3*c*d^2)))*ln(cos(f*x+e))/(a^2+b^2)^
3/f-(-a*d+b*c)*(a^5*b*B*d^2-3*a^6*C*d^2+a^4*b^2*d*(B*c-9*C*d)+a^3*b^3*B*(c
^2+3*d^2)-b^6*(c*(3*B*d+C*c)-A*(c^2-3*d^2))-a*b^5*(8*c*(A-C)*d+3*B*(c^2-2*
d^2))+a^2*b^4*(3*c^2*C+6*B*c*d-10*C*d^2-A*(3*c^2-d^2)))*ln(a+b*tan(f*x+e))
/b^4/(a^2+b^2)^3/f-d^2*(a^3*b*B*d-3*a^4*C*d-a*b^3*(2*A*c-3*B*d-2*C*c)+a^2*
b^2*(B*c-6*C*d)-b^4*(B*c+(2*A+C)*d))*tan(f*x+e)/b^3/(a^2+b^2)^2/f+1/2*(a^3
*b*B*d-3*a^4*C*d-b^4*(3*A*d+2*B*c)-a*b^3*(4*A*c-5*B*d-4*C*c)+a^2*b^2*(2*B*
c+(A-7*C)*d))*(c+d*tan(f*x+e))^2/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A
*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^3/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

3.69.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.56 (sec) , antiderivative size = 1409, normalized size of antiderivative = 1.77

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \frac{C(c+d\tan(e+fx))^3}{bf(a+b\tan(e+fx))^2} + \frac{b\left(-3a^2Abc^3+Ab^3c^3+a^3Bc^3-3ab^2Bc^3+3a^2bc^3C-b^3c^3C+3a^3Ac^2d-9aAb^2c^2d+9a^2bBc^2d-3b^3Bc^2d-3a^3c^2Cd+9ab^2c^2Cd+9a^2Abcd^2-3Ab\right.}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(a + b*Tan[e + f*x])^3,x]
```

output
```
(C*(c + d*Tan[e + f*x])^3)/(b*f*(a + b*Tan[e + f*x])^2) + ((b*(-3*a^2*A*b*
c^3 + A*b^3*c^3 + a^3*B*c^3 - 3*a*b^2*B*c^3 + 3*a^2*b*c^3*C - b^3*c^3*C +
3*a^3*A*c^2*d - 9*a*A*b^2*c^2*d + 9*a^2*b*B*c^2*d - 3*b^3*B*c^2*d - 3*a^3*
c^2*C*d + 9*a*b^2*c^2*C*d + 9*a^2*A*b*c*d^2 - 3*A*b^3*c*d^2 - 3*a^3*B*c*d^
2 + 9*a*b^2*B*c*d^2 - 9*a^2*b*c*C*d^2 + 3*b^3*c*C*d^2 - a^3*A*d^3 + 3*a*A*
b^2*d^3 - 3*a^2*b*B*d^3 + b^3*B*d^3 + a^3*C*d^3 - 3*a*b^2*C*d^3 + I*(-(a^3
*A*c^3) + 3*a*A*b^2*c^3 - 3*a^2*b*B*c^3 + b^3*B*c^3 + a^3*c^3*C - 3*a*b^2*
c^3*C - 9*a^2*A*b*c^2*d + 3*A*b^3*c^2*d + 3*a^3*B*c^2*d - 9*a*b^2*B*c^2*d
+ 9*a^2*b*c^2*C*d - 3*b^3*c^2*C*d + 3*a^3*A*c*d^2 - 9*a*A*b^2*c*d^2 + 9*a^
2*b*B*c*d^2 - 3*b^3*B*c*d^2 - 3*a^3*c*C*d^2 + 9*a*b^2*c*C*d^2 + 3*a^2*A*b*
d^3 - A*b^3*d^3 - a^3*B*d^3 + 3*a*b^2*B*d^3 - 3*a^2*b*C*d^3 + b^3*C*d^3))*
Log[I - Tan[e + f*x]])/(2*(a^2 + b^2)^3*f) - (b*(3*a^2*A*b*c^3 - A*b^3*c^3
 - a^3*B*c^3 + 3*a*b^2*B*c^3 - 3*a^2*b*c^3*C + b^3*c^3*C - 3*a^3*A*c^2*d +
 9*a*A*b^2*c^2*d - 9*a^2*b*B*c^2*d + 3*b^3*B*c^2*d + 3*a^3*c^2*C*d - 9*a*b
^2*c^2*C*d - 9*a^2*A*b*c*d^2 + 3*A*b^3*c*d^2 + 3*a^3*B*c*d^2 - 9*a*b^2*B*c
*d^2 + 9*a^2*b*c*C*d^2 - 3*b^3*c*C*d^2 + a^3*A*d^3 - 3*a*A*b^2*d^3 + 3*a^2
*b*B*d^3 - b^3*B*d^3 - a^3*C*d^3 + 3*a*b^2*C*d^3 + I*(-(a^3*A*c^3) + 3*a*A
*b^2*c^3 - 3*a^2*b*B*c^3 + b^3*B*c^3 + a^3*c^3*C - 3*a*b^2*c^3*C - 9*a^2*A
*b*c^2*d + 3*A*b^3*c^2*d + 3*a^3*B*c^2*d - 9*a*b^2*B*c^2*d + 9*a^2*b*c^2*C
*d - 3*b^3*c^2*C*d + 3*a^3*A*c*d^2 - 9*a*A*b^2*c*d^2 + 9*a^2*b*B*c*d^2 ...
```

3.69.3 Rubi [A] (verified)

Time = 4.15 (sec) , antiderivative size = 830, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4128, 3042, 4128, 3042, 4120, 27, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^2\left(\left(3Ca^2-bBa+Ab^2+2b^2C\right)d\tan^2(e+fx)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-3ad)+Ab(2ac+3bd)\right)}{(a+b\tan(e+fx))^2}d\cdots}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(c+d\tan(e+fx))^2\left(\left(3Ca^2-bBa+Ab^2+2b^2C\right)d\tan(e+fx)^2-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-3ad)+Ab(2ac+3bd)\right)}{(a+b\tan(e+fx))^2}d\cdots}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 4128$$

$$\frac{\int \frac{(c+d\tan(e+fx))\left(-2((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(ac+2bd)((bB-aC)(2bc-3ad)+Ab(2ac+3b\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(c+d\tan(e+fx))\left(-2((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(ac+2bd)((bB-aC)(2bc-3ad)+Ab(2ac+3b\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 4120$$

$$-\frac{\int \frac{2\left(3Cd^3a^5-bd^2(3cC+Bd)a^4+6b^2Cd^3a^3-b^3\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+9Cd^2c+3Bd^3\right)a^2-b^4\left(2Bc^3+6Adc^2-6Cdc^2-6Bd^2c-2Ad^3-Cd^3\right)a-\left(a^2+b^2\right)^2d^2(\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 27$$

3.69. $\int \frac{(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$-\frac{2\int \frac{3Cd^3a^5-bd^2(3cC+Bd)a^4+6b^2Cd^3a^3-b^3\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+9Cd^2c+3Bd^3\right)a^2-b^4\left(2Bc^3+6Adc^2-6Cdc^2-6Bd^2c-2Ad^3-Cd^3\right)a-\left(a^2+b^2\right)^2d^2(3}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$-\frac{2\int \frac{3Cd^3a^5-bd^2(3cC+Bd)a^4+6b^2Cd^3a^3-b^3\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+9Cd^2c+3Bd^3\right)a^2-b^4\left(2Bc^3+6Adc^2-6Cdc^2-6Bd^2c-2Ad^3-Cd^3\right)a-\left(a^2+b^2\right)^2d^2(3}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 4109

$$\frac{\frac{(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b^4(2Bc+3Ad))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2\left(-3Cda^4+bBda^3+b^2(Bc-6Cd)a^2-b^3(2A\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b^4(2Bc+3Ad))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2\left(-3Cda^4+bBda^3+b^2(Bc-6Cd)a^2-b^3(2A\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 3956

$$\frac{\frac{(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b^4(2Bc+3Ad))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2\left(-3Cda^4+bBda^3+b^2(Bc-6Cd)a^2-b^3(2A\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 4100

3.69. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\frac{(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b^4(2Bc+3Ad))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2\left(-3Cda^4+bBda^3+b^2(Bc-6Cd)a^2-b^3(2A\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 16

$$\frac{\frac{(-3Cda^4+bBda^3+b^2(2Bc+(A-7C)d)a^2-b^3(4Ac-4Cc-5Bd)a-b^4(2Bc+3Ad))(c+d\tan(e+fx))^2}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2\left(-3Cda^4+bBda^3+b^2(Bc-6Cd)a^2-b^3(2A\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^3}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

input
```
Int[((c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a +
b*Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^3)/(b*(a^2 + b^2)*f*(a
+ b*Tan[e + f*x])^2) + (((a^3*b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*
b^3*(4*A*c - 4*c*C - 5*B*d) + a^2*b^2*(2*B*c + (A - 7*C)*d))*(c + d*Tan[e
+ f*x])^2)/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])) + ((-2*(-((b^3*(a^3*(A*c
^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + 3*a*b^2*(c^3*C +
 3*B*c^2*d - 3*c*C*d^2 - B*d^3 - A*(c^3 - 3*c*d^2)) + 3*a^2*b*((A - C)*d*(
3*c^2 - d^2) + B*(c^3 - 3*c*d^2)) - b^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3
- 3*c*d^2)))*x)/(a^2 + b^2)) - (b^3*(3*a^2*b*(A*c^3 - c^3*C - 3*B*c^2*d -
3*A*c*d^2 + 3*c*C*d^2 + B*d^3) + b^3*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^
3 - A*(c^3 - 3*c*d^2)) - a^3*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2))
 + 3*a*b^2*((A - C)*d*(3*c^2 - d^2) + B*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]
])/((a^2 + b^2)*f) + ((b*c - a*d)*(a^5*b*B*d^2 - 3*a^6*C*d^2 + a^4*b^2*d*(
B*c - 9*C*d) + a^3*b^3*B*(c^2 + 3*d^2) - b^6*(c*(c*C + 3*B*d) - A*(c^2 - 3
*d^2)) - a*b^5*(8*c*(A - C)*d + 3*B*(c^2 - 2*d^2)) + a^2*b^4*(3*c^2*C + 6*
B*c*d - 10*C*d^2 - A*(3*c^2 - d^2)))*Log[a + b*Tan[e + f*x]])/(b*(a^2 + b^
2)*f)))/b - (2*d^2*(a^3*b*B*d - 3*a^4*C*d - a*b^3*(2*A*c - 2*c*C - 3*B*d)
+ a^2*b^2*(B*c - 6*C*d) - b^4*(B*c + (2*A + C)*d))*Tan[e + f*x])/(b*f))/(b
*(a^2 + b^2)))/(2*b*(a^2 + b^2))
```

3.69. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.69. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.69.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 1271, normalized size of antiderivative = 1.59

method	result	size
derivativedivides	Expression too large to display	1271
default	Expression too large to display	1271
norman	Expression too large to display	2076
parallelrisch	Expression too large to display	6687
risch	Expression too large to display	6825

input
```
int((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*d^3/b^3-1/2*(-A*a^3*b^2*d^3+3*A*a^2*b^3*c*d^2-3*A*a*b^4*
c^2*d+A*b^5*c^3+B*a^4*b*d^3-3*B*a^3*b^2*c*d^2+3*B*a^2*b^3*c^2*d-B*a*b^4*c^
3-C*a^5*d^3+3*C*a^4*b*c*d^2-3*C*a^3*b^2*c^2*d+C*a^2*b^3*c^3)/b^4/(a^2+b^2)
/(a+b*tan(f*x+e))^2-1/b^4*(A*a^4*b^2*d^3-3*A*a^2*b^4*c^2*d+3*A*a^2*b^4*d^3
+2*A*a*b^5*c^3-6*A*a*b^5*c*d^2+3*A*b^6*c^2*d-2*B*a^5*b*d^3+3*B*a^4*b^2*c*d
^2-4*B*a^3*b^3*d^3-B*a^2*b^4*c^3+9*B*a^2*b^4*c*d^2-6*B*a*b^5*c^2*d+B*b^6*c
^3+3*C*a^6*d^3-6*C*a^5*b*c*d^2+3*C*a^4*b^2*c^2*d+5*C*a^4*b^2*d^3-12*C*a^3*
b^3*c*d^2+9*C*a^2*b^4*c^2*d-2*C*a*b^5*c^3)/(a^2+b^2)^2/(a+b*tan(f*x+e))+(-
3*A*a^3*b^4*c^2*d+A*a^3*b^4*d^3+3*A*a^2*b^5*c^3-9*A*a^2*b^5*c*d^2+9*A*a*b^
6*c^2*d-3*A*a*b^6*d^3-A*b^7*c^3+3*A*b^7*c*d^2+B*a^6*b*d^3+3*B*a^4*b^3*d^3-
B*a^3*b^4*c^3+3*B*a^3*b^4*c*d^2-9*B*a^2*b^5*c^2*d+6*B*a^2*b^5*d^3+3*B*a*b^
6*c^3-9*B*a*b^6*c*d^2+3*B*b^7*c^2*d-3*C*a^7*d^3+3*C*a^6*b*c*d^2-9*C*a^5*b^
2*d^3+9*C*a^4*b^3*c*d^2+3*C*a^3*b^4*c^2*d-10*C*a^3*b^4*d^3-3*C*a^2*b^5*c^3
+18*C*a^2*b^5*c*d^2-9*C*a*b^6*c^2*d+C*b^7*c^3)/b^4/(a^2+b^2)^3*ln(a+b*tan(
f*x+e))+1/(a^2+b^2)^3*(1/2*(3*A*a^3*c^2*d-A*a^3*d^3-3*A*a^2*b*c^3+9*A*a^2*
b*c*d^2-9*A*a*b^2*c^2*d+3*A*a*b^2*d^3+A*b^3*c^3-3*A*b^3*c*d^2+B*a^3*c^3-3*
B*a^3*c*d^2+9*B*a^2*b*c^2*d-3*B*a^2*b*d^3-3*B*a*b^2*c^3+9*B*a*b^2*c*d^2-3*
B*b^3*c^2*d+B*b^3*d^3-3*C*a^3*c^2*d+C*a^3*d^3+3*C*a^2*b*c^3-9*C*a^2*b*c*d^
2+9*C*a*b^2*c^2*d-3*C*a*b^2*d^3-C*b^3*c^3+3*C*b^3*c*d^2)*ln(1+tan(f*x+e)^2
)+(A*a^3*c^3-3*A*a^3*c*d^2+9*A*a^2*b*c^2*d-3*A*a^2*b*d^3-3*A*a*b^2*c^3+...
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2549 vs. 2(794) = 1588.

Time = 1.44 (sec) , antiderivative size = 2549, normalized size of antiderivative = 3.19

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(C*a^6*b^3 + 3*C*a^4*b^5 + 3*C*a^2*b^7 + C*b^9)*d^3*tan(f*x + e)^3
- (3*C*a^4*b^5 - 5*B*a^3*b^6 + (7*A - 3*C)*a^2*b^7 + B*a*b^8 + A*b^9)*c^3
+ 3*(C*a^5*b^4 - 3*B*a^4*b^5 + 5*(A - C)*a^3*b^6 + 3*B*a^2*b^7 - A*a*b^8)*
c^2*d + 3*(C*a^6*b^3 + B*a^5*b^4 - (3*A - 7*C)*a^4*b^5 - 5*B*a^3*b^6 + 3*A
*a^2*b^7)*c*d^2 - (3*C*a^7*b^2 - B*a^6*b^3 - (A - 9*C)*a^5*b^4 - 7*B*a^4*b
^5 + 5*A*a^3*b^6)*d^3 + 2*(((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*
b^6 - B*a^2*b^7)*c^3 - 3*(B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A
 - C)*a^2*b^7)*c^2*d - 3*((A - C)*a^5*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^
6 - B*a^2*b^7)*c*d^2 + (B*a^5*b^4 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A -
 C)*a^2*b^7)*d^3)*f*x + ((C*a^4*b^5 - 3*B*a^3*b^6 + 5*(A - C)*a^2*b^7 + 3*
B*a*b^8 - A*b^9)*c^3 + 3*(C*a^5*b^4 + B*a^4*b^5 - (3*A - 7*C)*a^3*b^6 - 5*
B*a^2*b^7 + 3*A*a*b^8)*c^2*d - 3*(3*C*a^6*b^3 - B*a^5*b^4 - (A - 9*C)*a^4*
b^5 - 7*B*a^3*b^6 + 5*A*a^2*b^7)*c*d^2 + (9*C*a^7*b^2 - 3*B*a^6*b^3 + (A +
 23*C)*a^5*b^4 - 9*B*a^4*b^5 + (7*A + 12*C)*a^3*b^6 + 4*C*a*b^8)*d^3 + 2*(
((A - C)*a^3*b^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c^3 - 3*(B*a^3*b
^6 - 3*(A - C)*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*c^2*d - 3*((A - C)*a^3*b
^6 + 3*B*a^2*b^7 - 3*(A - C)*a*b^8 - B*b^9)*c*d^2 + (B*a^3*b^6 - 3*(A - C)
*a^2*b^7 - 3*B*a*b^8 + (A - C)*b^9)*d^3)*f*x)*tan(f*x + e)^2 - ((B*a^5*b^4
 - 3*(A - C)*a^4*b^5 - 3*B*a^3*b^6 + (A - C)*a^2*b^7)*c^3 + 3*((A - C)*a^5
*b^4 + 3*B*a^4*b^5 - 3*(A - C)*a^3*b^6 - B*a^2*b^7)*c^2*d - 3*(C*a^8*b ...
```

3.69.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*
x+e))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.40

$$\int \frac{(c+d\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*
b^2 - B*b^3)*c^3 - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c
^2*d - 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d^2 + (B*a^
3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d^3)*(f*x + e)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b^6 +
(A - C)*b^7)*c^3 + 3*((A - C)*a^3*b^4 + 3*B*a^2*b^5 - 3*(A - C)*a*b^6 - B*
b^7)*c^2*d - 3*(C*a^6*b + 3*C*a^4*b^3 + B*a^3*b^4 - 3*(A - 2*C)*a^2*b^5 -
3*B*a*b^6 + A*b^7)*c*d^2 + (3*C*a^7 - B*a^6*b + 9*C*a^5*b^2 - 3*B*a^4*b^3
- (A - 10*C)*a^3*b^4 - 6*B*a^2*b^5 + 3*A*a*b^6)*d^3)*log(b*tan(f*x + e) +
a)/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10) + ((B*a^3 - 3*(A - C)*a^2*b -
3*B*a*b^2 + (A - C)*b^3)*c^3 + 3*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^
2 - B*b^3)*c^2*d - 3*(B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c
*d^2 - ((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*d^3)*log(tan(f*
x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((C*a^4*b^3 - 3*B*a^3*
b^4 + (5*A - 3*C)*a^2*b^5 + B*a*b^6 + A*b^7)*c^3 + 3*(C*a^5*b^2 + B*a^4*b^
3 - (3*A - 5*C)*a^3*b^4 - 3*B*a^2*b^5 + A*a*b^6)*c^2*d - 3*(3*C*a^6*b - B*
a^5*b^2 - (A - 7*C)*a^4*b^3 - 5*B*a^3*b^4 + 3*A*a^2*b^5)*c*d^2 + (5*C*a^7
- 3*B*a^6*b + (A + 9*C)*a^5*b^2 - 7*B*a^4*b^3 + 5*A*a^3*b^4)*d^3 - 2*((B*a
^2*b^5 - 2*(A - C)*a*b^6 - B*b^7)*c^3 - 3*(C*a^4*b^3 - (A - 3*C)*a^2*b^5 -
 2*B*a*b^6 + A*b^7)*c^2*d + 3*(2*C*a^5*b^2 - B*a^4*b^3 + 4*C*a^3*b^4 - ...
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2441 vs. 2(794) = 1588.

Time = 1.36 (sec) , antiderivative size = 2441, normalized size of antiderivative = 3.06

$$\int \frac{(c+d\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
1/2*(2*C*d^3*tan(f*x + e)/b^3 + 2*(A*a^3*c^3 - C*a^3*c^3 + 3*B*a^2*b*c^3 -
 3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 - B*b^3*c^3 - 3*B*a^3*c^2*d + 9*A*a^2*b*c^2
*d - 9*C*a^2*b*c^2*d + 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3
*A*a^3*c*d^2 + 3*C*a^3*c*d^2 - 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b
^2*c*d^2 + 3*B*b^3*c*d^2 + B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 - 3*B
*a*b^2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) + (B*a^3*c^3 - 3*A*a^2*b*c^3 + 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 + A*b
^3*c^3 - C*b^3*c^3 + 3*A*a^3*c^2*d - 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d - 9*A
*a*b^2*c^2*d + 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 + 9*A*a^2*b
*c*d^2 - 9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 - 3*A*b^3*c*d^2 + 3*C*b^3*c*d^2
 - A*a^3*d^3 + C*a^3*d^3 - 3*B*a^2*b*d^3 + 3*A*a*b^2*d^3 - 3*C*a*b^2*d^3 +
 B*b^3*d^3)*log(tan(f*x + e)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
2*(B*a^3*b^4*c^3 - 3*A*a^2*b^5*c^3 + 3*C*a^2*b^5*c^3 - 3*B*a*b^6*c^3 + A*b
^7*c^3 - C*b^7*c^3 + 3*A*a^3*b^4*c^2*d - 3*C*a^3*b^4*c^2*d + 9*B*a^2*b^5*c
^2*d - 9*A*a*b^6*c^2*d + 9*C*a*b^6*c^2*d - 3*B*b^7*c^2*d - 3*C*a^6*b*c*d^2
 - 9*C*a^4*b^3*c*d^2 - 3*B*a^3*b^4*c*d^2 + 9*A*a^2*b^5*c*d^2 - 18*C*a^2*b^
5*c*d^2 + 9*B*a*b^6*c*d^2 - 3*A*b^7*c*d^2 + 3*C*a^7*d^3 - B*a^6*b*d^3 + 9*
C*a^5*b^2*d^3 - 3*B*a^4*b^3*d^3 - A*a^3*b^4*d^3 + 10*C*a^3*b^4*d^3 - 6*B*a
^2*b^5*d^3 + 3*A*a*b^6*d^3)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4 + 3*a^4*
b^6 + 3*a^2*b^8 + b^10) + (3*B*a^3*b^6*c^3*tan(f*x + e)^2 - 9*A*a^2*b^7...
```

3.69.9 Mupad [B] (verification not implemented)

Time = 17.39 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.47

$$\int \frac{(c+d\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{\ln(a+b\tan(e+fx))\,(b^3(3Ba^4d^3+9Cca^4d^2)-b^6(3Aad^3-3Bac^3-9Aac^2d+9Bacd^2+9C\cdots}{\cdots}$$
$$+\frac{\ln(\tan(e+fx)+1\mathrm{i})\,(Ac^3+Ad^3\,1\mathrm{i}-Bc^3\,1\mathrm{i}+Bd^3-Cc^3-Cd^3\,1\mathrm{i}-3Acd^2-Ac^2d\,3\mathrm{i}+Bcd^2\,3\mathrm{i}\cdots}{2f\,(-a^3\,1\mathrm{i}-3a^2b+ab^2\,3\mathrm{i}+b^3)}$$
$$-\frac{\frac{\tan(e+fx)\,(Bb^6c^3+3Ca^6d^3+2Aab^5c^3-2Ba^5bd^3-2Cab^5c^3+3Ab^6c^2d+3Aa^2b^4d^3+Aa^4b^2d^3-Ba^2b^4c^3-4Ba^3b^3d^3+5Ca^4b^2d^3\cdots}{a^4+2a^2b^2+\cdots}}{\cdots}$$
$$+\frac{\ln(\tan(e+fx)-\mathrm{i})\,(Ad^3-Bc^3-Cd^3-3Ac^2d+3Bcd^2+3Cc^2d+Ac^3\,1\mathrm{i}+Bd^3\,1\mathrm{i}-Cc^3\,1\mathrm{i}-\cdots}{2f\,(-a^3-a^2b\,3\mathrm{i}+3ab^2+b^3\,1\mathrm{i})}$$
$$+\frac{Cd^3\tan(e+fx)}{b^3f}$$

input
```
int(((c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(a +
b*tan(e + f*x))^3,x)
```

output
```
(log(tan(e + f*x) + 1i)*(A*c^3 + A*d^3*1i - B*c^3*1i + B*d^3 - C*c^3 - C*d
^3*1i - 3*A*c*d^2 - A*c^2*d*3i + B*c*d^2*3i - 3*B*c^2*d + 3*C*c*d^2 + C*c^
2*d*3i))/(2*f*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) - ((tan(e + f*x)*(B*b^6
*c^3 + 3*C*a^6*d^3 + 2*A*a*b^5*c^3 - 2*B*a^5*b*d^3 - 2*C*a*b^5*c^3 + 3*A*b
^6*c^2*d + 3*A*a^2*b^4*d^3 + A*a^4*b^2*d^3 - B*a^2*b^4*c^3 - 4*B*a^3*b^3*d
^3 + 5*C*a^4*b^2*d^3 - 3*A*a^2*b^4*c^2*d + 9*B*a^2*b^4*c*d^2 + 3*B*a^4*b^2
*c*d^2 + 9*C*a^2*b^4*c^2*d - 12*C*a^3*b^3*c*d^2 + 3*C*a^4*b^2*c^2*d - 6*A*
a*b^5*c*d^2 - 6*B*a*b^5*c^2*d - 6*C*a^5*b*c*d^2))/(a^4 + b^4 + 2*a^2*b^2)
+ (A*b^7*c^3 + 5*C*a^7*d^3 + B*a*b^6*c^3 - 3*B*a^6*b*d^3 + 5*A*a^2*b^5*c^3
 + 5*A*a^3*b^4*d^3 + A*a^5*b^2*d^3 - 3*B*a^3*b^4*c^3 - 7*B*a^4*b^3*d^3 - 3
*C*a^2*b^5*c^3 + C*a^4*b^3*c^3 + 9*C*a^5*b^2*d^3 - 9*A*a^2*b^5*c*d^2 - 9*A
*a^3*b^4*c^2*d + 3*A*a^4*b^3*c*d^2 - 9*B*a^2*b^5*c^2*d + 15*B*a^3*b^4*c*d^
2 + 3*B*a^4*b^3*c^2*d + 3*B*a^5*b^2*c*d^2 + 15*C*a^3*b^4*c^2*d - 21*C*a^4*
b^3*c*d^2 + 3*C*a^5*b^2*c^2*d + 3*A*a*b^6*c^2*d - 9*C*a^6*b*c*d^2)/(2*b*(a
^4 + b^4 + 2*a^2*b^2)))/(f*(a^2*b^3 + b^5*tan(e + f*x)^2 + 2*a*b^4*tan(e +
 f*x))) + (log(a + b*tan(e + f*x))*(b^3*(3*B*a^4*d^3 + 9*C*a^4*c*d^2) - b^
6*(3*A*a*d^3 - 3*B*a*c^3 - 9*A*a*c^2*d + 9*B*a*c*d^2 + 9*C*a*c^2*d) + b^5*
(3*A*a^2*c^3 + 6*B*a^2*d^3 - 3*C*a^2*c^3 - 9*A*a^2*c*d^2 - 9*B*a^2*c^2*d +
 18*C*a^2*c*d^2) + b^4*(A*a^3*d^3 - B*a^3*c^3 - 10*C*a^3*d^3 - 3*A*a^3*c^2
*d + 3*B*a^3*c*d^2 + 3*C*a^3*c^2*d) + b*(B*a^6*d^3 + 3*C*a^6*c*d^2) + b...
```

3.69. $\int \frac{(c+d\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.70 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

3.70.1 Optimal result

Integrand size = 45, antiderivative size = 337

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)-3a^2b(Bc-(A-C)d)+b^3(Bc-(A-C)d))\,x}{c^2+d^2}$$
$$-\frac{(3a^2b(Ac-cC+Bd)-b^3(Ac-cC+Bd)+a^3(Bc-(A-C)d)-3ab^2(Bc-(A-C)d))\log(\cos(e+}{(c^2+d^2)\,f}$$
$$-\frac{(bc-ad)^3\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{d^4\left(c^2+d^2\right)f}$$
$$+\frac{b(b(Ab+aB-bC)d^2+(bc-ad)(bcC-bBd-aCd))\tan(e+fx)}{d^3f}$$
$$-\frac{(bcC-bBd-aCd)(a+b\tan(e+fx))^2}{2d^2f}+\frac{C(a+b\tan(e+fx))^3}{3df}$$

output
```
(a^3*(A*c+B*d-C*c)-3*a*b^2*(A*c+B*d-C*c)-3*a^2*b*(B*c-(A-C)*d)+b^3*(B*c-(A
-C)*d))*x/(c^2+d^2)-(3*a^2*b*(A*c+B*d-C*c)-b^3*(A*c+B*d-C*c)+a^3*(B*c-(A-C
)*d)-3*a*b^2*(B*c-(A-C)*d))*ln(cos(f*x+e))/(c^2+d^2)/f-(-a*d+b*c)^3*(A*d^2
-B*c*d+C*c^2)*ln(c+d*tan(f*x+e))/d^4/(c^2+d^2)/f+b*(b*(A*b+B*a-C*b)*d^2+(-
a*d+b*c)*(-B*b*d-C*a*d+C*b*c))*tan(f*x+e)/d^3/f-1/2*(-B*b*d-C*a*d+C*b*c)*(
a+b*tan(f*x+e))^2/d^2/f+1/3*C*(a+b*tan(f*x+e))^3/d/f
```

3.70.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.86 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$= \frac{\frac{3(a+ib)^3(-iA+B+iC)d^2\log(i-\tan(e+fx))}{c+id}+\frac{3(a-ib)^3(iA+B-iC)d^2\log(i+\tan(e+fx))}{c-id}+\frac{6(-bc+ad)^3(c^2C-Bcd+Ad^2)\log(c+d\tan(e+f}{d^2(c^2+d^2)}}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x]),x]
```

output
```
((3*(a + I*b)^3*((-I)*A + B + I*C)*d^2*Log[I - Tan[e + f*x]])/(c + I*d) +
(3*(a - I*b)^3*(I*A + B - I*C)*d^2*Log[I + Tan[e + f*x]])/(c - I*d) + (6*(
-(b*c) + a*d)^3*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2
 + d^2)) + 6*b^2*(A*b + a*B - b*C)*d*Tan[e + f*x] - (6*b*(b*c - a*d)*(-(b*
c*C) + b*B*d + a*C*d)*Tan[e + f*x])/d - 3*(b*c*C - b*B*d - a*C*d)*(a + b*T
an[e + f*x])^2 + 2*C*d*(a + b*Tan[e + f*x])^3)/(6*d^2*f)
```

3.70.3 Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{c+d\tan(e+fx)}dx$$
$$\Big\downarrow \ 4130$$

$$\frac{\int -\frac{3(a+b\tan(e+fx))^2\left((bcC-adC-bBd)\tan^2(e+fx)-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{3d}+\frac{C(a+b\tan(e+fx))^3}{3df}$$

$$\Big\downarrow \; 27$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{\int \frac{(a+b\tan(e+fx))^2\left((bcC-adC-bBd)\tan^2(e+fx)-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{\int \frac{(a+b\tan(e+fx))^2\left((bcC-adC-bBd)\tan(e+fx)^2-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \; 4130$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{\frac{\int \frac{2(a+b\tan(e+fx))\left(-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-bBd)\right)\tan^2(e+fx)-\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)}{c+d\tan(e+fx)}dx}{2d}+}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{\frac{\int \frac{(a+b\tan(e+fx))\left(-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-bBd)\right)\tan^2(e+fx)-\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)}{c+d\tan(e+fx)}dx}{d}+}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{\frac{\int \frac{(a+b\tan(e+fx))\left(-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-bBd)\right)\tan(e+fx)^2-\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)}{c+d\tan(e+fx)}dx}{d}+}{d}$$

$$\Big\downarrow \; 4120$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-\frac{-\frac{\int -\frac{c\left(Cc^2-Bdc+(A-C)d^2\right)b^3-3acd(cC-Bd)b^2+3a^2cCd^2b-a^3Ad^3-\left(-\left(\left(Cc^3-Bdc^2+(A-C)d^2c+Bd^3\right)b^3\right)+3ad\left(Cc^2-Bdc+(A-C)d^2\right)b^2-3a^2d^2(cC-Bd)b\right.}{c+d\tan(e+fx)}}{d}}{d}$$

3.70. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

$\downarrow$ 25

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{\int \frac{c\left(Cc^2-Bdc+(A-C)d^2\right)b^3-3acd(cC-Bd)b^2+3a^2cCd^2b-a^3Ad^3-\left(-\left(\left(Cc^3-Bdc^2+(A-C)d^2c+Bd^3\right)b^3\right)+3ad\left(Cc^2-Bdc+(A-C)d^2\right)b^2-3a^2d^2(cC-Bd)b+a\cdots}{c+d\tan(e+fx)}}{d}}{d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{\int \frac{c\left(Cc^2-Bdc+(A-C)d^2\right)b^3-3acd(cC-Bd)b^2+3a^2cCd^2b-a^3Ad^3-\left(-\left(\left(Cc^3-Bdc^2+(A-C)d^2c+Bd^3\right)b^3\right)+3ad\left(Cc^2-Bdc+(A-C)d^2\right)b^2-3a^2d^2(cC-Bd)b+a\cdots}{c+d\tan(e+fx)}}{d}}{d}$$

$\downarrow$ 4109

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{-\frac{d^3\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)\int\tan(e+fx)dx}{c^2+d^2}+\frac{(bc-ad)^3\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan^2(e+fx)+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d^3x\left(a^3(A\cdots\right.}{\cdots}}{d}}{d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{-\frac{d^3\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)\int\tan(e+fx)dx}{c^2+d^2}+\frac{(bc-ad)^3\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d^3x\left(a^3(A\cdots\right.}{\cdots}}{d}}{d}$$

$\downarrow$ 3956

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{\frac{(bc-ad)^3\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{d^3\log(\cos(e+fx))\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f\left(c^2+d^2\right)}-\frac{d^3x\left(a^3(Ac\cdots\right.}{\cdots}}{d}}{d}$$

$\downarrow$ 4100

$$\frac{C(a+b\tan(e+fx))^3}{3df} - \frac{\frac{\frac{(bc-ad)^3\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df\left(c^2+d^2\right)}+\frac{d^3\log(\cos(e+fx))\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f\left(c^2+d^2\right)}-\cdots}{d}}{d}$$

3.70. $\displaystyle\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

$$\Big\downarrow 16$$

$$\frac{C(a+b\tan(e+fx))^3}{3df}-$$

$$\frac{\frac{\frac{d^3\log(\cos(e+fx))\left(a^3(Bc-d(A-C))+3a^2b(Ac+Bd-cC)-3ab^2(Bc-d(A-C))-b^3(Ac+Bd-cC)\right)}{f(c^2+d^2)}-\frac{d^3x\left(a^3(Ac+Bd-cC)-3a^2b(Bc-d(A-C))-3ab^2(Ac+Bd-cC)\right.}{c^2+d^2}}{d}}{d}$$

input
```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x]),x]
```

output
```
(C*(a + b*Tan[e + f*x])^3)/(3*d*f) - (((b*c*C - b*B*d - a*C*d)*(a + b*Tan[
e + f*x])^2)/(2*d*f) + ((-((d^3*(a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*
C + B*d) - 3*a^2*b*(B*c - (A - C)*d) + b^3*(B*c - (A - C)*d))*x)/(c^2 + d^
2)) + (d^3*(3*a^2*b*(A*c - c*C + B*d) - b^3*(A*c - c*C + B*d) + a^3*(B*c -
 (A - C)*d) - 3*a*b^2*(B*c - (A - C)*d))*Log[Cos[e + f*x]])/((c^2 + d^2)*f
) + ((b*c - a*d)^3*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d*(c^
2 + d^2)*f))/d - (b*(b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - b*B*d
- a*C*d))*Tan[e + f*x])/(d*f))/d)/d
```

3.70.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.70. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

3.70.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.48

method	result
norman	$\frac{(A a^3c+3A a^2bd-3Aa b^2c-A b^3d+B a^3d-3B a^2bc-3Ba b^2d+B b^3c-C a^3c-3C a^2bd+3Ca b^2c+C b^3d)x}{c^2+d^2}+\frac{(A b^2d^2+$
derivativedivides	$\frac{\frac{b\left(\frac{C b^2d^2\tan(fx+e)^3}{3}+\frac{B b^2d^2\tan(fx+e)^2}{2}+\frac{3Cab d^2\tan(fx+e)^2}{2}-\frac{C b^2cd\tan(fx+e)^2}{2}+\tan(fx+e)A b^2d^2+3\tan(fx+e)Bab d^2-\tan\right.}{d^3}}{}$
default	$\frac{\frac{b\left(\frac{C b^2d^2\tan(fx+e)^3}{3}+\frac{B b^2d^2\tan(fx+e)^2}{2}+\frac{3Cab d^2\tan(fx+e)^2}{2}-\frac{C b^2cd\tan(fx+e)^2}{2}+\tan(fx+e)A b^2d^2+3\tan(fx+e)Bab d^2-\tan\right.}{d^3}}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
(A*a^3*c+3*A*a^2*b*d-3*A*a*b^2*c-A*b^3*d+B*a^3*d-3*B*a^2*b*c-3*B*a*b^2*d+B
*b^3*c-C*a^3*c-3*C*a^2*b*d+3*C*a*b^2*c+C*b^3*d)/(c^2+d^2)*x+(A*b^2*d^2+3*B
*a*b*d^2-B*b^2*c*d+3*C*a^2*d^2-3*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)*b/f/d^3*ta
n(f*x+e)+1/3*C*b^3/d/f*tan(f*x+e)^3+1/2*b^2*(B*b*d+3*C*a*d-C*b*c)/d^2/f*ta
n(f*x+e)^2+(A*a^3*d^5-3*A*a^2*b*c*d^4+3*A*a*b^2*c^2*d^3-A*b^3*c^3*d^2-B*a^
3*c*d^4+3*B*a^2*b*c^2*d^3-3*B*a*b^2*c^3*d^2+B*b^3*c^4*d+C*a^3*c^2*d^3-3*C*
a^2*b*c^3*d^2+3*C*a*b^2*c^4*d-C*b^3*c^5)/(c^2+d^2)/d^4/f*ln(c+d*tan(f*x+e)
)-1/2*(A*a^3*d-3*A*a^2*b*c-3*A*a*b^2*d+A*b^3*c-B*a^3*c-3*B*a^2*b*d+3*B*a*b
^2*c+B*b^3*d-C*a^3*d+3*C*a^2*b*c+3*C*a*b^2*d-C*b^3*c)/f/(c^2+d^2)*ln(1+tan
(f*x+e)^2)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.86

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$=\frac{2\left(Cb^3c^2d^3+Cb^3d^5\right)\tan\left(fx+e\right)^3+6\left(\left(\left(A-C\right)a^3-3\,Ba^2b-3\left(A-C\right)ab^2+Bb^3\right)cd^4+\left(Ba^3+3\left(A-\right.\right.\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
1/6*(2*(C*b^3*c^2*d^3 + C*b^3*d^5)*tan(f*x + e)^3 + 6*(((A - C)*a^3 - 3*B*
a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^4 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*
b^2 - (A - C)*b^3)*d^5)*f*x - 3*(C*b^3*c^3*d^2 + C*b^3*c*d^4 - (3*C*a*b^2
+ B*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*d^5)*tan(f*x + e)^2 - 3*(C*b^3*c^5
- A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a^2*b)*c*
d^4)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 +
 1)) + 3*(C*b^3*c^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (3*C*a^2*b + 3
*B*a*b^2 + (A - C)*b^3)*c*d^4 - (C*a^3 + 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b
^3)*d^5)*log(1/(tan(f*x + e)^2 + 1)) + 6*(C*b^3*c^4*d - (3*C*a*b^2 + B*b^3
)*c^3*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 - (3*C*a*b^2 + B*b^3)*
c*d^4 + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^3)*d^5)*tan(f*x + e))/((c^2*d^4
 + d^6)*f)
```

3.70.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.49 (sec) , antiderivative size = 7096, normalized size of antiderivative = 21.06

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e)),x)
```

output
```
Piecewise((zoo*x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq
(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a**3*x + 3*A*a**2*b*log(tan(e + f*x)**2
 + 1)/(2*f) - 3*A*a*b**2*x + 3*A*a*b**2*tan(e + f*x)/f - A*b**3*log(tan(e
+ f*x)**2 + 1)/(2*f) + A*b**3*tan(e + f*x)**2/(2*f) + B*a**3*log(tan(e + f
*x)**2 + 1)/(2*f) - 3*B*a**2*b*x + 3*B*a**2*b*tan(e + f*x)/f - 3*B*a*b**2*
log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*tan(e + f*x)**2/(2*f) + B*b**3
*x + B*b**3*tan(e + f*x)**3/(3*f) - B*b**3*tan(e + f*x)/f - C*a**3*x + C*a
**3*tan(e + f*x)/f - 3*C*a**2*b*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a**2*
b*tan(e + f*x)**2/(2*f) + 3*C*a*b**2*x + C*a*b**2*tan(e + f*x)**3/f - 3*C*
a*b**2*tan(e + f*x)/f + C*b**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*tan
(e + f*x)**4/(4*f) - C*b**3*tan(e + f*x)**2/(2*f))/c, Eq(d, 0)), (3*I*A*a*
*3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 3*A*a**3*f*x/(6*d*f*t
an(e + f*x) - 6*I*d*f) + 3*I*A*a**3/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*A*a
**2*b*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*A*a**2*b*f*x/(
6*d*f*tan(e + f*x) - 6*I*d*f) - 9*A*a**2*b/(6*d*f*tan(e + f*x) - 6*I*d*f)
+ 9*I*A*a*b**2*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*A*a*b**
2*f*x/(6*d*f*tan(e + f*x) - 6*I*d*f) + 9*A*a*b**2*log(tan(e + f*x)**2 + 1)
*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*A*a*b**2*log(tan(e + f*
x)**2 + 1)/(6*d*f*tan(e + f*x) - 6*I*d*f) - 9*I*A*a*b**2/(6*d*f*tan(e + f*
x) - 6*I*d*f) - 9*A*b**3*f*x*tan(e + f*x)/(6*d*f*tan(e + f*x) - 6*I*d*f...
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$

$$= \frac{6\left(\left(\left((A-C)a^3-3\,Ba^2b-3\,(A-C)ab^2+Bb^3\right)c+\left(Ba^3+3\,(A-C)a^2b-3\,Bab^2-(A-C)b^3\right)d\right)(fx+e)}{c^2+d^2} - \frac{6\left(Cb^3c^5-Aa^3d^5-\left(3\,Cab^2+Bb^3\right)c^4d+(3\,C\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/6*(6*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c + (B*a^3 + 3
*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d)*(f*x + e)/(c^2 + d^2) - 6*(C*
b^3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d + (3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^3 + (B*a^3 + 3*A*a
^2*b)*c*d^4)*log(d*tan(f*x + e) + c)/(c^2*d^4 + d^6) + 3*((B*a^3 + 3*(A -
C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c - ((A - C)*a^3 - 3*B*a^2*b - 3*(A -
C)*a*b^2 + B*b^3)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + (2*C*b^3*d^2*ta
n(f*x + e)^3 - 3*(C*b^3*c*d - (3*C*a*b^2 + B*b^3)*d^2)*tan(f*x + e)^2 + 6*
(C*b^3*c^2 - (3*C*a*b^2 + B*b^3)*c*d + (3*C*a^2*b + 3*B*a*b^2 + (A - C)*b^
3)*d^2)*tan(f*x + e))/d^3)/f
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$

$$= \frac{6\,(Aa^3c-Ca^3c-3\,Ba^2bc-3\,Aab^2c+3\,Cab^2c+Bb^3c+Ba^3d+3\,Aa^2bd-3\,Ca^2bd-3\,Bab^2d-Ab^3d+Cb^3d)\,(fx+e)}{c^2+d^2}+\frac{3\,(Ba^3c+3\,Aa^2bc-3\,Ca^2bc-}{}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/6*(6*(A*a^3*c - C*a^3*c - 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c + B*b^
3*c + B*a^3*d + 3*A*a^2*b*d - 3*C*a^2*b*d - 3*B*a*b^2*d - A*b^3*d + C*b^3*
d)*(f*x + e)/(c^2 + d^2) + 3*(B*a^3*c + 3*A*a^2*b*c - 3*C*a^2*b*c - 3*B*a*
b^2*c - A*b^3*c + C*b^3*c - A*a^3*d + C*a^3*d + 3*B*a^2*b*d + 3*A*a*b^2*d
- 3*C*a*b^2*d - B*b^3*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 6*(C*b^3*c^
5 - 3*C*a*b^2*c^4*d - B*b^3*c^4*d + 3*C*a^2*b*c^3*d^2 + 3*B*a*b^2*c^3*d^2
+ A*b^3*c^3*d^2 - C*a^3*c^2*d^3 - 3*B*a^2*b*c^2*d^3 - 3*A*a*b^2*c^2*d^3 +
B*a^3*c*d^4 + 3*A*a^2*b*c*d^4 - A*a^3*d^5)*log(abs(d*tan(f*x + e) + c))/(c
^2*d^4 + d^6) + (2*C*b^3*d^2*tan(f*x + e)^3 - 3*C*b^3*c*d*tan(f*x + e)^2 +
 9*C*a*b^2*d^2*tan(f*x + e)^2 + 3*B*b^3*d^2*tan(f*x + e)^2 + 6*C*b^3*c^2*t
an(f*x + e) - 18*C*a*b^2*c*d*tan(f*x + e) - 6*B*b^3*c*d*tan(f*x + e) + 18*
C*a^2*b*d^2*tan(f*x + e) + 18*B*a*b^2*d^2*tan(f*x + e) + 6*A*b^3*d^2*tan(f
*x + e) - 6*C*b^3*d^2*tan(f*x + e))/d^3)/f
```

3.70.9 Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$

$$= \frac{\tan(e+f\,x)^2\left(\frac{B\,b^3+3\,C\,a\,b^2}{2\,d}-\frac{C\,b^3\,c}{2\,d^2}\right)}{f} - \frac{\tan(e+f\,x)\left(\frac{c\left(\frac{B\,b^3+3\,C\,a\,b^2}{d}-\frac{C\,b^3\,c}{d^2}\right)}{d}-\frac{3\,C\,a^2\,b+3\,B\,a\,b^2+A\,b^3}{d}+\frac{C\,b^3}{d}\right)}{f} - \frac{\ln(c+d\tan(e+f\,x))\,\left(d^4\,(B\,c\,a^3+3\,A\,b\,c\,a^2)-d^3\,(C\,a^3\,c^2+3\,B\,a^2\,b\,c^2+3\,A\,a\,b^2\,c^2)+d^2\,(3\,C\,a^2\,b\,c^3\right.}{f\,(c^2\,d^4+d^6)} - \frac{\ln(\tan(e+f\,x)+1\mathrm{i})\,(A\,a^3+A\,b^3\,1\mathrm{i}-B\,a^3\,1\mathrm{i}+B\,b^3-C\,a^3-C\,b^3\,1\mathrm{i}-3\,A\,a\,b^2-A\,a^2\,b\,3\mathrm{i}+B\,a\,b^2\,3\mathrm{i}}{2\,f\,(d+c\,1\mathrm{i})} - \frac{\ln(\tan(e+f\,x)-\mathrm{i})\,(A\,b^3-B\,a^3-C\,b^3-3\,A\,a^2\,b+3\,B\,a\,b^2+3\,C\,a^2\,b+A\,a^3\,1\mathrm{i}+B\,b^3\,1\mathrm{i}-C\,a^3\,1\mathrm{i}-}{2\,f\,(c+d\,1\mathrm{i})} + \frac{C\,b^3\tan(e+f\,x)^3}{3\,d\,f}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x)),x)
```

output
```
(tan(e + f*x)^2*((B*b^3 + 3*C*a*b^2)/(2*d) - (C*b^3*c)/(2*d^2)))/f - (tan(
e + f*x)*((c*((B*b^3 + 3*C*a*b^2)/d - (C*b^3*c)/d^2))/d - (A*b^3 + 3*B*a*b
^2 + 3*C*a^2*b)/d + (C*b^3)/d))/f - (log(c + d*tan(e + f*x))*(d^4*(B*a^3*c
 + 3*A*a^2*b*c) - d^3*(C*a^3*c^2 + 3*A*a*b^2*c^2 + 3*B*a^2*b*c^2) + d^2*(A
*b^3*c^3 + 3*B*a*b^2*c^3 + 3*C*a^2*b*c^3) - d*(B*b^3*c^4 + 3*C*a*b^2*c^4)
- A*a^3*d^5 + C*b^3*c^5))/(f*(d^6 + c^2*d^4)) - (log(tan(e + f*x) + 1i)*(A
*a^3 + A*b^3*1i - B*a^3*1i + B*b^3 - C*a^3 - C*b^3*1i - 3*A*a*b^2 - A*a^2*
b*3i + B*a*b^2*3i - 3*B*a^2*b + 3*C*a*b^2 + C*a^2*b*3i))/(2*f*(c*1i + d))
- (log(tan(e + f*x) - 1i)*(A*a^3*1i + A*b^3 - B*a^3 + B*b^3*1i - C*a^3*1i
- C*b^3 - A*a*b^2*3i - 3*A*a^2*b + 3*B*a*b^2 - B*a^2*b*3i + C*a*b^2*3i + 3
*C*a^2*b))/(2*f*(c + d*1i)) + (C*b^3*tan(e + f*x)^3)/(3*d*f)
```

3.71 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

3.71.1 Optimal result

Integrand size = 45, antiderivative size = 236

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx\\
&= \frac{\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)-2ab(Bc-(A-C)d)\right)x}{c^2+d^2}\\
&\quad-\frac{\left(2ab(Ac-cC+Bd)+a^2(Bc-(A-C)d)-b^2(Bc-(A-C)d)\right)\log(\cos(e+fx))}{(c^2+d^2)\,f}\\
&\quad+\frac{(bc-ad)^2\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{d^3\,(c^2+d^2)\,f}\\
&\quad-\frac{b(bcC-bBd-aCd)\tan(e+fx)}{d^2f}+\frac{C(a+b\tan(e+fx))^2}{2df}\end{aligned}$$

output
```
(a^2*(A*c+B*d-C*c)-b^2*(A*c+B*d-C*c)-2*a*b*(B*c-(A-C)*d))*x/(c^2+d^2)-(2*a
*b*(A*c+B*d-C*c)+a^2*(B*c-(A-C)*d)-b^2*(B*c-(A-C)*d))*ln(cos(f*x+e))/(c^2+
d^2)/f+(-a*d+b*c)^2*(A*d^2-B*c*d+C*c^2)*ln(c+d*tan(f*x+e))/d^3/(c^2+d^2)/f
-b*(-B*b*d-C*a*d+C*b*c)*tan(f*x+e)/d^2/f+1/2*C*(a+b*tan(f*x+e))^2/d/f
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$= \frac{\frac{(a+ib)^2(-iA+B+iC)d\log(i-\tan(e+fx))}{c+id}+\frac{(a-ib)^2(iA+B-iC)d\log(i+\tan(e+fx))}{c-id}+\frac{2(bc-ad)^2(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+}{2df}$$

input
```
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x]),x]
```

output
```
(((a + I*b)^2*((-I)*A + B + I*C)*d*Log[I - Tan[e + f*x]])/(c + I*d) + ((a
- I*b)^2*(I*A + B - I*C)*d*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(b*c - a*
d)^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) +
(2*b*(-(b*c*C) + b*B*d + a*C*d)*Tan[e + f*x])/d + C*(a + b*Tan[e + f*x])^2
)/(2*d*f)
```

3.71.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4130, 27, 3042, 4120, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{c+d\tan(e+fx)}dx$$
$$\Big\downarrow \text{4130}$$
$$\frac{\int -\frac{2(a+b\tan(e+fx))\left((bcC-adC-bBd)\tan^2(e+fx)-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{2d}+\frac{C(a+b\tan(e+fx))^2}{2df}$$

$$\Big\downarrow \ 27$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\int \frac{(a+b\tan(e+fx))\left((bcC-adC-bBd)\tan^2(e+fx)-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\int \frac{(a+b\tan(e+fx))\left((bcC-adC-bBd)\tan(e+fx)^2-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd\right)}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \ 4120$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{b\tan(e+fx)(-aCd-bBd+bcC)}{df}-\frac{\int -\frac{-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(\left(Cc^2-Bdc+(A-C)d^2\right)b^2-2ad(cC-Bd)b+a^2Cd^2\right)\tan^2(e+fx)-\left(Ba^2+2b(A-C)\right.}{c+d\tan(e+fx)}}{d}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{\int \frac{-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(\left(Cc^2-Bdc+(A-C)d^2\right)b^2-2ad(cC-Bd)b+a^2Cd^2\right)\tan^2(e+fx)-\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)}{c+d\tan(e+fx)}dx}{d}+\frac{b\tan(e+fx}{}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{\int \frac{-c(cC-Bd)b^2+2acCdb-a^2Ad^2-\left(\left(Cc^2-Bdc+(A-C)d^2\right)b^2-2ad(cC-Bd)b+a^2Cd^2\right)\tan(e+fx)^2-\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)}{c+d\tan(e+fx)}dx}{d}+\frac{b\tan(e+fx}{}}{d}$$

$$\Big\downarrow \ 4109$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{-\frac{d^2\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)\int\tan(e+fx)dx}{c^2+d^2}-\frac{(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan^2(e+fx)+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d^2x\left(a^2(Ac+Bd-cC)-2ab(Bc-\right.}{c^2+}}{d}}{d}$$

$$\Big\downarrow \ 3042$$

3.71. $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}dx$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{-\frac{d^2\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)\int\tan(e+fx)dx}{c^2+d^2}-\frac{(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d^2x\left(a^2(Ac+Bd-cC)-2ab(Bc-\cdots\right.}{c^2+\cdots}}{d}\cdots}{d}$$

$$\Big\downarrow \; 3956$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{-\frac{(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{d^2\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}-\frac{d^2x\left(a^2(Ac+Bd-cC)-2ab(Bc-\cdots\right.}{c^2+\cdots}}{d}\cdots}{d}$$

$$\Big\downarrow \; 4100$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{-\frac{(bc-ad)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df\left(c^2+d^2\right)}+\frac{d^2\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}-\frac{d^2x\left(a^2(Ac+Bd-c\cdots\right.}{\cdots}}{d}\cdots}{d}$$

$$\Big\downarrow \; 16$$

$$\frac{C(a+b\tan(e+fx))^2}{2df}-\frac{\frac{\frac{d^2\log(\cos(e+fx))\left(a^2(Bc-d(A-C))+2ab(Ac+Bd-cC)-b^2(Bc-d(A-C))\right)}{f\left(c^2+d^2\right)}-\frac{d^2x\left(a^2(Ac+Bd-cC)-2ab(Bc-d(A-C))-b^2(Ac+Bd-cC)\right)}{c^2+d^2}-\frac{(bc-ad)^2\left(Ad^2-Bcd\cdots\right.}{df\left(\cdots\right.}}{d}\cdots}{d}$$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x]),x]
```

output
```
(C*(a + b*Tan[e + f*x])^2)/(2*d*f) - ((-((d^2*(a^2*(A*c - c*C + B*d) - b^2
*(A*c - c*C + B*d) - 2*a*b*(B*c - (A - C)*d))*x)/(c^2 + d^2)) + (d^2*(2*a*
b*(A*c - c*C + B*d) + a^2*(B*c - (A - C)*d) - b^2*(B*c - (A - C)*d))*Log[C
os[e + f*x]])/((c^2 + d^2)*f) - ((b*c - a*d)^2*(c^2*C - B*c*d + A*d^2)*Log
[c + d*Tan[e + f*x]])/(d*(c^2 + d^2)*f))/d + (b*(b*c*C - b*B*d - a*C*d)*Ta
n[e + f*x])/(d*f))/d
```

3.71. $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.71.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB+2\tan(fx+e)Cad-\tan(fx+e)Cbc\right)}{d^2}+\frac{\frac{\left(-A a^2 d+2Aabc+A b^2 d+B a^2 c+2Babd-B b^2 c+C a^2 d-2\cdots\right.}{2}}{\cdots}}{\cdots}$
default	$\frac{\frac{b\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB+2\tan(fx+e)Cad-\tan(fx+e)Cbc\right)}{d^2}+\frac{\frac{\left(-A a^2 d+2Aabc+A b^2 d+B a^2 c+2Babd-B b^2 c+C a^2 d-2\cdots\right.}{2}}{\cdots}}{\cdots}$
norman	$\frac{(A a^2 c+2Aabd-A b^2 c+B a^2 d-2Babc-B b^2 d-C a^2 c-2Cabd+C b^2 c)x}{c^2+d^2}+\frac{b(bdB+2Cad-Cbc)\tan(fx+e)}{d^2 f}+\frac{C b^2 \tan\cdots}{2\cdots}$
parallelrisch	$-\frac{4Bxabc d^3 f-4C\tan(fx+e)ab c^2 d^2-2Ax a^2 c d^3 f-4Axab d^4 f+2Ax b^2 c d^3 f+2Cx a^2 c d^3 f+4Cxab d^4 f-2Cx b^2 c d^3 f\cdots}{\cdots}$
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
1/f*(b/d^2*(1/2*tan(f*x+e)^2*C*b*d+tan(f*x+e)*b*d*B+2*tan(f*x+e)*C*a*d-tan
(f*x+e)*C*b*c)+1/(c^2+d^2)*(1/2*(-A*a^2*d+2*A*a*b*c+A*b^2*d+B*a^2*c+2*B*a*
b*d-B*b^2*c+C*a^2*d-2*C*a*b*c-C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c+2*A*a*b
*d-A*b^2*c+B*a^2*d-2*B*a*b*c-B*b^2*d-C*a^2*c-2*C*a*b*d+C*b^2*c)*arctan(tan
(f*x+e)))+1/d^3*(A*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^2*c*d^3+2*B*a*b
*c^2*d^2-B*b^2*c^3*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/(c^2+d^2)*ln(c
+d*tan(f*x+e)))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$= \frac{2\left(\left(\left((A-C)a^2-2\,Bab-(A-C)b^2\right)cd^3+\left(Ba^2+2\,(A-C)ab-Bb^2\right)d^4\right)fx+\left(Cb^2c^2d^2+Cb^2d^4\right)\tan\left(f\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^3 + (B*a^2 + 2*(A - C)*a
*b - B*b^2)*d^4)*f*x + (C*b^2*c^2*d^2 + C*b^2*d^4)*tan(f*x + e)^2 + (C*b^2
*c^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2
*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x +
e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*b^2*c^4 - (2*C*a*b + B*b^2)*c^3*d + (
C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*c*d^3 + (C*a^2 + 2*B*
a*b + (A - C)*b^2)*d^4)*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b^2*c^3*d + C*b
^2*c*d^3 - (2*C*a*b + B*b^2)*c^2*d^2 - (2*C*a*b + B*b^2)*d^4)*tan(f*x + e)
)/((c^2*d^3 + d^5)*f)
```

3.71.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 4444, normalized size of antiderivative = 18.83

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e)),x)
```

output
```
Piecewise((zoo*x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq
(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a**2*x + A*a*b*log(tan(e + f*x)**2 + 1)
/f - A*b**2*x + A*b**2*tan(e + f*x)/f + B*a**2*log(tan(e + f*x)**2 + 1)/(2
*f) - 2*B*a*b*x + 2*B*a*b*tan(e + f*x)/f - B*b**2*log(tan(e + f*x)**2 + 1)
/(2*f) + B*b**2*tan(e + f*x)**2/(2*f) - C*a**2*x + C*a**2*tan(e + f*x)/f -
 C*a*b*log(tan(e + f*x)**2 + 1)/f + C*a*b*tan(e + f*x)**2/f + C*b**2*x + C
*b**2*tan(e + f*x)**3/(3*f) - C*b**2*tan(e + f*x)/f)/c, Eq(d, 0)), (I*A*a*
*2*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*a**2*f*x/(2*d*f*tan
(e + f*x) - 2*I*d*f) + I*A*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*A*a*b*f
*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - 2*I*A*a*b*f*x/(2*d*f*tan(
e + f*x) - 2*I*d*f) - 2*A*a*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A*b**2*f*
x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + A*b**2*f*x/(2*d*f*tan(e +
f*x) - 2*I*d*f) + A*b**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(
e + f*x) - 2*I*d*f) - I*A*b**2*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x
) - 2*I*d*f) - I*A*b**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*a**2*f*x*tan(e
+ f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*a**2*f*x/(2*d*f*tan(e + f*x) -
 2*I*d*f) - B*a**2/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*I*B*a*b*f*x*tan(e +
f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + 2*B*a*b*f*x/(2*d*f*tan(e + f*x) - 2*
I*d*f) + 2*B*a*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x)
 - 2*I*d*f) - 2*I*B*a*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - ...
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$= \frac{\frac{2\left(\left((A-C)a^2-2\,Bab-(A-C)b^2\right)c+\left(Ba^2+2\,(A-C)ab-Bb^2\right)d\right)(fx+e)}{c^2+d^2}+\frac{2\left(Cb^2c^4+Aa^2d^4-\left(2\,Cab+Bb^2\right)c^3d+\left(Ca^2+2\,Bab+Ab^2\right)c^2d^2-(Ba^2\right.}{c^2d^3+d^5}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c + (B*a^2 + 2*(A - C)*a*b -
 B*b^2)*d)*(f*x + e)/(c^2 + d^2) + 2*(C*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B
*b^2)*c^3*d + (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)
*log(d*tan(f*x + e) + c)/(c^2*d^3 + d^5) + ((B*a^2 + 2*(A - C)*a*b - B*b^2
)*c - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*d)*log(tan(f*x + e)^2 + 1)/(c^
2 + d^2) + (C*b^2*d*tan(f*x + e)^2 - 2*(C*b^2*c - (2*C*a*b + B*b^2)*d)*tan
(f*x + e))/d^2)/f
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$

$$= \frac{\frac{2\,(Aa^2c-Ca^2c-2\,Babc-Ab^2c+Cb^2c+Ba^2d+2\,Aabd-2\,Cabd-Bb^2d)(fx+e)}{c^2+d^2}+\frac{(Ba^2c+2\,Aabc-2\,Cabc-Bb^2c-Aa^2d+Ca^2d+2\,Babd+Ab^2d-}{c^2+d^2}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c - C*a^2*c - 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d + 2*A*
a*b*d - 2*C*a*b*d - B*b^2*d)*(f*x + e)/(c^2 + d^2) + (B*a^2*c + 2*A*a*b*c
- 2*C*a*b*c - B*b^2*c - A*a^2*d + C*a^2*d + 2*B*a*b*d + A*b^2*d - C*b^2*d)
*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*b^2*c^4 - 2*C*a*b*c^3*d - B*b^
2*c^3*d + C*a^2*c^2*d^2 + 2*B*a*b*c^2*d^2 + A*b^2*c^2*d^2 - B*a^2*c*d^3 -
2*A*a*b*c*d^3 + A*a^2*d^4)*log(abs(d*tan(f*x + e) + c))/(c^2*d^3 + d^5) +
(C*b^2*d*tan(f*x + e)^2 - 2*C*b^2*c*tan(f*x + e) + 4*C*a*b*d*tan(f*x + e)
+ 2*B*b^2*d*tan(f*x + e))/d^2)/f
```

3.71.9 Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$=\frac{\tan(e+f\,x)\,\left(\frac{B\,b^2+2\,C\,a\,b}{d}-\frac{C\,b^2\,c}{d^2}\right)}{f}$$
$$+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^2\,\left(C\,a^2\,c^2+2\,B\,a\,b\,c^2+A\,b^2\,c^2\right)-d\,\left(B\,b^2\,c^3+2\,C\,a\,b\,c^3\right)-d^3\,\left(B\,c\,a^2+2\,A\,b\,c\right.\right.}{f\,\left(c^2\,d^3+d^5\right)}$$
$$+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,b^2-A\,a^2+B\,a^2\,1\mathrm{i}-B\,b^2\,1\mathrm{i}+C\,a^2-C\,b^2+A\,a\,b\,2\mathrm{i}+2\,B\,a\,b-C\,a\,b\,2\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,a^2-B\,b^2+2\,A\,a\,b-2\,C\,a\,b-A\,a^2\,1\mathrm{i}+A\,b^2\,1\mathrm{i}+C\,a^2\,1\mathrm{i}-C\,b^2\,1\mathrm{i}+B\,a\,b\,2\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}$$
$$+\frac{C\,b^2\,{\tan(e+f\,x)}^2}{2\,d\,f}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x)),x)
```

output
```
(tan(e + f*x)*((B*b^2 + 2*C*a*b)/d - (C*b^2*c)/d^2))/f + (log(c + d*tan(e
+ f*x))*(d^2*(A*b^2*c^2 + C*a^2*c^2 + 2*B*a*b*c^2) - d*(B*b^2*c^3 + 2*C*a*
b*c^3) - d^3*(B*a^2*c + 2*A*a*b*c) + A*a^2*d^4 + C*b^2*c^4))/(f*(d^5 + c^2
*d^3)) + (log(tan(e + f*x) + 1i)*(A*b^2 - A*a^2 + B*a^2*1i - B*b^2*1i + C*
a^2 - C*b^2 + A*a*b*2i + 2*B*a*b - C*a*b*2i))/(2*f*(c*1i + d)) + (log(tan(
e + f*x) - 1i)*(A*b^2*1i - A*a^2*1i + B*a^2 - B*b^2 + C*a^2*1i - C*b^2*1i
+ 2*A*a*b + B*a*b*2i - 2*C*a*b))/(2*f*(c + d*1i)) + (C*b^2*tan(e + f*x)^2)
/(2*d*f)
```

3.72 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

3.72.1 Optimal result

Integrand size = 43, antiderivative size = 156

$$\begin{aligned}&\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx\\&=\frac{(a(Ac-cC+Bd)-b(Bc-(A-C)d))x}{c^2+d^2}\\&\quad-\frac{(Abc+aBc-bcC-aAd+bBd+aCd)\log(\cos(e+fx))}{(c^2+d^2)f}\\&\quad-\frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{d^2\left(c^2+d^2\right)f}+\frac{bC\tan(e+fx)}{df}\end{aligned}$$

output
```
(a*(A*c+B*d-C*c)-b*(B*c-(A-C)*d))*x/(c^2+d^2)-(-A*a*d+A*b*c+B*a*c+B*b*d+C*
a*d-C*b*c)*ln(cos(f*x+e))/(c^2+d^2)/f-(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)*ln(c+
d*tan(f*x+e))/d^2/(c^2+d^2)/f+b*C*tan(f*x+e)/d/f
```

3.72.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx\\&=\frac{\frac{(-ia+b)(A+iB-C)\log(i-\tan(e+fx))}{c+id}+\frac{(ia+b)(A-iB-C)\log(i+\tan(e+fx))}{c-id}+\frac{2(-bc+ad)\left(c^2C-Bcd+Ad^2\right)\log(c+d\tan(e+fx))}{d^2(c^2+d^2)}+\frac{2bC}{}}{2f}\end{aligned}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x]),x]
```

output
```
((((-I)*a + b)*(A + I*B - C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*a + b)
*(A - I*B - C)*Log[I + Tan[e + f*x]])/(c - I*d) + (2*(-(b*c) + a*d)*(c^2*C
 - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)) + (2*b*C*Tan[
e + f*x])/d)/(2*f)
```

3.72.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4120, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{c+d\tan(e+fx)}dx$$

$$\Big\downarrow \textbf{4120}$$

$$\frac{bC\tan(e+fx)}{df}-\frac{\int \frac{(bcC-adC-bBd)\tan^2(e+fx)-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{bC\tan(e+fx)}{df}-\frac{\int \frac{(bcC-adC-bBd)\tan(e+fx)^2-(Ab-Cb+aB)d\tan(e+fx)+bcC-aAd}{c+d\tan(e+fx)}dx}{d}$$

$$\Big\downarrow \textbf{4109}$$

$$\frac{bC\tan(e+fx)}{df}-\frac{\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)\int \frac{\tan^2(e+fx)+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d(-aAd+aBc+aCd+Abc+bBd-bcC)\int \tan(e+fx)dx}{c^2+d^2}-\frac{dx(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}}{d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{bC\tan(e+fx)}{df}-\frac{-\frac{d(-aAd+aBc+aCd+Abc+bBd-bcC)\int\tan(e+fx)dx}{c^2+d^2}+\frac{(bc-ad)(Ad^2-Bcd+c^2C)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{dx(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}}{d}$$

$\downarrow$ 3956

$$\frac{bC\tan(e+fx)}{df}-\frac{\frac{(bc-ad)(Ad^2-Bcd+c^2C)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{d\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f(c^2+d^2)}-\frac{dx(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}}{d}$$

$\downarrow$ 4100

$$\frac{bC\tan(e+fx)}{df}-\frac{\frac{(bc-ad)(Ad^2-Bcd+c^2C)\int\frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df(c^2+d^2)}+\frac{d\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f(c^2+d^2)}-\frac{dx(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}}{d}$$

$\downarrow$ 16

$$\frac{bC\tan(e+fx)}{df}-\frac{\frac{(bc-ad)(Ad^2-Bcd+c^2C)\log(c+d\tan(e+fx))}{df(c^2+d^2)}+\frac{d\log(\cos(e+fx))(-aAd+aBc+aCd+Abc+bBd-bcC)}{f(c^2+d^2)}-\frac{dx(a(Ac+Bd-cC)-b(Bc-d(A-C)))}{c^2+d^2}}{d}$$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*
Tan[e + f*x]),x]
```

output
```
-((-(((d*(a*(A*c - c*C + B*d) - b*(B*c - (A - C)*d))*x)/(c^2 + d^2)) + (d*(
A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*Log[Cos[e + f*x]])/((c^2 +
d^2)*f) + ((b*c - a*d)*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d
*(c^2 + d^2)*f))/d) + (b*C*Tan[e + f*x])/(d*f)
```

3.72.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\tan(fx+e)Cb}{d}+\frac{\frac{(-Aad+Abc+Bac+bdB+Cad-Cbc)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Aac+Abd+Bad-Bbc-Cac-Cbd)\arctan(\tan(fx+e))}{c^2+d^2}+}{f}$
default	$\frac{\frac{\tan(fx+e)Cb}{d}+\frac{\frac{(-Aad+Abc+Bac+bdB+Cad-Cbc)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Aac+Abd+Bad-Bbc-Cac-Cbd)\arctan(\tan(fx+e))}{c^2+d^2}+}{f}$
norman	$\frac{(Aac+Abd+Bad-Bbc-Cac-Cbd)x}{c^2+d^2}+\frac{bC\tan(fx+e)}{df}+\frac{(Aa\,d^3-Abc\,d^2-Bac\,d^2+Bb\,c^2d+Ca\,c^2d-Cb\,c^3)\ln(c+d\,\mathrm{ta}}{d^2f(c^2+d^2)}$
parallelrisch	$-\frac{-2Aac\,d^2fx-2Ab\,d^3fx-2Ba\,d^3fx+2Bbc\,d^2fx+2Cac\,d^2fx+2Cb\,d^3fx+A\ln\left(1+\tan(fx+e)^2\right)a\,d^3-A\ln\left(1+\tan(fa\right.}{}$
risch	$\frac{2ibBe}{df}+\frac{2iCae}{df}-\frac{2iCbcx}{d^2}-\frac{2idAax}{c^2+d^2}+\frac{2iAbcx}{c^2+d^2}+\frac{2iBacx}{c^2+d^2}-\frac{\ln\left(e^{2i(fx+e)}-\frac{id+c}{id-c}\right)Cb\,c^3}{d^2f(c^2+d^2)}-\frac{2iCbce}{d^2f}-\frac{2idAe}{f(c^2+}$

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,me
thod=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*b/d+1/(c^2+d^2)*(1/2*(-A*a*d+A*b*c+B*a*c+B*b*d+C*a*d-C*b
*c)*ln(1+tan(f*x+e)^2)+(A*a*c+A*b*d+B*a*d-B*b*c-C*a*c-C*b*d)*arctan(tan(f*
x+e)))+1/d^2*(A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*d-C*b*c^3)/(c^
2+d^2)*ln(c+d*tan(f*x+e)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.36

$$\int\frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$

$$=\frac{2\left(\left(((A-C)a-Bb)cd^2+(Ba+(A-C)b)d^3\right)fx-\left(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2\right)\log\left(\right.\right.}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
),x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a - B*b)*c*d^2 + (B*a + (A - C)*b)*d^3)*f*x - (C*b*c^3 -
A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)*log((d^2*tan(f*x + e)^2 +
 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + (C*b*c^3 + C*b*c*d^2 -
(C*a + B*b)*c^2*d - (C*a + B*b)*d^3)*log(1/(tan(f*x + e)^2 + 1)) + 2*(C*b*
c^2*d + C*b*d^3)*tan(f*x + e))/((c^2*d^2 + d^4)*f)
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2387, normalized size of antiderivative = 15.30

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
)),x)
```

output
```
Piecewise((zoo*x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c,
 0) & Eq(d, 0) & Eq(f, 0)), ((A*a*x + A*b*log(tan(e + f*x)**2 + 1)/(2*f) +
 B*a*log(tan(e + f*x)**2 + 1)/(2*f) - B*b*x + B*b*tan(e + f*x)/f - C*a*x +
 C*a*tan(e + f*x)/f - C*b*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*tan(e + f*x
)**2/(2*f))/c, Eq(d, 0)), (I*A*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*
I*d*f) + A*a*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A*a/(2*d*f*tan(e + f*x
) - 2*I*d*f) + A*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*A*b
*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) - A*b/(2*d*f*tan(e + f*x) - 2*I*d*f) +
 B*a*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*a*f*x/(2*d*f*ta
n(e + f*x) - 2*I*d*f) - B*a/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*B*b*f*x*tan
(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + B*b*f*x/(2*d*f*tan(e + f*x) - 2
*I*d*f) + B*b*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) -
2*I*d*f) - I*B*b*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) -
 I*B*b/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*a*f*x*tan(e + f*x)/(2*d*f*tan(
e + f*x) - 2*I*d*f) + C*a*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*a*log(tan
(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a*log(
tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C*a/(2*d*f*tan(e +
 f*x) - 2*I*d*f) - 3*C*b*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) +
 3*I*C*b*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*b*log(tan(e + f*x)**2 +
1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*b*log(tan(e + f*x)**...
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$

$$= \frac{\frac{2\,Cb\tan(fx+e)}{d} + \frac{2\,(((A-C)a-Bb)c+(Ba+(A-C)b)d)(fx+e)}{c^2+d^2} - \frac{2\,(Cbc^3-Aad^3-(Ca+Bb)c^2d+(Ba+Ab)cd^2)\log(d\tan(fx+e)+c)}{c^2d^2+d^4} + \frac{((B}{}}{2\,f}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
),x, algorithm="maxima")
```

output
```
1/2*(2*C*b*tan(f*x + e)/d + 2*(((A - C)*a - B*b)*c + (B*a + (A - C)*b)*d)*
(f*x + e)/(c^2 + d^2) - 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a +
A*b)*c*d^2)*log(d*tan(f*x + e) + c)/(c^2*d^2 + d^4) + ((B*a + (A - C)*b)*c
 - ((A - C)*a - B*b)*d)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2))/f
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$= \frac{\frac{2\,Cb\tan(fx+e)}{d} + \frac{2\,(Aac-Cac-Bbc+Bad+Abd-Cbd)(fx+e)}{c^2+d^2} + \frac{(Bac+Abc-Cbc-Aad+Cad+Bbd)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2} - \frac{2\,(Cbc^3-Cac}{}}{2\,f}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
),x, algorithm="giac")
```

output
```
1/2*(2*C*b*tan(f*x + e)/d + 2*(A*a*c - C*a*c - B*b*c + B*a*d + A*b*d - C*b
*d)*(f*x + e)/(c^2 + d^2) + (B*a*c + A*b*c - C*b*c - A*a*d + C*a*d + B*b*d
)*log(tan(f*x + e)^2 + 1)/(c^2 + d^2) - 2*(C*b*c^3 - C*a*c^2*d - B*b*c^2*d
 + B*a*c*d^2 + A*b*c*d^2 - A*a*d^3)*log(abs(d*tan(f*x + e) + c))/(c^2*d^2
+ d^4))/f
```

3.72.9 Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$=\frac{\ln(\tan(e+f\,x)-\mathrm{i})\,\left(A\,b+B\,a-C\,b-A\,a\,1\mathrm{i}+B\,b\,1\mathrm{i}+C\,a\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)}$$
$$+\frac{\ln(\tan(e+f\,x)+1\mathrm{i})\,\left(B\,b+A\,b\,1\mathrm{i}+B\,a\,1\mathrm{i}-A\,a+C\,a-C\,b\,1\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)}$$
$$-\frac{\ln(c+d\tan(e+f\,x))\,\left(d^2\,(A\,b\,c+B\,a\,c)-d\,(B\,b\,c^2+C\,a\,c^2)-A\,a\,d^3+C\,b\,c^3\right)}{f\,\left(c^2\,d^2+d^4\right)}$$
$$+\frac{C\,b\tan(e+f\,x)}{d\,f}$$

input

```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x)),x)
```

output

```
(log(tan(e + f*x) - 1i)*(A*b - A*a*1i + B*a + B*b*1i + C*a*1i - C*b))/(2*f
*(c + d*1i)) + (log(tan(e + f*x) + 1i)*(A*b*1i - A*a + B*a*1i + B*b + C*a
- C*b*1i))/(2*f*(c*1i + d)) - (log(c + d*tan(e + f*x))*(d^2*(A*b*c + B*a*c
) - d*(B*b*c^2 + C*a*c^2) - A*a*d^3 + C*b*c^3))/(f*(d^4 + c^2*d^2)) + (C*b
*tan(e + f*x))/(d*f)
```

3.73 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$

3.73.1 Optimal result

Integrand size = 33, antiderivative size = 99

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx = \frac{(Ac-cC+Bd)x}{c^2+d^2} - \frac{(Bc-(A-C)d)\log(\cos(e+fx))}{(c^2+d^2)\,f} + \frac{(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d\,(c^2+d^2)\,f}$$

output
```
(A*c+B*d-C*c)*x/(c^2+d^2)-(B*c-(A-C)*d)*ln(cos(f*x+e))/(c^2+d^2)/f+(A*d^2-
B*c*d+C*c^2)*ln(c+d*tan(f*x+e))/d/(c^2+d^2)/f
```

3.73.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx = \frac{\frac{(-iA+B+iC)\log(i-\tan(e+fx))}{c+id}+\frac{(iA+B-iC)\log(i+\tan(e+fx))}{c-id}+\frac{2(c^2C-Bcd+Ad^2)\log(c+d\tan(e+fx))}{d(c^2+d^2)}}{2f}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

output
```
((((-I)*A + B + I*C)*Log[I - Tan[e + f*x]])/(c + I*d) + ((I*A + B - I*C)*L
og[I + Tan[e + f*x]])/(c - I*d) + (2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan
[e + f*x]])/(d*(c^2 + d^2)))/(2*f)
```

3.73.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{c+d\tan(e+fx)}dx$$

$$\Big\downarrow \text{4109}$$

$$\frac{(Ad^2-Bcd+c^2C)\int \frac{\tan^2(e+fx)+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{(Bc-d(A-C))\int \tan(e+fx)dx}{c^2+d^2}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Bc-d(A-C))\int \tan(e+fx)dx}{c^2+d^2}+\frac{(Ad^2-Bcd+c^2C)\int \frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(Ad^2-Bcd+c^2C)\int \frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{(Bc-d(A-C))\log(\cos(e+fx))}{f(c^2+d^2)}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

$$\Big\downarrow \text{4100}$$

$$\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df(c^2+d^2)}-\frac{(Bc-d(A-C))\log(\cos(e+fx))}{f(c^2+d^2)}+\frac{x(Ac+Bd-cC)}{c^2+d^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{(Ad^2 - Bcd + c^2C)\log(c + d\tan(e + fx))}{df(c^2 + d^2)} - \frac{(Bc - d(A - C))\log(\cos(e + fx))}{f(c^2 + d^2)} + \frac{x(Ac + Bd - cC)}{c^2 + d^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x]),x]
```

output
```
((A*c - c*C + B*d)*x)/(c^2 + d^2) - ((B*c - (A - C)*d)*Log[Cos[e + f*x]])/
((c^2 + d^2)*f) + ((c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/(d*(c^
2 + d^2)*f)
```

3.73.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

3.73.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{(-Ad+Bc+Cd)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Ac+Bd-Cc)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}}{f}$
default	$\frac{\frac{\frac{(-Ad+Bc+Cd)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Ac+Bd-Cc)\arctan(\tan(fx+e))}{c^2+d^2}+\frac{\left(A\,d^2-Bcd+c^2C\right)\ln(c+d\tan(fx+e))}{\left(c^2+d^2\right)d}}{f}$
norman	$\frac{(Ac+Bd-Cc)x}{c^2+d^2}+\frac{(A\,d^2-Bcd+c^2C)\ln(c+d\tan(fx+e))}{d(c^2+d^2)f}-\frac{(Ad-Bc-Cd)\ln\left(1+\tan(fx+e)^2\right)}{2f(c^2+d^2)}$
parallelrisch	$-\frac{-2Axcdf-2Bx\,d^2f+2Cxcdf+A\ln\left(1+\tan(fx+e)^2\right)d^2-2A\ln(c+d\tan(fx+e))d^2-B\ln\left(1+\tan(fx+e)^2\right)cd+2B\ln(}{2(c^2+d^2)df}$
risch	$\frac{ixB}{id-c}-\frac{xA}{id-c}+\frac{xC}{id-c}-\frac{2idAx}{c^2+d^2}-\frac{2idAe}{(c^2+d^2)f}+\frac{2iBcx}{c^2+d^2}+\frac{2iBce}{(c^2+d^2)f}-\frac{2ic^2Cx}{(c^2+d^2)d}-\frac{2ic^2Ce}{(c^2+d^2)df}+\frac{2iCx}{d}+$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x,method=_RETURNVERBO
SE)
```

output
```
1/f*(1/(c^2+d^2)*(1/2*(-A*d+B*c+C*d)*ln(1+tan(f*x+e)^2)+(A*c+B*d-C*c)*arct
an(tan(f*x+e)))+(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/d*ln(c+d*tan(f*x+e)))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$$
$$=\frac{2\left((A-C)cd+Bd^2\right)fx+(Cc^2-Bcd+Ad^2)\log\left(\frac{d^2\tan(fx+e)^2+2\,cd\tan(fx+e)+c^2}{\tan(fx+e)^2+1}\right)-(Cc^2+Cd^2)\log\left(\frac{}{\tan(f}\right.}{2\left(c^2d+d^3\right)f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="f
ricas")
```

output
```
1/2*(2*((A - C)*c*d + B*d^2)*f*x + (C*c^2 - B*c*d + A*d^2)*log((d^2*tan(f*
x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*c^2 + C*d^
2)*log(1/(tan(f*x + e)^2 + 1)))/((c^2*d + d^3)*f)
```

3.73.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 966, normalized size of antiderivative = 9.76

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{c + d\tan(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(e) + C*tan(e)**2)/tan(e), Eq(c, 0) & Eq(d, 0)
& Eq(f, 0)), ((A*x + B*log(tan(e + f*x)**2 + 1)/(2*f) - C*x + C*tan(e + f*
x)/f)/c, Eq(d, 0)), (I*A*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) +
 A*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*A/(2*d*f*tan(e + f*x) - 2*I*d*f)
 + B*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*B*f*x/(2*d*f*tan(
e + f*x) - 2*I*d*f) - B/(2*d*f*tan(e + f*x) - 2*I*d*f) + I*C*f*x*tan(e + f
*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) + C*f*x/(2*d*f*tan(e + f*x) - 2*I*d*f)
+ C*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e + f*x) - 2*I*d*f) -
 I*C*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) - 2*I*d*f) - I*C/(2*d*f*
tan(e + f*x) - 2*I*d*f), Eq(c, -I*d)), (-I*A*f*x*tan(e + f*x)/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + A*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - I*A/(2*d*f*tan
(e + f*x) + 2*I*d*f) + B*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) +
 I*B*f*x/(2*d*f*tan(e + f*x) + 2*I*d*f) - B/(2*d*f*tan(e + f*x) + 2*I*d*f)
 - I*C*f*x*tan(e + f*x)/(2*d*f*tan(e + f*x) + 2*I*d*f) + C*f*x/(2*d*f*tan(
e + f*x) + 2*I*d*f) + C*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*d*f*tan(e
 + f*x) + 2*I*d*f) + I*C*log(tan(e + f*x)**2 + 1)/(2*d*f*tan(e + f*x) + 2*
I*d*f) + I*C/(2*d*f*tan(e + f*x) + 2*I*d*f), Eq(c, I*d)), (x*(A + B*tan(e)
 + C*tan(e)**2)/(c + d*tan(e)), Eq(f, 0)), (2*A*c*d*f*x/(2*c**2*d*f + 2*d*
*3*f) + 2*A*d**2*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*f) - A*d**2*
log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) - 2*B*c*d*log(c/d + ta...
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{c + d\tan(e + fx)}\,dx = \frac{\frac{2\,((A-C)c+Bd)(fx+e)}{c^2+d^2} + \frac{2\,(Cc^2-Bcd+Ad^2)\log(d\tan(fx+e)+c)}{c^2d+d^3} + \frac{(Bc-(A-C)d)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="m
axima")
```

output
```
1/2*(2*((A - C)*c + B*d)*(f*x + e)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)
*log(d*tan(f*x + e) + c)/(c^2*d + d^3) + (B*c - (A - C)*d)*log(tan(f*x + e
)^2 + 1)/(c^2 + d^2))/f
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx = \frac{\frac{2\,(Ac-Cc+Bd)(fx+e)}{c^2+d^2}+\frac{(Bc-Ad+Cd)\log\left(\tan(fx+e)^2+1\right)}{c^2+d^2}+\frac{2\,(Cc^2-Bcd+Ad^2)\log(|d\tan(fx+e)+c|)}{c^2d+d^3}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x, algorithm="g
iac")
```

output
```
1/2*(2*(A*c - C*c + B*d)*(f*x + e)/(c^2 + d^2) + (B*c - A*d + C*d)*log(tan
(f*x + e)^2 + 1)/(c^2 + d^2) + 2*(C*c^2 - B*c*d + A*d^2)*log(abs(d*tan(f*x
 + e) + c))/(c^2*d + d^3))/f
```

3.73.9 Mupad [B] (verification not implemented)

Time = 8.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx = \frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(C-A+B\,1\mathrm{i}\right)}{2\,f\,\left(d+c\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(c+d\,1\mathrm{i}\right)} + \frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{d\,f\,\left(c^2+d^2\right)}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x)),x)
```

output
```
(log(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(c*1i + d)) + (log(tan(e + f*
x) - 1i)*(B - A*1i + C*1i))/(2*f*(c + d*1i)) + (log(c + d*tan(e + f*x))*(A
*d^2 + C*c^2 - B*c*d))/(d*f*(c^2 + d^2))
```

3.73. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{c+d\tan(e+fx)}\,dx$

3.74 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$

3.74.1 Optimal result

Integrand size = 45, antiderivative size = 165

$$\begin{aligned}\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx &= \frac{(a(Ac-cC+Bd)+b(Bc-(A-C)d))x}{(a^2+b^2)(c^2+d^2)} \\ &\quad+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)(bc-ad)f} \\ &\quad-\frac{(c^2C-Bcd+Ad^2)\log(c\cos(e+fx)+d\sin(e+fx))}{(bc-ad)(c^2+d^2)f}\end{aligned}$$

output
```
(a*(A*c+B*d-C*c)+b*(B*c-(A-C)*d))*x/(a^2+b^2)/(c^2+d^2)+(A*b^2-a*(B*b-C*a)
)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)/(-a*d+b*c)/f-(A*d^2-B*c*d+C*c^2)
*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)/(c^2+d^2)/f
```

3.74.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.90

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx = -\frac{\frac{\left(Abc-aBc-bcC+aAd+bBd-aCd+\frac{\sqrt{-b^2}(bBc+b(-A+C)d+a(Ac-cC+Bd))}{b}\right)\log\left(\sqrt{-b^2}-b\tan(e+fx)\right)}{(a^2+b^2)(c^2+d^2)}+\frac{2(Ab^2+a(-bB+aC))\log(a+b\tan}{(a^2+b^2)(-bc+ad)}}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
 + d*Tan[e + f*x])),x]
```

output
```
-1/2*(((A*b*c - a*B*c - b*c*C + a*A*d + b*B*d - a*C*d + (Sqrt[-b^2]*(b*B*c
 + b*(-A + C)*d + a*(A*c - c*C + B*d)))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]
])/((a^2 + b^2)*(c^2 + d^2)) + (2*(A*b^2 + a*(-(b*B) + a*C))*Log[a + b*Tan
[e + f*x]])/((a^2 + b^2)*(-(b*c) + a*d)) + ((A*b*c - a*B*c - b*c*C + a*A*d
 + b*B*d - a*C*d + (b*(b*B*c + b*(-A + C)*d + a*(A*c - c*C + B*d)))/Sqrt[-
b^2])*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) + (2*(c^
2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/f
```

3.74.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{4134}$$

$$\frac{(Ab^2-a(bB-aC))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+\frac{x(a(Ac+Bd-cC)-bd(A-C)+bBc)}{(a^2+b^2)(c^2+d^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab^2-a(bB-aC))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+\frac{x(a(Ac+Bd-cC)-bd(A-C)+bBc)}{(a^2+b^2)(c^2+d^2)}$$

$$\Big\downarrow \text{4013}$$

$$\frac{x(a(Ac+Bd-cC)-bd(A-C)+bBc)}{(a^2+b^2)(c^2+d^2)}+\frac{(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)}-\frac{(Ad^2-Bcd+c^2C)\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*T
an[e + f*x])),x]
```

output
```
((b*B*c - b*(A - C)*d + a*(A*c - c*C + B*d))*x)/((a^2 + b^2)*(c^2 + d^2))
+ ((A*b^2 - a*(b*B - a*C))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b
^2)*(b*c - a*d)*f) - ((c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e
 + f*x]])/((b*c - a*d)*(c^2 + d^2)*f)
```

3.74.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.74.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-\frac{(A\,b^2-Bab+C\,a^2)\ln(a+b\tan(fx+e))}{(ad-bc)(a^2+b^2)}+\frac{\frac{(-Aad-Abc+Bac-bdB+Cad+Cbc)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Aac-Abd+Bad+Bbc-Cac+C}{(a^2+b^2)(c^2+d^2)}}{f}$
default	$\frac{-\frac{(A\,b^2-Bab+C\,a^2)\ln(a+b\tan(fx+e))}{(ad-bc)(a^2+b^2)}+\frac{\frac{(-Aad-Abc+Bac-bdB+Cad+Cbc)\ln\left(1+\tan(fx+e)^2\right)}{2}+(Aac-Abd+Bad+Bbc-Cac+C}{(a^2+b^2)(c^2+d^2)}}{f}$
norman	$\frac{(Aac-Abd+Bad+Bbc-Cac+Cbd)x}{(a^2+b^2)(c^2+d^2)}+\frac{(A\,d^2-Bcd+c^2C)\ln(c+d\tan(fx+e))}{f(a\,c^2d+a\,d^3-b\,c^3-bc\,d^2)}-\frac{(A\,b^2-Bab+C\,a^2)\ln(a+b\tan(fx+}{(ad-bc)(a^2+b^2)f}$
parallelrisch	$-\frac{-2C\ln(c+d\tan(fx+e))a^2c^2-2C\ln(c+d\tan(fx+e))b^2c^2+A\ln\left(1+\tan(fx+e)^2\right)a^2d^2-A\ln\left(1+\tan(fx+e)^2\right)b^2c^2+}{}$
risch	$-\frac{2iBabx}{a^3d-a^2bc+a\,b^2d-b^3c}-\frac{xA}{iad+ibc-ac+bd}+\frac{xC}{iad+ibc-ac+bd}-\frac{2ic^2Ce}{f(a\,c^2d+a\,d^3-b\,c^3-bc\,d^2)}+\frac{2iA\,b^2x}{a^3d-a^2bc+a\,b^2d-}$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)),x,me
thod=_RETURNVERBOSE)
```

output
```
1/f*(-(A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(a^2+b^
2)/(c^2+d^2)*(1/2*(-A*a*d-A*b*c+B*a*c-B*b*d+C*a*d+C*b*c)*ln(1+tan(f*x+e)^2
)+(A*a*c-A*b*d+B*a*d+B*b*c-C*a*c+C*b*d)*arctan(tan(f*x+e)))+(A*d^2-B*c*d+C
*c^2)/(a*d-b*c)/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.82

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$$
$$=\frac{2\left(\left(((A-C)ab+Bb^2)c^2-((A-C)a^2+(A-C)b^2)cd-(Ba^2-(A-C)ab)d^2\right)fx+((Ca^2-Bab+Ab^2\right.}{2\left((a^2b+\right.}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
),x, algorithm="fricas")
```

output
```
1/2*(2*(((A - C)*a*b + B*b^2)*c^2 - ((A - C)*a^2 + (A - C)*b^2)*c*d - (B*a
^2 - (A - C)*a*b)*d^2)*f*x + ((C*a^2 - B*a*b + A*b^2)*c^2 + (C*a^2 - B*a*b
 + A*b^2)*d^2)*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*
x + e)^2 + 1)) - ((C*a^2 + C*b^2)*c^2 - (B*a^2 + B*b^2)*c*d + (A*a^2 + A*b
^2)*d^2)*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)
^2 + 1)))/(((a^2*b + b^3)*c^3 - (a^3 + a*b^2)*c^2*d + (a^2*b + b^3)*c*d^2
- (a^3 + a*b^2)*d^3)*f)
```

3.74.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.90 (sec) , antiderivative size = 24052, normalized size of antiderivative = 145.77

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
)),x)
```

output
```
Piecewise(((2*A*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + 2*A*d**2*log(c/d + tan(e
 + f*x))/(2*c**2*d*f + 2*d**3*f) - A*d**2*log(tan(e + f*x)**2 + 1)/(2*c**2
*d*f + 2*d**3*f) - 2*B*c*d*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*d**3*f)
 + B*c*d*log(tan(e + f*x)**2 + 1)/(2*c**2*d*f + 2*d**3*f) + 2*B*d**2*f*x/(
2*c**2*d*f + 2*d**3*f) + 2*C*c**2*log(c/d + tan(e + f*x))/(2*c**2*d*f + 2*
d**3*f) - 2*C*c*d*f*x/(2*c**2*d*f + 2*d**3*f) + C*d**2*log(tan(e + f*x)**2
 + 1)/(2*c**2*d*f + 2*d**3*f))/a, Eq(b, 0)), ((2*A*a*b*f*x/(2*a**2*b*f + 2
*b**3*f) + 2*A*b**2*log(a/b + tan(e + f*x))/(2*a**2*b*f + 2*b**3*f) - A*b*
*2*log(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b**3*f) - 2*B*a*b*log(a/b + ta
n(e + f*x))/(2*a**2*b*f + 2*b**3*f) + B*a*b*log(tan(e + f*x)**2 + 1)/(2*a*
*2*b*f + 2*b**3*f) + 2*B*b**2*f*x/(2*a**2*b*f + 2*b**3*f) + 2*C*a**2*log(a
/b + tan(e + f*x))/(2*a**2*b*f + 2*b**3*f) - 2*C*a*b*f*x/(2*a**2*b*f + 2*b
**3*f) + C*b**2*log(tan(e + f*x)**2 + 1)/(2*a**2*b*f + 2*b**3*f))/c, Eq(d,
 0)), (I*A*c**2*f*x*tan(e + f*x)/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f +
 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) -
2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e + f*x) + 2*b*d**3*f) + A*c**2*f*x/(2*b
*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2*I*b*c**2*d*f*tan(e + f*x) + 2*b*c*
*2*d*f + 2*b*c*d**2*f*tan(e + f*x) - 2*I*b*c*d**2*f + 2*I*b*d**3*f*tan(e +
 f*x) + 2*b*d**3*f) + I*A*c**2/(2*b*c**3*f*tan(e + f*x) - 2*I*b*c**3*f + 2
*I*b*c**2*d*f*tan(e + f*x) + 2*b*c**2*d*f + 2*b*c*d**2*f*tan(e + f*x) -...
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.47

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))}\,dx$$
$$= \frac{\frac{2\,(((A-C)a+Bb)c+(Ba-(A-C)b)d)(fx+e)}{(a^2+b^2)c^2+(a^2+b^2)d^2} + \frac{2\,(Ca^2-Bab+Ab^2)\log(b\tan(fx+e)+a)}{(a^2b+b^3)c-(a^3+ab^2)d} - \frac{2\,(Cc^2-Bcd+Ad^2)\log(d\tan(fx+e)+c)}{bc^3-ac^2d+bcd^2-ad^3} + \frac{((Ba}{}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
),x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a + B*b)*c + (B*a - (A - C)*b)*d)*(f*x + e)/((a^2 + b^2)*
c^2 + (a^2 + b^2)*d^2) + 2*(C*a^2 - B*a*b + A*b^2)*log(b*tan(f*x + e) + a)
/((a^2*b + b^3)*c - (a^3 + a*b^2)*d) - 2*(C*c^2 - B*c*d + A*d^2)*log(d*tan
(f*x + e) + c)/(b*c^3 - a*c^2*d + b*c*d^2 - a*d^3) + ((B*a - (A - C)*b)*c
- ((A - C)*a + B*b)*d)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^2 + (a^2 + b
^2)*d^2))/f
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.62

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))}\,dx$$
$$= \frac{\frac{2\,(Aac-Cac+Bbc+Bad-Abd+Cbd)(fx+e)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2} + \frac{(Bac-Abc+Cbc-Aad+Cad-Bbd)\log\left(\tan(fx+e)^2+1\right)}{a^2c^2+b^2c^2+a^2d^2+b^2d^2} + \frac{2\,(Ca^2b-Bab^2+Ab^3)\log(|b\tan(f}{a^2b^2c+b^4c-a^3bd-ab^3d}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
),x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c - C*a*c + B*b*c + B*a*d - A*b*d + C*b*d)*(f*x + e)/(a^2*c^2
+ b^2*c^2 + a^2*d^2 + b^2*d^2) + (B*a*c - A*b*c + C*b*c - A*a*d + C*a*d -
B*b*d)*log(tan(f*x + e)^2 + 1)/(a^2*c^2 + b^2*c^2 + a^2*d^2 + b^2*d^2) + 2
*(C*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*tan(f*x + e) + a))/(a^2*b^2*c + b^4
*c - a^3*b*d - a*b^3*d) - 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(abs(d*tan(f*x
+ e) + c))/(b*c^3*d - a*c^2*d^2 + b*c*d^3 - a*d^4))/f
```

3.74.9 Mupad [B] (verification not implemented)

Time = 20.81 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.19

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}\,dx$$
$$=\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{f\,\left(a\,d-b\,c\right)\,\left(c^2+d^2\right)}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(C-A+B\,1\mathrm{i}\right)}{2\,f\,\left(a\,c\,1\mathrm{i}+a\,d+b\,c-b\,d\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(C\,a^2-B\,a\,b+A\,b^2\right)}{f\,\left(d\,a^3-c\,a^2\,b+d\,a\,b^2-c\,b^3\right)}-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A-C+B\,1\mathrm{i}\right)}{2\,f\,\left(a\,d-a\,c\,1\mathrm{i}+b\,c+b\,d\,1\mathrm{i}\right)}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))),x)
```

output
```
(log(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(a*c*1i + a*d + b*c - b*d*1i)
) - (log(tan(e + f*x) - 1i)*(A + B*1i - C))/(2*f*(a*d - a*c*1i + b*c + b*d
*1i)) - (log(a + b*tan(e + f*x))*(A*b^2 + C*a^2 - B*a*b))/(f*(a^3*d - b^3*
c - a^2*b*c + a*b^2*d)) + (log(c + d*tan(e + f*x))*(A*d^2 + C*c^2 - B*c*d)
)/(f*(a*d - b*c)*(c^2 + d^2))
```

3.75 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

3.75.1 Optimal result

Integrand size = 45, antiderivative size = 281

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$$
$$= \frac{(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A-C)d))\,x}{(a^2+b^2)^2\,(c^2+d^2)}$$
$$+ \frac{(2ab^3c(A-C)+2a^3bBd-a^4Cd+b^4(Bc-Ad)-a^2b^2(Bc+3Ad-Cd))\log(a\cos(e+fx)+b\sin(e}{(a^2+b^2)^2\,(bc-ad)^2 f}$$
$$+ \frac{d(c^2C-Bcd+Ad^2)\log(c\cos(e+fx)+d\sin(e+fx))}{(bc-ad)^2\,(c^2+d^2)\,f}$$
$$- \frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))}$$

output
```
(a^2*(A*c+B*d-C*c)-b^2*(A*c+B*d-C*c)+2*a*b*(B*c-(A-C)*d))*x/(a^2+b^2)^2/(c
^2+d^2)+(2*a*b^3*c*(A-C)+2*a^3*b*B*d-a^4*C*d+b^4*(-A*d+B*c)-a^2*b^2*(3*A*d
+B*c-C*d))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^2/f+d*(A*d
^2-B*c*d+C*c^2)*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)/f+(-A
*b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

3.75.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 572 vs. $2(281) = 562$.

Time = 7.29 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.04

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx =$$

$$-\frac{\frac{b(bc-ad)\left(2aAbc-a^2Bc+b^2Bc-2abcC+a^2Ad-Ab^2d+2abBd-a^2Cd+b^2Cd+\frac{\sqrt{-b^2}\left(a^2(Ac-cC+Bd)-b^2(Ac-cC+Bd)+2ab(Bc-(A-C)d)\right)}{b}\right)\log(}{2(a^2+b^2)(c^2+d^2)}}{}$$

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
(c + d*Tan[e + f*x])),x]
```

output
```
-(((b*(b*c - a*d)*(2*a*A*b*c - a^2*B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A
*b^2*d + 2*a*b*B*d - a^2*C*d + b^2*C*d + (Sqrt[-b^2]*(a^2*(A*c - c*C + B*d
) - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c - (A - C)*d)))/b)*Log[Sqrt[-b^2] -
b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(2*a*b^3*c*(A - C) + 2*a
^3*b*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))*Log[a
+ b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)) + (b*(b*c - a*d)*(2*a*A*b*c -
 a^2*B*c + b^2*B*c - 2*a*b*c*C + a^2*A*d - A*b^2*d + 2*a*b*B*d - a^2*C*d +
 b^2*C*d - (Sqrt[-b^2]*(a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) + 2*
a*b*(B*c - (A - C)*d)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/(2*(a^2 + b^2
)*(c^2 + d^2)) - (b*(a^2 + b^2)*d*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e
+ f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(a^2 + b^2)*(b*c - a*d)*f)) - (A*b^
2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

3.75.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4132, 25, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{\int -\frac{-Ada^2+bc(A-C)a-(Ab^2-a(bB-aC))d\tan^2(e+fx)+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(a^2+b^2)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-Ada^2+bc(A-C)a-(Ab^2-a(bB-aC))d\tan^2(e+fx)+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(a^2+b^2)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-Ada^2+bc(A-C)a-(Ab^2-a(bB-aC))d\tan(e+fx)^2+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(a^2+b^2)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 4134}$$

$$\frac{\frac{d(a^2+b^2)(Ad^2-Bcd+c^2C)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+\frac{(a^4(-C)d+2a^3bBd-a^2b^2(3Ad+Bc-Cd)+2ab^3c(A-C)+b^4(Bc-Ad))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}+ \cdots}{(a^2+b^2)(bc-ad)}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{d(a^2+b^2)(Ad^2-Bcd+c^2C)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+\frac{(a^4(-C)d+2a^3bBd-a^2b^2(3Ad+Bc-Cd)+2ab^3c(A-C)+b^4(Bc-Ad))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}+ \cdots}{(a^2+b^2)(bc-ad)}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 4013}$$

3.75. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx$

$$\frac{\frac{d(a^2+b^2)(Ad^2-Bcd+c^2C)\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)(bc-ad)}+\frac{x(bc-ad)(a^2(Ac+Bd-cC)+2ab(Bc-d(A-C))-b^2(Ac+Bd-cC))}{(a^2+b^2)(c^2+d^2)}+\frac{(a^4(-C)d+\cdots}{\cdots}}{(a^2+b^2)(bc-ad)}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d
*Tan[e + f*x])),x]
```

output
```
(((b*c - a*d)*(a^2*(A*c - c*C + B*d) - b^2*(A*c - c*C + B*d) + 2*a*b*(B*c
- (A - C)*d))*x)/((a^2 + b^2)*(c^2 + d^2)) + ((2*a*b^3*c*(A - C) + 2*a^3*b
*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))*Log[a*Cos[
e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f) + ((a^2 + b^2)*d*(
c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)*
(c^2 + d^2)*f))/((a^2 + b^2)*(b*c - a*d)) - (A*b^2 - a*(b*B - a*C))/((a^2
+ b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.75.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{\left(3A a^{2} b^{2} d-2A a b^{3} c+A b^{4} d-2a^{3} b B d+B a^{2} b^{2} c-B b^{4} c+a^{4} C d-C a^{2} b^{2} d+2C a b^{3} c\right) \ln\left(a+b\tan\left(fx+e\right)\right)}{\left(ad-bc\right)^{2} \left(a^{2}+b^{2}\right)^{2}}+\frac{A b^{2}-B a b+C a^{2}}{\left(ad-bc\right) \left(a^{2}+b^{2}\right) \left(a+b\tan\right.}$
default	$-\frac{\left(3A a^{2} b^{2} d-2A a b^{3} c+A b^{4} d-2a^{3} b B d+B a^{2} b^{2} c-B b^{4} c+a^{4} C d-C a^{2} b^{2} d+2C a b^{3} c\right) \ln\left(a+b\tan\left(fx+e\right)\right)}{\left(ad-bc\right)^{2} \left(a^{2}+b^{2}\right)^{2}}+\frac{A b^{2}-B a b+C a^{2}}{\left(ad-bc\right) \left(a^{2}+b^{2}\right) \left(a+b\tan\right.}$
norman	$\frac{\frac{a\left(A a^{2} c-2A a b d-A b^{2} c+B a^{2} d+2B a b c-B b^{2} d-C a^{2} c+2C a b d+C b^{2} c\right) x}{\left(a^{4}+2a^{2} b^{2}+b^{4}\right) \left(c^{2}+d^{2}\right)}+\frac{A b^{3}-B a b^{2}+C a^{2} b}{b f \left(ad-bc\right) \left(a^{2}+b^{2}\right)}+\frac{\left(A a^{2} c-2A a b d-A b^{2} c+B a^{2} d+2B\right.}{\left(a^{4}+2\right.}}{a+b\tan\left(fx+e\right)}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
1/f*(-(3*A*a^2*b^2*d-2*A*a*b^3*c+A*b^4*d-2*B*a^3*b*d+B*a^2*b^2*c-B*b^4*c+C
*a^4*d-C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^2/(a^2+b^2)^2*ln(a+b*tan(f*x+e))
+(A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))+1/(a^2+b^2)^2/(c
^2+d^2)*(1/2*(-A*a^2*d-2*A*a*b*c+A*b^2*d+B*a^2*c-2*B*a*b*d-B*b^2*c+C*a^2*d
+2*C*a*b*c-C*b^2*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c-2*A*a*b*d-A*b^2*c+B*a^2*d+
2*B*a*b*c-B*b^2*d-C*a^2*c+2*C*a*b*d+C*b^2*c)*arctan(tan(f*x+e)))+(A*d^2-B*
c*d+C*c^2)*d/(a*d-b*c)^2/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(280) = 560$.

Time = 1.08 (sec) , antiderivative size = 1345, normalized size of antiderivative = 4.79

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
-1/2*(2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c^3 - 2*(C*a^3*b^2 - B*a^2*b^3 + A*a
*b^4)*c^2*d + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*c*d^2 - 2*(C*a^3*b^2 - B*a^2
*b^3 + A*a*b^4)*d^3 - 2*(((A - C)*a^3*b^2 + 2*B*a^2*b^3 - (A - C)*a*b^4)*c
^3 - (2*(A - C)*a^4*b + 3*B*a^3*b^2 + B*a*b^4)*c^2*d + ((A - C)*a^5 + 3*(A
 - C)*a^3*b^2 + 2*B*a^2*b^3)*c*d^2 + (B*a^5 - 2*(A - C)*a^4*b - B*a^3*b^2)
*d^3)*f*x + ((B*a^3*b^2 - 2*(A - C)*a^2*b^3 - B*a*b^4)*c^3 + (C*a^5 - 2*B*
a^4*b + (3*A - C)*a^3*b^2 + A*a*b^4)*c^2*d + (B*a^3*b^2 - 2*(A - C)*a^2*b^
3 - B*a*b^4)*c*d^2 + (C*a^5 - 2*B*a^4*b + (3*A - C)*a^3*b^2 + A*a*b^4)*d^3
 + ((B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c^3 + (C*a^4*b - 2*B*a^3*b^2 + (
3*A - C)*a^2*b^3 + A*b^5)*c^2*d + (B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c*
d^2 + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3 + A*b^5)*d^3)*tan(f*x + e
))*log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1
)) - ((C*a^5 + 2*C*a^3*b^2 + C*a*b^4)*c^2*d - (B*a^5 + 2*B*a^3*b^2 + B*a*b
^4)*c*d^2 + (A*a^5 + 2*A*a^3*b^2 + A*a*b^4)*d^3 + ((C*a^4*b + 2*C*a^2*b^3
+ C*b^5)*c^2*d - (B*a^4*b + 2*B*a^2*b^3 + B*b^5)*c*d^2 + (A*a^4*b + 2*A*a^
2*b^3 + A*b^5)*d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x
+ e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*((C*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*c
^3 - (C*a^4*b - B*a^3*b^2 + A*a^2*b^3)*c^2*d + (C*a^3*b^2 - B*a^2*b^3 + A*
a*b^4)*c*d^2 - (C*a^4*b - B*a^3*b^2 + A*a^2*b^3)*d^3 + (((A - C)*a^2*b^3 +
 2*B*a*b^4 - (A - C)*b^5)*c^3 - (2*(A - C)*a^3*b^2 + 3*B*a^2*b^3 + B*b^...
```

3.75.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*
x+e)),x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.85

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$$
$$= \frac{\frac{2\left(\left(\left(A-C\right)a^2+2\,Bab-\left(A-C\right)b^2\right)c+\left(Ba^2-2\,(A-C)ab-Bb^2\right)d\right)(fx+e)}{(a^4+2\,a^2b^2+b^4)c^2+(a^4+2\,a^2b^2+b^4)d^2}-\frac{2\left(\left(Ba^2b^2-2\,(A-C)ab^3-Bb^4\right)c+\left(Ca^4-2\,Ba^3b+(3\,A-C)a^2b^2+Ab^4\right.\right.}{(a^4b^2+2\,a^2b^4+b^6)c^2-2\,(a^5b+2\,a^3b^3+ab^5)cd+(a^6+2\,a^4\right.}}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c + (B*a^2 - 2*(A - C)*a*b -
 B*b^2)*d)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4
)*d^2) - 2*((B*a^2*b^2 - 2*(A - C)*a*b^3 - B*b^4)*c + (C*a^4 - 2*B*a^3*b +
 (3*A - C)*a^2*b^2 + A*b^4)*d)*log(b*tan(f*x + e) + a)/((a^4*b^2 + 2*a^2*b
^4 + b^6)*c^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*d + (a^6 + 2*a^4*b^2 + a^2
*b^4)*d^2) + 2*(C*c^2*d - B*c*d^2 + A*d^3)*log(d*tan(f*x + e) + c)/(b^2*c^
4 - 2*a*b*c^3*d - 2*a*b*c*d^3 + a^2*d^4 + (a^2 + b^2)*c^2*d^2) + ((B*a^2 -
 2*(A - C)*a*b - B*b^2)*c - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d)*log(t
an(f*x + e)^2 + 1)/((a^4 + 2*a^2*b^2 + b^4)*c^2 + (a^4 + 2*a^2*b^2 + b^4)*
d^2) - 2*(C*a^2 - B*a*b + A*b^2)/((a^3*b + a*b^3)*c - (a^4 + a^2*b^2)*d +
((a^2*b^2 + b^4)*c - (a^3*b + a*b^3)*d)*tan(f*x + e)))/f
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(280) = 560$.

Time = 0.77 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.96

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$$
$$= \frac{\frac{2\left(Aa^2c-Ca^2c+2\,Babc-Ab^2c+Cb^2c+Ba^2d-2\,Aabd+2\,Cabd-Bb^2d\right)(fx+e)}{a^4c^2+2\,a^2b^2c^2+b^4c^2+a^4d^2+2\,a^2b^2d^2+b^4d^2}+\frac{\left(Ba^2c-2\,Aabc+2\,Cabc-Bb^2c-Aa^2d+Ca^2d-2\,Babd+Ab^2d-\right.}{a^4c^2+2\,a^2b^2c^2+b^4c^2+a^4d^2+2\,a^2b^2d^2+}}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c - C*a^2*c + 2*B*a*b*c - A*b^2*c + C*b^2*c + B*a^2*d - 2*A*
a*b*d + 2*C*a*b*d - B*b^2*d)*(f*x + e)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2
+ a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2) + (B*a^2*c - 2*A*a*b*c + 2*C*a*b*c -
B*b^2*c - A*a^2*d + C*a^2*d - 2*B*a*b*d + A*b^2*d - C*b^2*d)*log(tan(f*x +
 e)^2 + 1)/(a^4*c^2 + 2*a^2*b^2*c^2 + b^4*c^2 + a^4*d^2 + 2*a^2*b^2*d^2 +
b^4*d^2) - 2*(B*a^2*b^3*c - 2*A*a*b^4*c + 2*C*a*b^4*c - B*b^5*c + C*a^4*b*
d - 2*B*a^3*b^2*d + 3*A*a^2*b^3*d - C*a^2*b^3*d + A*b^5*d)*log(abs(b*tan(f
*x + e) + a))/(a^4*b^3*c^2 + 2*a^2*b^5*c^2 + b^7*c^2 - 2*a^5*b^2*c*d - 4*a
^3*b^4*c*d - 2*a*b^6*c*d + a^6*b*d^2 + 2*a^4*b^3*d^2 + a^2*b^5*d^2) + 2*(C
*c^2*d^2 - B*c*d^3 + A*d^4)*log(abs(d*tan(f*x + e) + c))/(b^2*c^4*d - 2*a*
b*c^3*d^2 + a^2*c^2*d^3 + b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 2*(B*a^2*
b^3*c*tan(f*x + e) - 2*A*a*b^4*c*tan(f*x + e) + 2*C*a*b^4*c*tan(f*x + e) -
 B*b^5*c*tan(f*x + e) + C*a^4*b*d*tan(f*x + e) - 2*B*a^3*b^2*d*tan(f*x + e
) + 3*A*a^2*b^3*d*tan(f*x + e) - C*a^2*b^3*d*tan(f*x + e) + A*b^5*d*tan(f*
x + e) - C*a^4*b*c + 2*B*a^3*b^2*c - 3*A*a^2*b^3*c + C*a^2*b^3*c - A*b^5*c
 + 2*C*a^5*d - 3*B*a^4*b*d + 4*A*a^3*b^2*d - B*a^2*b^3*d + 2*A*a*b^4*d)/((
a^4*b^2*c^2 + 2*a^2*b^4*c^2 + b^6*c^2 - 2*a^5*b*c*d - 4*a^3*b^3*c*d - 2*a*
b^5*c*d + a^6*d^2 + 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(b*tan(f*x + e) + a)))/f
```

3.75.9 Mupad [B] (verification not implemented)

Time = 60.06 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.40

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$$
$$=\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(a^2\,c-b^2\,c-2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,1\mathrm{i}+B-C\,1\mathrm{i}\right)}{2\,f\,\left(b^2\,c-a^2\,c+2\,a\,b\,d+a^2\,d\,1\mathrm{i}-b^2\,d\,1\mathrm{i}+a\,b\,c\,2\mathrm{i}\right)}$$
$$-\frac{\ln\left(a+b\tan(e+f\,x)\right)\,\left(C\,d\,a^4-2\,B\,d\,a^3\,b+\left(3\,A\,d+B\,c-C\,d\right)\,a^2\,b^2+\left(2\,C\,c-2\,A\,c\right)\,a\,b^3+\left(A\,d-B\right.\right.}{f\,\left(a^6\,d^2-2\,a^5\,b\,c\,d+a^4\,b^2\,c^2+2\,a^4\,b^2\,d^2-4\,a^3\,b^3\,c\,d+2\,a^2\,b^4\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d+b^6\,c^2\right)}$$
$$+\frac{C\,a^2-B\,a\,b+A\,b^2}{f\,\left(a\,d-b\,c\right)\,\left(a^2+b^2\right)\,\left(a+b\tan\left(e+f\,x\right)\right)}$$
$$+\frac{d\,\ln\left(c+d\tan(e+f\,x)\right)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{f\,{\left(a\,d-b\,c\right)}^2\,\left(c^2+d^2\right)}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))),x)
```

output
```
(log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(a^2*c + a^2*d*1i - b^2*c
- b^2*d*1i + a*b*c*2i - 2*a*b*d)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*
1i))/(2*f*(a^2*d*1i - a^2*c + b^2*c - b^2*d*1i + a*b*c*2i + 2*a*b*d)) - (l
og(a + b*tan(e + f*x))*(b^4*(A*d - B*c) + a^2*b^2*(3*A*d + B*c - C*d) + C*
a^4*d - a*b^3*(2*A*c - 2*C*c) - 2*B*a^3*b*d))/(f*(a^6*d^2 + b^6*c^2 + 2*a^
2*b^4*c^2 + a^4*b^2*c^2 + a^2*b^4*d^2 + 2*a^4*b^2*d^2 - 2*a*b^5*c*d - 2*a^
5*b*c*d - 4*a^3*b^3*c*d)) + (A*b^2 + C*a^2 - B*a*b)/(f*(a*d - b*c)*(a^2 +
b^2)*(a + b*tan(e + f*x))) + (d*log(c + d*tan(e + f*x))*(A*d^2 + C*c^2 - B
*c*d))/(f*(a*d - b*c)^2*(c^2 + d^2))
```

3.75. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}\,dx$

3.76 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

3.76.1 Optimal result

Integrand size = 45, antiderivative size = 477

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$$
$$= \frac{(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+Bd)+3a^2b(Bc-(A-C)d)-b^3(Bc-(A-C)d))\,x}{(a^2+b^2)^3\,(c^2+d^2)}$$
$$+ \frac{(3ab^5Bc^2-3a^5bBd^2+a^6Cd^2+3a^4b^2d(Bc+2Ad-Cd)+b^6(c(cC-Bd)-A(c^2-d^2))-a^3b^3(8c(A}{(a^2+b^2)^3\,(bc-}$$
$$- \frac{d^2(c^2C-Bcd+Ad^2)\log(c\cos(e+fx)+d\sin(e+fx))}{(bc-ad)^3\,(c^2+d^2)\,f}$$
$$- \frac{Ab^2-a(bB-aC)}{2\,(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))^2}$$
$$- \frac{2ab^3c(A-C)+2a^3bBd-a^4Cd+b^4(Bc-Ad)-a^2b^2(Bc+3Ad-Cd)}{(a^2+b^2)^2\,(bc-ad)^2f(a+b\tan(e+fx))}$$

output
```
(a^3*(A*c+B*d-C*c)-3*a*b^2*(A*c+B*d-C*c)+3*a^2*b*(B*c-(A-C)*d)-b^3*(B*c-(A
-C)*d))*x/(a^2+b^2)^3/(c^2+d^2)+(3*a*b^5*B*c^2-3*a^5*b*B*d^2+a^6*C*d^2+3*a
^4*b^2*d*(2*A*d+B*c-C*d)+b^6*(c*(-B*d+C*c)-A*(c^2-d^2))-a^3*b^3*(8*c*(A-C)
*d+B*(c^2-d^2))-3*a^2*b^4*(c*(2*B*d+C*c)-A*(c^2+d^2)))*ln(a*cos(f*x+e)+b*s
in(f*x+e))/(a^2+b^2)^3/(-a*d+b*c)^3/f-d^2*(A*d^2-B*c*d+C*c^2)*ln(c*cos(f*x
+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)/f+1/2*(-A*b^2+a*(B*b-C*a))/(a^2+b
^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^2+(-2*a*b^3*c*(A-C)-2*a^3*b*B*d+a^4*C*d-
b^4*(-A*d+B*c)+a^2*b^2*(3*A*d+B*c-C*d))/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*ta
n(f*x+e))
```

3.76.2 Mathematica [A] (verified)

Time = 8.92 (sec) , antiderivative size = 898, normalized size of antiderivative = 1.88

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = -\frac{Ab^2-a(bB-aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2}$$

$$-\frac{-\frac{-\frac{b(bc-ad)^2\left(3a^2Abc-Ab^3c-a^3Bc+3ab^2Bc-3a^2bcC+b^3cC+a^3Ad-3aAb^2d+3a^2bBd-b^3Bd-a^3Cd+3ab^2Cd+\frac{\sqrt{-b^2}\left(a^3(Ac-cC+Bd)-3ab^2(Ac-cC+B\right.}{}\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)}}{}}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*
(c + d*Tan[e + f*x])),x]
```

output
```
-1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])^2) - (-((-((b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3*B*c + 3*a*b^2*
B*c - 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2*b*B*d - b^3*B*
d - a^3*C*d + 3*a*b^2*C*d + (Sqrt[-b^2]*(a^3*(A*c - c*C + B*d) - 3*a*b^2*(
A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d)))/b)*
Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))) + (2*b*(3*a*b
^5*B*c^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c + 2*A*d - C*d) + b
^6*(c*(c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A - C)*d + B*(c^2 - d^2
)) - 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a + b*Tan[e + f*x]])
/((a^2 + b^2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(3*a^2*A*b*c - A*b^3*c - a^3
*B*c + 3*a*b^2*B*c - 3*a^2*b*c*C + b^3*c*C + a^3*A*d - 3*a*A*b^2*d + 3*a^2
*b*B*d - b^3*B*d - a^3*C*d + 3*a*b^2*C*d - (Sqrt[-b^2]*(a^3*(A*c - c*C + B
*d) - 3*a*b^2*(A*c - c*C + B*d) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (
A - C)*d)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2))
 - (2*b*(a^2 + b^2)^2*d^2*(c^2*C - B*c*d + A*d^2)*Log[c + d*Tan[e + f*x]])
/((b*c - a*d)*(c^2 + d^2)))/(b*(a^2 + b^2)*(b*c - a*d)*f)) - (-(a*(-2*a*(A
*b^2 - a*(b*B - a*C))*d + 2*b*(A*b - a*B - b*C)*(b*c - a*d))) - 2*b^2*(a*b
*c*(A - C) - a^2*A*d + b^2*(B*c - A*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b
*Tan[e + f*x])))/(2*(a^2 + b^2)*(b*c - a*d))
```

3.76.3 Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4132, 27, 3042, 4132, 25, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int -\frac{2\left(-Ada^2+bc(A-C)a-\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)\right)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{2f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-Ada^2+bc(A-C)a-\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{2f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-Ada^2+bc(A-C)a-\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2+b^2(Bc-Ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{2f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\int -\frac{Ad^2a^4-2bc(A-C)da^3-b^2\left(c(cC+3Bd)-A\left(c^2+2d^2\right)\right)a^2+2b^3Bc^2a-d\left(-Cda^4+2bBda^3-b^2(Bc+(3A-C)d)a^2+2b^3c(A-C)a+b^4(Bc-Ad)\right)\tan^2(e+fx)+b^4\cdots}{(a+b\tan(e+fx))(c+d\tan(e+fx))}}{\left(a^2+b^2\right)(bc-ad)}\cdots}{\left(a^2+b^2\right)(b\cdots}$$

$$\frac{Ab^2-a(bB-aC)}{2f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{Ad^2a^4-2bc(A-C)da^3-b^2\left(c(cC+3Bd)-A\left(c^2+2d^2\right)\right)a^2+2b^3Bc^2a-d\left(-Cda^4+2bBda^3-b^2(Bc+(3A-C)d)a^2+2b^3c(A-C)a+b^4(Bc-Ad)\right)\tan^2(e+fx)+b^4\left(c(c\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))}$$
$$(a^2+b^2)(bc-ad)$$
$$(a^2+b^2)(bc$$

$$\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}$$

↓ 3042

$$\int \frac{Ad^2a^4-2bc(A-C)da^3-b^2\left(c(cC+3Bd)-A\left(c^2+2d^2\right)\right)a^2+2b^3Bc^2a-d\left(-Cda^4+2bBda^3-b^2(Bc+(3A-C)d)a^2+2b^3c(A-C)a+b^4(Bc-Ad)\right)\tan(e+fx)^2+b^4\left(c(c\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))}$$
$$(a^2+b^2)(bc-ad)$$
$$(a^2+b^2)(bc$$

$$\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}$$

↓ 4134

$$-\frac{d^2\left(a^2+b^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{\left(c^2+d^2\right)(bc-ad)}+\frac{\left(a^6Cd^2-3a^5bBd^2+3a^4b^2d(2Ad+Bc-Cd)-a^3b^3\left(8cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(c(2Bd+cC)-A\left(c^2+d\right.\right.}{\left(a^2+b^2\right)(bc-ad)}$$
$$(a^2+b^2)(b$$

$$\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}$$

↓ 3042

$$-\frac{d^2\left(a^2+b^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{\left(c^2+d^2\right)(bc-ad)}+\frac{\left(a^6Cd^2-3a^5bBd^2+3a^4b^2d(2Ad+Bc-Cd)-a^3b^3\left(8cd(A-C)+B\left(c^2-d^2\right)\right)-3a^2b^4\left(c(2Bd+cC)-A\left(c^2+d\right.\right.}{\left(a^2+b^2\right)(bc-ad)}$$
$$(a^2+b^2)(b$$

$$\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}$$

↓ 4013

$$-\frac{d^2\left(a^2+b^2\right)^2\left(Ad^2-Bcd+c^2C\right)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)(bc-ad)}+\frac{x(bc-ad)^2\left(a^3(Ac+Bd-cC)+3a^2b(Bc-d(A-C))-3ab^2(Ac+Bd-cC)-b^3(Bc-d(A-C))\right)}{\left(a^2+b^2\right)\left(c^2+d^2\right)}+\frac{\left(a^6\right.}{}$$
$$($$

$$\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d
*Tan[e + f*x])),x]
```

3.76. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

output
```
-1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])^2) + ((((b*c - a*d)^2*(a^3*(A*c - c*C + B*d) - 3*a*b^2*(A*c - c*C + B*d
) + 3*a^2*b*(B*c - (A - C)*d) - b^3*(B*c - (A - C)*d))*x)/((a^2 + b^2)*(c^
2 + d^2)) + ((3*a*b^5*B*c^2 - 3*a^5*b*B*d^2 + a^6*C*d^2 + 3*a^4*b^2*d*(B*c
 + 2*A*d - C*d) + b^6*(c*(c*C - B*d) - A*(c^2 - d^2)) - a^3*b^3*(8*c*(A -
C)*d + B*(c^2 - d^2)) - 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + d^2)))*Log[a
*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f) - ((a^2 + b^2
)^2*d^2*(c^2*C - B*c*d + A*d^2)*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*
c - a*d)*(c^2 + d^2)*f))/((a^2 + b^2)*(b*c - a*d)) - (2*a*b^3*c*(A - C) +
2*a^3*b*B*d - a^4*C*d + b^4*(B*c - A*d) - a^2*b^2*(B*c + 3*A*d - C*d))/((a
^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d))
```

3.76.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.76.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{3A\,a^2b^2d-2Aa\,b^3c+A\,b^4d-2a^3bBd+B\,a^2b^2c-B\,b^4c+a^4Cd-C\,a^2b^2d+2Ca\,b^3c}{(ad-bc)^2\left(a^2+b^2\right)^2(a+b\tan(fx+e))}-\frac{\left(6A\,a^4b^2d^2-8A\,a^3b^3cd+3A\,a^2b^4c^2+3A\,a^2b^4d^2-\right.}{}}{}$
default	$\frac{\frac{3A\,a^2b^2d-2Aa\,b^3c+A\,b^4d-2a^3bBd+B\,a^2b^2c-B\,b^4c+a^4Cd-C\,a^2b^2d+2Ca\,b^3c}{(ad-bc)^2\left(a^2+b^2\right)^2(a+b\tan(fx+e))}-\frac{\left(6A\,a^4b^2d^2-8A\,a^3b^3cd+3A\,a^2b^4c^2+3A\,a^2b^4d^2-\right.}{}}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
1/f*((3*A*a^2*b^2*d-2*A*a*b^3*c+A*b^4*d-2*B*a^3*b*d+B*a^2*b^2*c-B*b^4*c+C*
a^4*d-C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^2/(a^2+b^2)^2/(a+b*tan(f*x+e))-(6
*A*a^4*b^2*d^2-8*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2+3*A*a^2*b^4*d^2-A*b^6*c^2+A
*b^6*d^2-3*B*a^5*b*d^2+3*B*a^4*b^2*c*d-B*a^3*b^3*c^2+B*a^3*b^3*d^2-6*B*a^2
*b^4*c*d+3*B*a*b^5*c^2-B*b^6*c*d+C*a^6*d^2-3*C*a^4*b^2*d^2+8*C*a^3*b^3*c*d
-3*C*a^2*b^4*c^2+C*b^6*c^2)/(a*d-b*c)^3/(a^2+b^2)^3*ln(a+b*tan(f*x+e))+1/2
*(A*b^2-B*a*b+C*a^2)/(a*d-b*c)/(a^2+b^2)/(a+b*tan(f*x+e))^2+1/(a^2+b^2)^3/
(c^2+d^2)*(1/2*(-A*a^3*d-3*A*a^2*b*c+3*A*a*b^2*d+A*b^3*c+B*a^3*c-3*B*a^2*b
*d-3*B*a*b^2*c+B*b^3*d+C*a^3*d+3*C*a^2*b*c-3*C*a*b^2*d-C*b^3*c)*ln(1+tan(f
*x+e)^2)+(A*a^3*c-3*A*a^2*b*d-3*A*a*b^2*c+A*b^3*d+B*a^3*d+3*B*a^2*b*c-3*B*
a*b^2*d-B*b^3*c-C*a^3*c+3*C*a^2*b*d+3*C*a*b^2*c-C*b^3*d)*arctan(tan(f*x+e)
))+(A*d^2-B*c*d+C*c^2)*d^2/(a*d-b*c)^3/(c^2+d^2)*ln(c+d*tan(f*x+e)))
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3643 vs. $2(475) = 950$.

Time = 3.39 (sec) , antiderivative size = 3643, normalized size of antiderivative = 7.64

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^3(c + d\tan(e + fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
-1/2*((3*C*a^4*b^4 - 5*B*a^3*b^5 + (7*A - 3*C)*a^2*b^6 + B*a*b^7 + A*b^8)*
c^4 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4*A - C)*a^3*b^5 + A*a*b^7)*c^3*d +
(5*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A + 2*C)*a^4*b^4 - 6*B*a^3*b^5 + (10*A - 3
*C)*a^2*b^6 + B*a*b^7 + A*b^8)*c^2*d^2 - 4*(2*C*a^5*b^3 - 3*B*a^4*b^4 + (4
*A - C)*a^3*b^5 + A*a*b^7)*c*d^3 + (5*C*a^6*b^2 - 7*B*a^5*b^3 + (9*A - C)*
a^4*b^4 - B*a^3*b^5 + 3*A*a^2*b^6)*d^4 - 2*(((A - C)*a^5*b^3 + 3*B*a^4*b^4
 - 3*(A - C)*a^3*b^5 - B*a^2*b^6)*c^4 - (3*(A - C)*a^6*b^2 + 8*B*a^5*b^3 -
 6*(A - C)*a^4*b^4 - (A - C)*a^2*b^6)*c^3*d + 3*((A - C)*a^7*b + 2*B*a^6*b
^2 + 2*B*a^4*b^4 - (A - C)*a^3*b^5)*c^2*d^2 - ((A - C)*a^8 + 6*(A - C)*a^6
*b^2 + 8*B*a^5*b^3 - 3*(A - C)*a^4*b^4)*c*d^3 - (B*a^8 - 3*(A - C)*a^7*b -
 3*B*a^6*b^2 + (A - C)*a^5*b^3)*d^4)*f*x - ((C*a^4*b^4 - 3*B*a^3*b^5 + 5*(
A - C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*c^4 - 4*(C*a^5*b^3 - 2*B*a^4*b^4 + (3*
A - 2*C)*a^3*b^5 + B*a^2*b^6)*c^3*d + (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A -
2*C)*a^4*b^4 - 2*B*a^3*b^5 + (6*A - 5*C)*a^2*b^6 + 3*B*a*b^7 - A*b^8)*c^2*
d^2 - 4*(C*a^5*b^3 - 2*B*a^4*b^4 + (3*A - 2*C)*a^3*b^5 + B*a^2*b^6)*c*d^3
+ (3*C*a^6*b^2 - 5*B*a^5*b^3 + (7*A - 3*C)*a^4*b^4 + B*a^3*b^5 + A*a^2*b^6
)*d^4 + 2*(((A - C)*a^3*b^5 + 3*B*a^2*b^6 - 3*(A - C)*a*b^7 - B*b^8)*c^4 -
 (3*(A - C)*a^4*b^4 + 8*B*a^3*b^5 - 6*(A - C)*a^2*b^6 - (A - C)*b^8)*c^3*d
 + 3*((A - C)*a^5*b^3 + 2*B*a^4*b^4 + 2*B*a^2*b^6 - (A - C)*a*b^7)*c^2*d^2
 - ((A - C)*a^6*b^2 + 6*(A - C)*a^4*b^4 + 8*B*a^3*b^5 - 3*(A - C)*a^2*b...
```

3.76.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^3 (c + d\tan(e + fx))}\, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*
x+e)),x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(475) = 950$.

Time = 0.38 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.30

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c + (B*a^3 - 3
*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d)*(f*x + e)/((a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - 2*((B*a
^3*b^3 - 3*(A - C)*a^2*b^4 - 3*B*a*b^5 + (A - C)*b^6)*c^2 - (3*B*a^4*b^2 -
 8*(A - C)*a^3*b^3 - 6*B*a^2*b^4 - B*b^6)*c*d - (C*a^6 - 3*B*a^5*b + 3*(2*
A - C)*a^4*b^2 + B*a^3*b^3 + 3*A*a^2*b^4 + A*b^6)*d^2)*log(b*tan(f*x + e)
+ a)/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*c^3 - 3*(a^7*b^2 + 3*a^5*b^4
 + 3*a^3*b^6 + a*b^8)*c^2*d + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*
c*d^2 - (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*d^3) - 2*(C*c^2*d^2 - B*c*
d^3 + A*d^4)*log(d*tan(f*x + e) + c)/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c*
d^4 - a^3*d^5 + (3*a^2*b + b^3)*c^3*d^2 - (a^3 + 3*a*b^2)*c^2*d^3) + ((B*a
^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c - ((A - C)*a^3 + 3*B*a^2
*b - 3*(A - C)*a*b^2 - B*b^3)*d)*log(tan(f*x + e)^2 + 1)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*c^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2) - ((C*a
^4*b - 3*B*a^3*b^2 + (5*A - 3*C)*a^2*b^3 + B*a*b^4 + A*b^5)*c - (3*C*a^5 -
 5*B*a^4*b + (7*A - C)*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4)*d - 2*((B*a^2*b^3
- 2*(A - C)*a*b^4 - B*b^5)*c + (C*a^4*b - 2*B*a^3*b^2 + (3*A - C)*a^2*b^3
+ A*b^5)*d)*tan(f*x + e))/((a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2 - 2*(a^7*b
+ 2*a^5*b^3 + a^3*b^5)*c*d + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^2 + ((a^4*b^4 +
 2*a^2*b^6 + b^8)*c^2 - 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*c*d + (a^6*b^2 ...
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2080 vs. $2(475) = 950$.

Time = 1.09 (sec) , antiderivative size = 2080, normalized size of antiderivative = 4.36

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e)),x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3*c - C*a^3*c + 3*B*a^2*b*c - 3*A*a*b^2*c + 3*C*a*b^2*c - B*b^
3*c + B*a^3*d - 3*A*a^2*b*d + 3*C*a^2*b*d - 3*B*a*b^2*d + A*b^3*d - C*b^3*
d)*(f*x + e)/(a^6*c^2 + 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2
+ 3*a^4*b^2*d^2 + 3*a^2*b^4*d^2 + b^6*d^2) + (B*a^3*c - 3*A*a^2*b*c + 3*C*
a^2*b*c - 3*B*a*b^2*c + A*b^3*c - C*b^3*c - A*a^3*d + C*a^3*d - 3*B*a^2*b*
d + 3*A*a*b^2*d - 3*C*a*b^2*d + B*b^3*d)*log(tan(f*x + e)^2 + 1)/(a^6*c^2
+ 3*a^4*b^2*c^2 + 3*a^2*b^4*c^2 + b^6*c^2 + a^6*d^2 + 3*a^4*b^2*d^2 + 3*a^
2*b^4*d^2 + b^6*d^2) - 2*(B*a^3*b^4*c^2 - 3*A*a^2*b^5*c^2 + 3*C*a^2*b^5*c^
2 - 3*B*a*b^6*c^2 + A*b^7*c^2 - C*b^7*c^2 - 3*B*a^4*b^3*c*d + 8*A*a^3*b^4*
c*d - 8*C*a^3*b^4*c*d + 6*B*a^2*b^5*c*d + B*b^7*c*d - C*a^6*b*d^2 + 3*B*a^
5*b^2*d^2 - 6*A*a^4*b^3*d^2 + 3*C*a^4*b^3*d^2 - B*a^3*b^4*d^2 - 3*A*a^2*b^
5*d^2 - A*b^7*d^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^4*c^3 + 3*a^4*b^6*c
^3 + 3*a^2*b^8*c^3 + b^10*c^3 - 3*a^7*b^3*c^2*d - 9*a^5*b^5*c^2*d - 9*a^3*
b^7*c^2*d - 3*a*b^9*c^2*d + 3*a^8*b^2*c*d^2 + 9*a^6*b^4*c*d^2 + 9*a^4*b^6*
c*d^2 + 3*a^2*b^8*c*d^2 - a^9*b*d^3 - 3*a^7*b^3*d^3 - 3*a^5*b^5*d^3 - a^3*
b^7*d^3) - 2*(C*c^2*d^3 - B*c*d^4 + A*d^5)*log(abs(d*tan(f*x + e) + c))/(b
^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 + b^3*c^3*d^3 - a^3*c^2*d^4 -
 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6) + (3*B*a^3*b^5*c^2*tan(f*x + e
)^2 - 9*A*a^2*b^6*c^2*tan(f*x + e)^2 + 9*C*a^2*b^6*c^2*tan(f*x + e)^2 - 9*
B*a*b^7*c^2*tan(f*x + e)^2 + 3*A*b^8*c^2*tan(f*x + e)^2 - 3*C*b^8*c^2*t...
```

3.76.9 Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 65819, normalized size of antiderivative = 137.99

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^3(c + d\tan(e + fx))}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^3*(c + d
*tan(e + f*x))),x)
```

output
```
-(((A*b^5*c - 3*C*a^5*d - 3*A*a*b^4*d + B*a*b^4*c + 5*B*a^4*b*d + C*a^4*b*
c + 5*A*a^2*b^3*c - 7*A*a^3*b^2*d - 3*B*a^3*b^2*c + B*a^2*b^3*d - 3*C*a^2*
b^3*c + C*a^3*b^2*d)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2
*b^2)) - (tan(e + f*x)*(A*b^5*d - B*b^5*c - 2*A*a*b^4*c + 2*C*a*b^4*c + C*
a^4*b*d + 3*A*a^2*b^3*d + B*a^2*b^3*c - 2*B*a^3*b^2*d - C*a^2*b^3*d))/((a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^4 + b^4 + 2*a^2*b^2)))/(a^2 + b^2*tan(e +
f*x)^2 + 2*a*b*tan(e + f*x)) - symsum(log(- (A^3*b^8*c^2*d^4 - 4*A^3*a^2*b
^6*d^6 - 7*A^3*a^4*b^4*d^6 - A^3*b^8*d^6 + A^2*C*b^8*d^6 - 3*A^3*a^2*b^6*c
^2*d^4 - B^3*a^3*b^5*c^2*d^4 - C^3*a^2*b^6*c^2*d^4 - 2*C^3*a^3*b^5*c^3*d^3
 + 7*C^3*a^4*b^4*c^2*d^4 + A^2*B*a*b^7*d^6 + A^2*B*b^8*c*d^5 + A^3*a*b^7*c
*d^5 + C^3*a^7*b*c*d^5 - A*B^2*a^2*b^6*d^6 - 3*A*B^2*a^6*b^2*d^6 + 2*A^2*B
*a^3*b^5*d^6 + 9*A^2*B*a^5*b^3*d^6 - A*C^2*a^2*b^6*d^6 - 4*A*C^2*a^4*b^4*d
^6 + A*C^2*a^6*b^2*d^6 + 5*A^2*C*a^2*b^6*d^6 + 11*A^2*C*a^4*b^4*d^6 - A^2*
C*a^6*b^2*d^6 + A*C^2*b^8*c^2*d^4 - 2*A^2*C*b^8*c^2*d^4 - B*C^2*b^8*c^3*d^
3 + B^2*C*b^8*c^2*d^4 + 9*A^3*a^3*b^5*c*d^5 - B^3*a*b^7*c^2*d^4 + B^3*a^2*
b^6*c*d^5 + B^3*a^4*b^4*c*d^5 + 2*C^3*a*b^7*c^3*d^3 - 3*C^3*a^5*b^3*c*d^5
+ A*B*C*a^7*b*d^6 - 2*A*B*C*b^8*c*d^5 + 3*A*B^2*a^2*b^6*c^2*d^4 - A*B^2*a^
4*b^4*c^2*d^4 + 3*A^2*B*a^3*b^5*c^2*d^4 - A*C^2*a^2*b^6*c^2*d^4 + 4*A*C^2*
a^3*b^5*c^3*d^3 - 14*A*C^2*a^4*b^4*c^2*d^4 + 5*A^2*C*a^2*b^6*c^2*d^4 - 2*A
^2*C*a^3*b^5*c^3*d^3 + 7*A^2*C*a^4*b^4*c^2*d^4 + 6*B*C^2*a^2*b^6*c^3*d^...
```

3.76. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))}\,dx$

3.77 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.77.1 Optimal result

Integrand size = 45, antiderivative size = 579

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx =$$

$$-\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3a^2b(2c(A-C)d-}{(c^2+d^2)^2}$$

$$+\frac{(3a^2b(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))+a^3(2c(A-C)d-B(}{(c^2+d^2)^2\,f}$$

$$+\frac{(bc-ad)^2\,(b(3c^4C-2Bc^3d+c^2(A+5C)d^2-4Bcd^3+3Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\log(c}{d^4\,(c^2+d^2)^2\,f}$$

$$+\frac{b^2(ad(3c^2C-Bcd+(A+2C)d^2)-b(3c^3C-2Bc^2d+c(A+2C)d^2-Bd^3))\tan(e+fx)}{d^3\,(c^2+d^2)\,f}$$

$$+\frac{b(3c^2C-2Bcd+(2A+C)d^2)\,(a+b\tan(e+fx))^2}{2d^2\,(c^2+d^2)\,f}$$

$$-\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^3}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

output
```
-(a^3*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^
2-d^2))-3*a^2*b*(2*c*(A-C)*d-B*(c^2-d^2))+b^3*(2*c*(A-C)*d-B*(c^2-d^2)))*x
/(c^2+d^2)^2+(3*a^2*b*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^3*(c^2*C-2*B*c*d
-C*d^2-A*(c^2-d^2))+a^3*(2*c*(A-C)*d-B*(c^2-d^2))-3*a*b^2*(2*c*(A-C)*d-B*(
c^2-d^2)))*ln(cos(f*x+e))/(c^2+d^2)^2/f+(-a*d+b*c)^2*(b*(3*c^4*C-2*B*c^3*d
+c^2*(A+5*C)*d^2-4*B*c*d^3+3*A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+
d*tan(f*x+e))/d^4/(c^2+d^2)^2/f+b^2*(a*d*(3*c^2*C-B*c*d+(A+2*C)*d^2)-b*(3*
c^3*C-2*B*c^2*d+c*(A+2*C)*d^2-B*d^3))*tan(f*x+e)/d^3/(c^2+d^2)/f+1/2*b*(3*
c^2*C-2*B*c*d+(2*A+C)*d^2)*(a+b*tan(f*x+e))^2/d^2/(c^2+d^2)/f-(A*d^2-B*c*d
+C*c^2)*(a+b*tan(f*x+e))^3/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

3.77.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.60 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx=\frac{C(a+b\tan(e+fx))^3}{2df(c+d\tan(e+fx))}$$

$$+\frac{\frac{(-3bcC+2bBd+3aCd)(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))}+\frac{2\left(-\frac{d^2\left(-3a^2Abc^2+Ab^3c^2-a^3Bc^2+3ab^2Bc^2+3a^2bc^2C-b^3c^2C+2a^3Acd-6aAb^2cd-6a^2bBcd+2b^3Bcd-\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^2,x]
```

output
```
(C*(a + b*Tan[e + f*x])^3)/(2*d*f*(c + d*Tan[e + f*x])) + (((-3*b*c*C + 2*
b*B*d + 3*a*C*d)*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*x])) + (2*(
-1/2*(d^2*(-3*a^2*A*b*c^2 + A*b^3*c^2 - a^3*B*c^2 + 3*a*b^2*B*c^2 + 3*a^2*
b*c^2*C - b^3*c^2*C + 2*a^3*A*c*d - 6*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*
B*c*d - 2*a^3*c*C*d + 6*a*b^2*c*C*d + 3*a^2*A*b*d^2 - A*b^3*d^2 + a^3*B*d^
2 - 3*a*b^2*B*d^2 - 3*a^2*b*C*d^2 + b^3*C*d^2 + I*(a^3*A*c^2 - 3*a*A*b^2*c
^2 - 3*a^2*b*B*c^2 + b^3*B*c^2 - a^3*c^2*C + 3*a*b^2*c^2*C + 6*a^2*A*b*c*d
 - 2*A*b^3*c*d + 2*a^3*B*c*d - 6*a*b^2*B*c*d - 6*a^2*b*c*C*d + 2*b^3*c*C*d
 - a^3*A*d^2 + 3*a*A*b^2*d^2 + 3*a^2*b*B*d^2 - b^3*B*d^2 + a^3*C*d^2 - 3*a
*b^2*C*d^2))*Log[I - Tan[e + f*x]])/((c^2 + d^2)^2*f) + (d^2*(3*a^2*A*b*c^
2 - A*b^3*c^2 + a^3*B*c^2 - 3*a*b^2*B*c^2 - 3*a^2*b*c^2*C + b^3*c^2*C - 2*
a^3*A*c*d + 6*a*A*b^2*c*d + 6*a^2*b*B*c*d - 2*b^3*B*c*d + 2*a^3*c*C*d - 6*
a*b^2*c*C*d - 3*a^2*A*b*d^2 + A*b^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 + 3*a^
2*b*C*d^2 - b^3*C*d^2 + I*(a^3*A*c^2 - 3*a*A*b^2*c^2 - 3*a^2*b*B*c^2 + b^3
*B*c^2 - a^3*c^2*C + 3*a*b^2*c^2*C + 6*a^2*A*b*c*d - 2*A*b^3*c*d + 2*a^3*B
*c*d - 6*a*b^2*B*c*d - 6*a^2*b*c*C*d + 2*b^3*c*C*d - a^3*A*d^2 + 3*a*A*b^2
*d^2 + 3*a^2*b*B*d^2 - b^3*B*d^2 + a^3*C*d^2 - 3*a*b^2*C*d^2))*Log[I + Tan
[e + f*x]])/(2*(c^2 + d^2)^2*f) + ((b*c - a*d)^2*(b*(3*c^4*C - 2*B*c^3*d +
 c^2*(A + 5*C)*d^2 - 4*B*c*d^3 + 3*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2
- d^2)))*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2*f) - ((b*c - a*d)^...
```

3.77.3 Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4128, 3042, 4130, 27, 3042, 4120, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(3Cc^2-2Bdc+(2A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+3bd)+(3bc-ad)(cC-Bd)\right)}{c+d\tan(e+fx)}dx}{d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(3Cc^2-2Bdc+(2A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+3bd)+(3bc-ad)(cC-Bd)\right)}{c+d\tan(e+fx)}dx}{d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 4130

$$\frac{\int -\frac{\frac{2(a+b\tan(e+fx))\left(c\left(3Cc^2-2Bdc+(2A+C)d^2\right)b^2-\left(ad\left(3Cc^2-Bdc+(A+2C)d^2\right)-b\left(3Cc^3-2Bdc^2+(A+2C)d^2c-Bd^3\right)\right)\tan^2(e+fx)b-ad(Ad(ac+3bd)+(3bc-\cdots}{c+d\tan(e+fx)}}{2d}}{d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)(a+b\tan(e+fx))^2}{2df}-\frac{\int \frac{(a+b\tan(e+fx))\left(c\left(3Cc^2-2Bdc+(2A+C)d^2\right)b^2-\left(ad\left(3Cc^2-Bdc+(A+2C)d^2\right)-b\left(3Cc^3-2Bdc^2+(A+\cdots\right.\right.\right.}{\cdots}}{\cdots}}{d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)(a+b\tan(e+fx))^2}{2df}-\frac{\int \frac{(a+b\tan(e+fx))\left(c\left(3Cc^2-2Bdc+(2A+C)d^2\right)b^2-\left(ad\left(3Cc^2-Bdc+(A+2C)d^2\right)-b\left(3Cc^3-2Bdc^2+(A+\cdots\right.\right.\right.}{\cdots}}{\cdots}}{d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 4120

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{\int \frac{c\left(3Cc^3-2Bdc^2+(A+2C)d^2c-Bd^3\right)b^3-3acd\left(2Cc^2-Bdc+(A+C)d^2\right)b^2+\left(c^2+d^2\right)\left(\left(3Cc^2-2Bdc+(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

3.77. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{\int \frac{c\left(3Cc^3-2Bdc^2+(A+2C)d^2c-Bd^3\right)b^3-3acd\left(2Cc^2-Bdc+(A+C)d^2\right)b^2+\left(c^2+d^2\right)\left(\left(3Cc^2-2Bdc+(A\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4109}$$

$$\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{-\frac{d^3\left(a^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-C)-B\left(c^2-d^2\cdots\right.\right.\right.}{c^2+d^2}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{-\frac{d^3\left(a^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-C)-B\left(c^2-d^2\cdots\right.\right.\right.}{c^2+d^2}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{\frac{(bc-ad)^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(c^2d^2(A+5C)+3Ad^4-2Bc^3d-4Bcd^3+3c^4C\right)\right)\int \frac{\tan(e+fx)\cdots}{c+d\tan(e+\cdots}}{c^2+d^2}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4100}$$

$$\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)(a+b\tan(e+fx))^2}{2df}-\frac{-\frac{\frac{(bc-ad)^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(c^2d^2(A+5C)+3Ad^4-2Bc^3d-4Bcd^3+3c^4C\right)\right)\int \frac{1}{c+d\tan(e+\cdots}}{df\left(c^2+d^2\right)}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{16}$$

3.77. $\displaystyle\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)(a+b\tan(e+fx))^2}{2df} - \frac{-\frac{d^3\log(\cos(e+fx))\left(a^3\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+3a^2b\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-3ab^2\left(2cd(A-\right.\right.}{f\left(c^2+d^2\right)}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

input
```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^2,x]
```

output
```
-(((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*(c + d
*Tan[e + f*x]))) + ((b*(3*c^2*C - 2*B*c*d + (2*A + C)*d^2)*(a + b*Tan[e +
f*x])^2)/(2*d*f) - (-((-((d^3*(a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2
)) - 3*a*b^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a^2*b*(2*c*(A -
 C)*d - B*(c^2 - d^2)) + b^3*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/(c^2 + d^
2)) + (d^3*(3*a^2*b*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^3*(c^2*C
 - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + a^3*(2*c*(A - C)*d - B*(c^2 - d^2))
- 3*a*b^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)
*f) + ((b*c - a*d)^2*(b*(3*c^4*C - 2*B*c^3*d + c^2*(A + 5*C)*d^2 - 4*B*c*d
^3 + 3*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f
*x]])/(d*(c^2 + d^2)*f))/d) - (b^2*(a*d*(3*c^2*C - B*c*d + (A + 2*C)*d^2)
- b*(3*c^3*C - 2*B*c^2*d + c*(A + 2*C)*d^2 - B*d^3))*Tan[e + f*x])/(d*f))/
d)/(d*(c^2 + d^2))
```

3.77.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.77. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.77.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{\frac{b^2\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB+3\tan(fx+e)Cad-2\tan(fx+e)Cbc\right)}{d^3}+\frac{\left(-2Aa^3cd+3Aa^2bc^2-3Aa^2bd^2+6Aab^2cd-Ab^3c^2+\cdots\right.}{}}{}$
default	$\frac{\frac{b^2\left(\frac{\tan(fx+e)^2 Cbd}{2}+\tan(fx+e)bdB+3\tan(fx+e)Cad-2\tan(fx+e)Cbc\right)}{d^3}+\frac{\left(-2Aa^3cd+3Aa^2bc^2-3Aa^2bd^2+6Aab^2cd-Ab^3c^2+\cdots\right.}{}}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(b^2/d^3*(1/2*tan(f*x+e)^2*C*b*d+tan(f*x+e)*b*d*B+3*tan(f*x+e)*C*a*d-2
*tan(f*x+e)*C*b*c)+1/(c^2+d^2)^2*(1/2*(-2*A*a^3*c*d+3*A*a^2*b*c^2-3*A*a^2*
b*d^2+6*A*a*b^2*c*d-A*b^3*c^2+A*b^3*d^2+B*a^3*c^2-B*a^3*d^2+6*B*a^2*b*c*d-
3*B*a*b^2*c^2+3*B*a*b^2*d^2-2*B*b^3*c*d+2*C*a^3*c*d-3*C*a^2*b*c^2+3*C*a^2*
b*d^2-6*C*a*b^2*c*d+C*b^3*c^2-C*b^3*d^2)*ln(1+tan(f*x+e)^2)+(A*a^3*c^2-A*a
^3*d^2+6*A*a^2*b*c*d-3*A*a*b^2*c^2+3*A*a*b^2*d^2-2*A*b^3*c*d+2*B*a^3*c*d-3
*B*a^2*b*c^2+3*B*a^2*b*d^2-6*B*a*b^2*c*d+B*b^3*c^2-B*b^3*d^2-C*a^3*c^2+C*a
^3*d^2-6*C*a^2*b*c*d+3*C*a*b^2*c^2-3*C*a*b^2*d^2+2*C*b^3*c*d)*arctan(tan(f
*x+e)))-1/d^4*(A*a^3*d^5-3*A*a^2*b*c*d^4+3*A*a*b^2*c^2*d^3-A*b^3*c^3*d^2-B
*a^3*c*d^4+3*B*a^2*b*c^2*d^3-3*B*a*b^2*c^3*d^2+B*b^3*c^4*d+C*a^3*c^2*d^3-3
*C*a^2*b*c^3*d^2+3*C*a*b^2*c^4*d-C*b^3*c^5)/(c^2+d^2)/(c+d*tan(f*x+e))+1/d
^4*(2*A*a^3*c*d^5-3*A*a^2*b*c^2*d^4+3*A*a^2*b*d^6-6*A*a*b^2*c*d^5+A*b^3*c^
4*d^2+3*A*b^3*c^2*d^4-B*a^3*c^2*d^4+B*a^3*d^6-6*B*a^2*b*c*d^5+3*B*a*b^2*c^
4*d^2+9*B*a*b^2*c^2*d^4-2*B*b^3*c^5*d-4*B*b^3*c^3*d^3-2*C*a^3*c*d^5+3*C*a^
2*b*c^4*d^2+9*C*a^2*b*c^2*d^4-6*C*a*b^2*c^5*d-12*C*a*b^2*c^3*d^3+3*C*b^3*c
^6+5*C*b^3*c^4*d^2)/(c^2+d^2)^2*ln(c+d*tan(f*x+e)))
```

3.77.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. 2(577) = 1154.

Time = 1.10 (sec) , antiderivative size = 1477, normalized size of antiderivative = 2.55

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
1/2*(3*C*b^3*c^5*d^2 - 2*A*a^3*d^7 - 2*(3*C*a*b^2 + B*b^3)*c^4*d^3 + 2*(3*
C*a^2*b + 3*B*a*b^2 + (A + C)*b^3)*c^3*d^4 - 2*(C*a^3 + 3*B*a^2*b + 3*A*a*
b^2)*c^2*d^5 + (2*B*a^3 + 6*A*a^2*b + C*b^3)*c*d^6 + (C*b^3*c^4*d^3 + 2*C*
b^3*c^2*d^5 + C*b^3*d^7)*tan(f*x + e)^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*
(A - C)*a*b^2 + B*b^3)*c^3*d^4 + 2*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 -
(A - C)*b^3)*c^2*d^5 - ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)
*c*d^6)*f*x - (3*C*b^3*c^5*d^2 + 6*C*b^3*c^3*d^4 + 3*C*b^3*c*d^6 - 2*(3*C*
a*b^2 + B*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^2*d^5 - 2*(3*C*a*b^2 + B*
b^3)*d^7)*tan(f*x + e)^2 + (3*C*b^3*c^7 - 2*(3*C*a*b^2 + B*b^3)*c^6*d + (3
*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^4*
d^3 - (B*a^3 + 3*(A - 3*C)*a^2*b - 9*B*a*b^2 - 3*A*b^3)*c^3*d^4 + 2*((A -
C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + (3*C
*b^3*c^6*d - 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 + (A +
 5*C)*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A - 3*C)*
a^2*b - 9*B*a*b^2 - 3*A*b^3)*c^2*d^5 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*
b^2)*c*d^6 + (B*a^3 + 3*A*a^2*b)*d^7)*tan(f*x + e))*log((d^2*tan(f*x + e)^
2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (3*C*b^3*c^7 - 2*(3*
C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 5*C)*b^3)*c^5*d^2 -
 4*(3*C*a*b^2 + B*b^3)*c^4*d^3 + (6*C*a^2*b + 6*B*a*b^2 + (2*A + C)*b^3)*c
^3*d^4 - 2*(3*C*a*b^2 + B*b^3)*c^2*d^5 + (3*C*a^2*b + 3*B*a*b^2 + (A - ...
```

3.77.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.02 (sec) , antiderivative size = 24300, normalized size of antiderivative = 41.97

$$\int \frac{(a+b\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**2,x)
```

output

```
Piecewise((zoo*x*(a + b*tan(e))**3*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
 Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a**3*x + 3*A*a**2*b*log(tan(e + f*x)
**2 + 1)/(2*f) - 3*A*a*b**2*x + 3*A*a*b**2*tan(e + f*x)/f - A*b**3*log(tan
(e + f*x)**2 + 1)/(2*f) + A*b**3*tan(e + f*x)**2/(2*f) + B*a**3*log(tan(e
+ f*x)**2 + 1)/(2*f) - 3*B*a**2*b*x + 3*B*a**2*b*tan(e + f*x)/f - 3*B*a*b*
*2*log(tan(e + f*x)**2 + 1)/(2*f) + 3*B*a*b**2*tan(e + f*x)**2/(2*f) + B*b
**3*x + B*b**3*tan(e + f*x)**3/(3*f) - B*b**3*tan(e + f*x)/f - C*a**3*x +
C*a**3*tan(e + f*x)/f - 3*C*a**2*b*log(tan(e + f*x)**2 + 1)/(2*f) + 3*C*a*
*2*b*tan(e + f*x)**2/(2*f) + 3*C*a*b**2*x + C*a*b**2*tan(e + f*x)**3/f - 3
*C*a*b**2*tan(e + f*x)/f + C*b**3*log(tan(e + f*x)**2 + 1)/(2*f) + C*b**3*
tan(e + f*x)**4/(4*f) - C*b**3*tan(e + f*x)**2/(2*f))/c**2, Eq(d, 0)), (-A
*a**3*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f
*x) - 4*d**2*f) + 2*I*A*a**3*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a**3*f*x/(4*d**2*f*tan(e + f*x)**2
 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*a**3*tan(e + f*x)/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a**3/(4*d**2*f*
tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 3*I*A*a**2*b*f*x*t
an(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) + 6*A*a**2*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*
tan(e + f*x) - 4*d**2*f) - 3*I*A*a**2*b*f*x/(4*d**2*f*tan(e + f*x)**2 -...
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.18

$$\int \frac{(a + b\tan(e + fx))^3 (A + B\tan(e + fx) + C\tan^2(e + fx))}{(c + d\tan(e + fx))^2}\, dx$$

$$= \frac{\frac{2\left(\left(\left((A-C)a^3-3\,Ba^2b-3\,(A-C)ab^2+Bb^3\right)c^2+2\left(Ba^3+3\,(A-C)a^2b-3\,Bab^2-(A-C)b^3\right)cd-\left((A-C)a^3-3\,Ba^2b-3\,(A-C)ab^2+Bb^3\right)d^2\right)(fx+e)}{c^4+2\,c^2d^2+d^4}}{}$$

input

```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^2 + 2*(B*a^3
 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c*d - ((A - C)*a^3 - 3*B*a^2
*b - 3*(A - C)*a*b^2 + B*b^3)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(
3*C*b^3*c^6 - 2*(3*C*a*b^2 + B*b^3)*c^5*d + (3*C*a^2*b + 3*B*a*b^2 + (A +
5*C)*b^3)*c^4*d^2 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^3 - (B*a^3 + 3*(A - 3*C)*a
^2*b - 9*B*a*b^2 - 3*A*b^3)*c^2*d^4 + 2*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b
^2)*c*d^5 + (B*a^3 + 3*A*a^2*b)*d^6)*log(d*tan(f*x + e) + c)/(c^4*d^4 + 2*
c^2*d^6 + d^8) + ((B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2
- 2*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d - (B*a^3 + 3*(
A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4
+ 2*c^2*d^2 + d^4) + 2*(C*b^3*c^5 - A*a^3*d^5 - (3*C*a*b^2 + B*b^3)*c^4*d
+ (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^2 - (C*a^3 + 3*B*a^2*b + 3*A*a*b^2
)*c^2*d^3 + (B*a^3 + 3*A*a^2*b)*c*d^4)/(c^3*d^4 + c*d^6 + (c^2*d^5 + d^7)*
tan(f*x + e)) + (C*b^3*d*tan(f*x + e)^2 - 2*(2*C*b^3*c - (3*C*a*b^2 + B*b^
3)*d)*tan(f*x + e))/d^3)/f
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(577) = 1154$.

Time = 1.14 (sec) , antiderivative size = 1327, normalized size of antiderivative = 2.29

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3*c^2 - C*a^3*c^2 - 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*
c^2 + B*b^3*c^2 + 2*B*a^3*c*d + 6*A*a^2*b*c*d - 6*C*a^2*b*c*d - 6*B*a*b^2*
c*d - 2*A*b^3*c*d + 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 + 3*B*a^2*b*d^2 +
3*A*a*b^2*d^2 - 3*C*a*b^2*d^2 - B*b^3*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^
4) + (B*a^3*c^2 + 3*A*a^2*b*c^2 - 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 - A*b^3*c^
2 + C*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d + 6*B*a^2*b*c*d + 6*A*a*b^2*c*d
- 6*C*a*b^2*c*d - 2*B*b^3*c*d - B*a^3*d^2 - 3*A*a^2*b*d^2 + 3*C*a^2*b*d^2
+ 3*B*a*b^2*d^2 + A*b^3*d^2 - C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*
c^2*d^2 + d^4) + 2*(3*C*b^3*c^6 - 6*C*a*b^2*c^5*d - 2*B*b^3*c^5*d + 3*C*a^
2*b*c^4*d^2 + 3*B*a*b^2*c^4*d^2 + A*b^3*c^4*d^2 + 5*C*b^3*c^4*d^2 - 12*C*a
*b^2*c^3*d^3 - 4*B*b^3*c^3*d^3 - B*a^3*c^2*d^4 - 3*A*a^2*b*c^2*d^4 + 9*C*a
^2*b*c^2*d^4 + 9*B*a*b^2*c^2*d^4 + 3*A*b^3*c^2*d^4 + 2*A*a^3*c*d^5 - 2*C*a
^3*c*d^5 - 6*B*a^2*b*c*d^5 - 6*A*a*b^2*c*d^5 + B*a^3*d^6 + 3*A*a^2*b*d^6)*
log(abs(d*tan(f*x + e) + c))/(c^4*d^4 + 2*c^2*d^6 + d^8) - 2*(3*C*b^3*c^6*
d*tan(f*x + e) - 6*C*a*b^2*c^5*d^2*tan(f*x + e) - 2*B*b^3*c^5*d^2*tan(f*x
+ e) + 3*C*a^2*b*c^4*d^3*tan(f*x + e) + 3*B*a*b^2*c^4*d^3*tan(f*x + e) + A
*b^3*c^4*d^3*tan(f*x + e) + 5*C*b^3*c^4*d^3*tan(f*x + e) - 12*C*a*b^2*c^3*
d^4*tan(f*x + e) - 4*B*b^3*c^3*d^4*tan(f*x + e) - B*a^3*c^2*d^5*tan(f*x +
e) - 3*A*a^2*b*c^2*d^5*tan(f*x + e) + 9*C*a^2*b*c^2*d^5*tan(f*x + e) + 9*B
*a*b^2*c^2*d^5*tan(f*x + e) + 3*A*b^3*c^2*d^5*tan(f*x + e) + 2*A*a^3*c*...
```

3.77.9 Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$$

$$= \frac{\tan(e+f\,x)\,\left(\frac{B\,b^3+3\,C\,a\,b^2}{d^2}-\frac{2\,C\,b^3\,c}{d^3}\right)}{f}$$

$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,a^3-A\,b^3+C\,b^3+3\,A\,a^2\,b-3\,B\,a\,b^2-3\,C\,a^2\,b+A\,a^3\,1\mathrm{i}+B\,b^3\,1\mathrm{i}-C\,a^3\,1\mathrm{i}\right.}{2\,f\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}$$

$$+\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^4\,\left(3\,A\,b^3\,c^2-B\,a^3\,c^2-3\,A\,a^2\,b\,c^2+9\,B\,a\,b^2\,c^2+9\,C\,a^2\,b\,c^2\right)-d^5\,\left(2\,C\,a^3\,c-2\right.\right.}{}$$

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,a^3-A\,b^3\,1\mathrm{i}+B\,a^3\,1\mathrm{i}+B\,b^3-C\,a^3+C\,b^3\,1\mathrm{i}-3\,A\,a\,b^2+A\,a^2\,b\,3\mathrm{i}-B\,a\,b^2\,3\mathrm{i}\right.}{2\,f\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}$$

$$-\frac{C\,a^3\,c^2\,d^3-B\,a^3\,c\,d^4+A\,a^3\,d^5-3\,C\,a^2\,b\,c^3\,d^2+3\,B\,a^2\,b\,c^2\,d^3-3\,A\,a^2\,b\,c\,d^4+3\,C\,a\,b^2\,c^4\,d-3\,B\,a\,b^2\,c^3}{d\,f\,\left(\tan\left(e+f\,x\right)\,d^4+c\,d^3\right)\,\left(c^2+d^2\right)}$$

$$+\frac{C\,b^3\,\tan(e+f\,x)^2}{2\,d^2\,f}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^2,x)
```

output
```
(tan(e + f*x)*((B*b^3 + 3*C*a*b^2)/d^2 - (2*C*b^3*c)/d^3))/f - (log(tan(e
+ f*x) + 1i)*(A*a^3*1i - A*b^3 + B*a^3 + B*b^3*1i - C*a^3*1i + C*b^3 - A*a
*b^2*3i + 3*A*a^2*b - 3*B*a*b^2 - B*a^2*b*3i + C*a*b^2*3i - 3*C*a^2*b))/(2
*f*(c*d*2i - c^2 + d^2)) + (log(c + d*tan(e + f*x))*(d^4*(3*A*b^3*c^2 - B*
a^3*c^2 - 3*A*a^2*b*c^2 + 9*B*a*b^2*c^2 + 9*C*a^2*b*c^2) - d^5*(2*C*a^3*c
- 2*A*a^3*c + 6*A*a*b^2*c + 6*B*a^2*b*c) - d^3*(4*B*b^3*c^3 + 12*C*a*b^2*c
^3) + d^6*(B*a^3 + 3*A*a^2*b) - d*(2*B*b^3*c^5 + 6*C*a*b^2*c^5) + d^2*(A*b
^3*c^4 + 5*C*b^3*c^4 + 3*B*a*b^2*c^4 + 3*C*a^2*b*c^4) + 3*C*b^3*c^6))/(f*(
d^8 + 2*c^2*d^6 + c^4*d^4)) - (log(tan(e + f*x) - 1i)*(A*a^3 - A*b^3*1i +
B*a^3*1i + B*b^3 - C*a^3 + C*b^3*1i - 3*A*a*b^2 + A*a^2*b*3i - B*a*b^2*3i
- 3*B*a^2*b + 3*C*a*b^2 - C*a^2*b*3i))/(2*f*(2*c*d - c^2*1i + d^2*1i)) - (
A*a^3*d^5 - C*b^3*c^5 - B*a^3*c*d^4 + B*b^3*c^4*d - A*b^3*c^3*d^2 + C*a^3*
c^2*d^3 + 3*A*a*b^2*c^2*d^3 - 3*B*a*b^2*c^3*d^2 + 3*B*a^2*b*c^2*d^3 - 3*C*
a^2*b*c^3*d^2 - 3*A*a^2*b*c*d^4 + 3*C*a*b^2*c^4*d)/(d*f*(c*d^3 + d^4*tan(e
 + f*x))*(c^2 + d^2)) + (C*b^3*tan(e + f*x)^2)/(2*d^2*f)
```

3.77. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.78 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.78.1 Optimal result

Integrand size = 45, antiderivative size = 417

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx =$$
$$-\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-2ab(2c(A-C)d-B(c}{(c^2+d^2)^2}$$
$$+\frac{(2ab(c^2C-2Bcd-Cd^2-A(c^2-d^2))+a^2(2c(A-C)d-B(c^2-d^2))-b^2(2c(A-C)d-B(c^2-d^2)}{(c^2+d^2)^2\,f}$$
$$-\frac{(bc-ad)\,(b(2c^4C-Bc^3d+4c^2Cd^2-3Bcd^3+2Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\log(c+d\tan(}{d^3\,(c^2+d^2)^2\,f}$$
$$+\frac{b^2(2c^2C-Bcd+(A+C)d^2)\tan(e+fx)}{d^2\,(c^2+d^2)\,f}-\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^2}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

output
```
-(a^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^
2))-2*a*b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(c^2+d^2)^2+(2*a*b*(c^2*C-2*B*c*d-C
*d^2-A*(c^2-d^2))+a^2*(2*c*(A-C)*d-B*(c^2-d^2))-b^2*(2*c*(A-C)*d-B*(c^2-d^
2)))*ln(cos(f*x+e))/(c^2+d^2)^2/f-(-a*d+b*c)*(b*(2*A*d^4-B*c^3*d-3*B*c*d^3
+2*C*c^4+4*C*c^2*d^2)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+d*tan(f*x+e))/
d^3/(c^2+d^2)^2/f+b^2*(2*c^2*C-B*c*d+(A+C)*d^2)*tan(f*x+e)/d^2/(c^2+d^2)/f
-(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.57 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{(a+ib)^2(-iA+B+iC)\log(i-\tan(e+fx))}{(c+id)^2}+\frac{(a-ib)^2(iA+B-iC)\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2(-bc+ad)\left(b\left(2c^4C-Bc^3d+4c^2Cd^2-3Bcd^3+2Ad^4\right)+\cdots}{d^3(c^2+\cdots}}{2f}$$

input
```
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^2,x]
```

output
```
(((a + I*b)^2*((-I)*A + B + I*C)*Log[I - Tan[e + f*x]])/(c + I*d)^2 + ((a
- I*b)^2*(I*A + B - I*C)*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*(-(b*c) +
 a*d)*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*c*d^3 + 2*A*d^4) + a*d^2*(
2*c*(A - C)*d + B*(-c^2 + d^2)))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)
^2) - (2*(b*c - a*d)^2*(2*c^2*C - B*c*d + (A + C)*d^2))/(d^3*(c^2 + d^2)*(
c + d*Tan[e + f*x])) + (2*C*(a + b*Tan[e + f*x])^2)/(d*(c + d*Tan[e + f*x]
)))/(2*f)
```

3.78.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4128, 3042, 4120, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(2Cc^2-Bdc+(A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+2bd)+(2bc-ad)(cC-Bd)\right)}{c+d\tan(e+fx)}dx}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(2Cc^2-Bdc+(A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+2bd)+(2bc-ad)(cC-Bd)\right)}{c+d\tan(e+fx)}dx}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 4120

$$\frac{\frac{b^2\tan(e+fx)\left(d^2(A+C)-Bcd+2c^2C\right)}{df}-\frac{\int \frac{c\left(2Cc^2-Bdc+(A+C)d^2\right)b^2+(2bcC-2adC-bBd)\left(c^2+d^2\right)\tan^2(e+fx)b-ad(Ad(ac+2bd)+(2bc-ad)(cC-Bd))-c}{c+d\tan(e+fx)}}{d}}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{b^2\tan(e+fx)\left(d^2(A+C)-Bcd+2c^2C\right)}{df}-\frac{\int \frac{c\left(2Cc^2-Bdc+(A+C)d^2\right)b^2+(2bcC-2adC-bBd)\left(c^2+d^2\right)\tan(e+fx)^2b-ad(Ad(ac+2bd)+(2bc-ad)(cC-Bd))-c}{c+d\tan(e+fx)}}{d}}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 4109

$$\frac{\frac{b^2\tan(e+fx)\left(d^2(A+C)-Bcd+2c^2C\right)}{df}-\frac{d^2\left(a^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)\int \tan(e+fx)}{c^2+d^2}}{}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{b^2\tan(e+fx)\left(d^2(A+C)-Bcd+2c^2C\right)}{df}-\frac{d^2\left(a^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)\int \tan(e+fx)}{c^2+d^2}}{}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

3.78. $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{b^2\tan(e+fx)(d^2(A+C)-Bcd+2c^2C)}{df}-\frac{\frac{(bc-ad)\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{d^2\log(\cos}{}}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 4100$$

$$\frac{\frac{b^2\tan(e+fx)(d^2(A+C)-Bcd+2c^2C)}{df}-\frac{\frac{(bc-ad)\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)\int\frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df\left(c^2+d^2\right)}}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{b^2\tan(e+fx)(d^2(A+C)-Bcd+2c^2C)}{df}-\frac{-\frac{d^2\log(\cos(e+fx))\left(a^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+2ab\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)-b^2\left(2cd(A-C)-B\left(c^2-d\right.\right.\right.}{f\left(c^2+d^2\right)}}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^2,x]
```

output
```
-(((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*(c + d
*Tan[e + f*x]))) + (-(((d^2*(a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))
 - b^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 2*a*b*(2*c*(A - C)*d -
B*(c^2 - d^2)))*x)/(c^2 + d^2) - (d^2*(2*a*b*(c^2*C - 2*B*c*d - C*d^2 - A*
(c^2 - d^2)) + a^2*(2*c*(A - C)*d - B*(c^2 - d^2)) - b^2*(2*c*(A - C)*d -
B*(c^2 - d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)*f) + ((b*c - a*d)*(b*(2*c^
4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*c*d^3 + 2*A*d^4) + a*d^2*(2*c*(A - C)*d
- B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d*(c^2 + d^2)*f))/d) + (b^2*(2
*c^2*C - B*c*d + (A + C)*d^2)*Tan[e + f*x])/(d*f))/(d*(c^2 + d^2))
```

3.78. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.78. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.78.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\tan(fx+e)C\,b^2}{d^2}+\frac{\frac{\left(-2A\,a^2cd+2Aab\,c^2-2Aab\,d^2+2A\,b^2cd+B\,a^2c^2-B\,a^2d^2+4Babcd-B\,b^2c^2+B\,b^2d^2+2C\,a^2cd-2Cab\,c^2+2Cab\,d^2-}{2}}{}$
default	$\frac{\tan(fx+e)C\,b^2}{d^2}+\frac{\frac{\left(-2A\,a^2cd+2Aab\,c^2-2Aab\,d^2+2A\,b^2cd+B\,a^2c^2-B\,a^2d^2+4Babcd-B\,b^2c^2+B\,b^2d^2+2C\,a^2cd-2Cab\,c^2+2Cab\,d^2-}{2}}{}$
norman	$\frac{\frac{c\left(A\,a^2c^2-A\,a^2d^2+4Aabcd-A\,b^2c^2+A\,b^2d^2+2B\,a^2cd-2Bab\,c^2+2Bab\,d^2-2B\,b^2cd-C\,a^2c^2+a^2C\,d^2-4Cabcd+C\,b^2c^2-C\,b^2d^2\right)x}{c^4+2c^2d^2+d^4}+}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*b^2/d^2+1/(c^2+d^2)^2*(1/2*(-2*A*a^2*c*d+2*A*a*b*c^2-2*A
*a*b*d^2+2*A*b^2*c*d+B*a^2*c^2-B*a^2*d^2+4*B*a*b*c*d-B*b^2*c^2+B*b^2*d^2+2
*C*a^2*c*d-2*C*a*b*c^2+2*C*a*b*d^2-2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)+(A*a^2*
c^2-A*a^2*d^2+4*A*a*b*c*d-A*b^2*c^2+A*b^2*d^2+2*B*a^2*c*d-2*B*a*b*c^2+2*B*
a*b*d^2-2*B*b^2*c*d-C*a^2*c^2+C*a^2*d^2-4*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)*a
rctan(tan(f*x+e)))-1/d^3*(A*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^2*c*d^
3+2*B*a*b*c^2*d^2-B*b^2*c^3*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/(c^2+
d^2)/(c+d*tan(f*x+e))+1/d^3*(2*A*a^2*c*d^4-2*A*a*b*c^2*d^3+2*A*a*b*d^5-2*A
*b^2*c*d^4-B*a^2*c^2*d^3+B*a^2*d^5-4*B*a*b*c*d^4+B*b^2*c^4*d+3*B*b^2*c^2*d
^3-2*C*a^2*c*d^4+2*C*a*b*c^4*d+6*C*a*b*c^2*d^3-2*C*b^2*c^5-4*C*b^2*c^3*d^2
)/(c^2+d^2)^2*ln(c+d*tan(f*x+e)))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(417) = 834$.

Time = 0.56 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.25

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx =$$

$$-\frac{2\,Cb^2c^4d^2+2\,Aa^2d^6-2\,(2\,Cab+Bb^2)c^3d^3+2\,(Ca^2+2\,Bab+Ab^2)c^2d^4-2\,(Ba^2+2\,Aab)cd^5-2\,(((}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*C*b^2*c^4*d^2 + 2*A*a^2*d^6 - 2*(2*C*a*b + B*b^2)*c^3*d^3 + 2*(C*a
^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 2*(B*a^2 + 2*A*a*b)*c*d^5 - 2*(((A - C)*a^
2 - 2*B*a*b - (A - C)*b^2)*c^3*d^3 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2
*d^4 - ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^5)*f*x - 2*(C*b^2*c^4*d^2
 + 2*C*b^2*c^2*d^4 + C*b^2*d^6)*tan(f*x + e)^2 + (2*C*b^2*c^6 + 4*C*b^2*c^
4*d^2 - (2*C*a*b + B*b^2)*c^5*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c^3*
d^3 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5
+ (2*C*b^2*c^5*d + 4*C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 +
2*(A - 3*C)*a*b - 3*B*b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d
^5 - (B*a^2 + 2*A*a*b)*d^6)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*
tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (2*C*b^2*c^6 + 4*C*b^2*c^4*d^2
 + 2*C*b^2*c^2*d^4 - (2*C*a*b + B*b^2)*c^5*d - 2*(2*C*a*b + B*b^2)*c^3*d^3
 - (2*C*a*b + B*b^2)*c*d^5 + (2*C*b^2*c^5*d + 4*C*b^2*c^3*d^3 + 2*C*b^2*c*
d^5 - (2*C*a*b + B*b^2)*c^4*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^4 - (2*C*a*b +
 B*b^2)*d^6)*tan(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(2*C*b^2*c^5*d
- (2*C*a*b + B*b^2)*c^4*d^2 + (C*a^2 + 2*B*a*b + (A + 2*C)*b^2)*c^3*d^3 -
(B*a^2 + 2*A*a*b)*c^2*d^4 + (A*a^2 + C*b^2)*c*d^5 + (((A - C)*a^2 - 2*B*a*
b - (A - C)*b^2)*c^2*d^4 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^5 - ((A -
 C)*a^2 - 2*B*a*b - (A - C)*b^2)*d^6)*f*x)*tan(f*x + e))/((c^4*d^4 + 2*c^2
*d^6 + d^8)*f*tan(f*x + e) + (c^5*d^3 + 2*c^3*d^5 + c*d^7)*f)
```

3.78.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 16225, normalized size of antiderivative = 38.91

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**2,x)
```

output
```
Piecewise((zoo*x*(a + b*tan(e))**2*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2,
 Eq(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a**2*x + A*a*b*log(tan(e + f*x)**2 +
 1)/f - A*b**2*x + A*b**2*tan(e + f*x)/f + B*a**2*log(tan(e + f*x)**2 + 1)
/(2*f) - 2*B*a*b*x + 2*B*a*b*tan(e + f*x)/f - B*b**2*log(tan(e + f*x)**2 +
 1)/(2*f) + B*b**2*tan(e + f*x)**2/(2*f) - C*a**2*x + C*a**2*tan(e + f*x)/
f - C*a*b*log(tan(e + f*x)**2 + 1)/f + C*a*b*tan(e + f*x)**2/f + C*b**2*x
+ C*b**2*tan(e + f*x)**3/(3*f) - C*b**2*tan(e + f*x)/f)/c**2, Eq(d, 0)), (
-A*a**2*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) + 2*I*A*a**2*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2
- 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a**2*f*x/(4*d**2*f*tan(e + f*x)*
*2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*a**2*tan(e + f*x)/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a**2/(4*d**2*
f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a*b*f*x*ta
n(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2
*f) + 4*A*a*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(
e + f*x) - 4*d**2*f) - 2*I*A*a*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*
f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a*b*tan(e + f*x)/(4*d**2*f*tan(e + f*x)
**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*b**2*f*x*tan(e + f*x)**2/(4*
d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*A*b**2*
f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - ...
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.18

$$\int \frac{(a + b\tan(e + fx))^2 (A + B\tan(e + fx) + C\tan^2(e + fx))}{(c + d\tan(e + fx))^2} \, dx$$
$$= \frac{\frac{2\,Cb^2\tan(fx+e)}{d^2} + \frac{2\,(((A-C)a^2 - 2\,Bab - (A-C)b^2)c^2 + 2\,(Ba^2 + 2\,(A-C)ab - Bb^2)cd - ((A-C)a^2 - 2\,Bab - (A-C)b^2)d^2)(fx+e)}{c^4 + 2\,c^2d^2 + d^4} - \frac{2\,(2\,Cb^2}{}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c
^2 + 2*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d - ((A - C)*a^2 - 2*B*a*b - (A -
 C)*b^2)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 + 4*C*b^2
*c^3*d^2 - (2*C*a*b + B*b^2)*c^4*d + (B*a^2 + 2*(A - 3*C)*a*b - 3*B*b^2)*c
^2*d^3 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*
log(d*tan(f*x + e) + c)/(c^4*d^3 + 2*c^2*d^5 + d^7) + ((B*a^2 + 2*(A - C)*
a*b - B*b^2)*c^2 - 2*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d - (B*a^2 +
2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4
) - 2*(C*b^2*c^4 + A*a^2*d^4 - (2*C*a*b + B*b^2)*c^3*d + (C*a^2 + 2*B*a*b
+ A*b^2)*c^2*d^2 - (B*a^2 + 2*A*a*b)*c*d^3)/(c^3*d^3 + c*d^5 + (c^2*d^4 +
d^6)*tan(f*x + e)))/f
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(417) = 834$.

Time = 0.78 (sec) , antiderivative size = 893, normalized size of antiderivative = 2.14

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2\,Cb^2\tan(fx+e)}{d^2} + \frac{2\,(Aa^2c^2-Ca^2c^2-2\,Babc^2-Ab^2c^2+Cb^2c^2+2\,Ba^2cd+4\,Aabcd-4\,Cabcd-2\,Bb^2cd-Aa^2d^2+Ca^2d^2+2\,Babd^2+Ab^2d^2-Cb^2}{c^4+2\,c^2d^2+d^4}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*C*b^2*tan(f*x + e)/d^2 + 2*(A*a^2*c^2 - C*a^2*c^2 - 2*B*a*b*c^2 - A
*b^2*c^2 + C*b^2*c^2 + 2*B*a^2*c*d + 4*A*a*b*c*d - 4*C*a*b*c*d - 2*B*b^2*c
*d - A*a^2*d^2 + C*a^2*d^2 + 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e
)/(c^4 + 2*c^2*d^2 + d^4) + (B*a^2*c^2 + 2*A*a*b*c^2 - 2*C*a*b*c^2 - B*b^2
*c^2 - 2*A*a^2*c*d + 2*C*a^2*c*d + 4*B*a*b*c*d + 2*A*b^2*c*d - 2*C*b^2*c*d
 - B*a^2*d^2 - 2*A*a*b*d^2 + 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 +
 1)/(c^4 + 2*c^2*d^2 + d^4) - 2*(2*C*b^2*c^5 - 2*C*a*b*c^4*d - B*b^2*c^4*d
 + 4*C*b^2*c^3*d^2 + B*a^2*c^2*d^3 + 2*A*a*b*c^2*d^3 - 6*C*a*b*c^2*d^3 - 3
*B*b^2*c^2*d^3 - 2*A*a^2*c*d^4 + 2*C*a^2*c*d^4 + 4*B*a*b*c*d^4 + 2*A*b^2*c
*d^4 - B*a^2*d^5 - 2*A*a*b*d^5)*log(abs(d*tan(f*x + e) + c))/(c^4*d^3 + 2*
c^2*d^5 + d^7) + 2*(2*C*b^2*c^5*d*tan(f*x + e) - 2*C*a*b*c^4*d^2*tan(f*x +
 e) - B*b^2*c^4*d^2*tan(f*x + e) + 4*C*b^2*c^3*d^3*tan(f*x + e) + B*a^2*c^
2*d^4*tan(f*x + e) + 2*A*a*b*c^2*d^4*tan(f*x + e) - 6*C*a*b*c^2*d^4*tan(f*
x + e) - 3*B*b^2*c^2*d^4*tan(f*x + e) - 2*A*a^2*c*d^5*tan(f*x + e) + 2*C*a
^2*c*d^5*tan(f*x + e) + 4*B*a*b*c*d^5*tan(f*x + e) + 2*A*b^2*c*d^5*tan(f*x
 + e) - B*a^2*d^6*tan(f*x + e) - 2*A*a*b*d^6*tan(f*x + e) + C*b^2*c^6 - C*
a^2*c^4*d^2 - 2*B*a*b*c^4*d^2 - A*b^2*c^4*d^2 + 3*C*b^2*c^4*d^2 + 2*B*a^2*
c^3*d^3 + 4*A*a*b*c^3*d^3 - 4*C*a*b*c^3*d^3 - 2*B*b^2*c^3*d^3 - 3*A*a^2*c^
2*d^4 + C*a^2*c^2*d^4 + 2*B*a*b*c^2*d^4 + A*b^2*c^2*d^4 - A*a^2*d^6)/((c^4
*d^3 + 2*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)))/f
```

3.78.9 Mupad [B] (verification not implemented)

Time = 33.60 (sec) , antiderivative size = 3958, normalized size of antiderivative = 9.49

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^2,x)
```

output
```
(log((2*C^2*b^4*c^5 - 2*C^2*a^2*b^2*c^5 + 4*C^2*b^4*c^3*d^2 - A*B*a^4*d^5
- 2*A*C*b^4*c^5 + B*C*a^4*d^5 + 2*A^2*a*b^3*d^5 - 2*A^2*a^3*b*d^5 - A^2*a^
4*c*d^4 + 2*B^2*a^3*b*d^5 - A^2*b^4*c*d^4 + B^2*a^4*c*d^4 + B^2*b^4*c*d^4
- C^2*a^4*c*d^4 + C^2*b^4*c*d^4 - 4*C^2*a^2*b^2*c^3*d^2 + 5*A*B*a^2*b^2*d^
5 + 2*A*C*a^2*b^2*c^5 + A*B*a^4*c^2*d^3 + 3*A*B*b^4*c^2*d^3 - B*C*a^2*b^2*
d^5 - 4*A*C*b^4*c^3*d^2 - B*C*a^4*c^2*d^3 - 3*B*C*b^4*c^2*d^3 + 2*B^2*a*b^
3*c^4*d - 2*C^2*a*b^3*c^4*d + 2*C^2*a^3*b*c^4*d - 2*A^2*a*b^3*c^2*d^3 + 6*
A^2*a^2*b^2*c*d^4 + 2*A^2*a^3*b*c^2*d^3 + 6*B^2*a*b^3*c^2*d^3 - 6*B^2*a^2*
b^2*c*d^4 - 2*B^2*a^3*b*c^2*d^3 - 6*C^2*a*b^3*c^2*d^3 + 4*C^2*a^2*b^2*c*d^
4 + 6*C^2*a^3*b*c^2*d^3 - 2*A*C*a*b^3*d^5 + 2*A*C*a^3*b*d^5 - 4*B*C*a*b^3*
c^5 + A*B*b^4*c^4*d + 2*A*C*a^4*c*d^4 - B*C*b^4*c^4*d - 8*A*B*a*b^3*c*d^4
+ 8*A*B*a^3*b*c*d^4 + 2*A*C*a*b^3*c^4*d - 2*A*C*a^3*b*c^4*d + 4*B*C*a*b^3*
c*d^4 - 8*B*C*a^3*b*c*d^4 - A*B*a^2*b^2*c^4*d + 8*A*C*a*b^3*c^2*d^3 - 10*A
*C*a^2*b^2*c*d^4 - 8*A*C*a^3*b*c^2*d^3 - 8*B*C*a*b^3*c^3*d^2 + 5*B*C*a^2*b
^2*c^4*d - 8*A*B*a^2*b^2*c^2*d^3 + 4*A*C*a^2*b^2*c^3*d^2 + 16*B*C*a^2*b^2*
c^2*d^3)/(d^2*(c^2 + d^2)^2) + ((a*1i - b)^2*((A*b^2*d^2 - A*a^2*d^2 + C*a
^2*d^2 - 8*C*b^2*c^2 - C*b^2*d^2 + 2*B*a*b*d^2 + 4*B*b^2*c*d + 8*C*a*b*c*d
)/d - (tan(e + f*x)*(3*B*a^2*d^5 - 5*B*b^2*d^5 - 4*C*b^2*c^5 + 6*A*a*b*d^5
 - 10*C*a*b*d^5 + 4*A*a^2*c*d^4 - 4*A*b^2*c*d^4 + 2*B*b^2*c^4*d - 4*C*a^2*
c*d^4 + 8*C*b^2*c*d^4 - B*a^2*c^2*d^3 + B*b^2*c^2*d^3 - 8*B*a*b*c*d^4 +...
```

3.78. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.79 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

3.79.1 Optimal result

Integrand size = 43, antiderivative size = 292

$$\begin{aligned}
&\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx\\
&=-\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b(2c(A-C)d-B(c^2-d^2)))\,x}{(c^2+d^2)^2}\\
&\quad-\frac{(a(Bc^2+2cCd-Bd^2)-b(c^2C-2Bcd-Cd^2)-A(2acd-b(c^2-d^2)))\log(\cos(e+fx))}{(c^2+d^2)^2\,f}\\
&\quad+\frac{(b(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\log(c+d\tan(e+fx))}{d^2\,(c^2+d^2)^2\,f}\\
&\quad+\frac{(bc-ad)\,(c^2C-Bcd+Ad^2)}{d^2\,(c^2+d^2)\,f(c+d\tan(e+fx))}
\end{aligned}$$

output
```
-(a*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(c^2+
d^2)^2-(a*(B*c^2-B*d^2+2*C*c*d)-b*(-2*B*c*d+C*c^2-C*d^2)-A*(2*a*c*d-b*(c^2
-d^2)))*ln(cos(f*x+e))/(c^2+d^2)^2/f+(b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3+A
*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(c+d*tan(f*x+e))/d^2/(c^2+d^2)^2/
f+(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{(-ia+b)(A+iB-C)\log(i-\tan(e+fx))}{(c+id)^2}+\frac{(ia+b)(A-iB-C)\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2\left(b\left(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4\right)+ad^2\left(2c(A-C)d+B\right.\right.}{d^2(c^2+d^2)^2}}{2f}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^2,x]
```

output
```
((((-I)*a + b)*(A + I*B - C)*Log[I - Tan[e + f*x]])/(c + I*d)^2 + ((I*a +
b)*(A - I*B - C)*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*(b*(c^4*C - c^2*(
A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d + B*(-c^2 + d^2))
)*Log[c + d*Tan[e + f*x]])/(d^2*(c^2 + d^2)^2) + (2*(b*c - a*d)*(c^2*C - B
*c*d + A*d^2))/(d^2*(c^2 + d^2)*(c + d*Tan[e + f*x])))/(2*f)
```

3.79.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^2}dx$$
$$\Big\downarrow \; 4118$$
$$\frac{\int \frac{bC\left(c^2+d^2\right)\tan^2(e+fx)+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b\left(Cc^2-Bdc+Ad^2\right)}{c+d\tan(e+fx)}dx}{d\left(c^2+d^2\right)}+$$
$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{bC(c^2+d^2)\tan(e+fx)^2+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{c+d\tan(e+fx)}dx}{d(c^2+d^2)}+\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 4109$$

$$\frac{-\frac{d(2aAcd-aB(c^2-d^2)-2acCd-Ab(c^2-d^2)+b(-2Bcd+c^2C-Cd^2))\int\tan(e+fx)dx}{c^2+d^2}+\frac{(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4}{c^2+d^2}}{d(c^2+d^2)}$$
$$\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{d(2aAcd-aB(c^2-d^2)-2acCd-Ab(c^2-d^2)+b(-2Bcd+c^2C-Cd^2))\int\tan(e+fx)dx}{c^2+d^2}+\frac{(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4}{c^2+d^2}}{d(c^2+d^2)}$$
$$\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C))\int\frac{\tan(e+fx)^2+1}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{d\log(\cos(e+fx))(2aAcd-aB(c^2-d^2)-2acCd-Ab(}{f(c^2+d^2)}}{d(c^2+d^2)}$$
$$\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 4100$$

$$\frac{\frac{(ad^2(2cd(A-C)-B(c^2-d^2))+b(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C))\int\frac{1}{c+d\tan(e+fx)}d(d\tan(e+fx))}{df(c^2+d^2)}+\frac{d\log(\cos(e+fx))(2aAcd-aB(c^2-d^2)-}{f}}{d(c^2+d^2)}$$
$$\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}$$

$$\Big\downarrow \; 16$$

$$\frac{(bc-ad)(Ad^2-Bcd+c^2C)}{d^2f(c^2+d^2)(c+d\tan(e+fx))}+$$
$$\frac{\frac{d\log(\cos(e+fx))(2aAcd-aB(c^2-d^2)-2acCd-Ab(c^2-d^2)+b(-2Bcd+c^2C-Cd^2))}{f(c^2+d^2)}-\frac{dx(a(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)-b(2cd(A-C)-B}{c^2+d^2}}{d(c^2+d^2)}$$

3.79. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*
Tan[e + f*x])^2,x]
```

output
```
(-((d*(a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b*(2*c*(A - C)*d - B*
(c^2 - d^2)))*x)/(c^2 + d^2)) + (d*(2*a*A*c*d - 2*a*c*C*d - A*b*(c^2 - d^2
) - a*B*(c^2 - d^2) + b*(c^2*C - 2*B*c*d - C*d^2))*Log[Cos[e + f*x]])/((c^
2 + d^2)*f) + ((b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*
(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/(d*(c^2 + d^2)*f
))/(d*(c^2 + d^2)) + ((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(d^2*(c^2 + d^2
)*f*(c + d*Tan[e + f*x]))
```

3.79.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.79.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\frac{\left(-2Aacd+Ab\,c^2-Ab\,d^2+Ba\,c^2-Ba\,d^2+2Bbcd+2Cacd-Cb\,c^2+Cb\,d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(Aa\,c^2-Aa\,d^2+2Abcd+2Bacd-Bb\,c^2+\cdots\right.}{\left(c^2+d^2\right)^2}}{\cdots}$
default	$\frac{\frac{\frac{\left(-2Aacd+Ab\,c^2-Ab\,d^2+Ba\,c^2-Ba\,d^2+2Bbcd+2Cacd-Cb\,c^2+Cb\,d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(Aa\,c^2-Aa\,d^2+2Abcd+2Bacd-Bb\,c^2+\cdots\right.}{\left(c^2+d^2\right)^2}}{\cdots}$
norman	$\frac{\frac{c\left(Aa\,c^2-Aa\,d^2+2Abcd+2Bacd-Bb\,c^2+Bb\,d^2-Ca\,c^2+Ca\,d^2-2Cbcd\right)x}{c^4+2c^2d^2+d^4}+\frac{d\left(Aa\,c^2-Aa\,d^2+2Abcd+2Bacd-Bb\,c^2+Bb\,d^2-Ca\,c^2+C\cdots\right.}{c^4+2c^2d^2+d^4}}{c+d\tan(fx+e)}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x,
method=_RETURNVERBOSE)
```

output
```
1/f*(1/(c^2+d^2)^2*(1/2*(-2*A*a*c*d+A*b*c^2-A*b*d^2+B*a*c^2-B*a*d^2+2*B*b*
c*d+2*C*a*c*d-C*b*c^2+C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2+2*A*b*c
*d+2*B*a*c*d-B*b*c^2+B*b*d^2-C*a*c^2+C*a*d^2-2*C*b*c*d)*arctan(tan(f*x+e))
)-(A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*d-C*b*c^3)/d^2/(c^2+d^2)/
(c+d*tan(f*x+e))+(2*A*a*c*d^3-A*b*c^2*d^2+A*b*d^4-B*a*c^2*d^2+B*a*d^4-2*B*
b*c*d^3-2*C*a*c*d^3+C*b*c^4+3*C*b*c^2*d^2)/(c^2+d^2)^2/d^2*ln(c+d*tan(f*x+
e)))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$$

$$= \frac{2\,Cbc^3d^2-2\,Aad^5-2\,(Ca+Bb)c^2d^3+2\,(Ba+Ab)cd^4+2\,(((A-C)a-Bb)c^3d^2+2\,(Ba+(A-C)b)}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^2,x, algorithm="fricas")
```

output
```
1/2*(2*C*b*c^3*d^2 - 2*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 2*(B*a + A*b)*c*d
^4 + 2*(((A - C)*a - B*b)*c^3*d^2 + 2*(B*a + (A - C)*b)*c^2*d^3 - ((A - C)
*a - B*b)*c*d^4)*f*x + (C*b*c^5 - (B*a + (A - 3*C)*b)*c^3*d^2 + 2*((A - C)
*a - B*b)*c^2*d^3 + (B*a + A*b)*c*d^4 + (C*b*c^4*d - (B*a + (A - 3*C)*b)*c
^2*d^3 + 2*((A - C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))*log((d
^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - (C*b
*c^5 + 2*C*b*c^3*d^2 + C*b*c*d^4 + (C*b*c^4*d + 2*C*b*c^2*d^3 + C*b*d^5)*t
an(f*x + e))*log(1/(tan(f*x + e)^2 + 1)) - 2*(C*b*c^4*d - A*a*c*d^4 - (C*a
 + B*b)*c^3*d^2 + (B*a + A*b)*c^2*d^3 - (((A - C)*a - B*b)*c^2*d^3 + 2*(B*
a + (A - C)*b)*c*d^4 - ((A - C)*a - B*b)*d^5)*f*x)*tan(f*x + e))/((c^4*d^3
 + 2*c^2*d^5 + d^7)*f*tan(f*x + e) + (c^5*d^2 + 2*c^3*d^4 + c*d^6)*f)
```

3.79.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 9721, normalized size of antiderivative = 33.29

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
))**2,x)
```

output
```
Piecewise((zoo*x*(a + b*tan(e))*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq
(c, 0) & Eq(d, 0) & Eq(f, 0)), ((A*a*x + A*b*log(tan(e + f*x)**2 + 1)/(2*f
) + B*a*log(tan(e + f*x)**2 + 1)/(2*f) - B*b*x + B*b*tan(e + f*x)/f - C*a*
x + C*a*tan(e + f*x)/f - C*b*log(tan(e + f*x)**2 + 1)/(2*f) + C*b*tan(e +
f*x)**2/(2*f))/c**2, Eq(d, 0)), (-A*a*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e
+ f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*a*f*x*tan(e + f*x)
/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*a*f*x
/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*a*tan
(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
+ 2*I*A*a/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
+ I*A*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) + 2*A*b*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I
*d**2*f*tan(e + f*x) - 4*d**2*f) - I*A*b*f*x/(4*d**2*f*tan(e + f*x)**2 - 8
*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*A*b*tan(e + f*x)/(4*d**2*f*tan(e +
f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*a*f*x*tan(e + f*x)**2/
(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*B*a*f*
x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**
2*f) - I*B*a*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d
**2*f) + I*B*a*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e +
 f*x) - 4*d**2*f) + B*b*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 -...
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$

$$= \frac{\frac{2\left(((A-C)a-Bb)c^2+2\,(Ba+(A-C)b)cd-((A-C)a-Bb)d^2\right)(fx+e)}{c^4+2\,c^2d^2+d^4} + \frac{2\left(Cbc^4-(Ba+(A-3\,C)b)c^2d^2+2\,((A-C)a-Bb)cd^3+(Ba+Ab)d^4\right)\log(}{c^4d^2+2\,c^2d^4+d^6}}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a - B*b)*c^2 + 2*(B*a + (A - C)*b)*c*d - ((A - C)*a - B*b
)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4 - (B*a + (A - 3*C)*b
)*c^2*d^2 + 2*((A - C)*a - B*b)*c*d^3 + (B*a + A*b)*d^4)*log(d*tan(f*x + e
) + c)/(c^4*d^2 + 2*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^2 - 2*((A - C)*a
 - B*b)*c*d - (B*a + (A - C)*b)*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*
d^2 + d^4) + 2*(C*b*c^3 - A*a*d^3 - (C*a + B*b)*c^2*d + (B*a + A*b)*c*d^2)
/(c^3*d^2 + c*d^4 + (c^2*d^3 + d^5)*tan(f*x + e)))/f
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{2\left(Aac^2-Cac^2-Bbc^2+2\,Bacd+2\,Abcd-2\,Cbcd-Aad^2+Cad^2+Bbd^2\right)(fx+e)}{c^4+2\,c^2d^2+d^4}+\frac{\left(Bac^2+Abc^2-Cbc^2-2\,Aacd+2\,Cacd+2\,Bbcd-Bad^2-Abd^2+\cdots\right.}{c^4+2\,c^2d^2+d^4}}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c^2 - C*a*c^2 - B*b*c^2 + 2*B*a*c*d + 2*A*b*c*d - 2*C*b*c*d -
A*a*d^2 + C*a*d^2 + B*b*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 + d^4) + (B*a*c^2
+ A*b*c^2 - C*b*c^2 - 2*A*a*c*d + 2*C*a*c*d + 2*B*b*c*d - B*a*d^2 - A*b*d^
2 + C*b*d^2)*log(tan(f*x + e)^2 + 1)/(c^4 + 2*c^2*d^2 + d^4) + 2*(C*b*c^4
- B*a*c^2*d^2 - A*b*c^2*d^2 + 3*C*b*c^2*d^2 + 2*A*a*c*d^3 - 2*C*a*c*d^3 -
2*B*b*c*d^3 + B*a*d^4 + A*b*d^4)*log(abs(d*tan(f*x + e) + c))/(c^4*d^2 + 2
*c^2*d^4 + d^6) - 2*(C*b*c^4*tan(f*x + e) - B*a*c^2*d^2*tan(f*x + e) - A*b
*c^2*d^2*tan(f*x + e) + 3*C*b*c^2*d^2*tan(f*x + e) + 2*A*a*c*d^3*tan(f*x +
 e) - 2*C*a*c*d^3*tan(f*x + e) - 2*B*b*c*d^3*tan(f*x + e) + B*a*d^4*tan(f*
x + e) + A*b*d^4*tan(f*x + e) + C*a*c^4 + B*b*c^4 - 2*B*a*c^3*d - 2*A*b*c^
3*d + 2*C*b*c^3*d + 3*A*a*c^2*d^2 - C*a*c^2*d^2 - B*b*c^2*d^2 + A*a*d^4)/(
(c^4*d + 2*c^2*d^3 + d^5)*(d*tan(f*x + e) + c)))/f
```

3.79.9 Mupad [B] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 1875, normalized size of antiderivative = 6.42

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x))^2,x)
```

output
```
(log(c + d*tan(e + f*x))*(d^4*(A*b + B*a) - d^3*(2*B*b*c - 2*A*a*c + 2*C*a
*c) - d^2*(A*b*c^2 + B*a*c^2 - 3*C*b*c^2) + C*b*c^4))/(f*(d^6 + 2*c^2*d^4
+ c^4*d^2)) - (log((A*B*b^2*d^4 - A*B*a^2*d^4 + B*C*a^2*d^4 + B*C*b^2*c^4
- A^2*a*b*d^4 + B^2*a*b*d^4 + C^2*a*b*c^4 - A^2*a^2*c*d^3 + A^2*b^2*c*d^3
+ B^2*a^2*c*d^3 - B^2*b^2*c*d^3 - C^2*a^2*c*d^3 + C^2*b^2*c*d^3 + A*B*a^2*
c^2*d^2 - A*B*b^2*c^2*d^2 - B*C*a^2*c^2*d^2 + 3*B*C*b^2*c^2*d^2 + A^2*a*b*
c^2*d^2 - B^2*a*b*c^2*d^2 + 3*C^2*a*b*c^2*d^2 - A*C*a*b*c^4 + A*C*a*b*d^4
+ 2*A*C*a^2*c*d^3 - 2*A*C*b^2*c*d^3 - 4*A*C*a*b*c^2*d^2 + 4*A*B*a*b*c*d^3
- 4*B*C*a*b*c*d^3)/(d*(c^2 + d^2)^2) + (tan(e + f*x)*(A^2*a^2*d^4 + B^2*b^
2*d^4 + C^2*a^2*d^4 + C^2*b^2*c^4 + C^2*b^2*d^4 + A^2*b^2*c^2*d^2 + B^2*a^
2*c^2*d^2 + 3*C^2*b^2*c^2*d^2 - 2*A*C*a^2*d^4 - A*C*b^2*c^4 - A*C*b^2*d^4
- 4*A*C*b^2*c^2*d^2 - 2*A*B*a*b*d^4 - B*C*a*b*c^4 + B*C*a*b*d^4 - 2*A*B*a^
2*c*d^3 + 2*A*B*b^2*c*d^3 + 2*B*C*a^2*c*d^3 - 2*B*C*b^2*c*d^3 - 2*A^2*a*b*
c*d^3 + 2*B^2*a*b*c*d^3 - 2*C^2*a*b*c*d^3 + 2*A*B*a*b*c^2*d^2 - 4*B*C*a*b*
c^2*d^2 + 4*A*C*a*b*c*d^3))/(d*(c^2 + d^2)^2) + ((a*1i + b)*(B*1i - A + C)
*(A*a*d - B*b*d - C*a*d - 4*C*b*c + (tan(e + f*x)*(3*A*b*d^4 + 3*B*a*d^4 +
 2*C*b*c^4 - 5*C*b*d^4 + 4*A*a*c*d^3 - 4*B*b*c*d^3 - 4*C*a*c*d^3 - A*b*c^2
*d^2 - B*a*c^2*d^2 + C*b*c^2*d^2))/(d*(c^2 + d^2)) + (d*(a*1i + b)*(4*c*d
- c^2*tan(e + f*x) + 3*d^2*tan(e + f*x))*(B*1i - A + C))/(c*1i + d)^2))/(2
*(c*1i + d)^2))*(A*a*1i + A*b + B*a - B*b*1i - C*a*1i - C*b))/(2*f*(c*d...
```

3.80 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx$

3.80.1 Optimal result

Integrand size = 33, antiderivative size = 140

$$\begin{aligned}\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx &= -\frac{(c^2C-2Bcd-Cd^2-A(c^2-d^2))\,x}{(c^2+d^2)^2} \\ &\quad+\frac{(2c(A-C)d-B(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^2 f} \\ &\quad-\frac{c^2C-Bcd+Ad^2}{d\,(c^2+d^2)\,f(c+d\tan(e+fx))}\end{aligned}$$

output
```
-(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))*x/(c^2+d^2)^2+(2*c*(A-C)*d-B*(c^2-d^2))
*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^2/f+(-A*d^2+B*c*d-C*c^2)/d/(c^2+d
^2)/f/(c+d*tan(f*x+e))
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx = \frac{\frac{B((-ic-d)\log(i-\tan(e+fx))+i(c+id)\log(i+\tan(e+fx))+2d\log(c+d\tan(e+fx)))}{c^2+d^2}-\frac{2C}{c+d\tan(e+fx)}+(Bc+(-A+C)d)\left(\frac{i\log(i\cdots}{(\cdots}\right.}{2df}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x
]
```

output
```
((B*(((-I)*c - d)*Log[I - Tan[e + f*x]] + I*(c + I*d)*Log[I + Tan[e + f*x]
] + 2*d*Log[c + d*Tan[e + f*x]]))/(c^2 + d^2) - (2*C)/(c + d*Tan[e + f*x])
 + (B*c + (-A + C)*d)*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2 - (I*Log[I +
Tan[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x]] + (c^2 + d
^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2))/(2*d*f)
```

3.80.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4111, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{x\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)}{c^2+d^2}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{x\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)}{c^2+d^2}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{\frac{(2cd(A-C)-B(c^2-d^2))\log(c\cos(e+fx)+d\sin(e+fx))}{f(c^2+d^2)}-\frac{x(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)}{c^2+d^2}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{df\left(c^2+d^2\right)\left(c+d\tan(e+fx)\right)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^2,x]
```

output
```
(-(((c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))*x)/(c^2 + d^2)) + ((2*c*(A -
 C)*d - B*(c^2 - d^2))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)*
f))/(c^2 + d^2) - (c^2*C - B*c*d + A*d^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e +
f*x]))
```

3.80.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.80.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{\frac{\left(-2Adc+B\,c^2-d^2B+2Ccd\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2\right)\arctan(\tan(fx+e))}{\left(c^2+d^2\right)^2}-\frac{A\,d^2-Bcd+c^2C}{\left(c^2+d^2\right)d(c+d\tan(fx+e))}}{f}$
default	$\frac{\frac{\frac{\left(-2Adc+B\,c^2-d^2B+2Ccd\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2\right)\arctan(\tan(fx+e))}{\left(c^2+d^2\right)^2}-\frac{A\,d^2-Bcd+c^2C}{\left(c^2+d^2\right)d(c+d\tan(fx+e))}}{f}$
norman	$\frac{\frac{c\left(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2\right)x}{c^4+2c^2d^2+d^4}+\frac{d\left(A\,c^2-A\,d^2+2Bcd-c^2C+C\,d^2\right)x\tan(fx+e)}{c^4+2c^2d^2+d^4}-\frac{A\,d^2-Bcd+c^2C}{\left(c^2+d^2\right)df}}{c+d\tan(fx+e)}+\frac{(2Adc-B\,c^2+d^2B-2}{f(c^4+2}$
parallelrisch	$-\frac{2A\,c^2d^2+2Ax\tan(fx+e)d^4f-2B\,c^3d-2Bc\,d^3+2C\,c^2d^2+2c^4C+2A\,d^4-2Cx\tan(fx+e)d^4f-2Ax\,c^3df+2Axc\,d^3f-4}{}$
risch	$\frac{4iCcde}{f(c^4+2c^2d^2+d^4)}-\frac{xA}{2icd-c^2+d^2}+\frac{xC}{2icd-c^2+d^2}+\frac{2iA\,d^2}{(id+c)f(-id+c)^2\left(-id\,e^{2i(fx+e)}+c\,e^{2i(fx+e)}+id+c\right)}-\frac{}{f(c^4-}$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x,method=_RETURNVER
BOSE)
```

output
```
1/f*(1/(c^2+d^2)^2*(1/2*(-2*A*c*d+B*c^2-B*d^2+2*C*c*d)*ln(1+tan(f*x+e)^2)+
(A*c^2-A*d^2+2*B*c*d-C*c^2+C*d^2)*arctan(tan(f*x+e)))-(A*d^2-B*c*d+C*c^2)/
(c^2+d^2)/d/(c+d*tan(f*x+e))+(2*A*c*d-B*c^2+B*d^2-2*C*c*d)/(c^2+d^2)^2*ln(
c+d*tan(f*x+e)))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.83

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx =$$

$$-\frac{2\,Cc^2d-2\,Bcd^2+2\,Ad^3-2\left((A-C)c^3+2\,Bc^2d-(A-C)cd^2\right)fx+\left(Bc^3-2\,(A-C)c^2d-Bcd^2+\right.}{2}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm=
"fricas")
```

output
```
-1/2*(2*C*c^2*d - 2*B*c*d^2 + 2*A*d^3 - 2*((A - C)*c^3 + 2*B*c^2*d - (A -
C)*c*d^2)*f*x + (B*c^3 - 2*(A - C)*c^2*d - B*c*d^2 + (B*c^2*d - 2*(A - C)*
c*d^2 - B*d^3)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e)
+ c^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^3 - B*c^2*d + A*c*d^2 + ((A - C)*c^2
*d + 2*B*c*d^2 - (A - C)*d^3)*f*x)*tan(f*x + e))/((c^4*d + 2*c^2*d^3 + d^5
)*f*tan(f*x + e) + (c^5 + 2*c^3*d^2 + c*d^4)*f)
```

3.80.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 4396, normalized size of antiderivative = 31.40

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(e) + C*tan(e)**2)/tan(e)**2, Eq(c, 0) & Eq(d,
0) & Eq(f, 0)), ((A*x + B*log(tan(e + f*x)**2 + 1)/(2*f) - C*x + C*tan(e +
 f*x)/f)/c**2, Eq(d, 0)), (-A*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**
2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A*f*x*tan(e + f*x)/(4*d**2*f
*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + A*f*x/(4*d**2*f*t
an(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - A*tan(e + f*x)/(4*d
**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*A/(4*d**
2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + I*B*f*x*tan(e
+ f*x)**2/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f)
+ 2*B*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
 - 4*d**2*f) - I*B*f*x/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x)
 - 4*d**2*f) + I*B*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan
(e + f*x) - 4*d**2*f) + C*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 -
8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 2*I*C*f*x*tan(e + f*x)/(4*d**2*f*tan
(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - C*f*x/(4*d**2*f*tan(e
 + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) - 3*C*tan(e + f*x)/(4*d**
2*f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f) + 2*I*C/(4*d**2*
f*tan(e + f*x)**2 - 8*I*d**2*f*tan(e + f*x) - 4*d**2*f), Eq(c, -I*d)), (-A
*f*x*tan(e + f*x)**2/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**2*f*tan(e + f*x) -
 4*d**2*f) - 2*I*A*f*x*tan(e + f*x)/(4*d**2*f*tan(e + f*x)**2 + 8*I*d**...
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.46

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^2}\, dx$$
$$= \frac{\frac{2\,((A-C)c^2+2\,Bcd-(A-C)d^2)(fx+e)}{c^4+2\,c^2d^2+d^4} - \frac{2\,(Bc^2-2\,(A-C)cd-Bd^2)\log(d\tan(fx+e)+c)}{c^4+2\,c^2d^2+d^4} + \frac{(Bc^2-2\,(A-C)cd-Bd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm=
"maxima")
```

output
```
1/2*(2*((A - C)*c^2 + 2*B*c*d - (A - C)*d^2)*(f*x + e)/(c^4 + 2*c^2*d^2 +
d^4) - 2*(B*c^2 - 2*(A - C)*c*d - B*d^2)*log(d*tan(f*x + e) + c)/(c^4 + 2*
c^2*d^2 + d^4) + (B*c^2 - 2*(A - C)*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(
c^4 + 2*c^2*d^2 + d^4) - 2*(C*c^2 - B*c*d + A*d^2)/(c^3*d + c*d^3 + (c^2*d
^2 + d^4)*tan(f*x + e)))/f
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(140) = 280.

Time = 0.54 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.08

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^2}\, dx$$
$$= \frac{\frac{2\,(Ac^2-Cc^2+2\,Bcd-Ad^2+Cd^2)(fx+e)}{c^4+2\,c^2d^2+d^4} + \frac{(Bc^2-2\,Acd+2\,Ccd-Bd^2)\log\left(\tan(fx+e)^2+1\right)}{c^4+2\,c^2d^2+d^4} - \frac{2\,(Bc^2d-2\,Acd^2+2\,Ccd^2-Bd^3)\log(|d\tan(fx}{c^4d+2\,c^2d^3+d^5}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,x, algorithm=
"giac")
```

output
```
1/2*(2*(A*c^2 - C*c^2 + 2*B*c*d - A*d^2 + C*d^2)*(f*x + e)/(c^4 + 2*c^2*d^
2 + d^4) + (B*c^2 - 2*A*c*d + 2*C*c*d - B*d^2)*log(tan(f*x + e)^2 + 1)/(c^
4 + 2*c^2*d^2 + d^4) - 2*(B*c^2*d - 2*A*c*d^2 + 2*C*c*d^2 - B*d^3)*log(abs
(d*tan(f*x + e) + c))/(c^4*d + 2*c^2*d^3 + d^5) + 2*(B*c^2*d^2*tan(f*x + e
) - 2*A*c*d^3*tan(f*x + e) + 2*C*c*d^3*tan(f*x + e) - B*d^4*tan(f*x + e) -
 C*c^4 + 2*B*c^3*d - 3*A*c^2*d^2 + C*c^2*d^2 - A*d^4)/((c^4*d + 2*c^2*d^3
+ d^5)*(d*tan(f*x + e) + c)))/f
```

3.80.9 Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.31

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^2}\,dx$$
$$=\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(-B\,c^2+\left(2\,A-2\,C\right)\,c\,d+B\,d^2\right)}{f\,\left(c^4+2\,c^2\,d^2+d^4\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A-C+B\,1\mathrm{i}\right)}{2\,f\,\left(-c^2\,1\mathrm{i}+2\,c\,d+d^2\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,1\mathrm{i}+B-C\,1\mathrm{i}\right)}{2\,f\,\left(-c^2+c\,d\,2\mathrm{i}+d^2\right)}-\frac{C\,c^2-B\,c\,d+A\,d^2}{d\,f\,\left(c^2+d^2\right)\,\left(c+d\tan\left(e+f\,x\right)\right)}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^2,x)
```

output
```
(log(c + d*tan(e + f*x))*(B*d^2 - B*c^2 + c*d*(2*A - 2*C)))/(f*(c^4 + d^4
+ 2*c^2*d^2)) - (log(tan(e + f*x) - 1i)*(A + B*1i - C))/(2*f*(2*c*d - c^2*
1i + d^2*1i)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*1i))/(2*f*(c*d*2i -
c^2 + d^2)) - (A*d^2 + C*c^2 - B*c*d)/(d*f*(c^2 + d^2)*(c + d*tan(e + f*x)
))
```

3.81 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

3.81.1 Optimal result

Integrand size = 45, antiderivative size = 293

$$\begin{aligned}
&\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx \\
&= -\frac{(a(c^2C-2Bcd-Cd^2-A(c^2-d^2))+b(2c(A-C)d-B(c^2-d^2)))\,x}{(a^2+b^2)\,(c^2+d^2)^2} \\
&\quad+\frac{b(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,(bc-ad)^2f} \\
&\quad-\frac{(b(c^4C-2Bc^3d+c^2(3A-C)d^2+Ad^4)-ad^2(2c(A-C)d-B(c^2-d^2)))\log(c\cos(e+fx)+d\sin(e}{(bc-ad)^2\,(c^2+d^2)^2\,f} \\
&\quad+\frac{c^2C-Bcd+Ad^2}{(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))}
\end{aligned}$$

output
```
-(a*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))+b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(a^2+
b^2)/(c^2+d^2)^2+b*(A*b^2-a*(B*b-C*a))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+
b^2)/(-a*d+b*c)^2/f-(b*(c^4*C-2*B*c^3*d+c^2*(3*A-C)*d^2+A*d^4)-a*d^2*(2*c*
(A-C)*d-B*(c^2-d^2)))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^2/(c^2+d^2)
^2/f+(A*d^2-B*c*d+C*c^2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 592 vs. $2(293) = 586$.

Time = 7.59 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.02

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx =$$

$$-\frac{-\frac{b(bc-ad)\left(Abc^2-aBc^2-bc^2C+2aAcd+2bBcd-2acCd-Abd^2+aBd^2+bCd^2-\frac{\sqrt{-b^2}\left(a\left(c^2C-2Bcd-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right.\right.\right.}{b}\right.}{2(a^2+b^2)(c^2+d^2)}}{}$$

$$-\frac{Ad^2-c(-cC+Bd)}{(-bc+ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
 + d*Tan[e + f*x])^2),x]
```

output
```
-((-1/2*(b*(b*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*
d - 2*a*c*C*d - A*b*d^2 + a*B*d^2 + b*C*d^2 - (Sqrt[-b^2]*(a*(c^2*C - 2*B*
c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[
Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)) + (b^2*(A*b^2 - a*
(b*B - a*C))*(c^2 + d^2)*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)
) - (b*(b*c - a*d)*(A*b*c^2 - a*B*c^2 - b*c^2*C + 2*a*A*c*d + 2*b*B*c*d -
2*a*c*C*d - A*b*d^2 + a*B*d^2 + b*C*d^2 + (Sqrt[-b^2]*(a*(c^2*C - 2*B*c*d
- C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt
[-b^2] + b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(b*(c^4*C - 2*B
*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)
))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(
c^2 + d^2)*f)) - (A*d^2 - c*(-(c*C) + B*d))/((-(b*c) + a*d)*(c^2 + d^2)*f*
(c + d*Tan[e + f*x]))
```

3.81.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4132, 25, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{\int -\frac{-b(Cc^2-Bdc+Ad^2)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab(c^2+d^2)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(c^2+d^2)(bc-ad)}+\frac{Ad^2-Bcd+c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{Ad^2-Bcd+c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{\int \frac{-b(Cc^2-Bdc+Ad^2)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab(c^2+d^2)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ad^2-Bcd+c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{\int \frac{-b(Cc^2-Bdc+Ad^2)\tan(e+fx)^2-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab(c^2+d^2)}{(a+b\tan(e+fx))(c+d\tan(e+fx))}dx}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \text{4134}$$

$$\frac{Ad^2-Bcd+c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{-\frac{b(c^2+d^2)(Ab^2-a(bB-aC))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}+\frac{(b(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C)-ad^2(2cd(A-C)-B(c^2-d^2)))\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ad^2-Bcd+c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{-\frac{b(c^2+d^2)(Ab^2-a(bB-aC))\int \frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{(a^2+b^2)(bc-ad)}+\frac{(b(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C)-ad^2(2cd(A-C)-B(c^2-d^2)))\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{(c^2+d^2)(bc-ad)}+}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \text{4013}$$

3.81. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx$

$$\frac{Ad^2 - Bcd + c^2C}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))} - \frac{-\frac{b(c^2+d^2)(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{f(a^2+b^2)(bc-ad)} + \frac{x(bc-ad)(a(-A(c^2-d^2)-2Bcd+c^2C-Cd^2)+b(2cd(A-C)-B(c^2-d^2)))}{(a^2+b^2)(c^2+d^2)} + \cdots}{(c^2+d^2)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*T
an[e + f*x])^2),x]
```

output
```
-((((b*c - a*d)*(a*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A -
 C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)) - (b*(A*b^2 - a*(b*B
- a*C))*(c^2 + d^2)*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*
c - a*d)*f) + ((b*(c^4*C - 2*B*c^3*d + c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*
(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b
*c - a*d)*(c^2 + d^2)*f))/((b*c - a*d)*(c^2 + d^2))) + (c^2*C - B*c*d + A*
d^2)/((b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.81.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\left(A b^2-Bab+C a^2\right) b\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)}+\frac{\frac{\left(-2Aacd-Ab c^2+Ab d^2+Ba c^2-Ba d^2-2Bbcd+2Cacd+Cb c^2-Cb d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}}{\left(a^2+b^2\right)}}{}$
default	$\frac{\frac{\left(A b^2-Bab+C a^2\right) b\ln(a+b\tan(fx+e))}{(ad-bc)^2\left(a^2+b^2\right)}+\frac{\frac{\left(-2Aacd-Ab c^2+Ab d^2+Ba c^2-Ba d^2-2Bbcd+2Cacd+Cb c^2-Cb d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}}{\left(a^2+b^2\right)}}{}$
norman	$\frac{\frac{\left(Aa c^2-Aa d^2-2Abcd+2Bacd+Bb c^2-Bb d^2-Ca c^2+Ca d^2+2Cbcd\right)cx}{\left(a^2+b^2\right)\left(c^4+2c^2d^2+d^4\right)}+\frac{\left(Aa c^2-Aa d^2-2Abcd+2Bacd+Bb c^2-Bb d^2-Ca c^2+Ca\right.}{\left(a^2+b^2\right)\left(c^4+2c^2d^2+d^4\right)}}{c+d\tan(fx+e)}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2,x,
method=_RETURNVERBOSE)
```

output
```
1/f*((A*b^2-B*a*b+C*a^2)*b/(a*d-b*c)^2/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(a^2
+b^2)/(c^2+d^2)^2*(1/2*(-2*A*a*c*d-A*b*c^2+A*b*d^2+B*a*c^2-B*a*d^2-2*B*b*c
*d+2*C*a*c*d+C*b*c^2-C*b*d^2)*ln(1+tan(f*x+e)^2)+(A*a*c^2-A*a*d^2-2*A*b*c*
d+2*B*a*c*d+B*b*c^2-B*b*d^2-C*a*c^2+C*a*d^2+2*C*b*c*d)*arctan(tan(f*x+e)))
+(2*A*a*c*d^3-3*A*b*c^2*d^2-A*b*d^4-B*a*c^2*d^2+B*a*d^4+2*B*b*c^3*d-2*C*a*
c*d^3-C*b*c^4+C*b*c^2*d^2)/(a*d-b*c)^2/(c^2+d^2)^2*ln(c+d*tan(f*x+e))-(A*d
^2-B*c*d+C*c^2)/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e)))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(291) = 582$.

Time = 1.11 (sec) , antiderivative size = 1275, normalized size of antiderivative = 4.35

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(C*a^2*b + C*b^3)*c^3*d^2 - 2*(C*a^3 + B*a^2*b + C*a*b^2 + B*b^3)*c
^2*d^3 + 2*(B*a^3 + A*a^2*b + B*a*b^2 + A*b^3)*c*d^4 - 2*(A*a^3 + A*a*b^2)
*d^5 + 2*(((A - C)*a*b^2 + B*b^3)*c^5 - 2*((A - C)*a^2*b + (A - C)*b^3)*c^
4*d + ((A - C)*a^3 - 3*B*a^2*b + 3*(A - C)*a*b^2 - B*b^3)*c^3*d^2 + 2*(B*a
^3 + B*a*b^2)*c^2*d^3 - ((A - C)*a^3 + B*a^2*b)*c*d^4)*f*x + ((C*a^2*b - B
*a*b^2 + A*b^3)*c^5 + 2*(C*a^2*b - B*a*b^2 + A*b^3)*c^3*d^2 + (C*a^2*b - B
*a*b^2 + A*b^3)*c*d^4 + ((C*a^2*b - B*a*b^2 + A*b^3)*c^4*d + 2*(C*a^2*b -
B*a*b^2 + A*b^3)*c^2*d^3 + (C*a^2*b - B*a*b^2 + A*b^3)*d^5)*tan(f*x + e))*
log((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)/(tan(f*x + e)^2 + 1))
- ((C*a^2*b + C*b^3)*c^5 - 2*(B*a^2*b + B*b^3)*c^4*d + (B*a^3 + (3*A - C)*
a^2*b + B*a*b^2 + (3*A - C)*b^3)*c^3*d^2 - 2*((A - C)*a^3 + (A - C)*a*b^2)
*c^2*d^3 - (B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*c*d^4 + ((C*a^2*b + C*b^3)*
c^4*d - 2*(B*a^2*b + B*b^3)*c^3*d^2 + (B*a^3 + (3*A - C)*a^2*b + B*a*b^2 +
 (3*A - C)*b^3)*c^2*d^3 - 2*((A - C)*a^3 + (A - C)*a*b^2)*c*d^4 - (B*a^3 -
 A*a^2*b + B*a*b^2 - A*b^3)*d^5)*tan(f*x + e))*log((d^2*tan(f*x + e)^2 + 2
*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*((C*a^2*b + C*b^3)*c^4*
d - (C*a^3 + B*a^2*b + C*a*b^2 + B*b^3)*c^3*d^2 + (B*a^3 + A*a^2*b + B*a*b
^2 + A*b^3)*c^2*d^3 - (A*a^3 + A*a*b^2)*c*d^4 - (((A - C)*a*b^2 + B*b^3)*c
^4*d - 2*((A - C)*a^2*b + (A - C)*b^3)*c^3*d^2 + ((A - C)*a^3 - 3*B*a^2*b
+ 3*(A - C)*a*b^2 - B*b^3)*c^2*d^3 + 2*(B*a^3 + B*a*b^2)*c*d^4 - ((A - ...
```

3.81.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))**2,x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.75

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{2\left(((A-C)a+Bb)c^2+2\left(Ba-(A-C)b\right)cd-((A-C)a+Bb)d^2\right)(fx+e)}{(a^2+b^2)c^4+2\left(a^2+b^2\right)c^2d^2+(a^2+b^2)d^4}+\frac{2\left(Ca^2b-Bab^2+Ab^3\right)\log(b\tan(fx+e)+a)}{(a^2b^2+b^4)c^2-2\left(a^3b+ab^3\right)cd+(a^4+a^2b^2)d^2}-\frac{2\left(Cbc^4-2\,Bbc^3d-2\,(A\right.}{b^2c^6-2\,abc^5d}}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a + B*b)*c^2 + 2*(B*a - (A - C)*b)*c*d - ((A - C)*a + B*b
)*d^2)*(f*x + e)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^
4) + 2*(C*a^2*b - B*a*b^2 + A*b^3)*log(b*tan(f*x + e) + a)/((a^2*b^2 + b^4
)*c^2 - 2*(a^3*b + a*b^3)*c*d + (a^4 + a^2*b^2)*d^2) - 2*(C*b*c^4 - 2*B*b*
c^3*d - 2*(A - C)*a*c*d^3 + (B*a + (3*A - C)*b)*c^2*d^2 - (B*a - A*b)*d^4)
*log(d*tan(f*x + e) + c)/(b^2*c^6 - 2*a*b*c^5*d - 4*a*b*c^3*d^3 - 2*a*b*c*
d^5 + a^2*d^6 + (a^2 + 2*b^2)*c^4*d^2 + (2*a^2 + b^2)*c^2*d^4) + ((B*a - (
A - C)*b)*c^2 - 2*((A - C)*a + B*b)*c*d - (B*a - (A - C)*b)*d^2)*log(tan(f
*x + e)^2 + 1)/((a^2 + b^2)*c^4 + 2*(a^2 + b^2)*c^2*d^2 + (a^2 + b^2)*d^4)
 + 2*(C*c^2 - B*c*d + A*d^2)/(b*c^4 - a*c^3*d + b*c^2*d^2 - a*c*d^3 + (b*c
^3*d - a*c^2*d^2 + b*c*d^3 - a*d^4)*tan(f*x + e)))/f
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(291) = 582$.

Time = 0.77 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.84

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{\frac{2\left(Aac^2-Cac^2+Bbc^2+2\,Bacd-2\,Abcd+2\,Cbcd-Aad^2+Cad^2-Bbd^2\right)(fx+e)}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+b^2d^4}+\frac{(Bac^2-Abc^2+Cbc^2-2\,Aacd+2\,Cacd-2\,Bbcd-Bad^2+Abd^2-}{a^2c^4+b^2c^4+2\,a^2c^2d^2+2\,b^2c^2d^2+a^2d^4+}}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c^2 - C*a*c^2 + B*b*c^2 + 2*B*a*c*d - 2*A*b*c*d + 2*C*b*c*d -
A*a*d^2 + C*a*d^2 - B*b*d^2)*(f*x + e)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2
+ 2*b^2*c^2*d^2 + a^2*d^4 + b^2*d^4) + (B*a*c^2 - A*b*c^2 + C*b*c^2 - 2*A*
a*c*d + 2*C*a*c*d - 2*B*b*c*d - B*a*d^2 + A*b*d^2 - C*b*d^2)*log(tan(f*x +
 e)^2 + 1)/(a^2*c^4 + b^2*c^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2 + a^2*d^4 +
b^2*d^4) + 2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*log(abs(b*tan(f*x + e) + a))/(a
^2*b^3*c^2 + b^5*c^2 - 2*a^3*b^2*c*d - 2*a*b^4*c*d + a^4*b*d^2 + a^2*b^3*d
^2) - 2*(C*b*c^4*d - 2*B*b*c^3*d^2 + B*a*c^2*d^3 + 3*A*b*c^2*d^3 - C*b*c^2
*d^3 - 2*A*a*c*d^4 + 2*C*a*c*d^4 - B*a*d^5 + A*b*d^5)*log(abs(d*tan(f*x +
e) + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3 + 2*b^2*c^4*d^3 - 4*a*b*
c^3*d^4 + 2*a^2*c^2*d^5 + b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 2*(C*b*c^
4*d*tan(f*x + e) - 2*B*b*c^3*d^2*tan(f*x + e) + B*a*c^2*d^3*tan(f*x + e) +
 3*A*b*c^2*d^3*tan(f*x + e) - C*b*c^2*d^3*tan(f*x + e) - 2*A*a*c*d^4*tan(f
*x + e) + 2*C*a*c*d^4*tan(f*x + e) - B*a*d^5*tan(f*x + e) + A*b*d^5*tan(f*
x + e) + 2*C*b*c^5 - C*a*c^4*d - 3*B*b*c^4*d + 2*B*a*c^3*d^2 + 4*A*b*c^3*d
^2 - 3*A*a*c^2*d^3 + C*a*c^2*d^3 - B*b*c^2*d^3 + 2*A*b*c*d^4 - A*a*d^5)/((
b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + 2*b^2*c^4*d^2 - 4*a*b*c^3*d^3 + 2*a^
2*c^2*d^4 + b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*(d*tan(f*x + e) + c)))/f
```

3.81.9 Mupad [B] (verification not implemented)

Time = 65.17 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.47

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx = \frac{\ln(\tan(e+f\,x)-\mathrm{i})\,(B-A\,1\mathrm{i}+C\,1\mathrm{i})}{2\,f\,(a\,c^2-a\,d^2-2\,b\,c\,d+b\,c^2\,1\mathrm{i}-b\,d^2\,1\mathrm{i}+a\,c\,d\,2\mathrm{i})} - \frac{\ln(\tan(e+f\,x)+1\mathrm{i})\,(A\,1\mathrm{i}+B-C\,1\mathrm{i})}{2\,f\,(a\,d^2-a\,c^2+2\,b\,c\,d+b\,c^2\,1\mathrm{i}-b\,d^2\,1\mathrm{i}+a\,c\,d\,2\mathrm{i})} + \frac{\ln(a+b\tan(e+f\,x))\,(C\,a^2\,b-B\,a\,b^2+A\,b^3)}{f\,(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2+a^2\,b^2\,d^2-2\,a\,b^3\,c\,d+b^4\,c^2)} - \frac{\ln(c+d\tan(e+f\,x))\,(C\,b\,c^4-2\,B\,b\,c^3\,d+(3\,A\,b+B\,a-C\,b)\,c^2\,d^2+(2\,C\,a-2\,A\,a)\,c\,d^3+(A\,b-B}{f\,(a^2\,c^4\,d^2+2\,a^2\,c^2\,d^4+a^2\,d^6-2\,a\,b\,c^5\,d-4\,a\,b\,c^3\,d^3-2\,a\,b\,c\,d^5+b^2\,c^6+2\,b^2\,c^4\,d^2+b^2\,c^2\,d^4)} - \frac{C\,c^2-B\,c\,d+A\,d^2}{f\,(a\,d-b\,c)\,(c^2+d^2)\,(c+d\tan(e+f\,x))}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))^2),x)
```

output
```
(log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(a*c^2 - a*d^2 + b*c^2*1i
- b*d^2*1i + a*c*d*2i - 2*b*c*d)) - (log(tan(e + f*x) + 1i)*(A*1i + B - C*
1i))/(2*f*(a*d^2 - a*c^2 + b*c^2*1i - b*d^2*1i + a*c*d*2i + 2*b*c*d)) + (l
og(a + b*tan(e + f*x))*(A*b^3 - B*a*b^2 + C*a^2*b))/(f*(a^4*d^2 + b^4*c^2
+ a^2*b^2*c^2 + a^2*b^2*d^2 - 2*a*b^3*c*d - 2*a^3*b*c*d)) - (log(c + d*tan
(e + f*x))*(d^4*(A*b - B*a) + c^2*d^2*(3*A*b + B*a - C*b) + C*b*c^4 - c*d^
3*(2*A*a - 2*C*a) - 2*B*b*c^3*d))/(f*(a^2*d^6 + b^2*c^6 + 2*a^2*c^2*d^4 +
a^2*c^4*d^2 + b^2*c^2*d^4 + 2*b^2*c^4*d^2 - 2*a*b*c*d^5 - 2*a*b*c^5*d - 4*
a*b*c^3*d^3)) - (A*d^2 + C*c^2 - B*c*d)/(f*(a*d - b*c)*(c^2 + d^2)*(c + d*
tan(e + f*x)))
```

3.81. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}\,dx$

3.82 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

3.82.1 Optimal result

Integrand size = 45, antiderivative size = 509

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx =$$
$$-\frac{(a^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))-b^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))+2ab(2c(A-C)d-B(c}{(a^2+b^2)^2(c^2+d^2)^2}$$
$$+\frac{b(3a^3bBd-2a^4Cd+b^4(Bc-2Ad)-a^2b^2(Bc+4Ad)+ab^3(2Ac-2cC+Bd))\log(a\cos(e+fx)+b\,s}{(a^2+b^2)^2(bc-ad)^3f}$$
$$+\frac{d(b(2c^4C-3Bc^3d+4Ac^2d^2-Bcd^3+2Ad^4)-ad^2(2c(A-C)d-B(c^2-d^2)))\log(c\cos(e+fx)+d\,s}{(bc-ad)^3(c^2+d^2)^2f}$$
$$-\frac{d(b^2c(cC-Bd)-abB(c^2+d^2)+a^2(2c^2C-Bcd+Cd^2)+A(a^2d^2+b^2(c^2+2d^2)))}{(a^2+b^2)(bc-ad)^2(c^2+d^2)f(c+d\tan(e+fx))}$$
$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

output
```
-(a^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^
2))+2*a*b*(2*c*(A-C)*d-B*(c^2-d^2)))*x/(a^2+b^2)^2/(c^2+d^2)^2+b*(3*a^3*b*
B*d-2*a^4*C*d+b^4*(-2*A*d+B*c)-a^2*b^2*(4*A*d+B*c)+a*b^3*(2*A*c+B*d-2*C*c)
)*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^2/(-a*d+b*c)^3/f+d*(b*(4*A*c^2*d
^2+2*A*d^4-3*B*c^3*d-B*c*d^3+2*C*c^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*ln(
c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)^2/f-d*(b^2*c*(-B*d+C*c)-
a*b*B*(c^2+d^2)+a^2*(-B*c*d+2*C*c^2+C*d^2)+A*(a^2*d^2+b^2*(c^2+2*d^2)))/(a
^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))+(-A*b^2+a*(B*b-C*a))/(a^
2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
```

3.82.2 Mathematica [A] (verified)

Time = 9.12 (sec) , antiderivative size = 984, normalized size of antiderivative = 1.93

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$$

$$= -\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$$-\frac{-\frac{b(bc-ad)^2\left(2aAbc^2-a^2Bc^2+b^2Bc^2-2abc^2C+2a^2Acd-2Ab^2cd+4abBcd-2a^2cCd+2b^2cCd-2aAbd^2+a^2Bd^2-b^2Bd^2+2abCd^2-\frac{\sqrt{-b^2}\left(a^2\left(c^2C-2Bcd-\cdots\right.\right.}{\cdots}\right.}{2\left(a^2+b^2\right)\left(c^2+d^2\right)\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
(c + d*Tan[e + f*x])^2),x]
```

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x]))) - (-(((b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b
^2*B*c^2 - 2*a*b*c^2*C + 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c
*C*d + 2*b^2*c*C*d - 2*a*A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 - (
Sqrt[-b^2]*(a^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2
*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/
b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b^2*(c
^2 + d^2)*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) - a^2*b^2*(B*c + 4*
A*d) + a*b^3*(2*A*c - 2*c*C + B*d))*Log[a + b*Tan[e + f*x]])/((a^2 + b^2)*
(b*c - a*d)) + (b*(b*c - a*d)^2*(2*a*A*b*c^2 - a^2*B*c^2 + b^2*B*c^2 - 2*a
*b*c^2*C + 2*a^2*A*c*d - 2*A*b^2*c*d + 4*a*b*B*c*d - 2*a^2*c*C*d + 2*b^2*c
*C*d - 2*a*A*b*d^2 + a^2*B*d^2 - b^2*B*d^2 + 2*a*b*C*d^2 + (Sqrt[-b^2]*(a^
2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d - C*d^2
 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d - B*(c^2 - d^2))))/b)*Log[Sqrt[-b
^2] + b*Tan[e + f*x]])/(2*(a^2 + b^2)*(c^2 + d^2)) - (b*(a^2 + b^2)*d*(b*(
2*c^4*C - 3*B*c^3*d + 4*A*c^2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C
)*d - B*(c^2 - d^2)))*Log[c + d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/
(b*(-(b*c) + a*d)*(c^2 + d^2)*f)) - (-(c*(-2*c*(A*b^2 - a*(b*B - a*C))*d +
 (A*b - a*B - b*C)*d*(b*c - a*d))) + d^2*(2*A*b^2*d - a*A*(b*c - a*d) - (b
*B - a*C)*(b*c + a*d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*...
```

3.82.3 Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4132, 3042, 4132, 25, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int \frac{2Adb^2+2\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-aA(bc-ad)-(bB-aC)(bc+ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{2Adb^2+2\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-aA(bc-ad)-(bB-aC)(bc+ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int -\frac{d^2(Ac-Cc+Bd)a^3-2Abd\left(c^2+d^2\right)a^2-b^2\left(Cc^3+2Cd^2c-Bd^3-A\left(c^3+2d^2c\right)\right)a-bd\left(Ad^2a^2+\left(2Cc^2-Bdc+Cd^2\right)a^2-bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(c^2+2\right.\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))} \quad \cdots}{\left(c^2+d^2\right)(bc-ad) \quad \cdots} \quad \cdots$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{d\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}-\int\frac{d^2(Ac-Cc+Bd)a^3-2Abd\left(c^2+d^2\right)a^2-b^2\left(Cc^3+2Cd^2c-Bd^3-\right.}{\cdots} \quad \cdots}{\cdots} \quad \cdots$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$\big\downarrow$ 3042

$$-\frac{\frac{d(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\int\frac{d^2(Ac-Cc+Bd)a^3-2Abd\left(c^2+d^2\right)a^2-b^2\left(Cc^3+2Cd^2c-Bd^3-\right.}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$\big\downarrow$ 4134

$$-\frac{\frac{d(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{\frac{d\left(a^2+b^2\right)\left(b\left(4Ac^2d^2+2Ad^4-3Bc^3d-Bcd^3+2c^4C\right)-ad^2\left(2cd(A-\right.\right.}{\left(c^2+d^2\right)(bc-ad)}}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$\big\downarrow$ 3042

$$-\frac{\frac{d(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{\frac{d\left(a^2+b^2\right)\left(b\left(4Ac^2d^2+2Ad^4-3Bc^3d-Bcd^3+2c^4C\right)-ad^2\left(2cd(A-\right.\right.}{\left(c^2+d^2\right)(bc-ad)}}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

$\big\downarrow$ 4013

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))}-$$

$$\frac{\frac{d(a^2Ad^2+a^2(-Bcd+2c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+2d^2)+b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))}-\frac{-\frac{x(bc-ad)^2\left(a^2\left(-A\left(c^2-d^2\right)-2Bcd+c^2C-Cd^2\right)+2ab\left(2cd(A-C)-\right.\right.}{\left(a^2+b^2\right)\left(c^2+d\right.}}{}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d
*Tan[e + f*x])^2),x]
```

3.82. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x]))) - (-((-(((b*c - a*d)^2*(a^2*(c^2*C - 2*B*c*d - C*d^
2 - A*(c^2 - d^2)) - b^2*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b
*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2))) + (b*(c^2
+ d^2)*(3*a^3*b*B*d - 2*a^4*C*d + b^4*(B*c - 2*A*d) - a^2*b^2*(B*c + 4*A*d
) + a*b^3*(2*A*c - 2*c*C + B*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/((a
^2 + b^2)*(b*c - a*d)*f) + ((a^2 + b^2)*d*(b*(2*c^4*C - 3*B*c^3*d + 4*A*c^
2*d^2 - B*c*d^3 + 2*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Log[c*
Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f))/((b*c - a*d)*
(c^2 + d^2))) + (d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + A*
b^2*(c^2 + 2*d^2) + a^2*(2*c^2*C - B*c*d + C*d^2)))/((b*c - a*d)*(c^2 + d^
2)*f*(c + d*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d))
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.82.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{b\left(4A\,a^{2}b^{2}d-2Aa\,b^{3}c+2A\,b^{4}d-3a^{3}bBd+B\,a^{2}b^{2}c-Ba\,b^{3}d-B\,b^{4}c+2a^{4}Cd+2Ca\,b^{3}c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^{3}\left(a^{2}+b^{2}\right)^{2}}-\frac{\left(A\,b^{2}-Bab+C\,a^{2}\right)}{(ad-bc)^{2}\left(a^{2}+b^{2}\right)(a+b\tan}}{}$
default	$\frac{\frac{b\left(4A\,a^{2}b^{2}d-2Aa\,b^{3}c+2A\,b^{4}d-3a^{3}bBd+B\,a^{2}b^{2}c-Ba\,b^{3}d-B\,b^{4}c+2a^{4}Cd+2Ca\,b^{3}c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^{3}\left(a^{2}+b^{2}\right)^{2}}-\frac{\left(A\,b^{2}-Bab+C\,a^{2}\right)}{(ad-bc)^{2}\left(a^{2}+b^{2}\right)(a+b\tan}}{}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*(4*A*a^2*b^2*d-2*A*a*b^3*c+2*A*b^4*d-3*B*a^3*b*d+B*a^2*b^2*c-B*a*b^
3*d-B*b^4*c+2*C*a^4*d+2*C*a*b^3*c)/(a*d-b*c)^3/(a^2+b^2)^2*ln(a+b*tan(f*x+
e))-(A*b^2-B*a*b+C*a^2)*b/(a*d-b*c)^2/(a^2+b^2)/(a+b*tan(f*x+e))+1/(a^2+b^
2)^2/(c^2+d^2)^2*(1/2*(-2*A*a^2*c*d-2*A*a*b*c^2+2*A*a*b*d^2+2*A*b^2*c*d+B*
a^2*c^2-B*a^2*d^2-4*B*a*b*c*d-B*b^2*c^2+B*b^2*d^2+2*C*a^2*c*d+2*C*a*b*c^2-
2*C*a*b*d^2-2*C*b^2*c*d)*ln(1+tan(f*x+e)^2)+(A*a^2*c^2-A*a^2*d^2-4*A*a*b*c
*d-A*b^2*c^2+A*b^2*d^2+2*B*a^2*c*d+2*B*a*b*c^2-2*B*a*b*d^2-2*B*b^2*c*d-C*a
^2*c^2+C*a^2*d^2+4*C*a*b*c*d+C*b^2*c^2-C*b^2*d^2)*arctan(tan(f*x+e)))+d*(2
*A*a*c*d^3-4*A*b*c^2*d^2-2*A*b*d^4-B*a*c^2*d^2+B*a*d^4+3*B*b*c^3*d+B*b*c*d
^3-2*C*a*c*d^3-2*C*b*c^4)/(a*d-b*c)^3/(c^2+d^2)^2*ln(c+d*tan(f*x+e))-(A*d^
2-B*c*d+C*c^2)*d/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e)))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4174 vs. $2(512) = 1024$.

Time = 3.54 (sec) , antiderivative size = 4174, normalized size of antiderivative = 8.20

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^6 - 2*(C*a^3*b^3 - B*a^2*b^4 + A*a
*b^5)*c^5*d + 4*(C*a^2*b^4 - B*a*b^5 + A*b^6)*c^4*d^2 + 2*(C*a^5*b + 2*B*a
^2*b^4 - (2*A - C)*a*b^5)*c^3*d^3 - 2*(C*a^6 + B*a^5*b + 2*C*a^4*b^2 + 2*B
*a^3*b^3 + 2*B*a*b^5 - A*b^6)*c^2*d^4 + 2*(B*a^6 + A*a^5*b + 2*B*a^4*b^2 +
 (2*A - C)*a^3*b^3 + 2*B*a^2*b^4)*c*d^5 - 2*(A*a^6 + 2*A*a^4*b^2 + A*a^2*b
^4)*d^6 - 2*(((A - C)*a^3*b^3 + 2*B*a^2*b^4 - (A - C)*a*b^5)*c^6 - (3*(A -
 C)*a^4*b^2 + 4*B*a^3*b^3 + (A - C)*a^2*b^4 + 2*B*a*b^5)*c^5*d + (3*(A - C
)*a^5*b + 8*(A - C)*a^3*b^3 + 4*B*a^2*b^4 + (A - C)*a*b^5)*c^4*d^2 - ((A -
 C)*a^6 - 4*B*a^5*b + 8*(A - C)*a^4*b^2 + 3*(A - C)*a^2*b^4)*c^3*d^3 - (2*
B*a^6 - (A - C)*a^5*b + 4*B*a^4*b^2 - 3*(A - C)*a^3*b^3)*c^2*d^4 + ((A - C
)*a^6 + 2*B*a^5*b - (A - C)*a^4*b^2)*c*d^5)*f*x - 2*((C*a^3*b^3 - B*a^2*b^
4 + A*a*b^5)*c^5*d + (B*a^3*b^3 - (A - 2*C)*a^2*b^4 + C*b^6)*c^4*d^2 - (C*
a^5*b + B*a^4*b^2 + 4*B*a^2*b^4 - (2*A - C)*a*b^5 + B*b^6)*c^3*d^3 + (B*a^
5*b + (A - 2*C)*a^4*b^2 + 4*B*a^3*b^3 + B*a*b^5 + A*b^6)*c^2*d^4 - (A*a^5*
b + (2*A - C)*a^3*b^3 + B*a^2*b^4)*c*d^5 - (C*a^4*b^2 - B*a^3*b^3 + A*a^2*
b^4)*d^6 + (((A - C)*a^2*b^4 + 2*B*a*b^5 - (A - C)*b^6)*c^5*d - (3*(A - C)
*a^3*b^3 + 4*B*a^2*b^4 + (A - C)*a*b^5 + 2*B*b^6)*c^4*d^2 + (3*(A - C)*a^4
*b^2 + 8*(A - C)*a^2*b^4 + 4*B*a*b^5 + (A - C)*b^6)*c^3*d^3 - ((A - C)*a^5
*b - 4*B*a^4*b^2 + 8*(A - C)*a^3*b^3 + 3*(A - C)*a*b^5)*c^2*d^4 - (2*B*a^5
*b - (A - C)*a^4*b^2 + 4*B*a^3*b^3 - 3*(A - C)*a^2*b^4)*c*d^5 + ((A - C...
```

3.82.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^2} \, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*
x+e))**2,x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(512) = 1024$.

Time = 0.41 (sec) , antiderivative size = 1185, normalized size of antiderivative = 2.33

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^2 + 2*(B*a^2 - 2*(A - C)*a
*b - B*b^2)*c*d - ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d^2)*(f*x + e)/((a
^4 + 2*a^2*b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2*a
^2*b^2 + b^4)*d^4) - 2*((B*a^2*b^3 - 2*(A - C)*a*b^4 - B*b^5)*c + (2*C*a^4
*b - 3*B*a^3*b^2 + 4*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*d)*log(b*tan(f*x + e)
+ a)/((a^4*b^3 + 2*a^2*b^5 + b^7)*c^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*c^
2*d + 3*(a^6*b + 2*a^4*b^3 + a^2*b^5)*c*d^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*
d^3) + 2*(2*C*b*c^4*d - 3*B*b*c^3*d^2 + (B*a + 4*A*b)*c^2*d^3 - (2*(A - C)
*a + B*b)*c*d^4 - (B*a - 2*A*b)*d^5)*log(d*tan(f*x + e) + c)/(b^3*c^7 - 3*
a*b^2*c^6*d + 3*a^2*b*c*d^6 - a^3*d^7 + (3*a^2*b + 2*b^3)*c^5*d^2 - (a^3 +
 6*a*b^2)*c^4*d^3 + (6*a^2*b + b^3)*c^3*d^4 - (2*a^3 + 3*a*b^2)*c^2*d^5) +
 ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^2 - 2*((A - C)*a^2 + 2*B*a*b - (A - C)
*b^2)*c*d - (B*a^2 - 2*(A - C)*a*b - B*b^2)*d^2)*log(tan(f*x + e)^2 + 1)/(
(a^4 + 2*a^2*b^2 + b^4)*c^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^2 + (a^4 + 2
*a^2*b^2 + b^4)*d^4) - 2*((C*a^2*b - B*a*b^2 + A*b^3)*c^3 + (C*a^3 + C*a*b
^2)*c^2*d - (B*a^3 - C*a^2*b + 2*B*a*b^2 - A*b^3)*c*d^2 + (A*a^3 + A*a*b^2
)*d^3 + ((2*C*a^2*b - B*a*b^2 + (A + C)*b^3)*c^2*d - (B*a^2*b + B*b^3)*c*d
^2 + ((A + C)*a^2*b - B*a*b^2 + 2*A*b^3)*d^3)*tan(f*x + e))/((a^3*b^2 + a*
b^4)*c^5 - 2*(a^4*b + a^2*b^3)*c^4*d + (a^5 + 2*a^3*b^2 + a*b^4)*c^3*d^2 -
 2*(a^4*b + a^2*b^3)*c^2*d^3 + (a^5 + a^3*b^2)*c*d^4 + ((a^2*b^3 + b^5)...
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2823 vs. $2(512) = 1024$.

Time = 1.07 (sec) , antiderivative size = 2823, normalized size of antiderivative = 5.55

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c^2 - C*a^2*c^2 + 2*B*a*b*c^2 - A*b^2*c^2 + C*b^2*c^2 + 2*B*
a^2*c*d - 4*A*a*b*c*d + 4*C*a*b*c*d - 2*B*b^2*c*d - A*a^2*d^2 + C*a^2*d^2
- 2*B*a*b*d^2 + A*b^2*d^2 - C*b^2*d^2)*(f*x + e)/(a^4*c^4 + 2*a^2*b^2*c^4
+ b^4*c^4 + 2*a^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2 + 2*b^4*c^2*d^2 + a^4*d^4 +
2*a^2*b^2*d^4 + b^4*d^4) + (B*a^2*c^2 - 2*A*a*b*c^2 + 2*C*a*b*c^2 - B*b^2*
c^2 - 2*A*a^2*c*d + 2*C*a^2*c*d - 4*B*a*b*c*d + 2*A*b^2*c*d - 2*C*b^2*c*d
- B*a^2*d^2 + 2*A*a*b*d^2 - 2*C*a*b*d^2 + B*b^2*d^2)*log(tan(f*x + e)^2 +
1)/(a^4*c^4 + 2*a^2*b^2*c^4 + b^4*c^4 + 2*a^4*c^2*d^2 + 4*a^2*b^2*c^2*d^2
+ 2*b^4*c^2*d^2 + a^4*d^4 + 2*a^2*b^2*d^4 + b^4*d^4) - 2*(B*a^2*b^4*c - 2*
A*a*b^5*c + 2*C*a*b^5*c - B*b^6*c + 2*C*a^4*b^2*d - 3*B*a^3*b^3*d + 4*A*a^
2*b^4*d - B*a*b^5*d + 2*A*b^6*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^4*c^3
 + 2*a^2*b^6*c^3 + b^8*c^3 - 3*a^5*b^3*c^2*d - 6*a^3*b^5*c^2*d - 3*a*b^7*c
^2*d + 3*a^6*b^2*c*d^2 + 6*a^4*b^4*c*d^2 + 3*a^2*b^6*c*d^2 - a^7*b*d^3 - 2
*a^5*b^3*d^3 - a^3*b^5*d^3) + 2*(2*C*b*c^4*d^2 - 3*B*b*c^3*d^3 + B*a*c^2*d
^4 + 4*A*b*c^2*d^4 - 2*A*a*c*d^5 + 2*C*a*c*d^5 - B*b*c*d^5 - B*a*d^6 + 2*A
*b*d^6)*log(abs(d*tan(f*x + e) + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*
b*c^5*d^3 + 2*b^3*c^5*d^3 - a^3*c^4*d^4 - 6*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^
5 + b^3*c^3*d^5 - 2*a^3*c^2*d^6 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^
8) + (B*a^2*b^3*c^4*d*tan(f*x + e)^2 - 2*A*a*b^4*c^4*d*tan(f*x + e)^2 + 2*
C*a*b^4*c^4*d*tan(f*x + e)^2 - B*b^5*c^4*d*tan(f*x + e)^2 - 2*B*a^3*b^2...
```

3.82.9 Mupad [B] (verification not implemented)

Time = 25.22 (sec) , antiderivative size = 73684, normalized size of antiderivative = 144.76

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))^2),x)
```

output
```
(symsum(log((tan(e + f*x)*(4*A^3*a^3*b^4*d^7 + B^3*a^2*b^5*d^7 + 4*A^3*b^7
*c^3*d^4 + 2*C^3*a^5*b^2*d^7 + B^3*b^7*c^2*d^5 + 2*C^3*b^7*c^5*d^2 + 4*A^2
*B*b^7*d^7 - 4*B^3*a^2*b^5*c^2*d^5 - 3*B^3*a^2*b^5*c^4*d^3 + 10*B^3*a^3*b^
4*c^3*d^4 - 3*B^3*a^4*b^3*c^2*d^5 - 4*A*B^2*a*b^6*d^7 - 4*A*B^2*b^7*c*d^6
+ 2*B^3*a*b^6*c*d^6 - 6*A*B^2*a^3*b^4*d^7 + 8*A^2*B*a^2*b^5*d^7 - 3*A^2*B*
a^4*b^3*d^7 + 4*A*C^2*a^3*b^4*d^7 - 4*A*C^2*a^5*b^2*d^7 - 8*A^2*C*a^3*b^4*
d^7 + 2*A^2*C*a^5*b^2*d^7 - 6*A*B^2*b^7*c^3*d^4 - 3*B*C^2*a^4*b^3*d^7 + 8*
A^2*B*b^7*c^2*d^5 - 3*A^2*B*b^7*c^4*d^3 + 4*A*C^2*b^7*c^3*d^4 - 4*A*C^2*b^
7*c^5*d^2 - 8*A^2*C*b^7*c^3*d^4 + 2*A^2*C*b^7*c^5*d^2 - 3*B*C^2*b^7*c^4*d^
3 - 4*A^3*a*b^6*c^2*d^5 - 4*A^3*a^2*b^5*c*d^6 + 6*B^3*a*b^6*c^3*d^4 + 6*B^
3*a^3*b^4*c*d^6 - 2*C^3*a*b^6*c^4*d^3 - 2*C^3*a^4*b^3*c*d^6 - 10*A*B^2*a^2
*b^5*c^3*d^4 - 10*A*B^2*a^3*b^4*c^2*d^5 + 18*A^2*B*a^2*b^5*c^2*d^5 + 2*B*C
^2*a^2*b^5*c^2*d^5 + 4*B*C^2*a^4*b^3*c^4*d^3 + 2*B^2*C*a^2*b^5*c^3*d^4 + 2
*B^2*C*a^2*b^5*c^5*d^2 + 2*B^2*C*a^3*b^4*c^2*d^5 - 6*B^2*C*a^3*b^4*c^4*d^3
 - 6*B^2*C*a^4*b^3*c^3*d^4 + 2*B^2*C*a^5*b^2*c^2*d^5 + 10*A*B*C*a^4*b^3*d^
7 + 10*A*B*C*b^7*c^4*d^3 - 8*A^2*B*a*b^6*c*d^6 - 2*A*B^2*a*b^6*c^2*d^5 + 6
*A*B^2*a*b^6*c^4*d^3 - 2*A*B^2*a^2*b^5*c*d^6 + 6*A*B^2*a^4*b^3*c*d^6 - 4*A
^2*B*a*b^6*c^3*d^4 - 4*A^2*B*a^3*b^4*c*d^6 - 4*A*C^2*a*b^6*c^2*d^5 + 4*A*C
^2*a*b^6*c^4*d^3 - 4*A*C^2*a^2*b^5*c*d^6 + 4*A*C^2*a^4*b^3*c*d^6 + 8*A^2*C
*a*b^6*c^2*d^5 - 2*A^2*C*a*b^6*c^4*d^3 + 8*A^2*C*a^2*b^5*c*d^6 - 2*A^2*...
```

3.82. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}\,dx$

3.83 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

3.83.1 Optimal result

Integrand size = 45, antiderivative size = 841

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx =$$

$$-\frac{(a^3(c^2C-2Bcd-Cd^2-A(c^2-d^2))-3ab^2(c^2C-2Bcd-Cd^2-A(c^2-d^2))+3a^2b(2c(A-C)d-}{(a^2+b^2)^3\,(c^2+d^2)^2}$$

$$-\frac{b(6a^5bBd^2-3a^6Cd^2-a^4b^2d(4Bc+(10A-C)d)-b^6(c(cC-2Bd)-A(c^2-3d^2))+ab^5(2c(A-C)}{}$$

$$-\frac{d^2(b(3c^4C-4Bc^3d+c^2(5A+C)d^2-2Bcd^3+3Ad^4)-ad^2(2c(A-C)d-B(c^2-d^2)))\log(c\cos(e+}{(bc-ad)^4\,(c^2+d^2)^2\,f}$$

$$-\frac{d(3a^3bBd(c^2+d^2)+ab^3(2Ac-2cC+Bd)\,(c^2+d^2)-a^4d(3c^2C-Bcd+(A+2C)d^2)-a^2b^2(Bc^3+}{(a^2+b^2)^2\,(bc-ad)^3\,(c^2+d^2)\,f(c+d}$$

$$-\frac{Ab^2-a(bB-aC)}{2\,(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$

$$-\frac{5a^3bBd-3a^4Cd+b^4(2Bc-3Ad)+ab^3(4Ac-4cC+Bd)-a^2b^2(2Bc+(7A-C)d)}{2\,(a^2+b^2)^2\,(bc-ad)^2f(a+b\tan(e+fx))(c+d\tan(e+fx))}$$

output
```
-(a^3*(c^2*C-2*B*c*d-C*d^2-A*(c^2-d^2))-3*a*b^2*(c^2*C-2*B*c*d-C*d^2-A*(c^
2-d^2))+3*a^2*b*(2*c*(A-C)*d-B*(c^2-d^2))-b^3*(2*c*(A-C)*d-B*(c^2-d^2)))*x
/(a^2+b^2)^3/(c^2+d^2)^2-b*(6*a^5*b*B*d^2-3*a^6*C*d^2-a^4*b^2*d*(4*B*c+(10
*A-C)*d)-b^6*(c*(-2*B*d+C*c)-A*(c^2-3*d^2))+a*b^5*(2*c*(A-C)*d-B*(3*c^2-d^
2))+3*a^2*b^4*(c*(2*B*d+C*c)-A*(c^2+3*d^2))+a^3*b^3*(10*c*(A-C)*d+B*(c^2+3
*d^2)))*ln(a*cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)^3/(-a*d+b*c)^4/f-d^2*(b*(3
*c^4*C-4*B*c^3*d+c^2*(5*A+C)*d^2-2*B*c*d^3+3*A*d^4)-a*d^2*(2*c*(A-C)*d-B*(
c^2-d^2)))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d^2)^2/f-d*(3*a
^3*b*B*d*(c^2+d^2)+a*b^3*(2*A*c+B*d-2*C*c)*(c^2+d^2)-a^4*d*(3*c^2*C-B*c*d+
(A+2*C)*d^2)-a^2*b^2*(4*A*c^2*d+6*A*d^3+B*c^3-B*c*d^2+2*C*c^2*d)-b^4*(d*(2
*A*c^2+3*A*d^2+C*c^2)-B*(c^3+2*c*d^2)))/(a^2+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)
/f/(c+d*tan(f*x+e))+1/2*(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*t
an(f*x+e))^2/(c+d*tan(f*x+e))+1/2*(-5*a^3*b*B*d+3*a^4*C*d-b^4*(-3*A*d+2*B*
c)-a*b^3*(4*A*c+B*d-4*C*c)+a^2*b^2*(2*B*c+(7*A-C)*d))/(a^2+b^2)^2/(-a*d+b*
c)^2/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))
```

3.83.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1758 vs. $2(841) = 1682$.

Time = 8.88 (sec) , antiderivative size = 1758, normalized size of antiderivative = 2.09

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$$
$$= -\frac{Ab^2 - a(bB - aC)}{2\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$
$$- \frac{-\frac{-a\left(-3a\left(Ab^2-a(bB-aC)\right)d+2b(Ab-aB-bC)(bc-ad)\right)+b^2\left(3Ab^2d-2aA(bc-ad)-(bB-aC)(2bc+ad)\right)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{-\frac{(bc-ad)^3\left(-b^2\left(-3a^2Abc^2+A\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*
(c + d*Tan[e + f*x])^2),x]
```

output
```
-1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])^2*(c + d*Tan[e + f*x])) - (-((-(a*(-3*a*(A*b^2 - a*(b*B - a*C))*d + 2*b
*(A*b - a*B - b*C)*(b*c - a*d))) + b^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b
*B - a*C)*(2*b*c + a*d)))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x]))) - (-((-(((b*c - a*d)^3*(-(b^2*(-3*a^2*A*b*c^2 + A*b
^3*c^2 + a^3*B*c^2 - 3*a*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c
*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c
*C*d + 3*a^2*A*b*d^2 - A*b^3*d^2 - a^3*B*d^2 + 3*a*b^2*B*d^2 - 3*a^2*b*C*d
^2 + b^3*C*d^2)) + Sqrt[-b^2]*(a^3*A*b*c^2 - 3*a*A*b^3*c^2 + 3*a^2*b^2*B*c
^2 - b^4*B*c^2 - a^3*b*c^2*C + 3*a*b^3*c^2*C - 6*a^2*A*b^2*c*d + 2*A*b^4*c
*d + 2*a^3*b*B*c*d - 6*a*b^3*B*c*d + 6*a^2*b^2*c*C*d - 2*b^4*c*C*d - a^3*A
*b*d^2 + 3*a*A*b^3*d^2 - 3*a^2*b^2*B*d^2 + b^4*B*d^2 + a^3*b*C*d^2 - 3*a*b
^3*C*d^2))*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2)))
- (2*b^2*(c^2 + d^2)*(6*a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10
*A - C)*d) - b^6*(c*(c*C - 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*
d - B*(3*c^2 - d^2)) + 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3
*b^3*(10*c*(A - C)*d + B*(c^2 + 3*d^2)))*Log[a + b*Tan[e + f*x]])/((a^2 +
b^2)*(b*c - a*d)) + ((b*c - a*d)^3*(b^2*(-3*a^2*A*b*c^2 + A*b^3*c^2 + a^3*
B*c^2 - 3*a*b^2*B*c^2 + 3*a^2*b*c^2*C - b^3*c^2*C - 2*a^3*A*c*d + 6*a*A*b^
2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b^2*c*C*d + 3*a...
```

3.83.3 Rubi [A] (verified)

Time = 5.32 (sec) , antiderivative size = 919, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {3042, 4132, 3042, 4132, 3042, 4132, 27, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int \frac{3Adb^2+3(Ab^2-a(bB-aC))d\tan^2(e+fx)-2aA(bc-ad)-(bB-aC)(2bc+ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}dx}{2(a^2+b^2)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3Adb^2+3(Ab^2-a(bB-aC))d\tan(e+fx)^2-2aA(bc-ad)-(bB-aC)(2bc+ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^2}dx}{2(a^2+b^2)(bc-ad)} - \frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{-3a^4Cd+5a^3bBd-a^2b^2(7Ad+2Bc-Cd)+ab^3(4Ac+Bd-4cC)+b^4(2Bc-3Ad)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{\int \frac{2\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)^2-2d\left(-3Cda^4+5b\right.}{}}{}}{} \frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{-3a^4Cd+5a^3bBd-a^2b^2(7Ad+2Bc-Cd)+ab^3(4Ac+Bd-4cC)+b^4(2Bc-3Ad)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{\int \frac{2\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)^2-2d\left(-3Cda^4+5b\right.}{}}{}}{} \frac{Ab^2-a(bB-aC)}{2f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^2(c+d\tan(e+fx))}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))} - \frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{\int -\frac{2\left(d^3(Ac-Cc+Bd)a^5-3Abd^2\left(c^2+d^2\right)a^4+b^2d\left(3Ac^3-3Cc^3+5Ad\right.}{}}{}}{}$$

$$\Big\downarrow \text{27}$$

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))} - \frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))} - \frac{-\frac{2d\left(-d\left(3Cc^2-Bdc+(A+2C)d^2\right)a^4+3bBd\left(c^2+d^2\right)a^3-b^2\left(Bc^3+4A\right.}{}}{}}{}$$

$$\Big\downarrow \text{3042}$$

3.83. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}-$$

$$\frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{-\frac{2d\left(-d\left(3Cc^2-Bdc+(A+2C)d^2\right)a^4+3bBd\left(c^2+d^2\right)a^3-b^2\left(Bc^3+4A\right.}{}}{}}{}$$

$\downarrow$ 4134

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}-$$

$$\frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{-\frac{2d\left(-d\left(3Cc^2-Bdc+(A+2C)d^2\right)a^4+3bBd\left(c^2+d^2\right)a^3-b^2\left(Bc^3+4A\right.}{}}{}}{}$$

$\downarrow$ 3042

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}-$$

$$\frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{-\frac{2d\left(-d\left(3Cc^2-Bdc+(A+2C)d^2\right)a^4+3bBd\left(c^2+d^2\right)a^3-b^2\left(Bc^3+4A\right.}{}}{}}{}$$

$\downarrow$ 4013

$$-\frac{Ab^2-a(bB-aC)}{2(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^2(c+d\tan(e+fx))}-$$

$$\frac{\frac{-3Cda^4+5bBda^3-b^2(2Bc+7Ad-Cd)a^2+b^3(4Ac-4Cc+Bd)a+b^4(2Bc-3Ad)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))}-\frac{-\frac{2d\left(-d\left(3Cc^2-Bdc+(A+2C)d^2\right)a^4+3bBd\left(c^2+d^2\right)a^3-b^2\left(Bc^3+4A\right.}{}}{}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^3*(c + d
*Tan[e + f*x])^2),x]
```

3.83. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

output
```
-1/2*(A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x
])^2*(c + d*Tan[e + f*x])) - ((5*a^3*b*B*d - 3*a^4*C*d + b^4*(2*B*c - 3*A*
d) + a*b^3*(4*A*c - 4*c*C + B*d) - a^2*b^2*(2*B*c + 7*A*d - C*d))/((a^2 +
b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])) - ((-2*(((b*
c - a*d)^3*(a^3*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - 3*a*b^2*(c^2*C
 - 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + 3*a^2*b*(2*c*(A - C)*d - B*(c^2 - d^
2)) - b^3*(2*c*(A - C)*d - B*(c^2 - d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)) +
(b*(c^2 + d^2)*(6*a^5*b*B*d^2 - 3*a^6*C*d^2 - a^4*b^2*d*(4*B*c + (10*A - C
)*d) - b^6*(c*(c*C - 2*B*d) - A*(c^2 - 3*d^2)) + a*b^5*(2*c*(A - C)*d - B*
(3*c^2 - d^2)) + 3*a^2*b^4*(c*(c*C + 2*B*d) - A*(c^2 + 3*d^2)) + a^3*b^3*(
10*c*(A - C)*d + B*(c^2 + 3*d^2)))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]])/(
(a^2 + b^2)*(b*c - a*d)*f) + ((a^2 + b^2)^2*d^2*(b*(3*c^4*C - 4*B*c^3*d +
c^2*(5*A + C)*d^2 - 2*B*c*d^3 + 3*A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 -
 d^2)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f))
)/((b*c - a*d)*(c^2 + d^2)) - (2*d*(3*a^3*b*B*d*(c^2 + d^2) + a*b^3*(2*A*c
 - 2*c*C + B*d)*(c^2 + d^2) - a^4*d*(3*c^2*C - B*c*d + (A + 2*C)*d^2) - a^
2*b^2*(B*c^3 + 4*A*c^2*d + 2*c^2*C*d - B*c*d^2 + 6*A*d^3) - b^4*(d*(2*A*c^
2 + c^2*C + 3*A*d^2) - B*(c^3 + 2*c*d^2))))/((b*c - a*d)*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])))/((a^2 + b^2)*(b*c - a*d)))/(2*(a^2 + b^2)*(b*c - a*d))
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.83.4 Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{b\left(4Aa^2b^2d-2Aab^3c+2Ab^4d-3a^3bBd+Ba^2b^2c-Bab^3d-Bb^4c+2a^4Cd+2Cab^3c\right)}{(ad-bc)^3\left(a^2+b^2\right)^2(a+b\tan(fx+e))}+\frac{b\left(10Aa^4b^2d^2-10Aa^3b^3cd+3Aa^2b^4c^2+9A\right.}{}$
default	$-\frac{b\left(4Aa^2b^2d-2Aab^3c+2Ab^4d-3a^3bBd+Ba^2b^2c-Bab^3d-Bb^4c+2a^4Cd+2Cab^3c\right)}{(ad-bc)^3\left(a^2+b^2\right)^2(a+b\tan(fx+e))}+\frac{b\left(10Aa^4b^2d^2-10Aa^3b^3cd+3Aa^2b^4c^2+9A\right.}{}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+e))^2,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b*(4*A*a^2*b^2*d-2*A*a*b^3*c+2*A*b^4*d-3*B*a^3*b*d+B*a^2*b^2*c-B*a*b
^3*d-B*b^4*c+2*C*a^4*d+2*C*a*b^3*c)/(a*d-b*c)^3/(a^2+b^2)^2/(a+b*tan(f*x+e
))+b*(10*A*a^4*b^2*d^2-10*A*a^3*b^3*c*d+3*A*a^2*b^4*c^2+9*A*a^2*b^4*d^2-2*
A*a*b^5*c*d-A*b^6*c^2+3*A*b^6*d^2-6*B*a^5*b*d^2+4*B*a^4*b^2*c*d-B*a^3*b^3*
c^2-3*B*a^3*b^3*d^2-6*B*a^2*b^4*c*d+3*B*a*b^5*c^2-B*a*b^5*d^2-2*B*b^6*c*d+
3*C*a^6*d^2-C*a^4*b^2*d^2+10*C*a^3*b^3*c*d-3*C*a^2*b^4*c^2+2*C*a*b^5*c*d+C
*b^6*c^2)/(a*d-b*c)^4/(a^2+b^2)^3*ln(a+b*tan(f*x+e))-1/2*(A*b^2-B*a*b+C*a^
2)*b/(a*d-b*c)^2/(a^2+b^2)/(a+b*tan(f*x+e))^2+1/(a^2+b^2)^3/(c^2+d^2)^2*(1
/2*(-2*A*a^3*c*d-3*A*a^2*b*c^2+3*A*a^2*b*d^2+6*A*a*b^2*c*d+A*b^3*c^2-A*b^3
*d^2+B*a^3*c^2-B*a^3*d^2-6*B*a^2*b*c*d-3*B*a*b^2*c^2+3*B*a*b^2*d^2+2*B*b^3
*c*d+2*C*a^3*c*d+3*C*a^2*b*c^2-3*C*a^2*b*d^2-6*C*a*b^2*c*d-C*b^3*c^2+C*b^3
*d^2)*ln(1+tan(f*x+e)^2)+(A*a^3*c^2-A*a^3*d^2-6*A*a^2*b*c*d-3*A*a*b^2*c^2+
3*A*a*b^2*d^2+2*A*b^3*c*d+2*B*a^3*c*d+3*B*a^2*b*c^2-3*B*a^2*b*d^2-6*B*a*b^
2*c*d-B*b^3*c^2+B*b^3*d^2-C*a^3*c^2+C*a^3*d^2+6*C*a^2*b*c*d+3*C*a*b^2*c^2-
3*C*a*b^2*d^2-2*C*b^3*c*d)*arctan(tan(f*x+e)))+d^2*(2*A*a*c*d^3-5*A*b*c^2*
d^2-3*A*b*d^4-B*a*c^2*d^2+B*a*d^4+4*B*b*c^3*d+2*B*b*c*d^3-2*C*a*c*d^3-3*C*
b*c^4-C*b*c^2*d^2)/(a*d-b*c)^4/(c^2+d^2)^2*ln(c+d*tan(f*x+e))-(A*d^2-B*c*d
+C*c^2)*d^2/(a*d-b*c)^3/(c^2+d^2)/(c+d*tan(f*x+e)))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9594 vs. 2(835) = 1670.

Time = 10.63 (sec) , antiderivative size = 9594, normalized size of antiderivative = 11.41

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^3(c + d\tan(e + fx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**3/(c+d*tan(f*
x+e))**2,x)
```

output
```
Timed out
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(835) = 1670$.

Time = 0.48 (sec) , antiderivative size = 2519, normalized size of antiderivative = 3.00

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c^2 + 2*(B*a^3
 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c*d - ((A - C)*a^3 + 3*B*a^2
*b - 3*(A - C)*a*b^2 - B*b^3)*d^2)*(f*x + e)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^2*d^2 + (a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6)*d^4) - 2*((B*a^3*b^4 - 3*(A - C)*a^2*b^5 - 3*B*a*b
^6 + (A - C)*b^7)*c^2 - 2*(2*B*a^4*b^3 - 5*(A - C)*a^3*b^4 - 3*B*a^2*b^5 -
 (A - C)*a*b^6 - B*b^7)*c*d - (3*C*a^6*b - 6*B*a^5*b^2 + (10*A - C)*a^4*b^
3 - 3*B*a^3*b^4 + 9*A*a^2*b^5 - B*a*b^6 + 3*A*b^7)*d^2)*log(b*tan(f*x + e)
 + a)/((a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*c^4 - 4*(a^7*b^3 + 3*a^5*b
^5 + 3*a^3*b^7 + a*b^9)*c^3*d + 6*(a^8*b^2 + 3*a^6*b^4 + 3*a^4*b^6 + a^2*b
^8)*c^2*d^2 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*c*d^3 + (a^10 +
3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6)*d^4) - 2*(3*C*b*c^4*d^2 - 4*B*b*c^3*d^3 +
 (B*a + (5*A + C)*b)*c^2*d^4 - 2*((A - C)*a + B*b)*c*d^5 - (B*a - 3*A*b)*d
^6)*log(d*tan(f*x + e) + c)/(b^4*c^8 - 4*a*b^3*c^7*d - 4*a^3*b*c*d^7 + a^4
*d^8 + 2*(3*a^2*b^2 + b^4)*c^6*d^2 - 4*(a^3*b + 2*a*b^3)*c^5*d^3 + (a^4 +
12*a^2*b^2 + b^4)*c^4*d^4 - 4*(2*a^3*b + a*b^3)*c^3*d^5 + 2*(a^4 + 3*a^2*b
^2)*c^2*d^6) + ((B*a^3 - 3*(A - C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*c^2 -
2*((A - C)*a^3 + 3*B*a^2*b - 3*(A - C)*a*b^2 - B*b^3)*c*d - (B*a^3 - 3*(A
- C)*a^2*b - 3*B*a*b^2 + (A - C)*b^3)*d^2)*log(tan(f*x + e)^2 + 1)/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c^4 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^...
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3115 vs. $2(835) = 1670$.

Time = 1.10 (sec) , antiderivative size = 3115, normalized size of antiderivative = 3.70

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^3(c + d\tan(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^3/(c+d*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^3*c^2 - C*a^3*c^2 + 3*B*a^2*b*c^2 - 3*A*a*b^2*c^2 + 3*C*a*b^2*
c^2 - B*b^3*c^2 + 2*B*a^3*c*d - 6*A*a^2*b*c*d + 6*C*a^2*b*c*d - 6*B*a*b^2*
c*d + 2*A*b^3*c*d - 2*C*b^3*c*d - A*a^3*d^2 + C*a^3*d^2 - 3*B*a^2*b*d^2 +
3*A*a*b^2*d^2 - 3*C*a*b^2*d^2 + B*b^3*d^2)*(f*x + e)/(a^6*c^4 + 3*a^4*b^2*
c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*c^2*d^2 + 6*a^2*
b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*a^2*b^4*d^4 + b^
6*d^4) + (B*a^3*c^2 - 3*A*a^2*b*c^2 + 3*C*a^2*b*c^2 - 3*B*a*b^2*c^2 + A*b^
3*c^2 - C*b^3*c^2 - 2*A*a^3*c*d + 2*C*a^3*c*d - 6*B*a^2*b*c*d + 6*A*a*b^2*
c*d - 6*C*a*b^2*c*d + 2*B*b^3*c*d - B*a^3*d^2 + 3*A*a^2*b*d^2 - 3*C*a^2*b*
d^2 + 3*B*a*b^2*d^2 - A*b^3*d^2 + C*b^3*d^2)*log(tan(f*x + e)^2 + 1)/(a^6*
c^4 + 3*a^4*b^2*c^4 + 3*a^2*b^4*c^4 + b^6*c^4 + 2*a^6*c^2*d^2 + 6*a^4*b^2*
c^2*d^2 + 6*a^2*b^4*c^2*d^2 + 2*b^6*c^2*d^2 + a^6*d^4 + 3*a^4*b^2*d^4 + 3*
a^2*b^4*d^4 + b^6*d^4) - 2*(B*a^3*b^5*c^2 - 3*A*a^2*b^6*c^2 + 3*C*a^2*b^6*
c^2 - 3*B*a*b^7*c^2 + A*b^8*c^2 - C*b^8*c^2 - 4*B*a^4*b^4*c*d + 10*A*a^3*b
^5*c*d - 10*C*a^3*b^5*c*d + 6*B*a^2*b^6*c*d + 2*A*a*b^7*c*d - 2*C*a*b^7*c*
d + 2*B*b^8*c*d - 3*C*a^6*b^2*d^2 + 6*B*a^5*b^3*d^2 - 10*A*a^4*b^4*d^2 + C
*a^4*b^4*d^2 + 3*B*a^3*b^5*d^2 - 9*A*a^2*b^6*d^2 + B*a*b^7*d^2 - 3*A*b^8*d
^2)*log(abs(b*tan(f*x + e) + a))/(a^6*b^5*c^4 + 3*a^4*b^7*c^4 + 3*a^2*b^9*
c^4 + b^11*c^4 - 4*a^7*b^4*c^3*d - 12*a^5*b^6*c^3*d - 12*a^3*b^8*c^3*d - 4
*a*b^10*c^3*d + 6*a^8*b^3*c^2*d^2 + 18*a^6*b^5*c^2*d^2 + 18*a^4*b^7*c^2...
```

3.83.9 Mupad [B] (verification not implemented)

Time = 44.53 (sec) , antiderivative size = 128667, normalized size of antiderivative = 152.99

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^3*(c + d
*tan(e + f*x))^2),x)
```

output
```
(symsum(log((24*A^3*a^3*b^7*d^9 + 27*A^3*a^5*b^5*d^9 + B^3*a^2*b^8*d^9 + 4
*B^3*a^4*b^6*d^9 + 7*B^3*a^6*b^4*d^9 + 3*A^3*b^10*c^3*d^6 - A^3*b^10*c^5*d
^4 + 4*B^3*b^10*c^2*d^7 + 6*B^3*b^10*c^4*d^5 + C^3*b^10*c^5*d^4 + 9*A^2*B*
b^10*d^9 + 9*A^3*a*b^9*d^9 + 16*A^3*a^2*b^8*c^3*d^6 + 3*A^3*a^2*b^8*c^5*d^
4 + 26*A^3*a^3*b^7*c^2*d^7 - 6*A^3*a^3*b^7*c^4*d^5 - 11*A^3*a^4*b^6*c^3*d^
6 + 31*A^3*a^5*b^5*c^2*d^7 + 5*B^3*a^2*b^8*c^2*d^7 + 6*B^3*a^2*b^8*c^4*d^5
 + 28*B^3*a^3*b^7*c^3*d^6 + 7*B^3*a^3*b^7*c^5*d^4 - 14*B^3*a^4*b^6*c^2*d^7
 - 20*B^3*a^4*b^6*c^4*d^5 + 19*B^3*a^5*b^5*c^3*d^6 + 9*B^3*a^6*b^4*c^2*d^7
 - 7*C^3*a^2*b^8*c^3*d^6 - 3*C^3*a^2*b^8*c^5*d^4 + C^3*a^3*b^7*c^2*d^7 + 1
5*C^3*a^3*b^7*c^4*d^5 + 6*C^3*a^3*b^7*c^6*d^3 - 28*C^3*a^4*b^6*c^3*d^6 - 2
4*C^3*a^4*b^6*c^5*d^4 - 4*C^3*a^5*b^5*c^2*d^7 + 3*C^3*a^6*b^4*c^3*d^6 - 9*
C^3*a^7*b^3*c^2*d^7 - 9*C^3*a^7*b^3*c^4*d^5 - 6*A*B^2*a*b^9*d^9 - 9*A^2*C*
a*b^9*d^9 - 12*A*B^2*b^10*c*d^8 + 4*B^3*a*b^9*c*d^8 - 20*A*B^2*a^3*b^7*d^9
 - 28*A*B^2*a^5*b^5*d^9 + 6*A*B^2*a^7*b^3*d^9 + 21*A^2*B*a^2*b^8*d^9 + 13*
A^2*B*a^4*b^6*d^9 - 27*A^2*B*a^6*b^4*d^9 - 3*A*C^2*a^3*b^7*d^9 - 9*A*C^2*a
^7*b^3*d^9 - 21*A^2*C*a^3*b^7*d^9 - 27*A^2*C*a^5*b^5*d^9 + 9*A^2*C*a^7*b^3
*d^9 - 17*A*B^2*b^10*c^3*d^6 + 3*A*B^2*b^10*c^5*d^4 + B*C^2*a^4*b^6*d^9 +
3*B*C^2*a^8*b^2*d^9 + 12*A^2*B*b^10*c^2*d^7 - 7*A^2*B*b^10*c^4*d^5 - B^2*C
*a^3*b^7*d^9 - 2*B^2*C*a^5*b^5*d^9 - 9*B^2*C*a^7*b^3*d^9 + 3*A*C^2*b^10*c^
3*d^6 - 3*A*C^2*b^10*c^5*d^4 - 6*A^2*C*b^10*c^3*d^6 + 3*A^2*C*b^10*c^5*...
```

3.83. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^3(c+d\tan(e+fx))^2}\,dx$

3.84 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.84.1 Optimal result

Integrand size = 45, antiderivative size = 804

$$\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{(3ab^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^3(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2}{(c^2+d^2)^3}$$

$$-\frac{(3a^2b(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)-b^3(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3}{(c^2}$$

$$-\frac{(bc-ad)\,(b^2(3c^6C-Bc^5d+9c^4Cd^2-3Bc^3d^3-c^2(A-10C)d^4-6Bcd^5+3Ad^6)+a^2d^3((A-C)d(3}{d^4\,(c^2+d^2)}$$

$$+\frac{b^2(b(3c^4C-Bc^3d+6c^2Cd^2-3Bcd^3+(2A+C)d^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\tan(e+fx)}{d^3\,(c^2+d^2)^2\,f}$$

$$-\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^3}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}$$

$$-\frac{(b(3c^4C-Bc^3d-c^2(A-7C)d^2-5Bcd^3+3Ad^4)+2ad^2(2c(A-C)d-B(c^2-d^2)))\,(a+b\tan(e+}{2d^2\,(c^2+d^2)^2\,f(c+d\tan(e+fx))}$$

output
```
-(3*a*b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^3*(c^3*C-3*B
*c^2*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))-3*a^2*b*((A-C)*d*(3*c^2-d^2)-B*(c^
3-3*c*d^2))+b^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*x/(c^2+d^2)^3-(3*a^
2*b*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)-b^3*(A*c^3-3*A*c*d^2
+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)-a^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)
)+3*a*b^2*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*ln(cos(f*x+e))/(c^2+d^2)^
3/f-(-a*d+b*c)*(b^2*(3*c^6*C-B*c^5*d+9*c^4*C*d^2-3*B*c^3*d^3-c^2*(A-10*C)*
d^4-6*B*c*d^5+3*A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))+a*b*d
^2*(8*c*(A-C)*d^3-B*(c^4+6*c^2*d^2-3*d^4)))*ln(c+d*tan(f*x+e))/d^4/(c^2+d^
2)^3/f+b^2*(b*(3*c^4*C-B*c^3*d+6*C*c^2*d^2-3*B*c*d^3+(2*A+C)*d^4)+a*d^2*(2
*c*(A-C)*d-B*(c^2-d^2)))*tan(f*x+e)/d^3/(c^2+d^2)^2/f-1/2*(A*d^2-B*c*d+C*c
^2)*(a+b*tan(f*x+e))^3/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1/2*(b*(3*c^4*C-B*
c^3*d-c^2*(A-7*C)*d^2-5*B*c*d^3+3*A*d^4)+2*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2))
)*(a+b*tan(f*x+e))^2/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

3.84.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.37 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$= \frac{\frac{(a+ib)^3(A+iB-C)\log(i-\tan(e+fx))}{(-ic+d)^3}+\frac{(a-ib)^3(A-iB-C)\log(i+\tan(e+fx))}{(ic+d)^3}+\frac{2(-bc+ad)\left(b^2\left(3c^6C-Bc^5d+9c^4Cd^2-3Bc^3d^3-c^2(A-10C\right.\right.}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^3,x]
```

output
```
(((a + I*b)^3*(A + I*B - C)*Log[I - Tan[e + f*x]])/((-I)*c + d)^3 + ((a -
I*b)^3*(A - I*B - C)*Log[I + Tan[e + f*x]])/(I*c + d)^3 + (2*(-(b*c) + a*d
)*(b^2*(3*c^6*C - B*c^5*d + 9*c^4*C*d^2 - 3*B*c^3*d^3 - c^2*(A - 10*C)*d^4
 - 6*B*c*d^5 + 3*A*d^6) + a^2*d^3*(-((A - C)*d*(-3*c^2 + d^2)) - B*(c^3 -
3*c*d^2)) - a*b*d^2*(8*c*(-A + C)*d^3 + B*(c^4 + 6*c^2*d^2 - 3*d^4)))*Log[
c + d*Tan[e + f*x]])/(d^4*(c^2 + d^2)^3) + ((b*c - a*d)^3*(3*c^2*C - B*c*d
 + (A + 2*C)*d^2))/(d^4*(c^2 + d^2)*(c + d*Tan[e + f*x])^2) + (2*C*(a + b*
Tan[e + f*x])^3)/(d*(c + d*Tan[e + f*x])^2) - (2*(b*c - a*d)^2*(b*(6*c^4*C
 - 2*B*c^3*d + c^2*(A + 11*C)*d^2 - 4*B*c*d^3 + 3*(A + C)*d^4) + a*d^2*(2*
c*(A - C)*d + B*(-c^2 + d^2))))/(d^4*(c^2 + d^2)^2*(c + d*Tan[e + f*x])))/
(2*f)
```

3.84.3 Rubi [A] (verified)

Time = 4.00 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4128, 3042, 4128, 3042, 4120, 27, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \ 4128$$

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(3Cc^2-Bdc+(A+2C)d^2\right)\tan^2(e+fx)+2d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(2ac+3bd)+(3bc-2ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^2}d}{2d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(3Cc^2-Bdc+(A+2C)d^2\right)\tan(e+fx)^2+2d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(2ac+3bd)+(3bc-2ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^2}d}{2d\left(c^2+d^2\right)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 4128$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(2((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+2bd)(Ad(2ac+3bd)+(3bc-2ad)(cC-Bd\right.}{c+}}{}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(e+fx))\Big(2((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+2bd)(Ad(2ac+3bd)+(3bc-2ad)(cC-Bd)}{c+}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\big\downarrow$ 4120

$$\frac{2b^2\tan(e+fx)\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(d^4(2A+C)-Bc^3d-3Bcd^3+3c^4C+6c^2Cd^2\right)\right)}{df}-\int -\frac{2\left(-c\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)b^3-(3bcC-\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\big\downarrow$ 27

$$2\int \frac{-c\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)b^3-(3bcC-3adC-bBd)\left(c^2+d^2\right)^2\tan^2(e+fx)b^2+3ad\left(Cc^4-(A-3C)d^2c^2-2Bd^3c+Ad^4\right)b^2+3a^2d^3\left(2c(A-C)\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\big\downarrow$ 3042

$$2\int \frac{-c\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)b^3-(3bcC-3adC-bBd)\left(c^2+d^2\right)^2\tan(e+fx)^2b^2+3ad\left(Cc^4-(A-3C)d^2c^2-2Bd^3c+Ad^4\right)b^2+3a^2d^3\left(2c(A-C)\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\big\downarrow$ 4109

$$\frac{2\left(a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)\right)\tan(e+fx)b^2}{df}+\frac{2\left(-\frac{\left(\left(Cc^3-3Bdc^2-3Cd^2c+Bd^3-A\left(c^3-3cd^2\right)\right)a^3-3b\left((A-C)\right.\right.}{}\right.}{}$$

$$\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$

$\big\downarrow$ 3042

3.84. $\displaystyle\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

$$\frac{\frac{2\left(a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)\right)\tan(e+fx)b^2}{df}+\frac{2\left(-\frac{\left(\left(Cc^3-3Bdc^2-3Cd^2c+Bd^3-A\left(c^3-3cd^2\right)\right)a^3-3b\left((A-C)\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$

$\downarrow$ 3956

$$\frac{\frac{2\left(a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)\right)\tan(e+fx)b^2}{df}+\frac{2\left(-\frac{\left(\left(Cc^3-3Bdc^2-3Cd^2c+Bd^3-A\left(c^3-3cd^2\right)\right)a^3-3b\left((A-C)\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$

$\downarrow$ 4100

$$\frac{\frac{2\left(a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)\right)\tan(e+fx)b^2}{df}+\frac{2\left(-\frac{\left(\left(Cc^3-3Bdc^2-3Cd^2c+Bd^3-A\left(c^3-3cd^2\right)\right)a^3-3b\left((A-C)\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$

$\downarrow$ 16

$$\frac{\frac{2\left(a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(3Cc^4-Bdc^3+6Cd^2c^2-3Bd^3c+(2A+C)d^4\right)\right)\tan(e+fx)b^2}{df}+\frac{2\left(-\frac{\left(\left(Cc^3-3Bdc^2-3Cd^2c+Bd^3-A\left(c^3-3cd^2\right)\right)a^3-3b\left((A-C)\right.\right.}{}\right.}{}}{}$$

$$\frac{\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^3}{2d\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$

input

```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^3,x]
```

3.84. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

output
```
-1/2*((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])^2) + (-(((b*(3*c^4*C - B*c^3*d - c^2*(A - 7*C)*d^2 - 5*B
*c*d^3 + 3*A*d^4) + 2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e
+ f*x])^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) + ((2*(-((d^3*(3*a*b^2*
(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + a^3*(c^3*C -
 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 3*a^2*b*((A - C)*d*(
3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + b^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3
- 3*c*d^2)))*x)/(c^2 + d^2)) - (d^3*(3*a^2*b*(A*c^3 - c^3*C + 3*B*c^2*d -
3*A*c*d^2 + 3*c*C*d^2 - B*d^3) - b^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^
2 + 3*c*C*d^2 - B*d^3) - a^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2))
 + 3*a*b^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]
])/((c^2 + d^2)*f) - ((b*c - a*d)*(b^2*(3*c^6*C - B*c^5*d + 9*c^4*C*d^2 -
3*B*c^3*d^3 - c^2*(A - 10*C)*d^4 - 6*B*c*d^5 + 3*A*d^6) + a^2*d^3*((A - C)
*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + a*b*d^2*(8*c*(A - C)*d^3 - B*(c^4
+ 6*c^2*d^2 - 3*d^4)))*Log[c + d*Tan[e + f*x]])/(d*(c^2 + d^2)*f)))/d + (2
*b^2*(b*(3*c^4*C - B*c^3*d + 6*c^2*C*d^2 - 3*B*c*d^3 + (2*A + C)*d^4) + a*
d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x])/(d*f))/(d*(c^2 + d^2)))
/(2*d*(c^2 + d^2))
```

3.84.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

3.84. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 1271, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	Expression too large to display	1271
default	Expression too large to display	1271
norman	Expression too large to display	2076
parallelrisch	Expression too large to display	6687
risch	Expression too large to display	6825

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(tan(f*x+e)*C*b^3/d^3+1/(c^2+d^2)^3*(1/2*(-3*A*a^3*c^2*d+A*a^3*d^3+3*A
*a^2*b*c^3-9*A*a^2*b*c*d^2+9*A*a*b^2*c^2*d-3*A*a*b^2*d^3-A*b^3*c^3+3*A*b^3
*c*d^2+B*a^3*c^3-3*B*a^3*c*d^2+9*B*a^2*b*c^2*d-3*B*a^2*b*d^3-3*B*a*b^2*c^3
+9*B*a*b^2*c*d^2-3*B*b^3*c^2*d+B*b^3*d^3+3*C*a^3*c^2*d-C*a^3*d^3-3*C*a^2*b
*c^3+9*C*a^2*b*c*d^2-9*C*a*b^2*c^2*d+3*C*a*b^2*d^3+C*b^3*c^3-3*C*b^3*c*d^2
)*ln(1+tan(f*x+e)^2)+(A*a^3*c^3-3*A*a^3*c*d^2+9*A*a^2*b*c^2*d-3*A*a^2*b*d^
3-3*A*a*b^2*c^3+9*A*a*b^2*c*d^2-3*A*b^3*c^2*d+A*b^3*d^3+3*B*a^3*c^2*d-B*a^
3*d^3-3*B*a^2*b*c^3+9*B*a^2*b*c*d^2-9*B*a*b^2*c^2*d+3*B*a*b^2*d^3+B*b^3*c^
3-3*B*b^3*c*d^2-C*a^3*c^3+3*C*a^3*c*d^2-9*C*a^2*b*c^2*d+3*C*a^2*b*d^3+3*C*
a*b^2*c^3-9*C*a*b^2*c*d^2+3*C*b^3*c^2*d-C*b^3*d^3)*arctan(tan(f*x+e)))-1/2
/d^4*(A*a^3*d^5-3*A*a^2*b*c*d^4+3*A*a*b^2*c^2*d^3-A*b^3*c^3*d^2-B*a^3*c*d^
4+3*B*a^2*b*c^2*d^3-3*B*a*b^2*c^3*d^2+B*b^3*c^4*d+C*a^3*c^2*d^3-3*C*a^2*b*
c^3*d^2+3*C*a*b^2*c^4*d-C*b^3*c^5)/(c^2+d^2)/(c+d*tan(f*x+e))^2-1/d^4*(2*A
*a^3*c*d^5-3*A*a^2*b*c^2*d^4+3*A*a^2*b*d^6-6*A*a*b^2*c*d^5+A*b^3*c^4*d^2+3
*A*b^3*c^2*d^4-B*a^3*c^2*d^4+B*a^3*d^6-6*B*a^2*b*c*d^5+3*B*a*b^2*c^4*d^2+9
*B*a*b^2*c^2*d^4-2*B*b^3*c^5*d-4*B*b^3*c^3*d^3-2*C*a^3*c*d^5+3*C*a^2*b*c^4
*d^2+9*C*a^2*b*c^2*d^4-6*C*a*b^2*c^5*d-12*C*a*b^2*c^3*d^3+3*C*b^3*c^6+5*C*
b^3*c^4*d^2)/(c^2+d^2)^2/(c+d*tan(f*x+e))+1/d^4*(3*A*a^3*c^2*d^5-A*a^3*d^7
-3*A*a^2*b*c^3*d^4+9*A*a^2*b*c*d^6-9*A*a*b^2*c^2*d^5+3*A*a*b^2*d^7+A*b^3*c
^3*d^4-3*A*b^3*c*d^6-B*a^3*c^3*d^4+3*B*a^3*c*d^6-9*B*a^2*b*c^2*d^5+3*B*...
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. 2(797) = 1594.

Time = 1.33 (sec) , antiderivative size = 2490, normalized size of antiderivative = 3.10

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
-1/2*(3*C*b^3*c^7*d^2 + A*a^3*d^9 - (3*C*a*b^2 + B*b^3)*c^6*d^3 - (3*C*a^2
*b + 3*B*a*b^2 + (A - 9*C)*b^3)*c^5*d^4 + (3*C*a^3 + 9*B*a^2*b + 3*(3*A -
7*C)*a*b^2 - 7*B*b^3)*c^4*d^5 - 5*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A
*b^3)*c^3*d^6 + ((7*A - 3*C)*a^3 - 9*B*a^2*b - 9*A*a*b^2)*c^2*d^7 + (B*a^3
 + 3*A*a^2*b)*c*d^8 - 2*(C*b^3*c^6*d^3 + 3*C*b^3*c^4*d^5 + 3*C*b^3*c^2*d^7
 + C*b^3*d^9)*tan(f*x + e)^3 - 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b
^2 + B*b^3)*c^5*d^4 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3
)*c^4*d^5 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c^3*d^6
- (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c^2*d^7)*f*x - (9*C*
b^3*c^7*d^2 - A*a^3*d^9 - 3*(3*C*a*b^2 + B*b^3)*c^6*d^3 + (3*C*a^2*b + 3*B
*a*b^2 + (A + 23*C)*b^3)*c^5*d^4 + (C*a^3 + 3*B*a^2*b + 3*(A - 9*C)*a*b^2
- 9*B*b^3)*c^4*d^5 - (3*B*a^3 + 3*(3*A - 7*C)*a^2*b - 21*B*a*b^2 - (7*A +
12*C)*b^3)*c^3*d^6 + 5*((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*c^2*d^7 + (3*
B*a^3 + 9*A*a^2*b + 4*C*b^3)*c*d^8 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A -
C)*a*b^2 + B*b^3)*c^3*d^6 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
C)*b^3)*c^2*d^7 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*
d^8 - (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^9)*f*x)*tan(f*
x + e)^2 + (3*C*b^3*c^9 + 9*C*b^3*c^7*d^2 - (3*C*a*b^2 + B*b^3)*c^8*d - 3*
(3*C*a*b^2 + B*b^3)*c^6*d^3 + (B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A -
10*C)*b^3)*c^5*d^4 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - 2*C)*a*b^2 + 2...
```

3.84.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1110, normalized size of antiderivative = 1.38

$$\int \frac{(a+b\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*
b^2 + B*b^3)*c^3 + 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*c
^2*d - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*c*d^2 - (B*a^
3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^3)*d^3)*(f*x + e)/(c^6 + 3*c^4
*d^2 + 3*c^2*d^4 + d^6) - 2*(3*C*b^3*c^7 + 9*C*b^3*c^5*d^2 - (3*C*a*b^2 +
B*b^3)*c^6*d - 3*(3*C*a*b^2 + B*b^3)*c^4*d^3 + (B*a^3 + 3*(A - C)*a^2*b -
3*B*a*b^2 - (A - 10*C)*b^3)*c^3*d^4 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A -
2*C)*a*b^2 + 2*B*b^3)*c^2*d^5 - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - A
*b^3)*c*d^6 + ((A - C)*a^3 - 3*B*a^2*b - 3*A*a*b^2)*d^7)*log(d*tan(f*x + e
) + c)/(c^6*d^4 + 3*c^4*d^6 + 3*c^2*d^8 + d^10) + ((B*a^3 + 3*(A - C)*a^2*
b - 3*B*a*b^2 - (A - C)*b^3)*c^3 - 3*((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*
a*b^2 + B*b^3)*c^2*d - 3*(B*a^3 + 3*(A - C)*a^2*b - 3*B*a*b^2 - (A - C)*b^
3)*c*d^2 + ((A - C)*a^3 - 3*B*a^2*b - 3*(A - C)*a*b^2 + B*b^3)*d^3)*log(ta
n(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (5*C*b^3*c^7 + A*a
^3*d^7 - 3*(3*C*a*b^2 + B*b^3)*c^6*d + (3*C*a^2*b + 3*B*a*b^2 + (A + 9*C)*
b^3)*c^5*d^2 + (C*a^3 + 3*B*a^2*b + 3*(A - 7*C)*a*b^2 - 7*B*b^3)*c^4*d^3 -
 (3*B*a^3 + 3*(3*A - 5*C)*a^2*b - 15*B*a*b^2 - 5*A*b^3)*c^3*d^4 + ((5*A -
3*C)*a^3 - 9*B*a^2*b - 9*A*a*b^2)*c^2*d^5 + (B*a^3 + 3*A*a^2*b)*c*d^6 + 2*
(3*C*b^3*c^6*d - 2*(3*C*a*b^2 + B*b^3)*c^5*d^2 + (3*C*a^2*b + 3*B*a*b^2 +
(A + 5*C)*b^3)*c^4*d^3 - 4*(3*C*a*b^2 + B*b^3)*c^3*d^4 - (B*a^3 + 3*(A ...
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2441 vs. 2(797) = 1594.

Time = 1.39 (sec) , antiderivative size = 2441, normalized size of antiderivative = 3.04

$$\int \frac{(a+b\tan(e+fx))^3 (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
1/2*(2*C*b^3*tan(f*x + e)/d^3 + 2*(A*a^3*c^3 - C*a^3*c^3 - 3*B*a^2*b*c^3 -
 3*A*a*b^2*c^3 + 3*C*a*b^2*c^3 + B*b^3*c^3 + 3*B*a^3*c^2*d + 9*A*a^2*b*c^2
*d - 9*C*a^2*b*c^2*d - 9*B*a*b^2*c^2*d - 3*A*b^3*c^2*d + 3*C*b^3*c^2*d - 3
*A*a^3*c*d^2 + 3*C*a^3*c*d^2 + 9*B*a^2*b*c*d^2 + 9*A*a*b^2*c*d^2 - 9*C*a*b
^2*c*d^2 - 3*B*b^3*c*d^2 - B*a^3*d^3 - 3*A*a^2*b*d^3 + 3*C*a^2*b*d^3 + 3*B
*a*b^2*d^3 + A*b^3*d^3 - C*b^3*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4
 + d^6) + (B*a^3*c^3 + 3*A*a^2*b*c^3 - 3*C*a^2*b*c^3 - 3*B*a*b^2*c^3 - A*b
^3*c^3 + C*b^3*c^3 - 3*A*a^3*c^2*d + 3*C*a^3*c^2*d + 9*B*a^2*b*c^2*d + 9*A
*a*b^2*c^2*d - 9*C*a*b^2*c^2*d - 3*B*b^3*c^2*d - 3*B*a^3*c*d^2 - 9*A*a^2*b
*c*d^2 + 9*C*a^2*b*c*d^2 + 9*B*a*b^2*c*d^2 + 3*A*b^3*c*d^2 - 3*C*b^3*c*d^2
 + A*a^3*d^3 - C*a^3*d^3 - 3*B*a^2*b*d^3 - 3*A*a*b^2*d^3 + 3*C*a*b^2*d^3 +
 B*b^3*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) -
2*(3*C*b^3*c^7 - 3*C*a*b^2*c^6*d - B*b^3*c^6*d + 9*C*b^3*c^5*d^2 - 9*C*a*b
^2*c^4*d^3 - 3*B*b^3*c^4*d^3 + B*a^3*c^3*d^4 + 3*A*a^2*b*c^3*d^4 - 3*C*a^2
*b*c^3*d^4 - 3*B*a*b^2*c^3*d^4 - A*b^3*c^3*d^4 + 10*C*b^3*c^3*d^4 - 3*A*a^
3*c^2*d^5 + 3*C*a^3*c^2*d^5 + 9*B*a^2*b*c^2*d^5 + 9*A*a*b^2*c^2*d^5 - 18*C
*a*b^2*c^2*d^5 - 6*B*b^3*c^2*d^5 - 3*B*a^3*c*d^6 - 9*A*a^2*b*c*d^6 + 9*C*a
^2*b*c*d^6 + 9*B*a*b^2*c*d^6 + 3*A*b^3*c*d^6 + A*a^3*d^7 - C*a^3*d^7 - 3*B
*a^2*b*d^7 - 3*A*a*b^2*d^7)*log(abs(d*tan(f*x + e) + c))/(c^6*d^4 + 3*c^4*
d^6 + 3*c^2*d^8 + d^10) + (9*C*b^3*c^7*d^2*tan(f*x + e)^2 - 9*C*a*b^2*c...
```

3.84.9 Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.46

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$= \frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(d^3\,\left(3\,B\,b^3\,c^4+9\,C\,a\,b^2\,c^4\right)-d^6\,\left(3\,A\,b^3\,c-3\,B\,a^3\,c-9\,A\,a^2\,b\,c+9\,B\,a\,b^2\,c+9\,C\,a\right.\right.}{}$$
$$+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,a^3+A\,b^3\,1\mathrm{i}-B\,a^3\,1\mathrm{i}+B\,b^3-C\,a^3-C\,b^3\,1\mathrm{i}-3\,A\,a\,b^2-A\,a^2\,b\,3\mathrm{i}+B\,a\,b^2\,3\mathrm{i}\right.}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$
$$-\frac{\frac{A\,a^3\,d^7+5\,C\,b^3\,c^7+B\,a^3\,c\,d^6-3\,B\,b^3\,c^6\,d+5\,A\,a^3\,c^2\,d^5+5\,A\,b^3\,c^3\,d^4+A\,b^3\,c^5\,d^2-3\,B\,a^3\,c^3\,d^4-7\,B\,b^3\,c^4\,d^3-3\,C\,a^3\,c^2\,d^5+C\,a^3\,c^4\,d^3+9\,C\,b^3\,c^5}{}}{}$$
$$+\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,b^3-B\,a^3-C\,b^3-3\,A\,a^2\,b+3\,B\,a\,b^2+3\,C\,a^2\,b+A\,a^3\,1\mathrm{i}+B\,b^3\,1\mathrm{i}-C\,a^3\,1\mathrm{i}-\right.}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}$$
$$+\frac{C\,b^3\tan(e+f\,x)}{d^3\,f}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^3,x)
```

output
```
(log(tan(e + f*x) + 1i)*(A*a^3 + A*b^3*1i - B*a^3*1i + B*b^3 - C*a^3 - C*b
^3*1i - 3*A*a*b^2 - A*a^2*b*3i + B*a*b^2*3i - 3*B*a^2*b + 3*C*a*b^2 + C*a^
2*b*3i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i + d^3)) - ((A*a^3*d^7 + 5*C*b^3
*c^7 + B*a^3*c*d^6 - 3*B*b^3*c^6*d + 5*A*a^3*c^2*d^5 + 5*A*b^3*c^3*d^4 + A
*b^3*c^5*d^2 - 3*B*a^3*c^3*d^4 - 7*B*b^3*c^4*d^3 - 3*C*a^3*c^2*d^5 + C*a^3
*c^4*d^3 + 9*C*b^3*c^5*d^2 - 9*A*a*b^2*c^2*d^5 + 3*A*a*b^2*c^4*d^3 - 9*A*a
^2*b*c^3*d^4 + 15*B*a*b^2*c^3*d^4 + 3*B*a*b^2*c^5*d^2 - 9*B*a^2*b*c^2*d^5
+ 3*B*a^2*b*c^4*d^3 - 21*C*a*b^2*c^4*d^3 + 15*C*a^2*b*c^3*d^4 + 3*C*a^2*b*
c^5*d^2 + 3*A*a^2*b*c*d^6 - 9*C*a*b^2*c^6*d)/(2*d*(c^4 + d^4 + 2*c^2*d^2))
 + (tan(e + f*x)*(B*a^3*d^6 + 3*C*b^3*c^6 + 3*A*a^2*b*d^6 + 2*A*a^3*c*d^5
- 2*B*b^3*c^5*d - 2*C*a^3*c*d^5 + 3*A*b^3*c^2*d^4 + A*b^3*c^4*d^2 - B*a^3*
c^2*d^4 - 4*B*b^3*c^3*d^3 + 5*C*b^3*c^4*d^2 - 3*A*a^2*b*c^2*d^4 + 9*B*a*b^
2*c^2*d^4 + 3*B*a*b^2*c^4*d^2 - 12*C*a*b^2*c^3*d^3 + 9*C*a^2*b*c^2*d^4 + 3
*C*a^2*b*c^4*d^2 - 6*A*a*b^2*c*d^5 - 6*B*a^2*b*c*d^5 - 6*C*a*b^2*c^5*d))/(
c^4 + d^4 + 2*c^2*d^2))/(f*(c^2*d^3 + d^5*tan(e + f*x)^2 + 2*c*d^4*tan(e +
 f*x))) + (log(c + d*tan(e + f*x))*(d^3*(3*B*b^3*c^4 + 9*C*a*b^2*c^4) - d^
6*(3*A*b^3*c - 3*B*a^3*c - 9*A*a^2*b*c + 9*B*a*b^2*c + 9*C*a^2*b*c) + d^5*
(3*A*a^3*c^2 + 6*B*b^3*c^2 - 3*C*a^3*c^2 - 9*A*a*b^2*c^2 - 9*B*a^2*b*c^2 +
 18*C*a*b^2*c^2) + d^4*(A*b^3*c^3 - B*a^3*c^3 - 10*C*b^3*c^3 - 3*A*a^2*b*c
^3 + 3*B*a*b^2*c^3 + 3*C*a^2*b*c^3) + d^7*(C*a^3 - A*a^3 + 3*A*a*b^2 + ...
```

3.84. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.85 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.85.1 Optimal result

Integrand size = 45, antiderivative size = 597

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))}{(c^2+d^2)^3}$$

$$-\frac{(2ab(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)-a^2((A-C)d(3c^2-d^2)-B(c^3-3cd^2))+b^2((A}{(c^2+d^2)^3\,f}$$

$$-\frac{(2abd^3(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)-b^2(c^6C+3c^4Cd^2+Bc^3d^3-3c^2(A-2C)d^4-}{d^3\,(c^2+d^2)^3\,f}$$

$$-\frac{(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^2}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}$$

$$+\frac{(bc-ad)\,(b(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))}{d^3\,(c^2+d^2)^2\,f(c+d\tan(e+fx))}$$

output
```
-(b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C-3*B*c^2
*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))-2*a*b*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*
d^2)))*x/(c^2+d^2)^3-(2*a*b*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d
^2)-a^2*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))+b^2*((A-C)*d*(3*c^2-d^2)-B*(
c^3-3*c*d^2)))*ln(cos(f*x+e))/(c^2+d^2)^3/f-(2*a*b*d^3*(A*c^3-3*A*c*d^2+3*
B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)-b^2*(c^6*C+3*c^4*C*d^2+B*c^3*d^3-3*c^2*(A-2
*C)*d^4-3*B*c*d^5+A*d^6)-a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2)))*ln
(c+d*tan(f*x+e))/d^3/(c^2+d^2)^3/f-1/2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e)
)^2/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-a*d+b*c)*(b*(c^4*C-c^2*(A-3*C)*d^2-
2*B*c*d^3+A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^3/(c^2+d^2)^2/f/(c+d*t
an(f*x+e))
```

3.85.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.98 (sec) , antiderivative size = 1044, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{(-2aAbc^3-a^2Bc^3+b^2Bc^3+2abc^3C+3a^2Ac^2d-3Ab^2c^2d-6abBc^2d-3a^2c^2Cd+3b^2c^2Cd+6aAb\cdots}{\cdots}$$

$$+\frac{(2aAbc^3+a^2Bc^3-b^2Bc^3-2abc^3C-3a^2Ac^2d+3Ab^2c^2d+6abBc^2d+3a^2c^2Cd-3b^2c^2Cd-6aAbcd\cdots}{\cdots}$$

$$-\frac{(2abd^3(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)-b^2(c^6C+3c^4Cd^2+Bc^3d^3-3c^2(A-2C)d^4-\cdots}{d^3\,(c^2+d^2)^3\,f}$$

$$-\frac{(bc-ad)^2\,(c^2C-Bcd+Ad^2)}{2d^3\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}$$

$$+\frac{(bc-ad)\,(b(2c^4C-Bc^3d+4c^2Cd^2-3Bcd^3+2Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))}{d^3\,(c^2+d^2)^2\,f(c+d\tan(e+fx))}$$

input
```
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^3,x]
```

output
```
-1/2*((-2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 + 2*a*b*c^3*C + 3*a^2*A*c^2*d
- 3*A*b^2*c^2*d - 6*a*b*B*c^2*d - 3*a^2*c^2*C*d + 3*b^2*c^2*C*d + 6*a*A*b*
c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 - 6*a*b*c*C*d^2 - a^2*A*d^3 + A*b^2*
d^3 + 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3 + I*(a^2*A*c^3 - A*b^2*c^3 - 2*a
*b*B*c^3 - a^2*c^3*C + b^2*c^3*C + 6*a*A*b*c^2*d + 3*a^2*B*c^2*d - 3*b^2*B
*c^2*d - 6*a*b*c^2*C*d - 3*a^2*A*c*d^2 + 3*A*b^2*c*d^2 + 6*a*b*B*c*d^2 + 3
*a^2*c*C*d^2 - 3*b^2*c*C*d^2 - 2*a*A*b*d^3 - a^2*B*d^3 + b^2*B*d^3 + 2*a*b
*C*d^3))*Log[I - Tan[e + f*x]])/((c^2 + d^2)^3*f) + ((2*a*A*b*c^3 + a^2*B*
c^3 - b^2*B*c^3 - 2*a*b*c^3*C - 3*a^2*A*c^2*d + 3*A*b^2*c^2*d + 6*a*b*B*c^
2*d + 3*a^2*c^2*C*d - 3*b^2*c^2*C*d - 6*a*A*b*c*d^2 - 3*a^2*B*c*d^2 + 3*b^
2*B*c*d^2 + 6*a*b*c*C*d^2 + a^2*A*d^3 - A*b^2*d^3 - 2*a*b*B*d^3 - a^2*C*d^
3 + b^2*C*d^3 + I*(a^2*A*c^3 - A*b^2*c^3 - 2*a*b*B*c^3 - a^2*c^3*C + b^2*c
^3*C + 6*a*A*b*c^2*d + 3*a^2*B*c^2*d - 3*b^2*B*c^2*d - 6*a*b*c^2*C*d - 3*a
^2*A*c*d^2 + 3*A*b^2*c*d^2 + 6*a*b*B*c*d^2 + 3*a^2*c*C*d^2 - 3*b^2*c*C*d^2
 - 2*a*A*b*d^3 - a^2*B*d^3 + b^2*B*d^3 + 2*a*b*C*d^3))*Log[I + Tan[e + f*x
]])/(2*(c^2 + d^2)^3*f) - ((2*a*b*d^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d
^2 + 3*c*C*d^2 - B*d^3) - b^2*(c^6*C + 3*c^4*C*d^2 + B*c^3*d^3 - 3*c^2*(A
- 2*C)*d^4 - 3*B*c*d^5 + A*d^6) - a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^
3 - 3*c*d^2)))*Log[c + d*Tan[e + f*x]])/(d^3*(c^2 + d^2)^3*f) - ((b*c - a*
d)^2*(c^2*C - B*c*d + A*d^2))/(2*d^3*(c^2 + d^2)*f*(c + d*Tan[e + f*x])...
```

3.85.3 Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {3042, 4128, 27, 3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{\int \frac{2(a+b\tan(e+fx))\left(bC\left(c^2+d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^2}dx}{2d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+b\tan(e+fx))\left(bC\left(c^2+d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^2}dx}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(e+fx))\left(bC\left(c^2+d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^2}dx}{d\left(c^2+d^2\right)}-\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\Big\downarrow$ 4118

$$\frac{\frac{\int \frac{C\left(c^2+d^2\right)^2\tan^2(e+fx)b^2+\left(Cc^4-(A-3C)d^2c^2-2Bd^3c+Ad^4\right)b^2+2ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)b-a^2d^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-d^2\left(\left(2c(A-C)d-B\right.\right.\cdots}{c+d\tan(e+fx)}\cdots}{d\left(c^2+d^2\right)}\cdots}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{C\left(c^2+d^2\right)^2\tan(e+fx)^2b^2+\left(Cc^4-(A-3C)d^2c^2-2Bd^3c+Ad^4\right)b^2+2ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)b-a^2d^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-d^2\left(\left(2c(A-C)d-B\right.\right.\cdots}{c+d\tan(e+fx)}\cdots}{d\left(c^2+d^2\right)}\cdots}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\Big\downarrow$ 4109

$$\frac{\frac{d^2\left(-\left(a^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)+2ab\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)+b^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)\int\tan(e+fx)dx}{c^2+d^2}-\frac{\left(-a^2d^3\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

3.85. $\displaystyle\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx$

$\downarrow$ 3042

$$\frac{d^2\left(-\left(a^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)+2ab\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)+b^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)\int\tan(e+fx)dx}{c^2+d^2}-\frac{\left(-a^2d^3\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\downarrow$ 3956

$$-\frac{\left(-a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)+2abd^3\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)-b^2\left(-3c^2d^4(A-2C)+Ad^6+Bc^3d^3-3Bcd^5+c^6C+3c^4Cd^2\right)\right)\int\frac{\mathrm{ta}}{\mathrm{c}}}{c^2+d^2}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\downarrow$ 4100

$$-\frac{\left(-a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)+2abd^3\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)-b^2\left(-3c^2d^4(A-2C)+Ad^6+Bc^3d^3-3Bcd^5+c^6C+3c^4Cd^2\right)\right)\int\frac{}{\mathrm{c}}}{df\left(c^2+d^2\right)}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$\downarrow$ 16

$$-\frac{d^2\log(\cos(e+fx))\left(-\left(a^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)+2ab\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)+b^2\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\right)}{f\left(c^2+d^2\right)}-\frac{d^2x\left(a\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^3,x]
```

3.85. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

output
```
-1/2*((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])^2) + ((-((d^2*(b^2*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^
2 + 3*c*C*d^2 - B*d^3) + a^2*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c
^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/(
c^2 + d^2)) - (d^2*(2*a*b*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d
^2 - B*d^3) - a^2*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) + b^2*((A
- C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[Cos[e + f*x]])/((c^2 + d^2)
*f) - ((2*a*b*d^3*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d
^3) - b^2*(c^6*C + 3*c^4*C*d^2 + B*c^3*d^3 - 3*c^2*(A - 2*C)*d^4 - 3*B*c*d
^5 + A*d^6) - a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*Log[c
 + d*Tan[e + f*x]])/(d*(c^2 + d^2)*f))/(d*(c^2 + d^2)) + ((b*c - a*d)*(b*(
c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*
(c^2 - d^2))))/(d^2*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/(d*(c^2 + d^2))
```

3.85.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

3.85. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.85. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.85.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\frac{\left(-3A\,a^2c^2d+A\,a^2d^3+2Aab\,c^3-6Aabc\,d^2+3A\,b^2c^2d-A\,b^2d^3+B\,a^2c^3-3B\,a^2c\,d^2+6Bab\,c^2d-2Bab\,d^3-B\,b^2c^3+3B\,b^2c\,d^2+3C\,a^2c^2\right.}{2}}{}$
default	$\frac{\frac{\left(-3A\,a^2c^2d+A\,a^2d^3+2Aab\,c^3-6Aabc\,d^2+3A\,b^2c^2d-A\,b^2d^3+B\,a^2c^3-3B\,a^2c\,d^2+6Bab\,c^2d-2Bab\,d^3-B\,b^2c^3+3B\,b^2c\,d^2+3C\,a^2c^2\right.}{2}}{}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/(c^2+d^2)^3*(1/2*(-3*A*a^2*c^2*d+A*a^2*d^3+2*A*a*b*c^3-6*A*a*b*c*d^
2+3*A*b^2*c^2*d-A*b^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2+6*B*a*b*c^2*d-2*B*a*b*d^
3-B*b^2*c^3+3*B*b^2*c*d^2+3*C*a^2*c^2*d-C*a^2*d^3-2*C*a*b*c^3+6*C*a*b*c*d^
2-3*C*b^2*c^2*d+C*b^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-3*A*a^2*c*d^2+6*A
*a*b*c^2*d-2*A*a*b*d^3-A*b^2*c^3+3*A*b^2*c*d^2+3*B*a^2*c^2*d-B*a^2*d^3-2*B
*a*b*c^3+6*B*a*b*c*d^2-3*B*b^2*c^2*d+B*b^2*d^3-C*a^2*c^3+3*C*a^2*c*d^2-6*C
*a*b*c^2*d+2*C*a*b*d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(tan(f*x+e)))-1/2*(A
*a^2*d^4-2*A*a*b*c*d^3+A*b^2*c^2*d^2-B*a^2*c*d^3+2*B*a*b*c^2*d^2-B*b^2*c^3
*d+C*a^2*c^2*d^2-2*C*a*b*c^3*d+C*b^2*c^4)/d^3/(c^2+d^2)/(c+d*tan(f*x+e))^2
-(2*A*a^2*c*d^4-2*A*a*b*c^2*d^3+2*A*a*b*d^5-2*A*b^2*c*d^4-B*a^2*c^2*d^3+B*
a^2*d^5-4*B*a*b*c*d^4+B*b^2*c^4*d+3*B*b^2*c^2*d^3-2*C*a^2*c*d^4+2*C*a*b*c^
4*d+6*C*a*b*c^2*d^3-2*C*b^2*c^5-4*C*b^2*c^3*d^2)/d^3/(c^2+d^2)^2/(c+d*tan(
f*x+e))+(3*A*a^2*c^2*d^4-A*a^2*d^6-2*A*a*b*c^3*d^3+6*A*a*b*c*d^5-3*A*b^2*c
^2*d^4+A*b^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5-6*B*a*b*c^2*d^4+2*B*a*b*d^6+B
*b^2*c^3*d^3-3*B*b^2*c*d^5-3*C*a^2*c^2*d^4+C*a^2*d^6+2*C*a*b*c^3*d^3-6*C*a
*b*c*d^5+C*b^2*c^6+3*C*b^2*c^4*d^2+6*C*b^2*c^2*d^4)/(c^2+d^2)^3/d^3*ln(c+d
*tan(f*x+e)))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(591) = 1182$.

Time = 0.65 (sec) , antiderivative size = 1618, normalized size of antiderivative = 2.71

$$\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
1/2*(C*b^2*c^6*d^2 - A*a^2*d^8 + (2*C*a*b + B*b^2)*c^5*d^3 - (3*C*a^2 + 6*
B*a*b + (3*A - 7*C)*b^2)*c^4*d^4 + 5*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d
^5 - ((7*A - 3*C)*a^2 - 6*B*a*b - 3*A*b^2)*c^2*d^6 - (B*a^2 + 2*A*a*b)*c*d
^7 + 2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^5*d^3 + 3*(B*a^2 + 2*(A -
C)*a*b - B*b^2)*c^4*d^4 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3*d^5
- (B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6)*f*x - (3*C*b^2*c^6*d^2 + A*a^2*
d^8 - (2*C*a*b + B*b^2)*c^5*d^3 - (C*a^2 + 2*B*a*b + (A - 9*C)*b^2)*c^4*d^
4 + (3*B*a^2 + 2*(3*A - 7*C)*a*b - 7*B*b^2)*c^3*d^5 - 5*((A - C)*a^2 - 2*B
*a*b - A*b^2)*c^2*d^6 - 3*(B*a^2 + 2*A*a*b)*c*d^7 - 2*(((A - C)*a^2 - 2*B*
a*b - (A - C)*b^2)*c^3*d^5 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6 - 3
*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^7 - (B*a^2 + 2*(A - C)*a*b - B*
b^2)*d^8)*f*x)*tan(f*x + e)^2 + (C*b^2*c^8 + 3*C*b^2*c^6*d^2 - (B*a^2 + 2*
(A - C)*a*b - B*b^2)*c^5*d^3 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c
^4*d^4 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*d^5 - ((A - C)*a^2 - 2*B*a*
b - A*b^2)*c^2*d^6 + (C*b^2*c^6*d^2 + 3*C*b^2*c^4*d^4 - (B*a^2 + 2*(A - C)
*a*b - B*b^2)*c^3*d^5 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^6
+ 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c*d^7 - ((A - C)*a^2 - 2*B*a*b - A*b^2
)*d^8)*tan(f*x + e)^2 + 2*(C*b^2*c^7*d + 3*C*b^2*c^5*d^3 - (B*a^2 + 2*(A -
 C)*a*b - B*b^2)*c^4*d^4 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^3*d
^5 + 3*(B*a^2 + 2*(A - C)*a*b - B*b^2)*c^2*d^6 - ((A - C)*a^2 - 2*B*a*b...
```

3.85.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 827, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$

$$= \frac{\frac{2\,(((A-C)a^2-2\,Bab-(A-C)b^2)c^3+3\,(Ba^2+2\,(A-C)ab-Bb^2)c^2d-3\,((A-C)a^2-2\,Bab-(A-C)b^2)cd^2-(Ba^2+2\,(A-C)ab-Bb^2)d^3)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 + 2*(A - C)*a
*b - B*b^2)*c^2*d - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2
 + 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^
6) + 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 - (B*a^2 + 2*(A - C)*a*b - B*b^2)*c^3*
d^3 + 3*((A - C)*a^2 - 2*B*a*b - (A - 2*C)*b^2)*c^2*d^4 + 3*(B*a^2 + 2*(A
- C)*a*b - B*b^2)*c*d^5 - ((A - C)*a^2 - 2*B*a*b - A*b^2)*d^6)*log(d*tan(f
*x + e) + c)/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d^9) + ((B*a^2 + 2*(A - C)
*a*b - B*b^2)*c^3 - 3*((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a
^2 + 2*(A - C)*a*b - B*b^2)*c*d^2 + ((A - C)*a^2 - 2*B*a*b - (A - C)*b^2)*
d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (3*C*b^
2*c^6 - A*a^2*d^6 - (2*C*a*b + B*b^2)*c^5*d - (C*a^2 + 2*B*a*b + (A - 7*C)
*b^2)*c^4*d^2 + (3*B*a^2 + 2*(3*A - 5*C)*a*b - 5*B*b^2)*c^3*d^3 - ((5*A -
3*C)*a^2 - 6*B*a*b - 3*A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b)*c*d^5 + 2*(2*C*b
^2*c^5*d + 4*C*b^2*c^3*d^3 - (2*C*a*b + B*b^2)*c^4*d^2 + (B*a^2 + 2*(A - 3
*C)*a*b - 3*B*b^2)*c^2*d^4 - 2*((A - C)*a^2 - 2*B*a*b - A*b^2)*c*d^5 - (B*
a^2 + 2*A*a*b)*d^6)*tan(f*x + e))/(c^6*d^3 + 2*c^4*d^5 + c^2*d^7 + (c^4*d^
5 + 2*c^2*d^7 + d^9)*tan(f*x + e)^2 + 2*(c^5*d^4 + 2*c^3*d^6 + c*d^8)*tan(
f*x + e)))/f
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1663 vs. $2(591) = 1182$.

Time = 1.05 (sec) , antiderivative size = 1663, normalized size of antiderivative = 2.79

$$\int \frac{(a + b\tan(e + fx))^2 (A + B\tan(e + fx) + C\tan^2(e + fx))}{(c + d\tan(e + fx))^3} dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c^3 - C*a^2*c^3 - 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*
a^2*c^2*d + 6*A*a*b*c^2*d - 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2
+ 3*C*a^2*c*d^2 + 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^
3 - 2*A*a*b*d^3 + 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*
c^2*d^4 + d^6) + (B*a^2*c^3 + 2*A*a*b*c^3 - 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*
a^2*c^2*d + 3*C*a^2*c^2*d + 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d
- 3*B*a^2*c*d^2 - 6*A*a*b*c*d^2 + 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^
3 - C*a^2*d^3 - 2*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 +
1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + 2*(C*b^2*c^6 + 3*C*b^2*c^4*d^2 -
B*a^2*c^3*d^3 - 2*A*a*b*c^3*d^3 + 2*C*a*b*c^3*d^3 + B*b^2*c^3*d^3 + 3*A*a^
2*c^2*d^4 - 3*C*a^2*c^2*d^4 - 6*B*a*b*c^2*d^4 - 3*A*b^2*c^2*d^4 + 6*C*b^2*
c^2*d^4 + 3*B*a^2*c*d^5 + 6*A*a*b*c*d^5 - 6*C*a*b*c*d^5 - 3*B*b^2*c*d^5 -
A*a^2*d^6 + C*a^2*d^6 + 2*B*a*b*d^6 + A*b^2*d^6)*log(abs(d*tan(f*x + e) +
c))/(c^6*d^3 + 3*c^4*d^5 + 3*c^2*d^7 + d^9) - (3*C*b^2*c^6*d*tan(f*x + e)^
2 + 9*C*b^2*c^4*d^3*tan(f*x + e)^2 - 3*B*a^2*c^3*d^4*tan(f*x + e)^2 - 6*A*
a*b*c^3*d^4*tan(f*x + e)^2 + 6*C*a*b*c^3*d^4*tan(f*x + e)^2 + 3*B*b^2*c^3*
d^4*tan(f*x + e)^2 + 9*A*a^2*c^2*d^5*tan(f*x + e)^2 - 9*C*a^2*c^2*d^5*tan(
f*x + e)^2 - 18*B*a*b*c^2*d^5*tan(f*x + e)^2 - 9*A*b^2*c^2*d^5*tan(f*x + e
)^2 + 18*C*b^2*c^2*d^5*tan(f*x + e)^2 + 9*B*a^2*c*d^6*tan(f*x + e)^2 + 18*
A*a*b*c*d^6*tan(f*x + e)^2 - 18*C*a*b*c*d^6*tan(f*x + e)^2 - 9*B*b^2*c*...
```

3.85.9 Mupad [B] (verification not implemented)

Time = 27.06 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{A\,a^2\,d^6-3\,C\,b^2\,c^6+B\,a^2\,c\,d^5+B\,b^2\,c^5\,d+5\,A\,a^2\,c^2\,d^4-3\,A\,b^2\,c^2\,d^4+A\,b^2\,c^4\,d^2-3\,B\,a^2\,c^3\,d^3+5\,B\,b^2\,c^3\,d^3-3\,C\,a^2\,c^2\,d^4+C\,a^2\,c^4\,d^2-7\,C\,b^2\,c^4\,d^2}{2\,d^3\,(c^4+2\,c^2\,d^2+d^4)}$$

$$-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(\frac{c^2\,\left(d^4\,\left(3\,A\,b^2-3\,A\,a^2+3\,C\,a^2-6\,C\,b^2+6\,B\,a\,b\right)+3\,C\,b^2\,d^4\right)-d^6\,\left(A\,b^2-A\,a^2+C\,a^2+2\,B\,a\,b\right)+C\,b^2\,d^6-c\,d^5\,(3}{c^6\,d^3+3\,c^4\,d^5+3\,c^2\,d^7+d^9}\right.}{f}$$

$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B\,a^2-B\,b^2+2\,A\,a\,b-2\,C\,a\,b-A\,a^2\,1\mathrm{i}+A\,b^2\,1\mathrm{i}+C\,a^2\,1\mathrm{i}-C\,b^2\,1\mathrm{i}+B\,a\,b\,2\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}$$

$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(A\,b^2-A\,a^2+B\,a^2\,1\mathrm{i}-B\,b^2\,1\mathrm{i}+C\,a^2-C\,b^2+A\,a\,b\,2\mathrm{i}+2\,B\,a\,b-C\,a\,b\,2\mathrm{i}\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^3,x)
```

output
```
- ((A*a^2*d^6 - 3*C*b^2*c^6 + B*a^2*c*d^5 + B*b^2*c^5*d + 5*A*a^2*c^2*d^4
- 3*A*b^2*c^2*d^4 + A*b^2*c^4*d^2 - 3*B*a^2*c^3*d^3 + 5*B*b^2*c^3*d^3 - 3*
C*a^2*c^2*d^4 + C*a^2*c^4*d^2 - 7*C*b^2*c^4*d^2 + 2*A*a*b*c*d^5 + 2*C*a*b*
c^5*d - 6*A*a*b*c^3*d^3 - 6*B*a*b*c^2*d^4 + 2*B*a*b*c^4*d^2 + 10*C*a*b*c^3
*d^3)/(2*d^3*(c^4 + d^4 + 2*c^2*d^2)) + (tan(e + f*x)*(B*a^2*d^5 - 2*C*b^2
*c^5 + 2*A*a*b*d^5 + 2*A*a^2*c*d^4 - 2*A*b^2*c*d^4 + B*b^2*c^4*d - 2*C*a^2
*c*d^4 - B*a^2*c^2*d^3 + 3*B*b^2*c^2*d^3 - 4*C*b^2*c^3*d^2 - 4*B*a*b*c*d^4
 + 2*C*a*b*c^4*d - 2*A*a*b*c^2*d^3 + 6*C*a*b*c^2*d^3))/(d^2*(c^4 + d^4 + 2
*c^2*d^2)))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) - (log(c +
 d*tan(e + f*x))*((c^2*(d^4*(3*A*b^2 - 3*A*a^2 + 3*C*a^2 - 6*C*b^2 + 6*B*a
*b) + 3*C*b^2*d^4) - d^6*(A*b^2 - A*a^2 + C*a^2 + 2*B*a*b) + C*b^2*d^6 - c
*d^5*(3*B*a^2 - 3*B*b^2 + 6*A*a*b - 6*C*a*b) + c^3*d^3*(B*a^2 - B*b^2 + 2*
A*a*b - 2*C*a*b))/(d^9 + 3*c^2*d^7 + 3*c^4*d^5 + c^6*d^3) - (C*b^2)/d^3))/
f - (log(tan(e + f*x) - 1i)*(A*b^2*1i - A*a^2*1i + B*a^2 - B*b^2 + C*a^2*1
i - C*b^2*1i + 2*A*a*b + B*a*b*2i - 2*C*a*b))/(2*f*(3*c*d^2 - c^2*d*3i - c
^3 + d^3*1i)) - (log(tan(e + f*x) + 1i)*(A*b^2 - A*a^2 + B*a^2*1i - B*b^2*
1i + C*a^2 - C*b^2 + A*a*b*2i + 2*B*a*b - C*a*b*2i))/(2*f*(c*d^2*3i - 3*c^
2*d - c^3*1i + d^3))
```

3.86 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

3.86.1 Optimal result

Integrand size = 43, antiderivative size = 352

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx =$$
$$-\frac{(a(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))-b((A-C)d(3c^2-d^2)-B(c^3-3cd^2)))\,x}{(c^2+d^2)^3}$$
$$+\frac{(b(c^3C-3Bc^2d-3cCd^2+Bd^3)-a(Bc^3+3c^2Cd-3Bcd^2-Cd^3)+A(ad(3c^2-d^2)-b(c^3-3cd^2))}{(c^2+d^2)^3 f}$$
$$+\frac{(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{2d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$
$$-\frac{b(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2))}{d^2\left(c^2+d^2\right)^2 f(c+d\tan(e+fx))}$$

output
```
-(a*(c^3*C-3*B*c^2*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))-b*((A-C)*d*(3*c^2-d^
2)-B*(c^3-3*c*d^2)))*x/(c^2+d^2)^3+(b*(-3*B*c^2*d+B*d^3+C*c^3-3*C*c*d^2)-a
*(B*c^3-3*B*c*d^2+3*C*c^2*d-C*d^3)+A*(a*d*(3*c^2-d^2)-b*(c^3-3*c*d^2)))*ln
(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^3/f+1/2*(-a*d+b*c)*(A*d^2-B*c*d+C*c^
2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3
+A*d^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e)
)
```

3.86.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx=-\frac{C(a+b\tan(e+fx))}{df(c+d\tan(e+fx))^2}$$

$$-\frac{\frac{bcC+bBd-aCd}{2df(c+d\tan(e+fx))^2}+\frac{\frac{\left(2c(Ab+aB-bC)d^2+2(bB-a(A-C))d^3\right)\left(-\frac{\log(i-\tan(e+fx))}{2(ic-d)^3}+\frac{\log(i+\tan(e+fx))}{2(ic+d)^3}+\frac{d\left(3c^2-d^2\right)\log(c+d\tan(e+fx))}{\left(c^2+d^2\right)^3}-\frac{}{2\left(c^2+d^2\right.}\right.}{d}}{}}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^3,x]
```

output
```
-((C*(a + b*Tan[e + f*x]))/(d*f*(c + d*Tan[e + f*x])^2)) - ((b*c*C + b*B*d
 - a*C*d)/(2*d*f*(c + d*Tan[e + f*x])^2) + (((2*c*(A*b + a*B - b*C)*d^2 +
2*(b*B - a*(A - C))*d^3)*(-1/2*Log[I - Tan[e + f*x]]/(I*c - d)^3 + Log[I +
 Tan[e + f*x]]/(2*(I*c + d)^3) + (d*(3*c^2 - d^2)*Log[c + d*Tan[e + f*x]])
/(c^2 + d^2)^3 - d/(2*(c^2 + d^2)*(c + d*Tan[e + f*x])^2) - (2*c*d)/((c^2
+ d^2)^2*(c + d*Tan[e + f*x]))))/d - 2*(A*b + a*B - b*C)*d*(((-1/2*I)*Log[
I - Tan[e + f*x]])/(c + I*d)^2 + ((I/2)*Log[I + Tan[e + f*x]])/(c - I*d)^2
 + (2*c*d*Log[c + d*Tan[e + f*x]])/(c^2 + d^2)^2 - d/((c^2 + d^2)*(c + d*T
an[e + f*x]))))/(2*d*f))/d
```

3.86.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3042, 4118, 3042, 4111, 25, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4118}$$

$$\frac{\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{(c+d\tan(e+fx))^2}dx}{d\,(c^2+d^2)}+\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{bC(c^2+d^2)\tan(e+fx)^2+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{(c+d\tan(e+fx))^2}dx}{d\,(c^2+d^2)}+\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{4111}$$

$$\frac{\frac{\int -\frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{ad^2}{}}{d\,(c^2+d^2)}$$

$$\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{ad}{}}{d\,(c^2+d^2)}$$

$$\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{ad}{}}{d\,(c^2+d^2)}$$

$$\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{-\frac{-\frac{d\left(aAd\left(3c^2-d^2\right)-a\left(Bc^3-3Bcd^2+3c^2Cd-Cd^3\right)-Ab\left(c^3-3cd^2\right)+b\left(-3Bc^2d+Bd^3+c^3C-3cCd^2\right)\right)\int\frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{dx\left(a\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-\right.\right.}{}}{c^2+d^2}}{d\,(c^2+d^2)}$$

$$\frac{(bc-ad)\,(Ad^2-Bcd+c^2C)}{2d^2f\,(c^2+d^2)\,(c+d\tan(e+fx))^2}$$

3.86. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{-\frac{d\left(aAd\left(3c^2-d^2\right)-a\left(Bc^3-3Bcd^2+3c^2Cd-Cd^3\right)-Ab\left(c^3-3cd^2\right)+b\left(-3Bc^2d+Bd^3+c^3C-3cCd^2\right)\right)\int\frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{dx\left(a\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-\right.\right.}{}}{c^2+d^2}}{d\left(c^2+d^2\right)}$$

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{2d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{2d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^2}+$$

$$\frac{-\frac{ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)}{df\left(c^2+d^2\right)(c+d\tan(e+fx))}-\frac{-\frac{d\left(aAd\left(3c^2-d^2\right)-a\left(Bc^3-3Bcd^2+3c^2Cd-Cd^3\right)-Ab\left(c^3-3cd^2\right)+b\left(-3Bc^2\right.\right.}{f\left(c^2+d^2\right)}}{}}{d\left(c^2+d^2\right)}$$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*
Tan[e + f*x])^3,x]
```

output
```
((b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(2*d^2*(c^2 + d^2)*f*(c + d*Tan[e +
f*x])^2) + (-((-((d*(b*(A - C)*d*(3*c^2 - d^2) - b*B*(c^3 - 3*c*d^2) + a*(
A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3))*x)/(c^2 + d^2)
) - (d*(a*A*d*(3*c^2 - d^2) - A*b*(c^3 - 3*c*d^2) + b*(c^3*C - 3*B*c^2*d -
 3*c*C*d^2 + B*d^3) - a*(B*c^3 + 3*c^2*C*d - 3*B*c*d^2 - C*d^3))*Log[c*Cos
[e + f*x] + d*Sin[e + f*x]])/((c^2 + d^2)*f))/(c^2 + d^2)) - (b*(c^4*C - c
^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2)))/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/(d*(c^2 + d^2))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.86. $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

3.86.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{\frac{\left(-3Aa\,c^2d+Aa\,d^3+Ab\,c^3-3Abc\,d^2+Ba\,c^3-3Bac\,d^2+3Bb\,c^2d-Bb\,d^3+3Ca\,c^2d-Ca\,d^3-Cb\,c^3+3Cbc\,d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(Aa\right.}{\left(c^2+d^2\right)^3}$
default	$\frac{\frac{\left(-3Aa\,c^2d+Aa\,d^3+Ab\,c^3-3Abc\,d^2+Ba\,c^3-3Bac\,d^2+3Bb\,c^2d-Bb\,d^3+3Ca\,c^2d-Ca\,d^3-Cb\,c^3+3Cbc\,d^2\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(Aa\right.}{\left(c^2+d^2\right)^3}$
norman	$\frac{\frac{\left(Aa\,c^3-3Aac\,d^2+3Ab\,c^2d-Ab\,d^3+3Ba\,c^2d-Ba\,d^3-Bb\,c^3+3Bbc\,d^2-Ca\,c^3+3Cac\,d^2-3Cb\,c^2d+Cb\,d^3\right)c^2x}{\left(c^4+2c^2d^2+d^4\right)\left(c^2+d^2\right)}+\frac{d^2\left(Aa\,c^3-3Aac\,d^2+3\right.}{}}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x,
method=_RETURNVERBOSE)
```

output
```
1/f*(1/(c^2+d^2)^3*(1/2*(-3*A*a*c^2*d+A*a*d^3+A*b*c^3-3*A*b*c*d^2+B*a*c^3-
3*B*a*c*d^2+3*B*b*c^2*d-B*b*d^3+3*C*a*c^2*d-C*a*d^3-C*b*c^3+3*C*b*c*d^2)*l
n(1+tan(f*x+e)^2)+(A*a*c^3-3*A*a*c*d^2+3*A*b*c^2*d-A*b*d^3+3*B*a*c^2*d-B*a
*d^3-B*b*c^3+3*B*b*c*d^2-C*a*c^3+3*C*a*c*d^2-3*C*b*c^2*d+C*b*d^3)*arctan(t
an(f*x+e)))+(3*A*a*c^2*d-A*a*d^3-A*b*c^3+3*A*b*c*d^2-B*a*c^3+3*B*a*c*d^2-3
*B*b*c^2*d+B*b*d^3-3*C*a*c^2*d+C*a*d^3+C*b*c^3-3*C*b*c*d^2)/(c^2+d^2)^3*ln
(c+d*tan(f*x+e))-1/2*(A*a*d^3-A*b*c*d^2-B*a*c*d^2+B*b*c^2*d+C*a*c^2*d-C*b*
c^3)/d^2/(c^2+d^2)/(c+d*tan(f*x+e))^2-(2*A*a*c*d^3-A*b*c^2*d^2+A*b*d^4-B*a
*c^2*d^2+B*a*d^4-2*B*b*c*d^3-2*C*a*c*d^3+C*b*c^4+3*C*b*c^2*d^2)/(c^2+d^2)^
2/d^2/(c+d*tan(f*x+e)))
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(350) = 700.

Time = 0.29 (sec) , antiderivative size = 897, normalized size of antiderivative = 2.55

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$

$$=\frac{Cbc^5-Aad^5-3\,(Ca+Bb)c^4d+5\,(Ba+(A-C)b)c^3d^2-((7\,A-3\,C)a-3\,Bb)c^2d^3-(Ba+Ab)cd^4+}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^3,x, algorithm="fricas")
```

output
```
1/2*(C*b*c^5 - A*a*d^5 - 3*(C*a + B*b)*c^4*d + 5*(B*a + (A - C)*b)*c^3*d^2
 - ((7*A - 3*C)*a - 3*B*b)*c^2*d^3 - (B*a + A*b)*c*d^4 + 2*(((A - C)*a - B
*b)*c^5 + 3*(B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - (B*a +
 (A - C)*b)*c^2*d^3)*f*x + (C*b*c^5 - A*a*d^5 + (C*a + B*b)*c^4*d - (3*B*a
 + (3*A - 7*C)*b)*c^3*d^2 + 5*((A - C)*a - B*b)*c^2*d^3 + 3*(B*a + A*b)*c*
d^4 + 2*(((A - C)*a - B*b)*c^3*d^2 + 3*(B*a + (A - C)*b)*c^2*d^3 - 3*((A -
 C)*a - B*b)*c*d^4 - (B*a + (A - C)*b)*d^5)*f*x)*tan(f*x + e)^2 - ((B*a +
(A - C)*b)*c^5 - 3*((A - C)*a - B*b)*c^4*d - 3*(B*a + (A - C)*b)*c^3*d^2 +
 ((A - C)*a - B*b)*c^2*d^3 + ((B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)*a - B
*b)*c^2*d^3 - 3*(B*a + (A - C)*b)*c*d^4 + ((A - C)*a - B*b)*d^5)*tan(f*x +
 e)^2 + 2*((B*a + (A - C)*b)*c^4*d - 3*((A - C)*a - B*b)*c^3*d^2 - 3*(B*a
+ (A - C)*b)*c^2*d^3 + ((A - C)*a - B*b)*c*d^4)*tan(f*x + e))*log((d^2*tan
(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) + 2*((C*a +
B*b)*c^5 - (2*B*a + (2*A - 3*C)*b)*c^4*d + 3*((A - C)*a - B*b)*c^3*d^2 + 3
*(B*a + (A - C)*b)*c^2*d^3 - ((3*A - 2*C)*a - 2*B*b)*c*d^4 - (B*a + A*b)*d
^5 + 2*(((A - C)*a - B*b)*c^4*d + 3*(B*a + (A - C)*b)*c^3*d^2 - 3*((A - C)
*a - B*b)*c^2*d^3 - (B*a + (A - C)*b)*c*d^4)*f*x)*tan(f*x + e))/((c^6*d^2
+ 3*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*d^3 + 3
*c^3*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c^2*d^6)
*f)
```

3.86.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: AttributeError}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.54

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$

$$= \frac{\frac{2\,(((A-C)a-Bb)c^3+3\,(Ba+(A-C)b)c^2d-3\,((A-C)a-Bb)cd^2-(Ba+(A-C)b)d^3)\,(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} - \frac{2\,((Ba+(A-C)b)c^3-3\,((A-C)a-Bb)c^2d-3\,(B}{c^6+3\,c^4}}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a - B*b)*c^3 + 3*(B*a + (A - C)*b)*c^2*d - 3*((A - C)*a -
 B*b)*c*d^2 - (B*a + (A - C)*b)*d^3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^
4 + d^6) - 2*((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b)*c^2*d - 3*(B*a +
 (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(d*tan(f*x + e) + c)/(c^6 +
3*c^4*d^2 + 3*c^2*d^4 + d^6) + ((B*a + (A - C)*b)*c^3 - 3*((A - C)*a - B*b
)*c^2*d - 3*(B*a + (A - C)*b)*c*d^2 + ((A - C)*a - B*b)*d^3)*log(tan(f*x +
 e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*b*c^5 + A*a*d^5 + (C*a
 + B*b)*c^4*d - (3*B*a + (3*A - 5*C)*b)*c^3*d^2 + ((5*A - 3*C)*a - 3*B*b)*
c^2*d^3 + (B*a + A*b)*c*d^4 + 2*(C*b*c^4*d - (B*a + (A - 3*C)*b)*c^2*d^3 +
 2*((A - C)*a - B*b)*c*d^4 + (B*a + A*b)*d^5)*tan(f*x + e))/(c^6*d^2 + 2*c
^4*d^4 + c^2*d^6 + (c^4*d^4 + 2*c^2*d^6 + d^8)*tan(f*x + e)^2 + 2*(c^5*d^3
 + 2*c^3*d^5 + c*d^7)*tan(f*x + e)))/f
```

3.86.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. 2(350) = 700.

Time = 0.81 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.86

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$

$$= \frac{\frac{2\,(Aac^3-Cac^3-Bbc^3+3\,Bac^2d+3\,Abc^2d-3\,Cbc^2d-3\,Aacd^2+3\,Cacd^2+3\,Bbcd^2-Bad^3-Abd^3+Cbd^3)\,(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} + \frac{(Bac^3+Abc^3-Cbc^3-3\,Aac^2d}{}}{}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c^3 - C*a*c^3 - B*b*c^3 + 3*B*a*c^2*d + 3*A*b*c^2*d - 3*C*b*c^
2*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 + 3*B*b*c*d^2 - B*a*d^3 - A*b*d^3 + C*b*d^
3)*(f*x + e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*a*c^3 + A*b*c^3 - C*
b*c^3 - 3*A*a*c^2*d + 3*C*a*c^2*d + 3*B*b*c^2*d - 3*B*a*c*d^2 - 3*A*b*c*d^
2 + 3*C*b*c*d^2 + A*a*d^3 - C*a*d^3 - B*b*d^3)*log(tan(f*x + e)^2 + 1)/(c^
6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*a*c^3*d + A*b*c^3*d - C*b*c^3*d -
3*A*a*c^2*d^2 + 3*C*a*c^2*d^2 + 3*B*b*c^2*d^2 - 3*B*a*c*d^3 - 3*A*b*c*d^3
+ 3*C*b*c*d^3 + A*a*d^4 - C*a*d^4 - B*b*d^4)*log(abs(d*tan(f*x + e) + c))/
(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7) + (3*B*a*c^3*d^4*tan(f*x + e)^2 + 3*
A*b*c^3*d^4*tan(f*x + e)^2 - 3*C*b*c^3*d^4*tan(f*x + e)^2 - 9*A*a*c^2*d^5*
tan(f*x + e)^2 + 9*C*a*c^2*d^5*tan(f*x + e)^2 + 9*B*b*c^2*d^5*tan(f*x + e)
^2 - 9*B*a*c*d^6*tan(f*x + e)^2 - 9*A*b*c*d^6*tan(f*x + e)^2 + 9*C*b*c*d^6
*tan(f*x + e)^2 + 3*A*a*d^7*tan(f*x + e)^2 - 3*C*a*d^7*tan(f*x + e)^2 - 3*
B*b*d^7*tan(f*x + e)^2 - 2*C*b*c^6*d*tan(f*x + e) + 8*B*a*c^4*d^3*tan(f*x
+ e) + 8*A*b*c^4*d^3*tan(f*x + e) - 14*C*b*c^4*d^3*tan(f*x + e) - 22*A*a*c
^3*d^4*tan(f*x + e) + 22*C*a*c^3*d^4*tan(f*x + e) + 22*B*b*c^3*d^4*tan(f*x
 + e) - 18*B*a*c^2*d^5*tan(f*x + e) - 18*A*b*c^2*d^5*tan(f*x + e) + 12*C*b
*c^2*d^5*tan(f*x + e) + 2*A*a*c*d^6*tan(f*x + e) - 2*C*a*c*d^6*tan(f*x + e
) - 2*B*b*c*d^6*tan(f*x + e) - 2*B*a*d^7*tan(f*x + e) - 2*A*b*d^7*tan(f*x
+ e) - C*b*c^7 - C*a*c^6*d - B*b*c^6*d + 6*B*a*c^5*d^2 + 6*A*b*c^5*d^2 ...
```

3.86.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx =$$
$$-\frac{\frac{A\,a\,d^5+C\,b\,c^5+A\,b\,c\,d^4+B\,a\,c\,d^4+B\,b\,c^4\,d+C\,a\,c^4\,d+5\,A\,a\,c^2\,d^3-3\,A\,b\,c^3\,d^2-3\,B\,a\,c^3\,d^2-3\,B\,b\,c^2\,d^3-3\,C\,a\,c^2\,d^3+5\,C\,b\,c^3\,d^2}{2\,d^2\,(c^4+2\,c^2\,d^2+d^4)}+\frac{\tan(e+f\,x)\,(}{}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\right.\right.}$$
$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,b+A\,b\,1\mathrm{i}+B\,a\,1\mathrm{i}-A\,a+C\,a-C\,b\,1\mathrm{i}\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(A\,b+B\,a-C\,b-A\,a\,1\mathrm{i}+B\,b\,1\mathrm{i}+C\,a\,1\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(\left(A\,b+B\,a-C\,b\right)\,c^3+\left(3\,B\,b-3\,A\,a+3\,C\,a\right)\,c^2\,d+\left(3\,C\,b-3\,B\,a-3\,A\,b\right)\,c\,d\right.}{f\,\left(c^6+3\,c^4\,d^2+3\,c^2\,d^4+d^6\right)}$$

input
```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x))^3,x)
```

output
```
- ((A*a*d^5 + C*b*c^5 + A*b*c*d^4 + B*a*c*d^4 + B*b*c^4*d + C*a*c^4*d + 5*
A*a*c^2*d^3 - 3*A*b*c^3*d^2 - 3*B*a*c^3*d^2 - 3*B*b*c^2*d^3 - 3*C*a*c^2*d^
3 + 5*C*b*c^3*d^2)/(2*d^2*(c^4 + d^4 + 2*c^2*d^2)) + (tan(e + f*x)*(A*b*d^
4 + B*a*d^4 + C*b*c^4 + 2*A*a*c*d^3 - 2*B*b*c*d^3 - 2*C*a*c*d^3 - A*b*c^2*
d^2 - B*a*c^2*d^2 + 3*C*b*c^2*d^2))/(d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 +
 d^2*tan(e + f*x)^2 + 2*c*d*tan(e + f*x))) - (log(tan(e + f*x) + 1i)*(A*b*
1i - A*a + B*a*1i + B*b + C*a - C*b*1i))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1i
 + d^3)) - (log(tan(e + f*x) - 1i)*(A*b - A*a*1i + B*a + B*b*1i + C*a*1i -
 C*b))/(2*f*(3*c*d^2 - c^2*d*3i - c^3 + d^3*1i)) - (log(c + d*tan(e + f*x)
)*(c^3*(A*b + B*a - C*b) - d^3*(B*b - A*a + C*a) + c^2*d*(3*B*b - 3*A*a +
3*C*a) - c*d^2*(3*A*b + 3*B*a - 3*C*b)))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*c^4
*d^2))
```

3.86. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.87 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

3.87.1 Optimal result

Integrand size = 33, antiderivative size = 209

$$\begin{aligned}
&\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx \\
&= -\frac{(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))\,x}{(c^2+d^2)^3} \\
&\quad+\frac{((A-C)d(3c^2-d^2)-B(c^3-3cd^2))\log(c\cos(e+fx)+d\sin(e+fx))}{(c^2+d^2)^3 f} \\
&\quad-\frac{c^2C-Bcd+Ad^2}{2d\,(c^2+d^2)\,f(c+d\tan(e+fx))^2}-\frac{2c(A-C)d-B(c^2-d^2)}{(c^2+d^2)^2\,f(c+d\tan(e+fx))}
\end{aligned}$$

output
```
-(c^3*C-3*B*c^2*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))*x/(c^2+d^2)^3+((A-C)*d*
(3*c^2-d^2)-B*(c^3-3*c*d^2))*ln(c*cos(f*x+e)+d*sin(f*x+e))/(c^2+d^2)^3/f+1
/2*(-A*d^2+B*c*d-C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-2*c*(A-C)*d+B*(
c^2-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

3.87.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.52 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.25

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{\frac{C}{(c+d\tan(e+fx))^2}+B\left(\frac{i\log(i-\tan(e+fx))}{(c+id)^2}-\frac{i\log(i+\tan(e+fx))}{(c-id)^2}+\frac{2d\left(-2c\log(c+d\tan(e+fx))+\frac{c^2+d^2}{c+d\tan(e+fx)}\right)}{(c^2+d^2)^2}\right)-(Bc+(-\cdots}{2\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x
]
```

output
```
-1/2*(C/(c + d*Tan[e + f*x])^2 + B*((I*Log[I - Tan[e + f*x]])/(c + I*d)^2
- (I*Log[I + Tan[e + f*x]])/(c - I*d)^2 + (2*d*(-2*c*Log[c + d*Tan[e + f*x
]] + (c^2 + d^2)/(c + d*Tan[e + f*x])))/(c^2 + d^2)^2) - (B*c + (-A + C)*d
)*((I*Log[I - Tan[e + f*x]])/(c + I*d)^3 - Log[I + Tan[e + f*x]]/(I*c + d)
^3 + (d*((-6*c^2 + 2*d^2)*Log[c + d*Tan[e + f*x]] + ((c^2 + d^2)*(5*c^2 +
d^2 + 4*c*d*Tan[e + f*x]))/(c + d*Tan[e + f*x])^2))/(c^2 + d^2)^3))/(d*f)
```

3.87.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4111, 3042, 4012, 25, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^2}dx}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^2}dx}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 4012$$

$$\frac{\frac{\int -\frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 4014$$

$$\frac{-\frac{-\frac{\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{x\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)}{c^2+d^2}}{c^2+d^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\int \frac{d-c\tan(e+fx)}{c+d\tan(e+fx)}dx}{c^2+d^2}-\frac{x\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)}{c^2+d^2}}{c^2+d^2}-\frac{2cd(A-C)-B\left(c^2-d^2\right)}{f\left(c^2+d^2\right)(c+d\tan(e+fx))}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

3.87. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$

$$\Big\downarrow \text{4013}$$

$$-\frac{-\frac{2cd(A-C)-B(c^2-d^2)}{f(c^2+d^2)(c+d\tan(e+fx))}-\frac{-\frac{\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\log(c\cos(e+fx)+d\sin(e+fx))}{f\left(c^2+d^2\right)}-\frac{x\left(Ac^3-3Acd^2+3Bc^2d-Bd^3-c^3C+3cCd^2\right)}{c^2+d^2}}{c^2+d^2}}{c^2+d^2}-\frac{Ad^2-Bcd+c^2C}{2df\left(c^2+d^2\right)(c+d\tan(e+fx))^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^3,x]
```

output
```
-1/2*(c^2*C - B*c*d + A*d^2)/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) + (-
((-(((A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3)*x)/(c^2 +
 d^2)) - (((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2))*Log[c*Cos[e + f*x]
 + d*Sin[e + f*x]])/((c^2 + d^2)*f))/(c^2 + d^2)) - (2*c*(A - C)*d - B*(c^
2 - d^2))/((c^2 + d^2)*f*(c + d*Tan[e + f*x])))/(c^2 + d^2)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.87.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{\left(-3A c^2 d+A d^3+B c^3-3Bc d^2+3C c^2 d-C d^3\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(A c^3-3Ac d^2+3B c^2 d-B d^3-c^3 C+3Cc d^2\right)\arctan(\tan(fx+e)}{\left(c^2+d^2\right)^3}}{f}$
default	$\frac{\frac{\frac{\left(-3A c^2 d+A d^3+B c^3-3Bc d^2+3C c^2 d-C d^3\right)\ln\left(1+\tan(fx+e)^2\right)}{2}+\left(A c^3-3Ac d^2+3B c^2 d-B d^3-c^3 C+3Cc d^2\right)\arctan(\tan(fx+e)}{\left(c^2+d^2\right)^3}}{f}$
norman	$\frac{\frac{\left(A c^3-3Ac d^2+3B c^2 d-B d^3-c^3 C+3Cc d^2\right)c^2 x}{\left(c^4+2c^2 d^2+d^4\right)\left(c^2+d^2\right)}+\frac{d^2\left(A c^3-3Ac d^2+3B c^2 d-B d^3-c^3 C+3Cc d^2\right)x\tan(fx+e)^2}{\left(c^4+2c^2 d^2+d^4\right)\left(c^2+d^2\right)}-\frac{5A c^2 d^3+A d^5-3}{2f c}}{(c+d}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x,method=_RETURNVER
BOSE)
```

output
```
1/f*(1/(c^2+d^2)^3*(1/2*(-3*A*c^2*d+A*d^3+B*c^3-3*B*c*d^2+3*C*c^2*d-C*d^3)
*ln(1+tan(f*x+e)^2)+(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)*arct
an(tan(f*x+e)))-1/2*(A*d^2-B*c*d+C*c^2)/(c^2+d^2)/d/(c+d*tan(f*x+e))^2-(2*
A*c*d-B*c^2+B*d^2-2*C*c*d)/(c^2+d^2)^2/(c+d*tan(f*x+e))+(3*A*c^2*d-A*d^3-B
*c^3+3*B*c*d^2-3*C*c^2*d+C*d^3)/(c^2+d^2)^3*ln(c+d*tan(f*x+e)))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(207) = 414$.

Time = 0.30 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.71

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{3\,Cc^4d - 5\,Bc^3d^2 + (7\,A - 3\,C)c^2d^3 + Bcd^4 + Ad^5 - 2\,((A-C)c^5 + 3\,Bc^4d - 3\,(A-C)c^3d^2 - Bc^2d^3}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm=
"fricas")
```

output
```
-1/2*(3*C*c^4*d - 5*B*c^3*d^2 + (7*A - 3*C)*c^2*d^3 + B*c*d^4 + A*d^5 - 2*
((A - C)*c^5 + 3*B*c^4*d - 3*(A - C)*c^3*d^2 - B*c^2*d^3)*f*x - (C*c^4*d -
 3*B*c^3*d^2 + 5*(A - C)*c^2*d^3 + 3*B*c*d^4 - A*d^5 + 2*((A - C)*c^3*d^2
+ 3*B*c^2*d^3 - 3*(A - C)*c*d^4 - B*d^5)*f*x)*tan(f*x + e)^2 + (B*c^5 - 3*
(A - C)*c^4*d - 3*B*c^3*d^2 + (A - C)*c^2*d^3 + (B*c^3*d^2 - 3*(A - C)*c^2
*d^3 - 3*B*c*d^4 + (A - C)*d^5)*tan(f*x + e)^2 + 2*(B*c^4*d - 3*(A - C)*c^
3*d^2 - 3*B*c^2*d^3 + (A - C)*c*d^4)*tan(f*x + e))*log((d^2*tan(f*x + e)^2
 + 2*c*d*tan(f*x + e) + c^2)/(tan(f*x + e)^2 + 1)) - 2*(C*c^5 - 2*B*c^4*d
+ 3*(A - C)*c^3*d^2 + 3*B*c^2*d^3 - (3*A - 2*C)*c*d^4 - B*d^5 + 2*((A - C)
*c^4*d + 3*B*c^3*d^2 - 3*(A - C)*c^2*d^3 - B*c*d^4)*f*x)*tan(f*x + e))/((c
^6*d^2 + 3*c^4*d^4 + 3*c^2*d^6 + d^8)*f*tan(f*x + e)^2 + 2*(c^7*d + 3*c^5*
d^3 + 3*c^3*d^5 + c*d^7)*f*tan(f*x + e) + (c^8 + 3*c^6*d^2 + 3*c^4*d^4 + c
^2*d^6)*f)
```

3.87.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.76

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^3}\,dx$$
$$= \frac{\frac{2\left((A-C)c^3+3\,Bc^2d-3\,(A-C)cd^2-Bd^3\right)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} - \frac{2\left(Bc^3-3\,(A-C)c^2d-3\,Bcd^2+(A-C)d^3\right)\log(d\tan(fx+e)+c)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} + \frac{(Bc^3-3\,(A-C)c^2d-3\,B}{c^6+}}{2\,f}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm=
"maxima")
```

output
```
1/2*(2*((A - C)*c^3 + 3*B*c^2*d - 3*(A - C)*c*d^2 - B*d^3)*(f*x + e)/(c^6
+ 3*c^4*d^2 + 3*c^2*d^4 + d^6) - 2*(B*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 +
(A - C)*d^3)*log(d*tan(f*x + e) + c)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) +
 (B*c^3 - 3*(A - C)*c^2*d - 3*B*c*d^2 + (A - C)*d^3)*log(tan(f*x + e)^2 +
1)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) - (C*c^4 - 3*B*c^3*d + (5*A - 3*C)*
c^2*d^2 + B*c*d^3 + A*d^4 - 2*(B*c^2*d^2 - 2*(A - C)*c*d^3 - B*d^4)*tan(f*
x + e))/(c^6*d + 2*c^4*d^3 + c^2*d^5 + (c^4*d^3 + 2*c^2*d^5 + d^7)*tan(f*x
 + e)^2 + 2*(c^5*d^2 + 2*c^3*d^4 + c*d^6)*tan(f*x + e)))/f
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(207) = 414$.

Time = 0.69 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.54

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^3}\,dx$$
$$= \frac{\frac{2\left(Ac^3-Cc^3+3\,Bc^2d-3\,Acd^2+3\,Ccd^2-Bd^3\right)(fx+e)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} + \frac{\left(Bc^3-3\,Ac^2d+3\,Cc^2d-3\,Bcd^2+Ad^3-Cd^3\right)\log\left(\tan(fx+e)^2+1\right)}{c^6+3\,c^4d^2+3\,c^2d^4+d^6} - \frac{2\,(Bc^3d-3\,Ac^2d}{}}{}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,x, algorithm=
"giac")
```

output
```
1/2*(2*(A*c^3 - C*c^3 + 3*B*c^2*d - 3*A*c*d^2 + 3*C*c*d^2 - B*d^3)*(f*x +
e)/(c^6 + 3*c^4*d^2 + 3*c^2*d^4 + d^6) + (B*c^3 - 3*A*c^2*d + 3*C*c^2*d -
3*B*c*d^2 + A*d^3 - C*d^3)*log(tan(f*x + e)^2 + 1)/(c^6 + 3*c^4*d^2 + 3*c^
2*d^4 + d^6) - 2*(B*c^3*d - 3*A*c^2*d^2 + 3*C*c^2*d^2 - 3*B*c*d^3 + A*d^4
- C*d^4)*log(abs(d*tan(f*x + e) + c))/(c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7
) + (3*B*c^3*d^3*tan(f*x + e)^2 - 9*A*c^2*d^4*tan(f*x + e)^2 + 9*C*c^2*d^4
*tan(f*x + e)^2 - 9*B*c*d^5*tan(f*x + e)^2 + 3*A*d^6*tan(f*x + e)^2 - 3*C*
d^6*tan(f*x + e)^2 + 8*B*c^4*d^2*tan(f*x + e) - 22*A*c^3*d^3*tan(f*x + e)
+ 22*C*c^3*d^3*tan(f*x + e) - 18*B*c^2*d^4*tan(f*x + e) + 2*A*c*d^5*tan(f*
x + e) - 2*C*c*d^5*tan(f*x + e) - 2*B*d^6*tan(f*x + e) - C*c^6 + 6*B*c^5*d
 - 14*A*c^4*d^2 + 11*C*c^4*d^2 - 7*B*c^3*d^3 - 3*A*c^2*d^4 - B*c*d^5 - A*d
^6)/((c^6*d + 3*c^4*d^3 + 3*c^2*d^5 + d^7)*(d*tan(f*x + e) + c)^2))/f
```

3.87.9 Mupad [B] (verification not implemented)

Time = 10.86 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.56

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^3}\,dx$$
$$= -\frac{\frac{\tan(e+f\,x)\,\left(B\,d^3+2\,A\,c\,d^2-B\,c^2\,d-2\,C\,c\,d^2\right)}{c^4+2\,c^2\,d^2+d^4}+\frac{A\,d^4+C\,c^4+5\,A\,c^2\,d^2-3\,C\,c^2\,d^2+B\,c\,d^3-3\,B\,c^3\,d}{2\,d\,\left(c^4+2\,c^2\,d^2+d^4\right)}}{f\,\left(c^2+2\,c\,d\tan\left(e+f\,x\right)+d^2\tan\left(e+f\,x\right)^2\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(B-A\,1\mathrm{i}+C\,1\mathrm{i}\right)}{2\,f\,\left(-c^3-c^2\,d\,3\mathrm{i}+3\,c\,d^2+d^3\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(c+d\tan(e+f\,x)\right)\,\left(B\,c^3+\left(3\,C-3\,A\right)\,c^2\,d-3\,B\,c\,d^2+\left(A-C\right)\,d^3\right)}{f\,\left(c^6+3\,c^4\,d^2+3\,c^2\,d^4+d^6\right)}$$
$$-\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(C-A+B\,1\mathrm{i}\right)}{2\,f\,\left(-c^3\,1\mathrm{i}-3\,c^2\,d+c\,d^2\,3\mathrm{i}+d^3\right)}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^3,x)
```

output
```
- ((tan(e + f*x)*(B*d^3 + 2*A*c*d^2 - B*c^2*d - 2*C*c*d^2))/(c^4 + d^4 + 2
*c^2*d^2) + (A*d^4 + C*c^4 + 5*A*c^2*d^2 - 3*C*c^2*d^2 + B*c*d^3 - 3*B*c^3
*d)/(2*d*(c^4 + d^4 + 2*c^2*d^2)))/(f*(c^2 + d^2*tan(e + f*x)^2 + 2*c*d*ta
n(e + f*x))) - (log(tan(e + f*x) - 1i)*(B - A*1i + C*1i))/(2*f*(3*c*d^2 -
c^2*d*3i - c^3 + d^3*1i)) - (log(c + d*tan(e + f*x))*(B*c^3 + d^3*(A - C)
- c^2*d*(3*A - 3*C) - 3*B*c*d^2))/(f*(c^6 + d^6 + 3*c^2*d^4 + 3*c^4*d^2))
- (log(tan(e + f*x) + 1i)*(B*1i - A + C))/(2*f*(c*d^2*3i - 3*c^2*d - c^3*1
i + d^3))
```

3.88 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

3.88.1 Optimal result

Integrand size = 45, antiderivative size = 487

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx = -\frac{(a(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))+b((A-C)d(3c^2-d^2)-B(c^3-3cd^2)))\,x}{(a^2+b^2)\,(c^2+d^2)^3} + \frac{b^2(Ab^2-a(bB-aC))\log(a\cos(e+fx)+b\sin(e+fx))}{(a^2+b^2)\,(bc-ad)^3f} - \frac{(b^2(c^6C-3Bc^5d+3c^4(2A-C)d^2+Bc^3d^3+3Ac^2d^4+Ad^6)+a^2d^3((A-C)d(3c^2-d^2)-B(c^3-3c}{(bc-ad)^3\,(c^2+d^2)^3} + \frac{c^2C-Bcd+Ad^2}{2(bc-ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))^2} + \frac{b(c^4C-2Bc^3d+c^2(3A-C)d^2+Ad^4)-ad^2(2c(A-C)d-B(c^2-d^2))}{(bc-ad)^2\,(c^2+d^2)^2\,f(c+d\tan(e+fx))}$$

output
```
-(a*(c^3*C-3*B*c^2*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))+b*((A-C)*d*(3*c^2-d^
2)-B*(c^3-3*c*d^2)))*x/(a^2+b^2)/(c^2+d^2)^3+b^2*(A*b^2-a*(B*b-C*a))*ln(a*
cos(f*x+e)+b*sin(f*x+e))/(a^2+b^2)/(-a*d+b*c)^3/f-(b^2*(c^6*C-3*B*c^5*d+3*
c^4*(2*A-C)*d^2+B*c^3*d^3+3*A*c^2*d^4+A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-
B*(c^3-3*c*d^2))-a*b*d^2*(8*c^3*(A-C)*d-B*(3*c^4-6*c^2*d^2-d^4)))*ln(c*cos
(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^3/(c^2+d^2)^3/f+1/2*(A*d^2-B*c*d+C*c^2)/(
-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(b*(c^4*C-2*B*c^3*d+c^2*(3*A-C)*d
^2+A*d^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/(-a*d+b*c)^2/(c^2+d^2)^2/f/(c+d
*tan(f*x+e))
```

3.88.2 Mathematica [A] (verified)

Time = 9.02 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.87

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$$
$$= -\frac{Ad^2-c(-cC+Bd)}{2(-bc+ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^2}$$
$$-\frac{-\frac{-\frac{b(bc-ad)^2\left(Abc^3-aBc^3-bc^3C+3aAc^2d+3bBc^2d-3ac^2Cd-3Abcd^2+3aBcd^2+3bcCd^2-aAd^3-bBd^3+aCd^3-\frac{\sqrt{-b^2}\left(a\left(c^3C-3Bc^2d-3cCd^2+Bd^3-A\left(\right.\right.\right.}{}\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)}}{}}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
 + d*Tan[e + f*x])^3),x]
```

output
```
-1/2*(A*d^2 - c*(-(c*C) + B*d))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^2) - (-((-((b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C + 3*a*A*
c^2*d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3*b*c*C*d^
2 - a*A*d^3 - b*B*d^3 + a*C*d^3 - (Sqrt[-b^2]*(a*(c^3*C - 3*B*c^2*d - 3*c*
C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) + b*((A - C)*d*(3*c^2 - d^2) - B*(c^3 -
 3*c*d^2))))/b)*Log[Sqrt[-b^2] - b*Tan[e + f*x]])/((a^2 + b^2)*(c^2 + d^2)
)) + (2*b^3*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)^2*Log[a + b*Tan[e + f*x]])
/((a^2 + b^2)*(b*c - a*d)) - (b*(b*c - a*d)^2*(A*b*c^3 - a*B*c^3 - b*c^3*C
 + 3*a*A*c^2*d + 3*b*B*c^2*d - 3*a*c^2*C*d - 3*A*b*c*d^2 + 3*a*B*c*d^2 + 3
*b*c*C*d^2 - a*A*d^3 - b*B*d^3 + a*C*d^3 + (Sqrt[-b^2]*(b*(A - C)*d*(3*c^2
 - d^2) - b*B*(c^3 - 3*c*d^2) - a*(A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 +
 3*c*C*d^2 - B*d^3)))/b)*Log[Sqrt[-b^2] + b*Tan[e + f*x]])/((a^2 + b^2)*(c
^2 + d^2)) - (2*b*(b^2*(c^6*C - 3*B*c^5*d + 3*c^4*(2*A - C)*d^2 + B*c^3*d^
3 + 3*A*c^2*d^4 + A*d^6) + a^2*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c
*d^2)) - a*b*d^2*(8*c^3*(A - C)*d - B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log[c +
d*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)))/(b*(-(b*c) + a*d)*(c^2 + d^2)*
f)) - (-2*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)) - c*(2*d*(b*c
- a*d)*(B*c - (A - C)*d) - 2*b*c*(c^2*C - B*c*d + A*d^2)))/((-(b*c) + a*d)
*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/(2*(-(b*c) + a*d)*(c^2 + d^2))
```

3.88.3 Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{\int -\frac{2\left(-b\left(Cc^2-Bdc+Ad^2\right)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)\right)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx}{2\left(c^2+d^2\right)(bc-ad)}+\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan(e+fx)^2-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-$$

$$\frac{\frac{\int \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-b\left(b\left(Cc^4-2Bdc^3+(3A-C)d^2c^2+Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\tan^2(e+fx)+A\left(2abdc^3-b^2\left(c^2+d^2\right)^2-\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))}}{\left(c^2+d^2\right)(bc-ad)}}{\left(c^2+d^2\right)(bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{\frac{\int \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-b\left(b\left(Cc^4-2Bdc^3+(3A-C)d^2c^2+Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\tan(e+fx)^2+A\left(2abdc^3-b^2\left(c^2+d^2\right)^2-\cdots\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))}}{(c^2+d^2)(bc-ad)}\cdots}{\left(c^2+d^2\right)(bc\cdots}$$

$\downarrow$ 4134

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{\left(a^2+b^2\right)(bc-ad)}+\frac{\left(a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-abd^2\left(8c^3d(A-C)-B\left(3c^4-6c^2d^2-d^4\right)\right)+b^2\left(3c^4d^2(2A-C)+\cdots\right.\right.}{\left(c^2+d^2\right)(bc-ad)}\cdots}{(c^2+d^2)(\cdots}$$

$\downarrow$ 3042

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{b-a\tan(e+fx)}{a+b\tan(e+fx)}dx}{\left(a^2+b^2\right)(bc-ad)}+\frac{\left(a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-abd^2\left(8c^3d(A-C)-B\left(3c^4-6c^2d^2-d^4\right)\right)+b^2\left(3c^4d^2(2A-C)+\cdots\right.\right.}{\left(c^2+d^2\right)(bc-ad)}\cdots}{(c^2+d^2)(\cdots}$$

$\downarrow$ 4013

$$\frac{Ad^2-Bcd+c^2C}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}-\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\log(a\cos(e+fx)+b\sin(e+fx))}{f\left(a^2+b^2\right)(bc-ad)}+\frac{x(bc-ad)^2\left(a\left(-A\left(c^3-3cd^2\right)-3Bc^2d+Bd^3+c^3C-3cCd^2\right)+b\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\cdots\right.\right.}{\left(a^2+b^2\right)\left(c^2+d^2\right)}\cdots}{\cdots}$$

input

```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*T
an[e + f*x])^3),x]
```

3.88. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

output
```
(c^2*C - B*c*d + A*d^2)/(2*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^
2) - ((((b*c - a*d)^2*(a*(c^3*C - 3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 -
 3*c*d^2)) + b*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b
^2)*(c^2 + d^2)) - (b^2*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)^2*Log[a*Cos[e
+ f*x] + b*Sin[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f) + ((b^2*(c^6*C - 3*B
*c^5*d + 3*c^4*(2*A - C)*d^2 + B*c^3*d^3 + 3*A*c^2*d^4 + A*d^6) + a^2*d^3*
((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - a*b*d^2*(8*c^3*(A - C)*d -
 B*(3*c^4 - 6*c^2*d^2 - d^4)))*Log[c*Cos[e + f*x] + d*Sin[e + f*x]])/((b*c
 - a*d)*(c^2 + d^2)*f))/((b*c - a*d)*(c^2 + d^2)) - (b*(c^4*C - 2*B*c^3*d
+ c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))/((b*
c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])))/((b*c - a*d)*(c^2 + d^2))
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.88.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-\frac{\left(A\,b^{2}-Bab+C\,a^{2}\right)b^{2}\ln(a+b\tan(fx+e))}{(ad-bc)^{3}\left(a^{2}+b^{2}\right)}+\frac{\frac{\left(-3Aa\,c^{2}d+Aa\,d^{3}-Ab\,c^{3}+3Abc\,d^{2}+Ba\,c^{3}-3Bac\,d^{2}-3Bb\,c^{2}d+Bb\,d^{3}+3Ca\,c^{2}d-Ca\,d^{3}}{2}}{}}{}$
default	$\frac{-\frac{\left(A\,b^{2}-Bab+C\,a^{2}\right)b^{2}\ln(a+b\tan(fx+e))}{(ad-bc)^{3}\left(a^{2}+b^{2}\right)}+\frac{\frac{\left(-3Aa\,c^{2}d+Aa\,d^{3}-Ab\,c^{3}+3Abc\,d^{2}+Ba\,c^{3}-3Bac\,d^{2}-3Bb\,c^{2}d+Bb\,d^{3}+3Ca\,c^{2}d-Ca\,d^{3}}{2}}{}}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^3,x,
method=_RETURNVERBOSE)
```

output
```
1/f*(-(A*b^2-B*a*b+C*a^2)*b^2/(a*d-b*c)^3/(a^2+b^2)*ln(a+b*tan(f*x+e))+1/(
a^2+b^2)/(c^2+d^2)^3*(1/2*(-3*A*a*c^2*d+A*a*d^3-A*b*c^3+3*A*b*c*d^2+B*a*c^
3-3*B*a*c*d^2-3*B*b*c^2*d+B*b*d^3+3*C*a*c^2*d-C*a*d^3+C*b*c^3-3*C*b*c*d^2)
*ln(1+tan(f*x+e)^2)+(A*a*c^3-3*A*a*c*d^2-3*A*b*c^2*d+A*b*d^3+3*B*a*c^2*d-B
*a*d^3+B*b*c^3-3*B*b*c*d^2-C*a*c^3+3*C*a*c*d^2+3*C*b*c^2*d-C*b*d^3)*arctan
(tan(f*x+e)))-(2*A*a*c*d^3-3*A*b*c^2*d^2-A*b*d^4-B*a*c^2*d^2+B*a*d^4+2*B*b
*c^3*d-2*C*a*c*d^3-C*b*c^4+C*b*c^2*d^2)/(a*d-b*c)^2/(c^2+d^2)^2/(c+d*tan(f
*x+e))+(3*A*a^2*c^2*d^4-A*a^2*d^6-8*A*a*b*c^3*d^3+6*A*b^2*c^4*d^2+3*A*b^2*
c^2*d^4+A*b^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5+3*B*a*b*c^4*d^2-6*B*a*b*c^2*
d^4-B*a*b*d^6-3*B*b^2*c^5*d+B*b^2*c^3*d^3-3*C*a^2*c^2*d^4+C*a^2*d^6+8*C*a*
b*c^3*d^3+C*b^2*c^6-3*C*b^2*c^4*d^2)/(a*d-b*c)^3/(c^2+d^2)^3*ln(c+d*tan(f*
x+e))-1/2*(A*d^2-B*c*d+C*c^2)/(a*d-b*c)/(c^2+d^2)/(c+d*tan(f*x+e))^2)
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3496 vs. $2(485) = 970$.

Time = 4.01 (sec) , antiderivative size = 3496, normalized size of antiderivative = 7.18

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^3,x, algorithm="fricas")
```

output
```
1/2*(5*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (8*C*a^3*b + 7*B*a^2*b^2 + 8*C*a*b^3
+ 7*B*b^4)*c^5*d^3 + (3*C*a^4 + 12*B*a^3*b + (9*A + 2*C)*a^2*b^2 + 12*B*a*
b^3 + (9*A - C)*b^4)*c^4*d^4 - (5*B*a^4 + 4*(4*A - C)*a^3*b + 6*B*a^2*b^2
+ 4*(4*A - C)*a*b^3 + B*b^4)*c^3*d^5 + ((7*A - 3*C)*a^4 + (10*A - 3*C)*a^2
*b^2 + 3*A*b^4)*c^2*d^6 + (B*a^4 - 4*A*a^3*b + B*a^2*b^2 - 4*A*a*b^3)*c*d^
7 + (A*a^4 + A*a^2*b^2)*d^8 + 2*(((A - C)*a*b^3 + B*b^4)*c^8 - 3*((A - C)*
a^2*b^2 + (A - C)*b^4)*c^7*d + 3*((A - C)*a^3*b - 2*B*a^2*b^2 + 2*(A - C)*
a*b^3 - B*b^4)*c^6*d^2 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a*b^3 - (A - C)*b^
4)*c^5*d^3 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (A - C)*a*b^3)*c^4
*d^4 + 3*((A - C)*a^4 + (A - C)*a^2*b^2)*c^3*d^5 + (B*a^4 - (A - C)*a^3*b)
*c^2*d^6)*f*x - (3*(C*a^2*b^2 + C*b^4)*c^6*d^2 - (4*C*a^3*b + 5*B*a^2*b^2
+ 4*C*a*b^3 + 5*B*b^4)*c^5*d^3 + (C*a^4 + 8*B*a^3*b + (7*A - 2*C)*a^2*b^2
+ 8*B*a*b^3 + (7*A - 3*C)*b^4)*c^4*d^4 - (3*B*a^4 + 4*(3*A - 2*C)*a^3*b +
2*B*a^2*b^2 + 4*(3*A - 2*C)*a*b^3 - B*b^4)*c^3*d^5 + (5*(A - C)*a^4 - 4*B*
a^3*b + (6*A - 5*C)*a^2*b^2 - 4*B*a*b^3 + A*b^4)*c^2*d^6 + 3*(B*a^4 + B*a^
2*b^2)*c*d^7 - (A*a^4 + A*a^2*b^2)*d^8 - 2*(((A - C)*a*b^3 + B*b^4)*c^6*d^
2 - 3*((A - C)*a^2*b^2 + (A - C)*b^4)*c^5*d^3 + 3*((A - C)*a^3*b - 2*B*a^2
*b^2 + 2*(A - C)*a*b^3 - B*b^4)*c^4*d^4 - ((A - C)*a^4 - 8*B*a^3*b - 8*B*a
*b^3 - (A - C)*b^4)*c^3*d^5 - 3*(B*a^4 + 2*(A - C)*a^3*b + 2*B*a^2*b^2 + (
A - C)*a*b^3)*c^2*d^6 + 3*((A - C)*a^4 + (A - C)*a^2*b^2)*c*d^7 + (B*a^...
```

3.88.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^3}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))**3,x)
```

output
```
Exception raised: NotImplementedError >> no valid subset found
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(485) = 970.

Time = 0.38 (sec) , antiderivative size = 1078, normalized size of antiderivative = 2.21

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a + B*b)*c^3 + 3*(B*a - (A - C)*b)*c^2*d - 3*((A - C)*a +
 B*b)*c*d^2 - (B*a - (A - C)*b)*d^3)*(f*x + e)/((a^2 + b^2)*c^6 + 3*(a^2 +
 b^2)*c^4*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + 2*(C*a^2*b^2 -
B*a*b^3 + A*b^4)*log(b*tan(f*x + e) + a)/((a^2*b^3 + b^5)*c^3 - 3*(a^3*b^2
 + a*b^4)*c^2*d + 3*(a^4*b + a^2*b^3)*c*d^2 - (a^5 + a^3*b^2)*d^3) - 2*(C*
b^2*c^6 - 3*B*b^2*c^5*d + 3*B*a^2*c*d^5 + 3*(B*a*b + (2*A - C)*b^2)*c^4*d^
2 - (B*a^2 + 8*(A - C)*a*b - B*b^2)*c^3*d^3 + 3*((A - C)*a^2 - 2*B*a*b + A
*b^2)*c^2*d^4 - ((A - C)*a^2 + B*a*b - A*b^2)*d^6)*log(d*tan(f*x + e) + c)
/(b^3*c^9 - 3*a*b^2*c^8*d + 3*a^2*b*c*d^8 - a^3*d^9 + 3*(a^2*b + b^3)*c^7*
d^2 - (a^3 + 9*a*b^2)*c^6*d^3 + 3*(3*a^2*b + b^3)*c^5*d^4 - 3*(a^3 + 3*a*b
^2)*c^4*d^5 + (9*a^2*b + b^3)*c^3*d^6 - 3*(a^3 + a*b^2)*c^2*d^7) + ((B*a -
 (A - C)*b)*c^3 - 3*((A - C)*a + B*b)*c^2*d - 3*(B*a - (A - C)*b)*c*d^2 +
((A - C)*a + B*b)*d^3)*log(tan(f*x + e)^2 + 1)/((a^2 + b^2)*c^6 + 3*(a^2 +
 b^2)*c^4*d^2 + 3*(a^2 + b^2)*c^2*d^4 + (a^2 + b^2)*d^6) + (3*C*b*c^5 - A*
a*d^5 - (C*a + 5*B*b)*c^4*d + (3*B*a + (7*A - C)*b)*c^3*d^2 - ((5*A - 3*C)
*a + B*b)*c^2*d^3 - (B*a - 3*A*b)*c*d^4 + 2*(C*b*c^4*d - 2*B*b*c^3*d^2 - 2
*(A - C)*a*c*d^4 + (B*a + (3*A - C)*b)*c^2*d^3 - (B*a - A*b)*d^5)*tan(f*x
+ e))/(b^2*c^8 - 2*a*b*c^7*d - 4*a*b*c^5*d^3 - 2*a*b*c^3*d^5 + a^2*c^2*d^6
 + (a^2 + 2*b^2)*c^6*d^2 + (2*a^2 + b^2)*c^4*d^4 + (b^2*c^6*d^2 - 2*a*b*c^
5*d^3 - 4*a*b*c^3*d^5 - 2*a*b*c*d^7 + a^2*d^8 + (a^2 + 2*b^2)*c^4*d^4 +...
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. 2(485) = 970.

Time = 1.04 (sec) , antiderivative size = 2078, normalized size of antiderivative = 4.27

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a*c^3 - C*a*c^3 + B*b*c^3 + 3*B*a*c^2*d - 3*A*b*c^2*d + 3*C*b*c^
2*d - 3*A*a*c*d^2 + 3*C*a*c*d^2 - 3*B*b*c*d^2 - B*a*d^3 + A*b*d^3 - C*b*d^
3)*(f*x + e)/(a^2*c^6 + b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^
2*d^4 + 3*b^2*c^2*d^4 + a^2*d^6 + b^2*d^6) + (B*a*c^3 - A*b*c^3 + C*b*c^3
- 3*A*a*c^2*d + 3*C*a*c^2*d - 3*B*b*c^2*d - 3*B*a*c*d^2 + 3*A*b*c*d^2 - 3*
C*b*c*d^2 + A*a*d^3 - C*a*d^3 + B*b*d^3)*log(tan(f*x + e)^2 + 1)/(a^2*c^6
+ b^2*c^6 + 3*a^2*c^4*d^2 + 3*b^2*c^4*d^2 + 3*a^2*c^2*d^4 + 3*b^2*c^2*d^4
+ a^2*d^6 + b^2*d^6) + 2*(C*a^2*b^3 - B*a*b^4 + A*b^5)*log(abs(b*tan(f*x +
 e) + a))/(a^2*b^4*c^3 + b^6*c^3 - 3*a^3*b^3*c^2*d - 3*a*b^5*c^2*d + 3*a^4
*b^2*c*d^2 + 3*a^2*b^4*c*d^2 - a^5*b*d^3 - a^3*b^3*d^3) - 2*(C*b^2*c^6*d -
 3*B*b^2*c^5*d^2 + 3*B*a*b*c^4*d^3 + 6*A*b^2*c^4*d^3 - 3*C*b^2*c^4*d^3 - B
*a^2*c^3*d^4 - 8*A*a*b*c^3*d^4 + 8*C*a*b*c^3*d^4 + B*b^2*c^3*d^4 + 3*A*a^2
*c^2*d^5 - 3*C*a^2*c^2*d^5 - 6*B*a*b*c^2*d^5 + 3*A*b^2*c^2*d^5 + 3*B*a^2*c
*d^6 - A*a^2*d^7 + C*a^2*d^7 - B*a*b*d^7 + A*b^2*d^7)*log(abs(d*tan(f*x +
e) + c))/(b^3*c^9*d - 3*a*b^2*c^8*d^2 + 3*a^2*b*c^7*d^3 + 3*b^3*c^7*d^3 -
a^3*c^6*d^4 - 9*a*b^2*c^6*d^4 + 9*a^2*b*c^5*d^5 + 3*b^3*c^5*d^5 - 3*a^3*c^
4*d^6 - 9*a*b^2*c^4*d^6 + 9*a^2*b*c^3*d^7 + b^3*c^3*d^7 - 3*a^3*c^2*d^8 -
3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10) + (3*C*b^2*c^6*d^2*tan(f*x + e
)^2 - 9*B*b^2*c^5*d^3*tan(f*x + e)^2 + 9*B*a*b*c^4*d^4*tan(f*x + e)^2 + 18
*A*b^2*c^4*d^4*tan(f*x + e)^2 - 9*C*b^2*c^4*d^4*tan(f*x + e)^2 - 3*B*a^...
```

3.88.9 Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 65817, normalized size of antiderivative = 135.15

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))^3),x)
```

output
```
(symsum(log(- root(480*a^9*b*c^7*d^11*f^4 + 480*a*b^9*c^11*d^7*f^4 + 360*a
^9*b*c^9*d^9*f^4 + 360*a^9*b*c^5*d^13*f^4 + 360*a*b^9*c^13*d^5*f^4 + 360*a
*b^9*c^9*d^9*f^4 + 144*a^9*b*c^11*d^7*f^4 + 144*a^9*b*c^3*d^15*f^4 + 144*a
*b^9*c^15*d^3*f^4 + 144*a*b^9*c^7*d^11*f^4 + 48*a^7*b^3*c*d^17*f^4 + 48*a^
3*b^7*c^17*d*f^4 + 24*a^9*b*c^13*d^5*f^4 + 24*a^5*b^5*c^17*d*f^4 + 24*a^5*
b^5*c*d^17*f^4 + 24*a*b^9*c^5*d^13*f^4 + 24*a^9*b*c*d^17*f^4 + 24*a*b^9*c^
17*d*f^4 + 3920*a^5*b^5*c^9*d^9*f^4 - 3360*a^6*b^4*c^8*d^10*f^4 - 3360*a^4
*b^6*c^10*d^8*f^4 - 3024*a^6*b^4*c^10*d^8*f^4 + 3024*a^5*b^5*c^11*d^7*f^4
+ 3024*a^5*b^5*c^7*d^11*f^4 - 3024*a^4*b^6*c^8*d^10*f^4 + 2320*a^7*b^3*c^9
*d^9*f^4 + 2320*a^3*b^7*c^9*d^9*f^4 - 2240*a^6*b^4*c^6*d^12*f^4 - 2240*a^4
*b^6*c^12*d^6*f^4 + 2160*a^7*b^3*c^7*d^11*f^4 + 2160*a^3*b^7*c^11*d^7*f^4
- 1624*a^6*b^4*c^12*d^6*f^4 - 1624*a^4*b^6*c^6*d^12*f^4 + 1488*a^7*b^3*c^1
1*d^7*f^4 + 1488*a^3*b^7*c^7*d^11*f^4 + 1344*a^5*b^5*c^13*d^5*f^4 + 1344*a
^5*b^5*c^5*d^13*f^4 - 1320*a^8*b^2*c^8*d^10*f^4 - 1320*a^2*b^8*c^10*d^8*f^
4 + 1200*a^7*b^3*c^5*d^13*f^4 + 1200*a^3*b^7*c^13*d^5*f^4 - 1060*a^8*b^2*c
^6*d^12*f^4 - 1060*a^2*b^8*c^12*d^6*f^4 - 948*a^8*b^2*c^10*d^8*f^4 - 948*a
^2*b^8*c^8*d^10*f^4 - 840*a^6*b^4*c^4*d^14*f^4 - 840*a^4*b^6*c^14*d^4*f^4
+ 528*a^7*b^3*c^13*d^5*f^4 + 528*a^3*b^7*c^5*d^13*f^4 - 480*a^8*b^2*c^4*d^
14*f^4 - 480*a^6*b^4*c^14*d^4*f^4 - 480*a^4*b^6*c^4*d^14*f^4 - 480*a^2*b^8
*c^14*d^4*f^4 - 368*a^8*b^2*c^12*d^6*f^4 + 368*a^7*b^3*c^3*d^15*f^4 + 3...
```

3.88. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}\,dx$

3.89 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

3.89.1 Optimal result

Integrand size = 45, antiderivative size = 861

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx =$$

$$-\frac{(b^2(Ac^3-c^3C+3Bc^2d-3Acd^2+3cCd^2-Bd^3)+a^2(c^3C-3Bc^2d-3cCd^2+Bd^3-A(c^3-3cd^2))}{(a^2+b^2)^2(c^2+d^2)^3}$$

$$+\frac{b^2(4a^3bBd-3a^4Cd+b^4(Bc-3Ad)+2ab^3(Ac-cC+Bd)-a^2b^2(Bc+(5A+C)d))\log(a\cos(e+f}{(a^2+b^2)^2(bc-ad)^4f}$$

$$+\frac{d(b^2(3c^6C-6Bc^5d+c^4(10A-C)d^2-3Bc^3d^3+9Ac^2d^4-Bcd^5+3Ad^6)+a^2d^3((A-C)d(3c^2-d^2)}{(bc-ad)}$$

$$-\frac{d(b^2c(cC-Bd)-2abB(c^2+d^2)+a^2(3c^2C-Bcd+2Cd^2)+A(a^2d^2+b^2(2c^2+3d^2)))}{2(a^2+b^2)(bc-ad)^2(c^2+d^2)f(c+d\tan(e+fx))^2}$$

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$-\frac{d(b^3c(2c^3C-3Bc^2d-Bd^3)+a^2b(3c^4C-3Bc^3d+2c^2Cd^2-Bcd^3+Cd^4)+a^3d^2(2cCd+B(c^2-d^2)}{(a^2+b^2)(bc-ad)^3(c^2+}$$

output
```
-(b^2*(A*c^3-3*A*c*d^2+3*B*c^2*d-B*d^3-C*c^3+3*C*c*d^2)+a^2*(c^3*C-3*B*c^2
*d-3*C*c*d^2+B*d^3-A*(c^3-3*c*d^2))+2*a*b*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*
d^2)))*x/(a^2+b^2)^2/(c^2+d^2)^3+b^2*(4*a^3*b*B*d-3*a^4*C*d+b^4*(-3*A*d+B*
c)+2*a*b^3*(A*c+B*d-C*c)-a^2*b^2*(B*c+(5*A+C)*d))*ln(a*cos(f*x+e)+b*sin(f*
x+e))/(a^2+b^2)^2/(-a*d+b*c)^4/f+d*(b^2*(3*c^6*C-6*B*c^5*d+c^4*(10*A-C)*d^
2-3*B*c^3*d^3+9*A*c^2*d^4-B*c*d^5+3*A*d^6)+a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*
(c^3-3*c*d^2))-2*a*b*d^2*(c*(A-C)*d*(5*c^2+d^2)-B*(2*c^4-3*c^2*d^2-d^4)))*
ln(c*cos(f*x+e)+d*sin(f*x+e))/(-a*d+b*c)^4/(c^2+d^2)^3/f-1/2*d*(b^2*c*(-B*
d+C*c)-2*a*b*B*(c^2+d^2)+a^2*(-B*c*d+3*C*c^2+2*C*d^2)+A*(a^2*d^2+b^2*(2*c^
2+3*d^2)))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^2+(-A*b^2+a
*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^2-d*(
b^3*c*(-3*B*c^2*d-B*d^3+2*C*c^3)+a^2*b*(-3*B*c^3*d-B*c*d^3+3*C*c^4+2*C*c^2
*d^2+C*d^4)+a^3*d^2*(2*C*c*d+B*(c^2-d^2))+a*b^2*(2*c*C*d^3-B*(c^4+c^2*d^2+
2*d^4))-A*(2*a^3*c*d^3+2*a*b^2*c*d^3-2*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+6*c^
2*d^2+3*d^4)))/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

3.89.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1732 vs. $2(861) = 1722$.

Time = 8.63 (sec) , antiderivative size = 1732, normalized size of antiderivative = 2.01

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$$

$$= -\frac{Ab^2 - a(bB - aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$-\frac{-\frac{-c(-3c(Ab^2-a(bB-aC))d+(Ab-aB-bC)d(bc-ad))+d^2(3Ab^2d-aA(bc-ad)-(bB-aC)(bc+2ad))}{2(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))^2} - \frac{-\frac{(bc-ad)^3\left(-b^2\left(2aAbc^3-a^2Bc^3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
(c + d*Tan[e + f*x])^3),x]
```

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x])^2)) - (-1/2*(-(c*(-3*c*(A*b^2 - a*(b*B - a*C))*d + (A
*b - a*B - b*C)*d*(b*c - a*d))) + d^2*(3*A*b^2*d - a*A*(b*c - a*d) - (b*B
- a*C)*(b*c + 2*a*d)))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^
2) - (-((-(((b*c - a*d)^3*(-(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c^3 - 2*
a*b*c^3*C + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c^2*C*d
+ 3*b^2*c^2*C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6*a*b*c*
C*d^2 - a^2*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3)) + Sq
rt[-b^2]*(-(a^2*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*C - b^3*c
^3*C + 6*a*A*b^2*c^2*d - 3*a^2*b*B*c^2*d + 3*b^3*B*c^2*d - 6*a*b^2*c^2*C*d
 + 3*a^2*A*b*c*d^2 - 3*A*b^3*c*d^2 + 6*a*b^2*B*c*d^2 - 3*a^2*b*c*C*d^2 + 3
*b^3*c*C*d^2 - 2*a*A*b^2*d^3 + a^2*b*B*d^3 - b^3*B*d^3 + 2*a*b^2*C*d^3))*L
og[Sqrt[-b^2] - b*Tan[e + f*x]])/(b*(a^2 + b^2)*(c^2 + d^2))) - (2*b^3*(c^
2 + d^2)^2*(4*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c
*C + B*d) - a^2*b^2*(B*c + (5*A + C)*d))*Log[a + b*Tan[e + f*x]])/((a^2 +
b^2)*(b*c - a*d)) + ((b*c - a*d)^3*(b^2*(2*a*A*b*c^3 - a^2*B*c^3 + b^2*B*c
^3 - 2*a*b*c^3*C + 3*a^2*A*c^2*d - 3*A*b^2*c^2*d + 6*a*b*B*c^2*d - 3*a^2*c
^2*C*d + 3*b^2*c^2*C*d - 6*a*A*b*c*d^2 + 3*a^2*B*c*d^2 - 3*b^2*B*c*d^2 + 6
*a*b*c*C*d^2 - a^2*A*d^3 + A*b^2*d^3 - 2*a*b*B*d^3 + a^2*C*d^3 - b^2*C*d^3
) + Sqrt[-b^2]*(-(a^2*A*b*c^3) + A*b^3*c^3 - 2*a*b^2*B*c^3 + a^2*b*c^3*...
```

3.89.3 Rubi [A] (verified)

Time = 5.33 (sec) , antiderivative size = 932, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {3042, 4132, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int \frac{3Adb^2+3\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-aA(bc-ad)-(bB-aC)(bc+2ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{3Adb^2+3(Ab^2-a(bB-aC))d\tan(e+fx)^2-aA(bc-ad)-(bB-aC)(bc+2ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^3}dx}{(a^2+b^2)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\int -\frac{2\left(d^2(Ac-Cc+Bd)a^3-b(2A+C)d\left(c^2+d^2\right)a^2+b^2(Ac-Cc+Bd)\left(c^2+2d^2\right)a-bd\left(Ad^2a^2+\left(3Cc^2-Bdc+2Cd^2\right)a^2-2bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(2c^2+3\right.\right.\right.}{(a+b\tan(e+fx))(c+d\tan(e+fx))^2}}{2(c^2+d^2)(bc-ad)}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{d(a^2Ad^2+a^2(-Bcd+3c^2C+2Cd^2)-2abB(c^2+d^2)+Ab^2(2c^2+3d^2)+b^2c(cC-Bd))}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}-\frac{\int \frac{d^2(Ac-Cc+Bd)a^3-b(2A+C)d\left(c^2+d^2\right)a^2+b^2(Ac-Cc+Bd}{}}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{d(a^2Ad^2+a^2(-Bcd+3c^2C+2Cd^2)-2abB(c^2+d^2)+Ab^2(2c^2+3d^2)+b^2c(cC-Bd))}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}-\frac{\int \frac{d^2(Ac-Cc+Bd)a^3-b(2A+C)d\left(c^2+d^2\right)a^2+b^2(Ac-Cc+Bd}{}}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \; 4132$$

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-$$

$$\frac{\frac{d(Ad^2a^2+(3Cc^2-Bdc+2Cd^2)a^2-2bB(c^2+d^2)a+b^2c(cC-Bd)+Ab^2(2c^2+3d^2))}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}-\frac{\int \frac{d^3\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^4-bd^2\left(3Cc^3-4Bdc^2+Cd^2c\right.}{}}{}}{}$$

$$\Big\downarrow \; 3042$$

3.89. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-$$

$$\frac{d\left(Ad^2a^2+\left(3Cc^2-Bdc+2Cd^2\right)a^2-2bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(2c^2+3d^2\right)\right)}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}-\frac{\int\frac{d^3\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^4-bd^2\left(3Cc^3-4Bdc^2+Cd^2c\right.}{}}{}$$

$\downarrow$ 4134

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-$$

$$\frac{d\left(Ad^2a^2+\left(3Cc^2-Bdc+2Cd^2\right)a^2-2bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(2c^2+3d^2\right)\right)}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}-\frac{\frac{\left(\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)a^2-2b\left((A-C)d\left(3c^2-\right.\right.\right.}{(}}{}$$

$\downarrow$ 3042

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-$$

$$\frac{d\left(Ad^2a^2+\left(3Cc^2-Bdc+2Cd^2\right)a^2-2bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(2c^2+3d^2\right)\right)}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}-\frac{\frac{\left(\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)a^2-2b\left((A-C)d\left(3c^2-\right.\right.\right.}{(}}{}$$

$\downarrow$ 4013

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^2}-$$

$$\frac{d\left(Ad^2a^2+\left(3Cc^2-Bdc+2Cd^2\right)a^2-2bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(2c^2+3d^2\right)\right)}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}-\frac{\frac{\left(\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)a^2-2b\left((A-C)d\left(3c^2-\right.\right.\right.}{(}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d
*Tan[e + f*x])^3),x]
```

3.89. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x])^2)) - ((d*(a^2*A*d^2 + b^2*c*(c*C - B*d) - 2*a*b*B*(c
^2 + d^2) + A*b^2*(2*c^2 + 3*d^2) + a^2*(3*c^2*C - B*c*d + 2*C*d^2)))/(2*(
b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - ((((b*c - a*d)^3*(a^2*(
A*c^3 - c^3*C + 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 - B*d^3) + b^2*(c^3*C -
3*B*c^2*d - 3*c*C*d^2 + B*d^3 - A*(c^3 - 3*c*d^2)) - 2*a*b*((A - C)*d*(3*c
^2 - d^2) - B*(c^3 - 3*c*d^2)))*x)/((a^2 + b^2)*(c^2 + d^2)) + (b^2*(c^2 +
 d^2)^2*(4*a^3*b*B*d - 3*a^4*C*d + b^4*(B*c - 3*A*d) + 2*a*b^3*(A*c - c*C
+ B*d) - a^2*b^2*(B*c + (5*A + C)*d))*Log[a*Cos[e + f*x] + b*Sin[e + f*x]]
)/((a^2 + b^2)*(b*c - a*d)*f) + ((a^2 + b^2)*d*(b^2*(3*c^6*C - 6*B*c^5*d +
 c^4*(10*A - C)*d^2 - 3*B*c^3*d^3 + 9*A*c^2*d^4 - B*c*d^5 + 3*A*d^6) + a^2
*d^3*((A - C)*d*(3*c^2 - d^2) - B*(c^3 - 3*c*d^2)) - 2*a*b*d^2*(c*(A - C)*
d*(5*c^2 + d^2) - B*(2*c^4 - 3*c^2*d^2 - d^4)))*Log[c*Cos[e + f*x] + d*Sin
[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f))/((b*c - a*d)*(c^2 + d^2)) - (d*(b
^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) + a^2*b*(3*c^4*C - 3*B*c^3*d + 2*c^2*C*
d^2 - B*c*d^3 + C*d^4) + a^3*d^2*(2*c*C*d + B*(c^2 - d^2)) + a*b^2*(2*c*C*
d^3 - B*(c^4 + c^2*d^2 + 2*d^4)) - A*(2*a^3*c*d^3 + 2*a*b^2*c*d^3 - 2*a^2*
b*d^2*(2*c^2 + d^2) - b^3*(c^4 + 6*c^2*d^2 + 3*d^4))))/((b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])))/((b*c - a*d)*(c^2 + d^2)))/((a^2 + b^2)*(b*
c - a*d))
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.89. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

3.89.4 Maple [A] (verified)

Time = 5.79 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{b^2\left(5Aa^2b^2d-2Aab^3c+3Ab^4d-4a^3bBd+Ba^2b^2c-2Bab^3d-Bb^4c+3a^4Cd+Ca^2b^2d+2Cab^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^4\left(a^2+b^2\right)^2}+\frac{\left(Ab^2\right.}{(ad-bc)^3\left(a\right.}$
default	$-\frac{b^2\left(5Aa^2b^2d-2Aab^3c+3Ab^4d-4a^3bBd+Ba^2b^2c-2Bab^3d-Bb^4c+3a^4Cd+Ca^2b^2d+2Cab^3c\right)\ln(a+b\tan(fx+e))}{(ad-bc)^4\left(a^2+b^2\right)^2}+\frac{\left(Ab^2\right.}{(ad-bc)^3\left(a\right.}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^3,
x,method=_RETURNVERBOSE)
```

output
```
1/f*(-b^2*(5*A*a^2*b^2*d-2*A*a*b^3*c+3*A*b^4*d-4*B*a^3*b*d+B*a^2*b^2*c-2*B
*a*b^3*d-B*b^4*c+3*C*a^4*d+C*a^2*b^2*d+2*C*a*b^3*c)/(a*d-b*c)^4/(a^2+b^2)^
2*ln(a+b*tan(f*x+e))+(A*b^2-B*a*b+C*a^2)*b^2/(a*d-b*c)^3/(a^2+b^2)/(a+b*ta
n(f*x+e))+1/(a^2+b^2)^2/(c^2+d^2)^3*(1/2*(-3*A*a^2*c^2*d+A*a^2*d^3-2*A*a*b
*c^3+6*A*a*b*c*d^2+3*A*b^2*c^2*d-A*b^2*d^3+B*a^2*c^3-3*B*a^2*c*d^2-6*B*a*b
*c^2*d+2*B*a*b*d^3-B*b^2*c^3+3*B*b^2*c*d^2+3*C*a^2*c^2*d-C*a^2*d^3+2*C*a*b
*c^3-6*C*a*b*c*d^2-3*C*b^2*c^2*d+C*b^2*d^3)*ln(1+tan(f*x+e)^2)+(A*a^2*c^3-
3*A*a^2*c*d^2-6*A*a*b*c^2*d+2*A*a*b*d^3-A*b^2*c^3+3*A*b^2*c*d^2+3*B*a^2*c^
2*d-B*a^2*d^3+2*B*a*b*c^3-6*B*a*b*c*d^2-3*B*b^2*c^2*d+B*b^2*d^3-C*a^2*c^3+
3*C*a^2*c*d^2+6*C*a*b*c^2*d-2*C*a*b*d^3+C*b^2*c^3-3*C*b^2*c*d^2)*arctan(ta
n(f*x+e)))-d*(2*A*a*c*d^3-4*A*b*c^2*d^2-2*A*b*d^4-B*a*c^2*d^2+B*a*d^4+3*B*
b*c^3*d+B*b*c*d^3-2*C*a*c*d^3-2*C*b*c^4)/(a*d-b*c)^3/(c^2+d^2)^2/(c+d*tan(
f*x+e))+d*(3*A*a^2*c^2*d^4-A*a^2*d^6-10*A*a*b*c^3*d^3-2*A*a*b*c*d^5+10*A*b
^2*c^4*d^2+9*A*b^2*c^2*d^4+3*A*b^2*d^6-B*a^2*c^3*d^3+3*B*a^2*c*d^5+4*B*a*b
*c^4*d^2-6*B*a*b*c^2*d^4-2*B*a*b*d^6-6*B*b^2*c^5*d-3*B*b^2*c^3*d^3-B*b^2*c
*d^5-3*C*a^2*c^2*d^4+C*a^2*d^6+10*C*a*b*c^3*d^3+2*C*a*b*c*d^5+3*C*b^2*c^6-
C*b^2*c^4*d^2)/(a*d-b*c)^4/(c^2+d^2)^3*ln(c+d*tan(f*x+e))-1/2*(A*d^2-B*c*d
+C*c^2)*d/(a*d-b*c)^2/(c^2+d^2)/(c+d*tan(f*x+e))^2)
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9567 vs. $2(862) = 1724$.

Time = 12.03 (sec) , antiderivative size = 9567, normalized size of antiderivative = 11.11

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*
x+e))**3,x)
```

output
```
Timed out
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2537 vs. $2(862) = 1724$.

Time = 0.55 (sec) , antiderivative size = 2537, normalized size of antiderivative = 2.95

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c^3 + 3*(B*a^2 - 2*(A - C)*a
*b - B*b^2)*c^2*d - 3*((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*c*d^2 - (B*a^2
 - 2*(A - C)*a*b - B*b^2)*d^3)*(f*x + e)/((a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*
(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d^4 + (a^4
 + 2*a^2*b^2 + b^4)*d^6) - 2*((B*a^2*b^4 - 2*(A - C)*a*b^5 - B*b^6)*c + (3
*C*a^4*b^2 - 4*B*a^3*b^3 + (5*A + C)*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*d)*log
(b*tan(f*x + e) + a)/((a^4*b^4 + 2*a^2*b^6 + b^8)*c^4 - 4*(a^5*b^3 + 2*a^3
*b^5 + a*b^7)*c^3*d + 6*(a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*c^2*d^2 - 4*(a^7*b
 + 2*a^5*b^3 + a^3*b^5)*c*d^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d^4) + 2*(3*C*
b^2*c^6*d - 6*B*b^2*c^5*d^2 + (4*B*a*b + (10*A - C)*b^2)*c^4*d^3 - (B*a^2
+ 10*(A - C)*a*b + 3*B*b^2)*c^3*d^4 + 3*((A - C)*a^2 - 2*B*a*b + 3*A*b^2)*
c^2*d^5 + (3*B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d^6 - ((A - C)*a^2 + 2*B*a*b
 - 3*A*b^2)*d^7)*log(d*tan(f*x + e) + c)/(b^4*c^10 - 4*a*b^3*c^9*d - 4*a^3
*b*c*d^9 + a^4*d^10 + 3*(2*a^2*b^2 + b^4)*c^8*d^2 - 4*(a^3*b + 3*a*b^3)*c^
7*d^3 + (a^4 + 18*a^2*b^2 + 3*b^4)*c^6*d^4 - 12*(a^3*b + a*b^3)*c^5*d^5 +
(3*a^4 + 18*a^2*b^2 + b^4)*c^4*d^6 - 4*(3*a^3*b + a*b^3)*c^3*d^7 + 3*(a^4
+ 2*a^2*b^2)*c^2*d^8) + ((B*a^2 - 2*(A - C)*a*b - B*b^2)*c^3 - 3*((A - C)*
a^2 + 2*B*a*b - (A - C)*b^2)*c^2*d - 3*(B*a^2 - 2*(A - C)*a*b - B*b^2)*c*d
^2 + ((A - C)*a^2 + 2*B*a*b - (A - C)*b^2)*d^3)*log(tan(f*x + e)^2 + 1)/((
a^4 + 2*a^2*b^2 + b^4)*c^6 + 3*(a^4 + 2*a^2*b^2 + b^4)*c^4*d^2 + 3*(a^4...
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3115 vs. $2(862) = 1724$.

Time = 1.10 (sec) , antiderivative size = 3115, normalized size of antiderivative = 3.62

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
1/2*(2*(A*a^2*c^3 - C*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3 + C*b^2*c^3 + 3*B*
a^2*c^2*d - 6*A*a*b*c^2*d + 6*C*a*b*c^2*d - 3*B*b^2*c^2*d - 3*A*a^2*c*d^2
+ 3*C*a^2*c*d^2 - 6*B*a*b*c*d^2 + 3*A*b^2*c*d^2 - 3*C*b^2*c*d^2 - B*a^2*d^
3 + 2*A*a*b*d^3 - 2*C*a*b*d^3 + B*b^2*d^3)*(f*x + e)/(a^4*c^6 + 2*a^2*b^2*
c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*c^4*d^2 + 3*a^4*
c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*a^2*b^2*d^6 + b^
4*d^6) + (B*a^2*c^3 - 2*A*a*b*c^3 + 2*C*a*b*c^3 - B*b^2*c^3 - 3*A*a^2*c^2*
d + 3*C*a^2*c^2*d - 6*B*a*b*c^2*d + 3*A*b^2*c^2*d - 3*C*b^2*c^2*d - 3*B*a^
2*c*d^2 + 6*A*a*b*c*d^2 - 6*C*a*b*c*d^2 + 3*B*b^2*c*d^2 + A*a^2*d^3 - C*a^
2*d^3 + 2*B*a*b*d^3 - A*b^2*d^3 + C*b^2*d^3)*log(tan(f*x + e)^2 + 1)/(a^4*
c^6 + 2*a^2*b^2*c^6 + b^4*c^6 + 3*a^4*c^4*d^2 + 6*a^2*b^2*c^4*d^2 + 3*b^4*
c^4*d^2 + 3*a^4*c^2*d^4 + 6*a^2*b^2*c^2*d^4 + 3*b^4*c^2*d^4 + a^4*d^6 + 2*
a^2*b^2*d^6 + b^4*d^6) - 2*(B*a^2*b^5*c - 2*A*a*b^6*c + 2*C*a*b^6*c - B*b^
7*c + 3*C*a^4*b^3*d - 4*B*a^3*b^4*d + 5*A*a^2*b^5*d + C*a^2*b^5*d - 2*B*a*
b^6*d + 3*A*b^7*d)*log(abs(b*tan(f*x + e) + a))/(a^4*b^5*c^4 + 2*a^2*b^7*c
^4 + b^9*c^4 - 4*a^5*b^4*c^3*d - 8*a^3*b^6*c^3*d - 4*a*b^8*c^3*d + 6*a^6*b
^3*c^2*d^2 + 12*a^4*b^5*c^2*d^2 + 6*a^2*b^7*c^2*d^2 - 4*a^7*b^2*c*d^3 - 8*
a^5*b^4*c*d^3 - 4*a^3*b^6*c*d^3 + a^8*b*d^4 + 2*a^6*b^3*d^4 + a^4*b^5*d^4)
 + 2*(3*C*b^2*c^6*d^2 - 6*B*b^2*c^5*d^3 + 4*B*a*b*c^4*d^4 + 10*A*b^2*c^4*d
^4 - C*b^2*c^4*d^4 - B*a^2*c^3*d^5 - 10*A*a*b*c^3*d^5 + 10*C*a*b*c^3*d^...
```

3.89.9 Mupad [B] (verification not implemented)

Time = 43.73 (sec) , antiderivative size = 128666, normalized size of antiderivative = 149.44

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))^3),x)
```

output
```
(((2*A*b^4*c^6 - A*a^4*d^6 - 2*B*a*b^3*c^6 - B*a^4*c*d^5 - A*a^2*b^2*d^6 -
 5*A*a^4*c^2*d^4 + 2*C*a^2*b^2*c^6 + 2*A*b^4*c^2*d^4 + 4*A*b^4*c^4*d^2 + 3
*B*a^4*c^3*d^3 + 3*C*a^4*c^2*d^4 - C*a^4*c^4*d^2 + 9*A*a*b^3*c^3*d^3 + 9*A
*a^3*b*c^3*d^3 - 5*B*a*b^3*c^2*d^4 - 11*B*a*b^3*c^4*d^2 - B*a^2*b^2*c*d^5
- 3*B*a^3*b*c^2*d^4 - 7*B*a^3*b*c^4*d^2 + C*a*b^3*c^3*d^3 + C*a^3*b*c^3*d^
3 - 5*A*a^2*b^2*c^2*d^4 + 3*B*a^2*b^2*c^3*d^3 + 5*C*a^2*b^2*c^2*d^4 + 3*C*
a^2*b^2*c^4*d^2 + 5*A*a*b^3*c*d^5 + 5*A*a^3*b*c*d^5 + 5*C*a*b^3*c^5*d + 5*
C*a^3*b*c^5*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2
*c^4 + a^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) + (ta
n(e + f*x)*(3*A*a*b^3*d^6 - 2*B*a^4*d^6 + 3*A*a^3*b*d^6 - 4*A*a^4*c*d^5 +
9*A*b^4*c*d^5 + 4*A*b^4*c^5*d + 4*C*a^4*c*d^5 + 5*C*b^4*c^5*d - 2*B*a^2*b^
2*d^6 + 17*A*b^4*c^3*d^3 + 2*B*a^4*c^2*d^4 - 3*B*b^4*c^2*d^4 - 7*B*b^4*c^4
*d^2 + C*b^4*c^3*d^3 + 3*A*a*b^3*c^2*d^4 + A*a^2*b^2*c*d^5 + 3*A*a^3*b*c^2
*d^4 - 11*B*a*b^3*c^3*d^3 - 3*B*a^3*b*c^3*d^3 + 3*C*a*b^3*c^2*d^4 + 3*C*a*
b^3*c^4*d^2 + 8*C*a^2*b^2*c*d^5 + 9*C*a^2*b^2*c^5*d + 3*C*a^3*b*c^2*d^4 +
3*C*a^3*b*c^4*d^2 + 9*A*a^2*b^2*c^3*d^3 - B*a^2*b^2*c^2*d^4 - 7*B*a^2*b^2*
c^4*d^2 + 9*C*a^2*b^2*c^3*d^3 - 7*B*a*b^3*c*d^5 - 4*B*a*b^3*c^5*d - 3*B*a^
3*b*c*d^5))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^2*c^
4 + a^2*d^4 + b^2*c^4 + b^2*d^4 + 2*a^2*c^2*d^2 + 2*b^2*c^2*d^2)) + (tan(e
 + f*x)^2*(3*A*b^4*d^6 - 2*B*a*b^3*d^6 - B*a^3*b*d^6 - B*b^4*c*d^5 + 2*...
```

3.89. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^3}\,dx$

3.90 $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}(A+B\tan(e$

3.90.1 Optimal result

Integrand size = 47, antiderivative size = 464

$$
\begin{aligned}
&\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx \\
&= -\frac{(a-ib)^3(iA+B-iC)\sqrt{c-id}\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} \\
&\quad+\frac{(a+ib)^3(iA-B-iC)\sqrt{c+id}\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} \\
&\quad+\frac{2(a^3B-3ab^2B+3a^2b(A-C)-b^3(A-C))\sqrt{c+d\tan(e+fx)}}{f} \\
&\quad+\frac{2(40a^3Cd^3-6a^2bd^2(16cC-45Bd)+9ab^2d(8c^2C-14Bcd+35(A-C)d^2)-b^3(16c^3C-24Bc^2d+4}{315d^4f} \\
&\quad+\frac{2b(21b(Ab+aB-bC)d^2+4(bc-ad)(2bcC-3bBd-2aCd))\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{105d^3f} \\
&\quad-\frac{2(2bcC-3bBd-2aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{21d^2f} \\
&\quad+\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}
\end{aligned}
$$

output
```
-(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*
d)^(1/2)/f+(a+I*b)^3*(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))*(c+I*d)^(1/2)/f+2*(B*a^3-3*B*a*b^2+3*a^2*b*(A-C)-b^3*(A-C))*(c+d*tan(
f*x+e))^(1/2)/f+2/315*(40*a^3*C*d^3-6*a^2*b*d^2*(-45*B*d+16*C*c)+9*a*b^2*d
*(8*c^2*C-14*B*c*d+35*(A-C)*d^2)-b^3*(16*c^3*C-24*B*c^2*d+42*c*(A-C)*d^2+1
05*B*d^3))*(c+d*tan(f*x+e))^(3/2)/d^4/f+2/105*b*(21*b*(A*b+B*a-C*b)*d^2+4*
(-a*d+b*c)*(-3*B*b*d-2*C*a*d+2*C*b*c))*tan(f*x+e)*(c+d*tan(f*x+e))^(3/2)/d
^3/f-2/21*(-3*B*b*d-2*C*a*d+2*C*b*c)*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(
3/2)/d^2/f+2/9*C*(a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)/d/f
```

3.90.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1232 vs. $2(464) = 928$.

Time = 6.55 (sec) , antiderivative size = 1232, normalized size of antiderivative = 2.66

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$= \frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}$$

$$+ \frac{2\left(-\frac{3(2bcC-3bBd-2aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df} + \frac{2\left(\frac{3b\left(21b(Ab+aB-bC)d^2+4(bc-ad)(2bcC-3bBd-2aCd)\right)\tan(e+fx)(c+d}{10df}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f
*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2))/(9*d*f) + (2*((-3*
(2*b*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^
(3/2))/(7*d*f) + (2*((3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b
*c*C - 3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(10*d*
f) - (2*((2*((-15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C
 - 3*b*B*d - 2*a*C*d)))/8 + b*((-315*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/
8 + (3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d -
2*a*C*d)))/4))*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((I/2)*((-15*a*d*(a^2
*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C + 9*B*d)))/8
+ (3*b*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - 3*b*B*d -
2*a*C*d)))/4 + (15*a*d*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*
C - 3*b*B*d - 2*a*C*d)))/8 + ((5*I)/2)*d*((63*a*(a^2*B - b^2*B + 2*a*b*(A
- C))*d^2)/4 + (3*b*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b
*d*(16*c*C + 9*B*d)))/4 - (3*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)
*(2*b*c*C - 3*b*B*d - 2*a*C*d)))/4) - b*((-315*(a^2*B - b^2*B + 2*a*b*(A -
 C))*d^3)/8 + (3*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C -
3*b*B*d - 2*a*C*d)))/4))*((2*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*
x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*x]]))/f - ((I/2)*(
(-15*a*d*(a^2*(21*A - 13*C)*d^2 + 4*b^2*c*(2*c*C - 3*B*d) - a*b*d*(16*c*C
+ 9*B*d)))/8 + (3*b*c*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*...
```

3.90.3 Rubi [A] (warning: unable to verify)

Time = 3.44 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.03, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.426$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4130$$

$$\frac{2\int -\frac{3}{2}(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left((2bcC-2adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e-\cdots\right.}{9d}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\int(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left((2bcC-2adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx)\cdots\right.}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\int(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left((2bcC-2adC-3bBd)\tan(e+fx)^2-3(Ab-Cb+aB)d\tan(e+fx)\cdots\right.}{3d}$$

$$\Big\downarrow \; 4130$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(4c(2cC-3Bd)b^2-ad(16cC+9Bd)b+a^2(21A-13C)d^2+\left(21b(Ab-Cb+aB)d^2+4(bc-ad)(2bcC-2adC-\cdots\right.\right.}{7d}\cdots}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(4c(2cC-3Bd)b^2-ad(16cC+9Bd)b+a^2(2\cdots\right.}{\cdots}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(4c(2cC-3Bd)b^2-ad(16cC+9Bd)b+a^2(2\cdots\right.}{\cdots}}{3d}$$

$$\Big\downarrow \; 4120$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC)\cdots\right.}{5df}\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

3.90. $\int(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\sqrt{c+d\tan(e+fx)}\,(-2c(8Cc^2-12Bdc+21(A-C)d^2)b^3+18acd(4cC-7Bd)b^2-3a^2d^2(3\ldots$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\sqrt{c+d\tan(e+fx)}\,(-2c(8Cc^2-12Bdc+21(A-C)d^2)b^3+18acd(4cC-7Bd)b^2-3a^2d^2(3\ldots$$

$\downarrow$ 4113

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\sqrt{c+d\tan(e+fx)}\,(105(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C))d^3\tan(e+fx)-105(-((\ldots$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\sqrt{c+d\tan(e+fx)}\,(105(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C))d^3\tan(e+fx)-105(-((\ldots$$

$\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\frac{105((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3(Bc+(A-C)d))d^3\ldots}{\sqrt{\ldots}}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\int\frac{105((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3(Bc+(A-C)d))d^3\ldots}{\sqrt{\ldots}}$$

$\downarrow$ 4022

3.90. $\int(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

$\big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

$\big\downarrow$ 4020

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

$\big\downarrow$ 25

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

$\big\downarrow$ 73

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

$\big\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}}{9df}-$$

$$\frac{2(-2aCd-3bBd+2bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{3/2}\left(21bd^2(aB+Ab-bC)+4(bc-ad)(-2aCd-3bBd+2bcC\right.}{5df}$$

3.90. $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

input
```
Int[(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] +
C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2))/(9*d*f) - ((2*(2*b
*c*C - 3*b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2
))/(7*d*f) - ((2*b*(21*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C -
3*b*B*d - 2*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*f) + ((1
05*(a - I*b)^3*(A - I*B - C)*Sqrt[c - I*d]*d^3*ArcTan[Tan[e + f*x]/Sqrt[c
- I*d]])/f + (105*(a + I*b)^3*(A + I*B - C)*Sqrt[c + I*d]*d^3*ArcTan[Tan[e
 + f*x]/Sqrt[c + I*d]])/f + (210*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^
3*(A - C))*d^3*Sqrt[c + d*Tan[e + f*x]])/f + (2*(40*a^3*C*d^3 - 6*a^2*b*d^
2*(16*c*C - 45*B*d) + 9*a*b^2*d*(8*c^2*C - 14*B*c*d + 35*(A - C)*d^2) - b^
3*(16*c^3*C - 24*B*c^2*d + 42*c*(A - C)*d^2 + 105*B*d^3))*(c + d*Tan[e + f
*x])^(3/2))/(3*d*f))/(5*d))/(7*d))/(3*d)
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.90. $\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.90.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4472 vs. $2(424) = 848$.

Time = 0.53 (sec) , antiderivative size = 4473, normalized size of antiderivative = 9.64

method	result	size
parts	Expression too large to display	4473
derivativedivides	Expression too large to display	6661
default	Expression too large to display	6661

input
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x,method=_RETURNVERBOSE)
```

output
```
1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a^3+1/f*d/(2*(c
^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/
2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b^3-1/f*d/(2*(c^2+d^2)^(1/
2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/
2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a^3-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/
2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2))*B*b^3+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((
(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2))*C*a^3-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
)*C*a^3+2/f/d*A*a*b^2*(c+d*tan(f*x+e))^(3/2)+2/f/d*B*a^2*b*(c+d*tan(f*x+e)
)^(3/2)-12/5/f/d^3*C*a*b^2*c*(c+d*tan(f*x+e))^(5/2)-1/4/f/d*ln(d*tan(f*x+e
)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a^3+1/4/f/d*ln(d*tan(f*x+e
)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*
A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a^3*c-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f
*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b^3+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f
*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c^2+d...
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35153 vs. $2(414) = 828$.

Time = 10.87 (sec) , antiderivative size = 35153, normalized size of antiderivative = 75.76

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.90.6 Sympy [F]

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\big(A+B\tan(e+fx)+C\tan^2(e+fx)\big)\,dx = \int (a+b\tan{(e+fx)})^3\sqrt{c+d\tan{(e+fx)}}\big(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\big)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*ta
n(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

3.90.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\big(A+B\tan(e+fx)+C\tan^2(e+fx)\big)\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.90.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\big(A+B\tan(e+fx)+C\tan^2(e+fx)\big)\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x)
+ C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.91 $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}(A+B\tan(e$

3.91.1 Optimal result

Integrand size = 47, antiderivative size = 325

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= -\frac{(a-ib)^2(B+i(A-C))\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} - \frac{(a+ib)^2(B-i(A-C))\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} + \frac{2(a^2B-b^2B+2ab(A-C))\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(20a^2Cd^2-14abd(2cC-5Bd)+b^2(8c^2C-14Bcd+35(A-C)d^2))(c+d\tan(e+fx))^{3/2}}{105d^3f} - \frac{2b(4bcC-7bBd-4aCd)\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{35d^2f} + \frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}$$

output
```
-(a-I*b)^2*(B+I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*
d)^(1/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))*(c+I*d)^(1/2)/f+2*(B*a^2-B*b^2+2*a*b*(A-C))*(c+d*tan(f*x+e))^(1/2)/f+
2/105*(20*a^2*C*d^2-14*a*b*d*(-5*B*d+2*C*c)+b^2*(8*c^2*C-14*B*c*d+35*(A-C)
*d^2))*(c+d*tan(f*x+e))^(3/2)/d^3/f-2/35*b*(-7*B*b*d-4*C*a*d+4*C*b*c)*tan(
f*x+e)*(c+d*tan(f*x+e))^(3/2)/d^2/f+2/7*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+
e))^(3/2)/d/f
```

3.91.2 Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.97

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{2\Big(\left(20a^2Cd^2+14abd(-2cC+5Bd)+b^2(8c^2C-14Bcd+35(A-C)d^2)\right)(c+d\tan(e+fx))^{3/2}+3bd(-$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f
*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*((20*a^2*C*d^2 + 14*a*b*d*(-2*c*C + 5*B*d) + b^2*(8*c^2*C - 14*B*c*d +
35*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2) + 3*b*d*(-4*b*c*C + 7*b*B*d +
4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*C*d^2*(a + b*Tan[e +
 f*x])^2*(c + d*Tan[e + f*x])^(3/2) + (105*(a - I*b)^2*(I*A + B - I*C)*d^3
*(-(Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[
c + d*Tan[e + f*x]]))/2 + (105*(a + I*b)^2*((-I)*A + B + I*C)*d^3*(-(Sqrt[
c + I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan
[e + f*x]]))/2))/(105*d^3*f)
```

3.91.3 Rubi [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.362$, Rules used = {3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left((4bcC-4adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+\right.}{7d}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}$$

$$\Big\downarrow \ 27$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left((4bcC-4adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+fx)\right.}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left((4bcC-4adC-7bBd)\tan(e+fx)^2-7(Ab-Cb+aB)d\tan(e+fx)\right.}{7d}$$

$$\Big\downarrow \ 4120$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{2\int -\frac{1}{2}\sqrt{c+d\tan(e+fx)}\left(-2c(4cC-7Bd)b^2+28acCdb-5a^2(7A-3C)d^2-\left((8Cc^2-14\right.\right.}{}}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{\int\sqrt{c+d\tan(e+fx)}\left(-2c(4cC-7Bd)b^2+28acCdb-5a^2(7A-3C)d^2-\left((8Cc^2-14Bdc+35(A-C)d^2)b^2-14ad(2cC-5Bd)b+20a^2Cd^2\right)\tan^2(e+fx)\right.}{5d}}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{\int\sqrt{c+d\tan(e+fx)}\left(-2c(4cC-7Bd)b^2+28acCdb-5a^2(7A-3C)d^2-\left((8Cc^2-14Bdc+35(A-C)d^2)b^2-14ad(2cC-5Bd)b+20a^2Cd^2\right)\tan(e+fx)^2\right.}{5d}}{7d}$$

$$\Big\downarrow \ 4113$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{\int\sqrt{c+d\tan(e+fx)}\left(35\left(-((A-C)a^2)+2bBa+b^2(A-C)\right)d^2-35\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2c\right.}{3}}{5d}}{7d}$$

$$\Big\downarrow \ 3042$$

3.91. $\int(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{\int \sqrt{c+d\tan(e+fx)}\left(35\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-35\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2c\right.}{3}}{5d}}{7d}$$

$\Big\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\int \frac{-35\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-35\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)\right)\tan(e+fx)d^2}{\sqrt{c+d\tan(e+fx)}}dx-\frac{2(c+d\tan(e+fx)}{}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\int \frac{-35\left((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)d^2-35\left((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)\right)\tan(e+fx)d^2}{\sqrt{c+d\tan(e+fx)}}dx-\frac{2(c+d\tan(e+fx)}{}}{5d}$$

$\Big\downarrow$ 4022

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{-\frac{35}{2}d^2(a+ib)^2(c+id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{35}{2}d^2(a-ib)^2(c-id)(A-iB-}{}}{7}$$

$\Big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{-\frac{35}{2}d^2(a+ib)^2(c+id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{35}{2}d^2(a-ib)^2(c-id)(A-iB-}{}}{7}$$

$\Big\downarrow$ 4020

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{-\frac{35id^2(a-ib)^2(c-id)(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{35id}{}}{}$$

$\Big\downarrow$ 25

3.91. $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{\frac{35id^2(a-ib)^2(c-id)(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{35id^2(a-}{}$$

$\downarrow$ 73

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{-\frac{35d(a-ib)^2(c-id)(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{35d(a+ib)^2(c+id)}{}}{}$$

$\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}{7df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-7bBd+4bcC)(c+d\tan(e+fx))^{3/2}}{5df}+\frac{-\frac{2(c+d\tan(e+fx))^{3/2}\left(20a^2Cd^2-14abd(2cC-5Bd)+b^2\left(35d^2(A-C)-14Bcd+8c^2C\right)\right)}{3df}-\frac{70d}{}}{}}{7d}$$

input
```
Int[(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] +
C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2))/(7*d*f) - ((2*b*(4
*b*c*C - 7*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*
f) + ((-35*(a - I*b)^2*(A - I*B - C)*Sqrt[c - I*d]*d^2*ArcTan[Tan[e + f*x]
/Sqrt[c - I*d]])/f - (35*(a + I*b)^2*(A + I*B - C)*Sqrt[c + I*d]*d^2*ArcTa
n[Tan[e + f*x]/Sqrt[c + I*d]])/f - (70*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2
*Sqrt[c + d*Tan[e + f*x]])/f - (2*(20*a^2*C*d^2 - 14*a*b*d*(2*c*C - 5*B*d)
 + b^2*(8*c^2*C - 14*B*c*d + 35*(A - C)*d^2))*(c + d*Tan[e + f*x])^(3/2))/
(3*d*f))/(5*d))/(7*d)
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.91 $\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(291) = 582$.

Time = 0.16 (sec) , antiderivative size = 3353, normalized size of antiderivative = 10.32

method	result	size
parts	Expression too large to display	3353
derivativedivides	Expression too large to display	4775
default	Expression too large to display	4775

input
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x,method=_RETURNVERBOSE)
```

output
```
4/3/f/d*B*a*b*(c+d*tan(f*x+e))^(3/2)-1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*a
rctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2))*A*a^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(
c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c
)^(1/2))*A*b^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1
/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*
a^2+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*a^2-1/f*d/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)
^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A*b^2-1/f*d/(2*(c^2+d^2)
^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)
^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*C*a^2+1/2/f/d*ln(d*tan(f*x+e)+c+(c+
d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b-1/2/f/d*ln(d*tan(f*x+e)+c+(c+
d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*B*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)*a*b*c-1/2/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*(c^2+d^2)^(1/2)*a*b+1/2/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)*a*b*c+2/f*b*(B*b+2*C*a)/d^2*(1/5*(c+d*tan(f*x+e))^(5/2)-1/3*...
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23984 vs. 2(281) = 562.

Time = 4.69 (sec) , antiderivative size = 23984, normalized size of antiderivative = 73.80

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\big(A+B\tan(e+fx)+C\tan^2(e+fx)\big)\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.91.6 Sympy [F]

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan{(e+fx)})^2\sqrt{c+d\tan{(e+fx)}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*ta
n(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

3.91.7 Maxima [F]

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan{(fx+e)}^2+B\tan{(fx+e)}+A\right)(b\tan{(fx+e)}+a)^2\sqrt{d\tan{(fx+e)}+c}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^2*s
qrt(d*tan(f*x + e) + c), x)
```

3.91.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x)
+ C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.92 $\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}(A+B\tan(e+$

3.92.1 Optimal result

Integrand size = 45, antiderivative size = 224

$$\begin{aligned}
&\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx \\
&= -\frac{(ia+b)(A-iB-C)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} \\
&\quad+\frac{(ia-b)(A+iB-C)\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} \\
&\quad+\frac{2(Ab+aB-bC)\sqrt{c+d\tan(e+fx)}}{f} \\
&\quad-\frac{2(2bcC-5bBd-5aCd)(c+d\tan(e+fx))^{3/2}}{15d^2f} \\
&\quad+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}
\end{aligned}$$

output
```
-(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(
1/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+
I*d)^(1/2)/f+2*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(1/2)/f-2/15*(-5*B*b*d-5*C*a
*d+2*C*b*c)*(c+d*tan(f*x+e))^(3/2)/d^2/f+2/5*b*C*tan(f*x+e)*(c+d*tan(f*x+e
))^(3/2)/d/f
```

3.92.2 Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{\frac{2(-2bcC+5bBd+5aCd)(c+d\tan(e+fx))^{3/2}}{d}+6bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}+15(ia+b)(A-iB-C)d\Big(-$$

input
```
Integrate[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
((2*(-2*b*c*C + 5*b*B*d + 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/d + 6*b*C*T
an[e + f*x]*(c + d*Tan[e + f*x])^(3/2) + 15*(I*a + b)*(A - I*B - C)*d*(-(S
qrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]) + Sqrt[c + d
*Tan[e + f*x]]) + 15*((-I)*a + b)*(A + I*B - C)*d*(-(Sqrt[c + I*d]*ArcTanh
[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]) + Sqrt[c + d*Tan[e + f*x]]))/(15
*d*f)
```

3.92.3 Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.311$, Rules used = {3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-$$
$$\frac{2\int\frac{1}{2}\sqrt{c+d\tan(e+fx)}\left((2bcC-5adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)+2bcC-5aAd\right)}{5d}$$

$\Big\downarrow$ 27

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\sqrt{c+d\tan(e+fx)}\left((2bcC-5adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)+2bcC-5aAd\right)dx}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\sqrt{c+d\tan(e+fx)}\left((2bcC-5adC-5bBd)\tan(e+fx)^2-5(Ab-Cb+aB)d\tan(e+fx)+2bcC-5aAd\right)dx}{5d}$$

$\Big\downarrow$ 4113

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\sqrt{c+d\tan(e+fx)}(5(bB-a(A-C))d-5(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-5aCd-5bBd+2bcC)(c+d\tan(e+fx}{3df}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\sqrt{c+d\tan(e+fx)}(5(bB-a(A-C))d-5(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-5aCd-5bBd+2bcC)(c+d\tan(e+fx}{3df}}{5d}$$

$\Big\downarrow$ 4011

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\frac{5d(bBc+b(A-C)d-a(Ac-Cc-Bd))-5d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{10d(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(-5aC}{}}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\int\frac{5d(bBc+b(A-C)d-a(Ac-Cc-Bd))-5d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{10d(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(-5aC}{}}{5d}$$

$\Big\downarrow$ 4022

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{-\frac{5}{2}d(a+ib)(c+id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{5}{2}d(a-ib)(c-id)(A-iB-C)\int\frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-\frac{10d(}{}}{5d}$$

3.92. $\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{-\frac{5}{2}d(a+ib)(c+id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{5}{2}d(a-ib)(c-id)(A-iB-C)\int\frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-\frac{10d(}{}}{5d}$$

$$\Big\downarrow \text{4020}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{-\frac{5id(a-ib)(c-id)(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{5id(a+ib)(c+id)(A+iB-C)\int-\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+f}}}{2f}}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{\frac{5id(a-ib)(c-id)(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{5id(a+ib)(c+id)(A+iB-C)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-}{2f}}{5d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{-\frac{5(a+ib)(c+id)(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{5(a-ib)(c-id)(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-}{5d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{3/2}}{5df}-\frac{-\frac{5d(a-ib)\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}-\frac{5d(a+ib)\sqrt{c+id}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f}-\frac{10d(aB+Ab-bC)\sqrt{c+d\tan(e+fx)}}{f}+}{5d}$$

input
```
Int[(a + b*Tan[e + f*x])*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

output
```
(2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(3/2))/(5*d*f) - ((-5*(a - I*b)*(
A - I*B - C)*Sqrt[c - I*d]*d*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f - (5*(a
 + I*b)*(A + I*B - C)*Sqrt[c + I*d]*d*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/
f - (10*(A*b + a*B - b*C)*d*Sqrt[c + d*Tan[e + f*x]])/f + (2*(2*b*c*C - 5*
b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f))/(5*d)
```

3.92. $\int(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.92.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2217 vs. $2(194) = 388$.

Time = 0.18 (sec) , antiderivative size = 2218, normalized size of antiderivative = 9.90

method	result	size
parts	Expression too large to display	2218
derivativedivides	Expression too large to display	3028
default	Expression too large to display	3028

input
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x,method=_RETURNVERBOSE)
```

output
```
1/f*(A*b+B*a)*(2*(c+d*tan(f*x+e))^(1/2)+1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*
ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^
2+d^2)^(1/2))+((c^2+d^2)^(1/2)-c)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2
*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2
*c)^(1/2))-1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*
x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+(-(c^2+d^2)^(1/
2)+c)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c
^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+2/3/f/d*B*b*(c+d
*tan(f*x+e))^(3/2)+2/3/f/d*C*a*(c+d*tan(f*x+e))^(3/2)-1/4/f/d*ln((c+d*tan(
f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2)
)*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-1/4/f/d*ln((c+d*tan(f*
x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*
C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+1/f*d/(2*(c^2+d^2)^(1/2)
-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2)
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*B*b+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*a
rctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2
)^(1/2)-2*c)^(1/2))*C*a+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/
2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)*b*c+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*
tan(f*x+e)-c-(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+1/4/f...
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12410 vs. $2(187) = 374$.

Time = 1.66 (sec) , antiderivative size = 12410, normalized size of antiderivative = 55.40

$$\int (a + b\tan(e + fx))\sqrt{c + d\tan(e + fx)}\left(A + B\tan(e + fx) + C\tan^2(e + fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.92.6 Sympy [F]

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan{(e+fx)})\sqrt{c+d\tan{(e+fx)}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f
*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x)
 + C*tan(e + f*x)**2), x)
```

3.92.7 Maxima [F]

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (C\tan{(fx+e)}^2+B\tan{(fx+e)}+A)(b\tan{(fx+e)}+a)\sqrt{d\tan{(fx+e)}+c}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)*sqr
t(d*tan(f*x + e) + c), x)
```

3.92.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.92.9 Mupad [B] (verification not implemented)

Time = 57.65 (sec) , antiderivative size = 22955, normalized size of antiderivative = 102.48

$$\int (a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) +
C*tan(e + f*x)^2),x)
```

output
```
((2*B*a*d - 4*C*a*c)/(d*f) + (4*C*a*c)/(d*f))*(c + d*tan(e + f*x))^(1/2) +
 ((2*B*b*d - 6*C*b*c)/(3*d^2*f) + (4*C*b*c)/(3*d^2*f))*(c + d*tan(e + f*x)
)^(3/2) + (c + d*tan(e + f*x))^(1/2)*(2*c*((2*B*b*d - 6*C*b*c)/(d^2*f) + (
4*C*b*c)/(d^2*f)) + (2*A*b*d^2 + 6*C*b*c^2 - 4*B*b*c*d)/(d^2*f) - (2*C*b*(
d^4*f + c^2*d^2*f))/(d^4*f^2)) - atan(((((8*(4*A*b*d^4*f^2 - 4*C*b*d^4*f^2
 + 4*A*b*c^2*d^2*f^2 - 4*C*b*c^2*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f
*x))^(1/2)*((A^2*b^2*c)/(4*f^2) - (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 -
 C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c
^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 - 6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2
*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 +
 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B
^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/
(4*f^4) - (B^2*b^2*c)/(4*f^2) + (C^2*b^2*c)/(4*f^2) - (A*B*b^2*d)/(2*f^2)
- (A*C*b^2*c)/(2*f^2) + (B*C*b^2*d)/(2*f^2))^(1/2))*((A^2*b^2*c)/(4*f^2) -
 (4*A*C^3*b^4*d^2*f^4 - B^4*b^4*d^2*f^4 - C^4*b^4*d^2*f^4 - A^4*b^4*d^2*f^
4 + 4*A^3*C*b^4*d^2*f^4 - 4*A^2*B^2*b^4*c^2*f^4 + 2*A^2*B^2*b^4*d^2*f^4 -
6*A^2*C^2*b^4*d^2*f^4 - 4*B^2*C^2*b^4*c^2*f^4 + 2*B^2*C^2*b^4*d^2*f^4 + 4*
A*B^3*b^4*c*d*f^4 - 4*A^3*B*b^4*c*d*f^4 + 4*B*C^3*b^4*c*d*f^4 - 4*B^3*C*b^
4*c*d*f^4 + 8*A*B^2*C*b^4*c^2*f^4 - 4*A*B^2*C*b^4*d^2*f^4 - 12*A*B*C^2*b^4
*c*d*f^4 + 12*A^2*B*C*b^4*c*d*f^4)^(1/2)/(4*f^4) - (B^2*b^2*c)/(4*f^2) ...
```

3.93 $\int \sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e$

3.93.1 Optimal result

Integrand size = 35, antiderivative size = 155

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= -\frac{(iA+B-iC)\sqrt{c-id}\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} - \frac{(B-i(A-C))\sqrt{c+id}\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} + \frac{2B\sqrt{c+d\tan(e+fx)}}{f} + \frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/f
-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/f
+2*B*(c+d*tan(f*x+e))^(1/2)/f+2/3*C*(c+d*tan(f*x+e))^(3/2)/d/f
```

3.93.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{-3i(A-iB-C)\sqrt{c-id}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+3i(A+iB-C)\sqrt{c+id}\,d\,\mathrm{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)+}{3df}$$

input
```
Integrate[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)
,x]
```

output
```
((-3*I)*(A - I*B - C)*Sqrt[c - I*d]*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqr
t[c - I*d]] + (3*I)*(A + I*B - C)*Sqrt[c + I*d]*d*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c + I*d]] + 2*Sqrt[c + d*Tan[e + f*x]]*(c*C + 3*B*d + C*d*Ta
n[e + f*x]))/(3*d*f)
```

3.93.3 Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$
$$\Big\downarrow \text{4113}$$
$$\int (A-C+B\tan(e+fx))\sqrt{c+d\tan(e+fx)}dx+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$
$$\Big\downarrow \text{3042}$$
$$\int (A-C+B\tan(e+fx))\sqrt{c+d\tan(e+fx)}dx+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{Ac - Cc - Bd + (Bc + (A - C)d)\tan(e + fx)}{\sqrt{c + d\tan(e + fx)}}dx + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ac - Cc - Bd + (Bc + (A - C)d)\tan(e + fx)}{\sqrt{c + d\tan(e + fx)}}dx + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(c + id)(A + iB - C)\int \frac{1 - i\tan(e + fx)}{\sqrt{c + d\tan(e + fx)}}dx + \frac{1}{2}(c - id)(A - iB - C)\int \frac{i\tan(e + fx) + 1}{\sqrt{c + d\tan(e + fx)}}dx + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(c + id)(A + iB - C)\int \frac{1 - i\tan(e + fx)}{\sqrt{c + d\tan(e + fx)}}dx + \frac{1}{2}(c - id)(A - iB - C)\int \frac{i\tan(e + fx) + 1}{\sqrt{c + d\tan(e + fx)}}dx + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(c - id)(A - iB - C)\int -\frac{1}{(1 - i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e + fx))}{2f} - \frac{i(c + id)(A + iB - C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e + fx))}{2f} + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(c - id)(A - iB - C)\int \frac{1}{(1 - i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e + fx))}{2f} + \frac{i(c + id)(A + iB - C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e + fx))}{2f} + \frac{2B\sqrt{c + d\tan(e + fx)}}{f} + \frac{2C(c + d\tan(e + fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{73}$$

3.93. $\int \sqrt{c + d\tan(e + fx)}(A + B\tan(e + fx) + C\tan^2(e + fx))\ dx$

$$\frac{(c+id)(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(c-id)(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{\sqrt{c+id}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f}+\frac{2B\sqrt{c+d\tan(e+fx)}}{f}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3df}$$

input
```
Int[Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
((A - I*B - C)*Sqrt[c - I*d]*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f + ((A +
 I*B - C)*Sqrt[c + I*d]*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f + (2*B*Sqrt[
c + d*Tan[e + f*x]])/f + (2*C*(c + d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1311 vs. $2(130) = 260$.

Time = 0.13 (sec) , antiderivative size = 1312, normalized size of antiderivative = 8.46

method	result	size
parts	Expression too large to display	1312
derivativedivides	Expression too large to display	1472
default	Expression too large to display	1472

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETUR
NVERBOSE)
```

output
```
1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+
e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)-1/f*
d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c
+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*A-1/4/f/d*ln((c+d*tan
(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2
))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*
x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)+1/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arcta
n((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1
/2)-2*c)^(1/2))*A+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2
+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c+
B/f*(2*(c+d*tan(f*x+e))^(1/2)+1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*ln((c+d*ta
n(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/
2))+((c^2+d^2)^(1/2)-c)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)
^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))
-1/4*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2
)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))+(-(c^2+d^2)^(1/2)+c)/(2*(
c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1
/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)))+C*(2/3/f/d*(c+d*tan(f*x+e)
)^(3/2)-1/4/f/d*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*ln((c+d*t...
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2588 vs. $2(123) = 246$.

Time = 0.35 (sec) , antiderivative size = 2588, normalized size of antiderivative = 16.70

$$\int \sqrt{c + d\tan(e + fx)}\left(A + B\tan(e + fx) + C\tan^2(e + fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="fricas")
```

output
```
-1/6*(3*d*f*sqrt(-(f^2*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c^2 + 4*(A
^3*B - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4 - 2*A^2*B
^2 + B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*C)*d^2)/f
^4) + (A^2 - B^2 - 2*A*C + C^2)*c - 2*(A*B - B*C)*d)/f^2)*log((2*(A^3*B +
A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B + B^3)*C)*c + (A^4 - B^4 - 4*A^3*C +
6*A^2*C^2 - 4*A*C^3 + C^4)*d)*sqrt(d*tan(f*x + e) + c) + ((A - C)*f^3*sqrt
(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c^2 + 4*(A^3*B - A*B^3 + 3*A*B*C^2 -
B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4 - 2*A^2*B^2 + B^4 - 4*A*C^3 + C^4 +
2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*C)*d^2)/f^4) + (2*(A*B^2 - B^2*C)*c
+ (A^2*B - B^3 - 2*A*B*C + B*C^2)*d)*f)*sqrt(-(f^2*sqrt(-(4*(A^2*B^2 - 2*A
*B^2*C + B^2*C^2)*c^2 + 4*(A^3*B - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B -
B^3)*C)*c*d + (A^4 - 2*A^2*B^2 + B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2
 - 4*(A^3 - A*B^2)*C)*d^2)/f^4) + (A^2 - B^2 - 2*A*C + C^2)*c - 2*(A*B - B
*C)*d)/f^2)) - 3*d*f*sqrt(-(f^2*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c
^2 + 4*(A^3*B - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4
- 2*A^2*B^2 + B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*
C)*d^2)/f^4) + (A^2 - B^2 - 2*A*C + C^2)*c - 2*(A*B - B*C)*d)/f^2)*log((2*
(A^3*B + A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B + B^3)*C)*c + (A^4 - B^4 - 4
*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^4)*d)*sqrt(d*tan(f*x + e) + c) - ((A - C)
*f^3*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c^2 + 4*(A^3*B - A*B^3 + ...
```

3.93.6 Sympy [F]

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \sqrt{c+d\tan{(e+fx)}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
, x)
```

3.93.7 Maxima [F]

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)
, x)
```

3.93.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="giac")
```

output
```
Timed out
```

3.93.9 Mupad [B] (verification not implemented)

Time = 16.04 (sec) , antiderivative size = 1199, normalized size of antiderivative = 7.74

$$\int \sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$= 2\,\mathrm{atanh}\left(\frac{32\,B^2\,d^4\,\sqrt{\frac{B^2\,c}{4\,f^2}-\frac{\sqrt{-B^4\,d^2\,f^4}}{4\,f^4}}\,\sqrt{c+d\tan\left(e+f\,x\right)}}{\frac{16\,B\,d^4\,\sqrt{-B^4\,d^2\,f^4}}{f^3}+\frac{16\,B\,c^2\,d^2\,\sqrt{-B^4\,d^2\,f^4}}{f^3}} - \frac{32\,c\,d^2\,\sqrt{\frac{B^2\,c}{4\,f^2}-\frac{\sqrt{-B^4\,d^2\,f^4}}{4\,f^4}}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{-B^4\,d^2\,f^4}}{\frac{16\,B\,d^4\,\sqrt{-B^4\,d^2\,f^4}}{f}+\frac{16\,B\,c^2\,d^2\,\sqrt{-B^4\,d^2\,f^4}}{f}}\right)\sqrt{-\frac{\sqrt{-B^4\,d^2\,f^4}-B^2\,c\,f^2}{4\,f^4}}$$

$$- 2\,\mathrm{atanh}\left(\frac{32\,B^2\,d^4\,\sqrt{\frac{\sqrt{-B^4\,d^2\,f^4}}{4\,f^4}+\frac{B^2\,c}{4\,f^2}}\,\sqrt{c+d\tan\left(e+f\,x\right)}}{\frac{16\,B\,d^4\,\sqrt{-B^4\,d^2\,f^4}}{f^3}+\frac{16\,B\,c^2\,d^2\,\sqrt{-B^4\,d^2\,f^4}}{f^3}} + \frac{32\,c\,d^2\,\sqrt{\frac{\sqrt{-B^4\,d^2\,f^4}}{4\,f^4}+\frac{B^2\,c}{4\,f^2}}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\sqrt{-B^4\,d^2\,f^4}}{\frac{16\,B\,d^4\,\sqrt{-B^4\,d^2\,f^4}}{f}+\frac{16\,B\,c^2\,d^2\,\sqrt{-B^4\,d^2\,f^4}}{f}}\right)\sqrt{\frac{\sqrt{-B^4\,d^2\,f^4}+B^2\,c\,f^2}{4\,f^4}}$$

$$-\mathrm{atanh}\left(\frac{f^3\,\sqrt{-\frac{\sqrt{-A^4\,d^2\,f^4}+A^2\,c\,f^2}{f^4}}\,\left(\frac{16\,\left(A^2\,d^4-A^2\,c^2\,d^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^2}+\frac{16\,c\,d^2\,\left(\sqrt{-A^4\,d^2\,f^4}+A^2\,c\,f^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^4}\right.}{16\,\left(A^3\,c^2\,d^3+A^3\,d^5\right)}\right.$$

$$-\mathrm{atanh}\left(\frac{f^3\,\sqrt{\frac{\sqrt{-A^4\,d^2\,f^4}-A^2\,c\,f^2}{f^4}}\,\left(\frac{16\,\left(A^2\,d^4-A^2\,c^2\,d^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^2}-\frac{16\,c\,d^2\,\left(\sqrt{-A^4\,d^2\,f^4}-A^2\,c\,f^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^4}\right)}{16\,\left(A^3\,c^2\,d^3+A^3\,d^5\right)}\right.$$

$$+\mathrm{atanh}\left(\frac{f^3\,\sqrt{-\frac{\sqrt{-C^4\,d^2\,f^4}+C^2\,c\,f^2}{f^4}}\,\left(\frac{16\,\left(C^2\,d^4-C^2\,c^2\,d^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^2}+\frac{16\,c\,d^2\,\left(\sqrt{-C^4\,d^2\,f^4}+C^2\,c\,f^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^4}\right.}{16\,\left(C^3\,c^2\,d^3+C^3\,d^5\right)}\right.$$

$$+\mathrm{atanh}\left(\frac{f^3\,\sqrt{\frac{\sqrt{-C^4\,d^2\,f^4}-C^2\,c\,f^2}{f^4}}\,\left(\frac{16\,\left(C^2\,d^4-C^2\,c^2\,d^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^2}-\frac{16\,c\,d^2\,\left(\sqrt{-C^4\,d^2\,f^4}-C^2\,c\,f^2\right)\,\sqrt{c+d\tan(e+f\,x)}}{f^4}\right)}{16\,\left(C^3\,c^2\,d^3+C^3\,d^5\right)}\right.$$

$$+\frac{2\,B\,\sqrt{c+d\tan\left(e+f\,x\right)}}{f}+\frac{2\,C\,{\left(c+d\tan(e+f\,x)\right)}^{3/2}}{3\,d\,f}$$

input
```
int((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
2*atanh((32*B^2*d^4*((B^2*c)/(4*f^2) - (-B^4*d^2*f^4)^(1/2)/(4*f^4))^(1/2)
*(c + d*tan(e + f*x))^(1/2))/((16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f^3 + (16*B*
c^2*d^2*(-B^4*d^2*f^4)^(1/2))/f^3) - (32*c*d^2*((B^2*c)/(4*f^2) - (-B^4*d^
2*f^4)^(1/2)/(4*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-B^4*d^2*f^4)^(1/2
))/((16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f + (16*B*c^2*d^2*(-B^4*d^2*f^4)^(1/2)
)/f))*(-((-B^4*d^2*f^4)^(1/2) - B^2*c*f^2)/(4*f^4))^(1/2) - 2*atanh((32*B^
2*d^4*((-B^4*d^2*f^4)^(1/2)/(4*f^4) + (B^2*c)/(4*f^2))^(1/2)*(c + d*tan(e
+ f*x))^(1/2))/((16*B*d^4*(-B^4*d^2*f^4)^(1/2))/f^3 + (16*B*c^2*d^2*(-B^4*
d^2*f^4)^(1/2))/f^3) + (32*c*d^2*((-B^4*d^2*f^4)^(1/2)/(4*f^4) + (B^2*c)/(
4*f^2))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(-B^4*d^2*f^4)^(1/2))/((16*B*d^4*
(-B^4*d^2*f^4)^(1/2))/f + (16*B*c^2*d^2*(-B^4*d^2*f^4)^(1/2))/f))*(((-B^4*
d^2*f^4)^(1/2) + B^2*c*f^2)/(4*f^4))^(1/2) - atanh((f^3*(-((-A^4*d^2*f^4)^
(1/2) + A^2*c*f^2)/f^4)^(1/2)*((16*(A^2*d^4 - A^2*c^2*d^2)*(c + d*tan(e +
f*x))^(1/2))/f^2 + (16*c*d^2*((-A^4*d^2*f^4)^(1/2) + A^2*c*f^2)*(c + d*tan
(e + f*x))^(1/2))/f^4))/(16*(A^3*d^5 + A^3*c^2*d^3)))*(-((-A^4*d^2*f^4)^(1
/2) + A^2*c*f^2)/f^4)^(1/2) - atanh((f^3*(((-A^4*d^2*f^4)^(1/2) - A^2*c*f^
2)/f^4)^(1/2)*((16*(A^2*d^4 - A^2*c^2*d^2)*(c + d*tan(e + f*x))^(1/2))/f^2
 - (16*c*d^2*((-A^4*d^2*f^4)^(1/2) - A^2*c*f^2)*(c + d*tan(e + f*x))^(1/2)
)/f^4))/(16*(A^3*d^5 + A^3*c^2*d^3)))*(((-A^4*d^2*f^4)^(1/2) - A^2*c*f^2)/
f^4)^(1/2) + atanh((f^3*(-((-C^4*d^2*f^4)^(1/2) + C^2*c*f^2)/f^4)^(1/2)...
```

3.93. $\int \sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

3.94 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.94.1 Optimal result

Integrand size = 47, antiderivative size = 234

$$\begin{aligned}\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx &= -\frac{(iA+B-iC)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f} \\ &\quad+\frac{(iA-B-iC)\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)f} \\ &\quad-\frac{2(Ab^2-a(bB-aC))\,\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}\,(a^2+b^2)\,f}+\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}\end{aligned}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(
a-I*b)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*d)
^(1/2)/(a+I*b)/f-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1
/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2)/(a^2+b^2)/f+2*C*(c+d*tan(f*
x+e))^(1/2)/b/f
```

3.94.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$
$$= \frac{b^{3/2}(-ia+b)(A-iB-C)\sqrt{c-id}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+b^{3/2}(ia+b)(A+iB-C)\sqrt{c+id}\text{arctanh}\left(\right.}{b^{3/}}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x]),x]
```

output
```
(b^(3/2)*((-I)*a + b)*(A - I*B - C)*Sqrt[c - I*d]*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c - I*d]] + b^(3/2)*(I*a + b)*(A + I*B - C)*Sqrt[c + I*d]*Ar
cTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b*B) + a*C
))*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a
*d]] + 2*Sqrt[b]*(a^2 + b^2)*C*Sqrt[c + d*Tan[e + f*x]])/(b^(3/2)*(a^2 + b
^2)*f)
```

3.94.3 Rubi [A] (warning: unable to verify)

Time = 1.76 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.319$, Rules used = {3042, 4130, 27, 3042, 4136, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan(e+fx)^2)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int \frac{(bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(bcC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

$\Big\downarrow$ 4136

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\int \frac{b(bBc+b(A-C)d+a(Ac-Cc-Bd))-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}}{b}$$

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{b(bBc+b(A-C)d+a(Ac-Cc-Bd))-b(Abc-aBc-bCc-aAd-bBd+aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}}{b}$$

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}$$

$\Big\downarrow$ 4022

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+$$

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{1}{2}b(a-ib)(c+id)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}b(a+ib)(c-id)(A-iB-C)}{a^2+b^2}}{b}$$

$\Big\downarrow$ 3042

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+$$

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{1}{2}b(a-ib)(c+id)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}b(a+ib)(c-id)(A-iB-C)}{a^2+b^2}}{b}$$

$\Big\downarrow$ 4020

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+$$

$$\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{ib(a+ib)(c-id)(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{ib(a-}{}}{a^2+b^2}}{b}$$

$\Big\downarrow$ 25

3.94. $\quad \displaystyle\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{ib(a-ib)(c+id)(A+iB-C)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}-\frac{ib(a+}{}}{a^2+b^2}}{b}$$

$\downarrow$ 73

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{b(a-ib)(c+id)(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{b(a+ib)(c-id)(}{}}{a^2+b^2}}{b}$$

$\downarrow$ 221

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{b(a+ib)\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{b(a-ib)\sqrt{c+id}(A+iB-C)\arctan\left(\frac{\tan}{\sqrt{}}\right.}{f}}{a^2+b^2}}{b}$$

$\downarrow$ 4117

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f(a^2+b^2)}+\frac{\frac{b(a+ib)\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{b(a-ib)\sqrt{c+id}(A+iB-C)\,a}{f}}{a^2+b^2}}{b}$$

$\downarrow$ 73

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df(a^2+b^2)}+\frac{\frac{b(a+ib)\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{b(a-ib)\sqrt{c+id}(A+iB-C)}{f}}{a^2+b^2}}{b}$$

$\downarrow$ 221

$$\frac{2C\sqrt{c+d\tan(e+fx)}}{bf}+\frac{-\frac{2\sqrt{bc-ad}(Ab^2-a(bB-aC))\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f(a^2+b^2)}+\frac{\frac{b(a+ib)\sqrt{c-id}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{b(a-ib)\sqrt{c+id}(A+iB-C)\arctan\left(\frac{\tan}{\sqrt{}}\right.}{f}}{a^2+b^2}}{b}$$

3.94. $\int\frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x]),x]
```

output
```
((((a + I*b)*b*(A - I*B - C)*Sqrt[c - I*d]*ArcTan[Tan[e + f*x]/Sqrt[c - I*
d]])/f + ((a - I*b)*b*(A + I*B - C)*Sqrt[c + I*d]*ArcTan[Tan[e + f*x]/Sqrt
[c + I*d]])/f)/(a^2 + b^2) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 +
 b^2)*f))/b + (2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.94. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.94. $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3575 vs. $2(200) = 400$.

Time = 0.15 (sec) , antiderivative size = 3576, normalized size of antiderivative = 15.28

method	result	size
derivativedivides	Expression too large to display	3576
default	Expression too large to display	3576

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x,method=_RETURNVERBOSE)
```

output
```
2*C*(c+d*tan(f*x+e))^(1/2)/b/f-1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)
*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*A*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)*a*c+1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c
^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1
/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-2/f*b/(a^2+b^2)/((a*d-b*c)*b)^(1/2)*arcta
n(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^(1/2))*B*a*c-1/4/f/(a^2+b^2)/d*ln
(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+
d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a+1/4/f/(a^2+b
^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c-1/4/f/(a^2+b^2)/d*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^
2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*b-1/4/f/(a^2
+b^2)/d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)+(c^2+d^2)^(1/2))*C*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*a*c+1/4/f/(a^2+b^2)/
d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(
c^2+d^2)^(1/2))*A*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*a-2/f*b/(a
^2+b^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(c+d*tan(f*x+e))^(1/2)/((a*d-b*c)*b)^
(1/2))*A*a*d-1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*B*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)*b*c-1/4/f/(a^2+b^2)/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2...
```

3.94.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.94.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e)),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x)), x)
```

3.94.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.94.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.94.9 Mupad [B] (verification not implemented)

Time = 33.93 (sec) , antiderivative size = 62245, normalized size of antiderivative = 266.00

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x)),x)
```

output
```
atan(((((((32*(4*C*a*b^8*d^11*f^4 - 4*C*b^9*c*d^10*f^4 + 8*C*a^3*b^6*d^11*
f^4 + 4*C*a^5*b^4*d^11*f^4 - 4*C*b^9*c^3*d^8*f^4 + 4*C*a*b^8*c^2*d^9*f^4 -
 8*C*a^2*b^7*c*d^10*f^4 - 4*C*a^4*b^5*c*d^10*f^4 - 8*C*a^2*b^7*c^3*d^8*f^4
 + 8*C*a^3*b^6*c^2*d^9*f^4 - 4*C*a^4*b^5*c^3*d^8*f^4 + 4*C*a^5*b^4*c^2*d^9
*f^4))/(b*f^5) - (32*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^2*c*f^2 - 8*C^
2*b^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16
*b^4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*
C^2*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2)*(16*b^10*d^
10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 - 16*a^6*b^4*d^10*f^4 +
 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^4*b^6*c^2*d^8*f^4 - 8*
a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7*c*d^9*f^4 + 24*a^5*b^
5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*((((8*C^2*a^2*c*f^2 - 8*C^2*b
^2*c*f^2 + 16*C^2*a*b*d*f^2)^2/4 - (C^4*c^2 + C^4*d^2)*(16*a^4*f^4 + 16*b^
4*f^4 + 32*a^2*b^2*f^4))^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 - 8*C^2
*a*b*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^4)))^(1/2) - (32*(c + d*t
an(e + f*x))^(1/2)*(14*C^2*a*b^7*d^11*f^2 - 2*C^2*a^5*b^3*d^11*f^2 - 10*C^
2*b^8*c^3*d^8*f^2 - 4*C^2*a^3*b^5*d^11*f^2 - 16*C^2*a^7*b*d^11*f^2 + 8*C^2
*a^8*c*d^10*f^2 - 6*C^2*b^8*c*d^10*f^2 + 18*C^2*a*b^7*c^2*d^9*f^2 + 12*C^2
*a^2*b^6*c*d^10*f^2 + 2*C^2*a^4*b^4*c*d^10*f^2 + 24*C^2*a^6*b^2*c*d^10*f^2
 - 16*C^2*a^7*b*c^2*d^9*f^2 + 4*C^2*a^2*b^6*c^3*d^8*f^2 + 4*C^2*a^3*b^5...
```

3.94. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

3.95 $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.95.1 Optimal result

Integrand size = 47, antiderivative size = 317

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$$
$$= -\frac{(iA+B-iC)\sqrt{c-id}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}$$
$$-\frac{(B-i(A-C))\sqrt{c+id}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2f}$$
$$-\frac{(a^3bBd+a^4Cd+b^4(2Bc+Ad)+ab^3(4Ac-4cC-3Bd)-a^2b^2(2Bc+3Ad-5Cd))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{3/2}(a^2+b^2)^2\sqrt{bc-ad}f}$$
$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f(a+b\tan(e+fx))}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))*(c-I*d)^(1/2)/(
a-I*b)^2/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))*(c+I*
d)^(1/2)/(a+I*b)^2/f-(a^3*b*B*d+a^4*C*d+b^4*(A*d+2*B*c)+a*b^3*(4*A*c-3*B*d
-4*C*c)-a^2*b^2*(3*A*d+2*B*c-5*C*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2
)/(-a*d+b*c)^(1/2))/b^(3/2)/(a^2+b^2)^2/f/(-a*d+b*c)^(1/2)-(A*b^2-a*(B*b-C
*a))*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.95.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 764 vs. 2(317) = 634.

Time = 6.46 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.41

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = -\frac{2C\sqrt{c+d\tan(e+fx)}}{bf(a+b\tan(e+fx))}$$

$$-\frac{2\left(-\frac{\frac{i\sqrt{c-id}\left(\frac{1}{2}b(bc-ad)\left(a^2(Ac-cC-Bd)-b^2(Ac-cC-Bd)+2ab(Bc+(A-C)d)\right)+\frac{1}{2}ib(bc-ad)\left(2ab(Ac-cC-Bd)-a^2(Bc+(A-C)d)+b^2(Bc+(A-C)d)\right)\right)}{(-c+id)f}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x])^2,x]
```

output
```
(-2*C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])) - (2*(-((((I*Sq
rt[c - I*d]*((b*(b*c - a*d)*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d)
 + 2*a*b*(B*c + (A - C)*d)))/2 + (I/2)*b*(b*c - a*d)*(2*a*b*(A*c - c*C - B
*d) - a^2*(B*c + (A - C)*d) + b^2*(B*c + (A - C)*d)))*ArcTanh[Sqrt[c + d*T
an[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(b*c -
a*d)*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) + 2*a*b*(B*c + (A - C)
*d)))/2 - (I/2)*b*(b*c - a*d)*(2*a*b*(A*c - c*C - B*d) - a^2*(B*c + (A - C
)*d) + b^2*(B*c + (A - C)*d)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I
*d]])/((-c - I*d)*f))/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*(-1/4*(a^2*(A*b^2 -
 a*b*B - a^2*C - 2*b^2*C)*d*(b*c - a*d)) + (a*b^2*(b*c - a*d)*(A*b*c - a*B
*c - b*c*C - a*A*d - b*B*d + a*C*d))/2 + (b^2*(b*c - a*d)*(a^2*C*d + b^2*(
2*B*c + A*d) + a*b*(2*A*c - 2*c*C - B*d)))/4)*ArcTanh[(Sqrt[b]*Sqrt[c + d*
Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f))/(
(a^2 + b^2)*(b*c - a*d))) - (((b^2*(-(A*b*c) + 2*b*c*C - a*C*d))/2 - a*(-1
/2*(b^2*(B*c + (A - C)*d)) - (a*(b*c*C - b*B*d - a*C*d))/2))*Sqrt[c + d*Ta
n[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))))/b
```

3.95.3 Rubi [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.340, Rules used = {3042, 4128, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-bBa+Ab^2-2b^2C\right)d\tan^2(e+fx)\right)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(bc-\frac{ad}{2}\right)+2Ab\left(ac+\frac{bd}{2}\right)}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-bBa+Ab^2-2b^2C\right)d\tan^2(e+fx)\right)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-ad)+Ab(2ac+bd)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{2b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-bBa+Ab^2-2b^2C\right)d\tan(e+fx)^2\right)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-ad)+Ab(2ac+bd)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{2b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{ 4136}$$

$$\frac{\frac{\int \frac{2\left(b\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{(a^4Cd+a^3bB\cdots}{\cdots}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{b\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{(a^4Cd+a^3bBd\cdots}{\cdots}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{2\int \frac{b\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{(a^4Cd+a^3bBd\cdots}{\cdots}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4022

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{1}{2}b(a-ib)^2(c+id)(A+iB\cdots\right.}{\cdots}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{1}{2}b(a-ib)^2(c+id)(A+iB\cdots\right.}{\cdots}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 4020

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{ib(a+ib)^2(c-id)(A-iB-C\cdots}{\cdots}\right.}{\cdots}}{2b\left(a^2+b^2\right)}$$

3.95. $$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$\downarrow$ 25

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{ib(a-ib)^2(c+id)(A+iB-C}{}\right.}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{b(a-ib)^2(c+id)(A+iB-C)}{}\right.}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{2\left(\frac{b(a-ib)^2\sqrt{c+id}(A+iB-C)}{f}\right.}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{\frac{\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f(a^2+b^2)}+\frac{2\left(\frac{b(a-ib)^2\sqrt{c+id}}{}\right.}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{\frac{2\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df(a^2+b^2)}+\frac{2\left(\frac{b(a-ib)^2\sqrt{c+id}}{}\right.}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 221

3.95. $\int\frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)}+$$

$$\frac{-\frac{2\left(a^4Cd+a^3bBd-a^2b^2(3Ad+2Bc-5Cd)+ab^3(4Ac-3Bd-4cC)+b^4(Ad+2Bc)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f(a^2+b^2)\sqrt{bc-ad}}+\frac{2\left(\frac{b(a-ib)^2\sqrt{c+id}(A+iB-C)}{f}\right.}{}}{2b\left(a^2+b^2\right)}$$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x])^2,x]
```

output
```
((2*(((a + I*b)^2*b*(A - I*B - C)*Sqrt[c - I*d]*ArcTan[Tan[e + f*x]/Sqrt[c
 - I*d]])/f + ((a - I*b)^2*b*(A + I*B - C)*Sqrt[c + I*d]*ArcTan[Tan[e + f*
x]/Sqrt[c + I*d]])/f))/(a^2 + b^2) - (2*(a^3*b*B*d + a^4*C*d + b^4*(2*B*c
+ A*d) + a*b^3*(4*A*c - 4*c*C - 3*B*d) - a^2*b^2*(2*B*c + 3*A*d - 5*C*d))*
ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2
 + b^2)*Sqrt[b*c - a*d]*f))/(2*b*(a^2 + b^2)) - ((A*b^2 - a*(b*B - a*C))*S
qrt[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.95. $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5777 vs. 2(284) = 568.

Time = 0.13 (sec) , antiderivative size = 5778, normalized size of antiderivative = 18.23

method	result	size
derivativedivides	Expression too large to display	5778
default	Expression too large to display	5778

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.95.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.95.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{c+d\tan{(e+fx)}}(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**2,x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x))**2, x)
```

3.95.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.95.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="giac")
```

output
```
Timed out
```

3.95.9 Mupad [B] (verification not implemented)

Time = 42.93 (sec) , antiderivative size = 138318, normalized size of antiderivative = 436.33

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^2,x)
```

output
```
atan(((((8*(156*B^3*a^2*b^9*d^12*f^2 - 16*B^3*a^4*b^7*d^12*f^2 - 120*B^3*a
^6*b^5*d^12*f^2 + 48*B^3*a^8*b^3*d^12*f^2 + 12*B^3*b^11*c^2*d^10*f^2 + 12*
B^3*b^11*c^4*d^8*f^2 - 4*B^3*a^10*b*d^12*f^2 - 124*B^3*a*b^10*c*d^11*f^2 -
 124*B^3*a*b^10*c^3*d^9*f^2 + 224*B^3*a^3*b^8*c*d^11*f^2 + 200*B^3*a^5*b^6
*c*d^11*f^2 - 128*B^3*a^7*b^4*c*d^11*f^2 + 20*B^3*a^9*b^2*c*d^11*f^2 - 4*B
^3*a^10*b*c^2*d^10*f^2 + 44*B^3*a^2*b^9*c^2*d^10*f^2 - 112*B^3*a^2*b^9*c^4
*d^8*f^2 + 224*B^3*a^3*b^8*c^3*d^9*f^2 - 40*B^3*a^4*b^7*c^2*d^10*f^2 - 24*
B^3*a^4*b^7*c^4*d^8*f^2 + 200*B^3*a^5*b^6*c^3*d^9*f^2 - 40*B^3*a^6*b^5*c^2
*d^10*f^2 + 80*B^3*a^6*b^5*c^4*d^8*f^2 - 128*B^3*a^7*b^4*c^3*d^9*f^2 + 28*
B^3*a^8*b^3*c^2*d^10*f^2 - 20*B^3*a^8*b^3*c^4*d^8*f^2 + 20*B^3*a^9*b^2*c^3
*d^9*f^2))/(a^8*f^5 + b^8*f^5 + 4*a^2*b^6*f^5 + 6*a^4*b^4*f^5 + 4*a^6*b^2*
f^5) + (((8*(80*B*a*b^14*d^11*f^4 - 48*B*b^15*c*d^10*f^4 + 384*B*a^3*b^12*
d^11*f^4 + 720*B*a^5*b^10*d^11*f^4 + 640*B*a^7*b^8*d^11*f^4 + 240*B*a^9*b^
6*d^11*f^4 - 16*B*a^13*b^2*d^11*f^4 - 48*B*b^15*c^3*d^8*f^4 + 80*B*a*b^14*
c^2*d^9*f^4 - 224*B*a^2*b^13*c*d^10*f^4 - 400*B*a^4*b^11*c*d^10*f^4 - 320*
B*a^6*b^9*c*d^10*f^4 - 80*B*a^8*b^7*c*d^10*f^4 + 32*B*a^10*b^5*c*d^10*f^4
+ 16*B*a^12*b^3*c*d^10*f^4 - 224*B*a^2*b^13*c^3*d^8*f^4 + 384*B*a^3*b^12*c
^2*d^9*f^4 - 400*B*a^4*b^11*c^3*d^8*f^4 + 720*B*a^5*b^10*c^2*d^9*f^4 - 320
*B*a^6*b^9*c^3*d^8*f^4 + 640*B*a^7*b^8*c^2*d^9*f^4 - 80*B*a^8*b^7*c^3*d^8*
f^4 + 240*B*a^9*b^6*c^2*d^9*f^4 + 32*B*a^10*b^5*c^3*d^8*f^4 + 16*B*a^12...
```

3.96 $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.96.1 Optimal result

Integrand size = 47, antiderivative size = 543

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= -\frac{(A-iB-C)\sqrt{c-id}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}$$
$$+\frac{(A+iB-C)\sqrt{c+id}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}$$
$$+\frac{(3a^5bBd^2+a^6Cd^2-3a^4b^2d(4Bc+5Ad-6Cd)-3a^2b^4(8Ac^2-8c^2C-16Bcd-6Ad^2+5Cd^2)+2a^3 \cdots}{\cdots}$$
$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{2b(a^2+b^2)f(a+b\tan(e+fx))^2}$$
$$-\frac{(3a^3bBd+a^4Cd+b^4(4Bc+Ad)+ab^3(8Ac-8cC-5Bd)-a^2b^2(4Bc+7Ad-9Cd))\sqrt{c+d\tan(e \cdots}{4b(a^2+b^2)^2(bc-ad)f(a+b\tan(e+fx))}$$

output
```
1/4*(3*a^5*b*B*d^2+a^6*C*d^2-3*a^4*b^2*d*(5*A*d+4*B*c-6*C*d)-3*a^2*b^4*(8*
A*c^2-6*A*d^2-16*B*c*d-8*C*c^2+5*C*d^2)+2*a^3*b^3*(20*c*(A-C)*d+B*(4*c^2-1
3*d^2))-3*a*b^5*(8*c*(A-C)*d+B*(8*c^2-d^2))-b^6*(4*c*(B*d+2*C*c)-A*(8*c^2+
d^2)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(a
^2+b^2)^3/(-a*d+b*c)^(3/2)/f-(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I
*d)^(1/2))*(c-I*d)^(1/2)/(I*a+b)^3/f+(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1
/2)/(c+I*d)^(1/2))*(c+I*d)^(1/2)/(I*a-b)^3/f-1/2*(A*b^2-a*(B*b-C*a))*(c+d*
tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2-1/4*(3*a^3*b*B*d+a^4*C*
d+b^4*(A*d+4*B*c)+a*b^3*(8*A*c-5*B*d-8*C*c)-a^2*b^2*(7*A*d+4*B*c-9*C*d))*(
c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

3.96.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2819 vs. $2(543) = 1086$.

Time = 6.72 (sec) , antiderivative size = 2819, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x])^3,x]
```

output
```
(-2*C*Sqrt[c + d*Tan[e + f*x]])/(3*b*f*(a + b*Tan[e + f*x])^2) - (2*(-1/2*
(((b^2*(-3*A*b*c + 4*b*c*C - a*C*d))/2 - a*((-3*b^2*(B*c + (A - C)*d))/2 -
 (a*(b*c*C - 3*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*
(b*c - a*d)*f*(a + b*Tan[e + f*x])^2) - (-((((I*Sqrt[c - I*d]*(b*(b*c - a*
d)*((3*b*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b
*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*
d)*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) + a*((3*(
b*c - a*d)*((b^2*d)/2 - a*(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*
(4*A*c - 4*c*C - B*d)))/4 + (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B -
a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c
*C - a*A*d - b*B*d + a*C*d)) - (d*((3*b^2*(b*c - a*d)*(a^2*C*d + b^2*(4*B*
c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4 - a*((3*a*(3*A*b^2 - 3*a*b*B - a^
2*C - 4*b^2*C)*d*(b*c - a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C
 - a*A*d - b*B*d + a*C*d))))/2) - I*(a*(b*c - a*d)*((3*b*(3*A*b^2 - 3*a*b*
B - a^2*C - 4*b^2*C)*d*(b*c - a*d))/4 + 3*a*b*(b*c - a*d)*(A*b*c - a*B*c -
 b*c*C - a*A*d - b*B*d + a*C*d) + (3*b*(b*c - a*d)*(a^2*C*d + b^2*(4*B*c +
 A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4) - b*((3*(b*c - a*d)*((b^2*d)/2 - a*
(b*c - a*d))*(a^2*C*d + b^2*(4*B*c + A*d) + a*b*(4*A*c - 4*c*C - B*d)))/4
+ (-(b*c) + (a*d)/2)*((3*a*(3*A*b^2 - 3*a*b*B - a^2*C - 4*b^2*C)*d*(b*c -
a*d))/4 - 3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*...
```

3.96.3 Rubi [A] (warning: unable to verify)

Time = 4.28 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.404$, Rules used = {3042, 4128, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-3bBa+3Ab^2-4b^2C\right)d\tan^2(e+fx)\right)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(2bc-\frac{ad}{2}\right)+Ab(4ac+bd)}{2(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}dx}{2b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 27

$$\frac{\int \frac{-\left(\left(-Ca^2-3bBa+3Ab^2-4b^2C\right)d\tan^2(e+fx)\right)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-ad)+Ab(4ac+bd)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}dx}{4b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{-\left(\left(-Ca^2-3bBa+3Ab^2-4b^2C\right)d\tan(e+fx)^2\right)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-ad)+Ab(4ac+bd)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}dx}{4b\left(a^2+b^2\right)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 4132

$$-\frac{\int -\frac{-d\left(Cda^4+3bBda^3-b^2(4Bc+7Ad-9Cd)a^2+b^3(8Ac-8Cc-5Bd)a+b^4(4Bc+Ad)\right)\tan^2(e+fx)-8b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc\ldots\right.}{2(a+b\tan(e+fx)\ldots}}{(a^2+b^2)(\ldots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 27

$$\frac{\int \frac{-d\left(Cda^4+3bBda^3-b^2(4Bc+7Ad-9Cd)a^2+b^3(8Ac-8Cc-5Bd)a+b^4(4Bc+Ad)\right)\tan^2(e+fx)-8b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A\ldots\right.}{(a+b\tan(e+fx))\sqrt{c+\ldots}}}{2(a^2+b^2)(bc\ldots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

3.96. $\displaystyle\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\int \frac{-d\left(Cda^4+3bBda^3-b^2(4Bc+7Ad-9Cd)a^2+b^3(8Ac-8Cc-5Bd)a+b^4(4Bc+Ad)\right)\tan(e+fx)^2-8b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-}{(a+b\tan(e+fx))\sqrt{c+}}}{2(a^2+b^2)(bc-}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}$$

$\downarrow$ 4136

$$\frac{\int \frac{8\left(b(bc-ad)\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(Bc+(A-C}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}$$

$\downarrow$ 27

$$\frac{8\int \frac{b(bc-ad)\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(Bc+(A-C}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{8\int \frac{b(bc-ad)\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(Bc+(A-C}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}$$

$\downarrow$ 4022

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\,(a^2+b^2)\,(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd)\right.}{}$$

$\downarrow$ 3042

3.96. $\displaystyle\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd\right)}{}}{}}{}$$

$\downarrow$ 4020

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd\right)}{}}{}}{}$$

$\downarrow$ 25

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd\right)}{}}{}}{}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd\right)}{}}{}}{}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd\right)}{}}{}}{}$$

$\downarrow$ 4117

3.96. $\displaystyle\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd)\right.}{}}{}$$

↓ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{-\frac{2\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6C\right.}{}}{}$$

↓ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(a^4Cd+3a^3bBd-a^2b^2(7Ad+4Bc-9Cd)+ab^3(8Ac-5Bd-8cC)+b^4(Ad+4Bc)\right)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))}+\frac{\frac{2\left(a^6Cd^2+3a^5bBd^2-3a^4b^2d(5Ad+4Bc-6Cd)\right.}{}}{}$$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*(
a + b*Tan[e + f*x])^2) + (((8*(((a + I*b)^3*b*(A - I*B - C)*Sqrt[c - I*d]*
(b*c - a*d)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f + ((a - I*b)^3*b*(A + I*
B - C)*Sqrt[c + I*d]*(b*c - a*d)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f))/(
a^2 + b^2) + (2*(3*a^5*b*B*d^2 + a^6*C*d^2 - 3*a^4*b^2*d*(4*B*c + 5*A*d -
6*C*d) - 3*a^2*b^4*(8*A*c^2 - 8*c^2*C - 16*B*c*d - 6*A*d^2 + 5*C*d^2) + 2*
a^3*b^3*(20*c*(A - C)*d + B*(4*c^2 - 13*d^2)) - 3*a*b^5*(8*c*(A - C)*d + B
*(8*c^2 - d^2)) - b^6*(4*c*(2*c*C + B*d) - A*(8*c^2 + d^2)))*ArcTanh[(Sqrt
[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*Sqrt[
b*c - a*d]*f))/(2*(a^2 + b^2)*(b*c - a*d)) - ((3*a^3*b*B*d + a^4*C*d + b^4
*(4*B*c + A*d) + a*b^3*(8*A*c - 8*c*C - 5*B*d) - a^2*b^2*(4*B*c + 7*A*d -
9*C*d))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e
+ f*x])))/(4*b*(a^2 + b^2))
```

3.96. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.96. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.96. $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9796 vs. 2(503) = 1006.

Time = 0.15 (sec) , antiderivative size = 9797, normalized size of antiderivative = 18.04

method	result	size
derivativedivides	Expression too large to display	9797
default	Expression too large to display	9797

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{c+d\tan{(e+fx)}}(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**3,x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x))**3, x)
```

3.96.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$$
$$= \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.96.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="giac")
```

output
```
Timed out
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^3,x)
```

output
```
\text{Hanged}
```

3.97 $\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e$

3.97.1 Optimal result

Integrand size = 47, antiderivative size = 550

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx = \frac{(ia+b)^3(A-iB-C)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$
$$+\frac{(a+ib)^3(iA-B-iC)(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$
$$+\frac{2(3a^2b(Ac-cC-Bd)-b^3(Ac-cC-Bd)+a^3(Bc+(A-C)d)-3ab^2(Bc+(A-C)d))\sqrt{c+d\tan(e}}{f}$$
$$+\frac{2(a^3B-3ab^2B+3a^2b(A-C)-b^3(A-C))(c+d\tan(e+fx))^{3/2}}{3f}$$
$$+\frac{2(168a^3Cd^3-2a^2bd^2(192cC-847Bd)+33ab^2d(8c^2C-18Bcd+63(A-C)d^2)-b^3(48c^3C-88Bc^2d+}{3465d^4f}$$
$$+\frac{2b(99b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-11bBd-6aCd))\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{693d^3f}$$
$$-\frac{2(6bcC-11bBd-6aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{99d^2f}$$
$$+\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}$$

output
```
(I*a+b)^3*(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(
1/2))/f+(a+I*b)^3*(I*A-B-I*C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)
/(c+I*d)^(1/2))/f+2*(3*a^2*b*(A*c-B*d-C*c)-b^3*(A*c-B*d-C*c)+a^3*(B*c+(A-C
)*d)-3*a*b^2*(B*c+(A-C)*d))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(B*a^3-3*B*a*b^2+
3*a^2*b*(A-C)-b^3*(A-C))*(c+d*tan(f*x+e))^(3/2)/f+2/3465*(168*a^3*C*d^3-2*
a^2*b*d^2*(-847*B*d+192*C*c)+33*a*b^2*d*(8*c^2*C-18*B*c*d+63*(A-C)*d^2)-b^
3*(48*c^3*C-88*B*c^2*d+198*c*(A-C)*d^2+693*B*d^3))*(c+d*tan(f*x+e))^(5/2)/
d^4/f+2/693*b*(99*b*(A*b+B*a-C*b)*d^2+4*(-a*d+b*c)*(-11*B*b*d-6*C*a*d+6*C*
b*c))*tan(f*x+e)*(c+d*tan(f*x+e))^(5/2)/d^3/f-2/99*(-11*B*b*d-6*C*a*d+6*C*
b*c)*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/11*C*(a+b*tan(f*x+e
))^3*(c+d*tan(f*x+e))^(5/2)/d/f
```

3.97.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1290 vs. $2(550) = 1100$.

Time = 6.57 (sec) , antiderivative size = 1290, normalized size of antiderivative = 2.35

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}$$

$$+ \frac{2\left(\frac{(-6bcC+11bBd+6aCd)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} + \frac{2\left(\frac{b\left(99b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-11bBd-6aCd)\right)\tan(e+fx)(c+d\tan(e\cdots}{14df}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
 f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2))/(11*d*f) + (2*(((-
6*b*c*C + 11*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^
(5/2))/(9*d*f) + (2*((b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c
*C - 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(14*d*f
) - (2*((2*((-7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C -
 11*b*B*d - 6*a*C*d)))/8 + b*((-693*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/8
 + (c*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d - 6*
a*C*d)))/4))*(c + d*Tan[e + f*x])^(5/2))/(5*d*f) + ((I/2)*((-7*a*d*(3*a^2*
(33*A - 25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8
 + (b*c*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 11*b*B*d -
6*a*C*d)))/4 + (7*a*d*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C
 - 11*b*B*d - 6*a*C*d)))/8 + ((7*I)/2)*d*((99*a*(a^2*B - b^2*B + 2*a*b*(A
- C))*d^2)/4 + (b*(3*a^2*(33*A - 25*C)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*
b*d*(48*c*C + 55*B*d)))/4 - (b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)
*(6*b*c*C - 11*b*B*d - 6*a*C*d)))/4) - b*((-693*(a^2*B - b^2*B + 2*a*b*(A
- C))*d^3)/8 + (c*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 1
1*b*B*d - 6*a*C*d)))/4))*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2
*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*
d) + 2*Sqrt[c + d*Tan[e + f*x]])))/f - ((I/2)*((-7*a*d*(3*a^2*(33*A - 25*C
)*d^2 + 4*b^2*c*(6*c*C - 11*B*d) - a*b*d*(48*c*C + 55*B*d)))/8 + (b*c*(...
```

3.97.3 Rubi [A] (warning: unable to verify)

Time = 4.50 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.468, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left((6bcC-6adC-11bBd)\tan^2(e+fx)-11(Ab-Cb+aB)d\tan\right.}{11d}$$
$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}$$

$\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left((6bcC-6adC-11bBd)\tan^2(e+fx)-11(Ab-Cb+aB)d\tan(e\right.}{11d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left((6bcC-6adC-11bBd)\tan(e+fx)^2-11(Ab-Cb+aB)d\tan(e\right.}{11d}$$

$\downarrow$ 4130

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(4c(6cC-11Bd)b^2-ad(48cC+55Bd)b+3a^2(33A-25C)d^2+(99b(Ab-Cb+aB)d^2+4(bc-ad)(6bcC-6\right.}{9d}}{11d}$$

$\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(4c(6cC-11Bd)b^2-ad(48cC+55Bd)b\right.}{}}{11d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(4c(6cC-11Bd)b^2-ad(48cC+55Bd)b\right.}{}}{11d}$$

$\downarrow$ 4120

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$
$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}$$

$\downarrow$ 27

3.97.

$$\int(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int (c+d\tan(e+fx))^{3/2}\left(-2c\left(24Cc^2-44Bdc+99(A-C)d^2\right)b^3+66acd(4cC-9Bd)b^2-a^2\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int (c+d\tan(e+fx))^{3/2}\left(-2c\left(24Cc^2-44Bdc+99(A-C)d^2\right)b^3+66acd(4cC-9Bd)b^2-a^2\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 4113

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int (c+d\tan(e+fx))^{3/2}\left(693\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-693(\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int (c+d\tan(e+fx))^{3/2}\left(693\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-693(\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int \sqrt{c+d\tan(e+fx)}\left(693\left((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df} - \frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df} - \frac{\int \sqrt{c+d\tan(e+fx)}\left(693\left((Ac-Cc-Bd)a^3-3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a+b^3\ldots}{\ldots}}{\ldots}$$

$\downarrow$ 4011

3.97.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\int \frac{-693\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3+3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2\right.}{}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\int \frac{-693\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3+3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2\right.}{}}{}$$

$$\Big\downarrow \text{4022}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}$$

$$\Big\downarrow \text{4020}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}$$

$$\Big\downarrow \text{25}$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}$$

$$\Big\downarrow \text{73}$$

3.97.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}}{}$$

$\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^3(c+d\tan(e+fx))^{5/2}}{11df}-$$

$$\frac{\frac{2(-6aCd-11bBd+6bcC)(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(c+d\tan(e+fx))^{5/2}\left(99bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-11bBd+6bc\right.}{7df}}{}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3*(c + d*Tan[e + f*x])^(5/2))/(11*d*f) - ((2*(6*
b*c*C - 11*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5
/2))/(9*d*f) - ((2*b*(99*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C
- 11*b*B*d - 6*a*C*d))*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(7*d*f) +
((693*(a - I*b)^3*(A - I*B - C)*(c - I*d)^(3/2)*d^3*ArcTan[Tan[e + f*x]/Sq
rt[c - I*d]])/f + (693*(a + I*b)^3*(A + I*B - C)*(c + I*d)^(3/2)*d^3*ArcTa
n[Tan[e + f*x]/Sqrt[c + I*d]])/f + (1386*d^3*(3*a^2*b*(A*c - c*C - B*d) -
b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d))
*Sqrt[c + d*Tan[e + f*x]])/f + (462*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) -
 b^3*(A - C))*d^3*(c + d*Tan[e + f*x])^(3/2))/f + (2*(168*a^3*C*d^3 - 2*a^
2*b*d^2*(192*c*C - 847*B*d) + 33*a*b^2*d*(8*c^2*C - 18*B*c*d + 63*(A - C)*
d^2) - b^3*(48*c^3*C - 88*B*c^2*d + 198*c*(A - C)*d^2 + 693*B*d^3))*(c + d
*Tan[e + f*x])^(5/2))/(5*d*f))/(7*d))/(9*d))/(11*d)
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.97.
$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 10951 vs. $2(507) = 1014$.

Time = 0.34 (sec) , antiderivative size = 10952, normalized size of antiderivative = 19.91

method	result	size
parts	Expression too large to display	10952
derivativedivides	Expression too large to display	11056
default	Expression too large to display	11056

input
```
int((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84950 vs. $2(498) = 996$.

Time = 194.27 (sec) , antiderivative size = 84950, normalized size of antiderivative = 154.45

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.97.6 Sympy [F]

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int (a+b\tan{(e+fx)})^3\,(c+d\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*ta
n(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

3.97.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.97.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^3(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^3*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x)
+ C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.98 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2} \left(A+B\tan(e\right.$

3.98.1 Optimal result

Integrand size = 47, antiderivative size = 396

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$

$$-\frac{(a-ib)^2(B+i(A-C))(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$
$$+\frac{(a+ib)^2(iA-B-iC)(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$
$$+\frac{2(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\sqrt{c+d\tan(e+fx)}}{f}$$
$$+\frac{2(a^2B-b^2B+2ab(A-C))\,(c+d\tan(e+fx))^{3/2}}{3f}$$
$$+\frac{2(28a^2Cd^2-18abd(2cC-7Bd)+b^2(8c^2C-18Bcd+63(A-C)d^2))\,(c+d\tan(e+fx))^{5/2}}{315d^3f}$$
$$-\frac{2b(4bcC-9bBd-4aCd)\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{63d^2f}$$
$$+\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}$$

3.98.
$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
-(a-I*b)^2*(B+I*(A-C))*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d
)^(1/2))/f+(a+I*b)^2*(I*A-B-I*C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1
/2)/(c+I*d)^(1/2))/f+2*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-
C)*d))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(B*a^2-B*b^2+2*a*b*(A-C))*(c+d*tan(f*x
+e))^(3/2)/f+2/315*(28*a^2*C*d^2-18*a*b*d*(-7*B*d+2*C*c)+b^2*(8*c^2*C-18*B
*c*d+63*(A-C)*d^2))*(c+d*tan(f*x+e))^(5/2)/d^3/f-2/63*b*(-9*B*b*d-4*C*a*d+
4*C*b*c)*tan(f*x+e)*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/9*C*(a+b*tan(f*x+e))^2*
(c+d*tan(f*x+e))^(5/2)/d/f
```

3.98.2 Mathematica [A] (verified)

Time = 6.43 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.29

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}+\frac{2\left(\frac{b(-4bcC+9bBd+4aCd)\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-\frac{2\left(\frac{\left(-28a^2Cd^2+18abd(2cC-7Bd)-b^2\left(8c^2C-18Bcd+63(A-C)d^2\right)\right)(c+d\tan(e+fx))^{5/2}}{10df}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e +
 f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2))/(9*d*f) + (2*((b*(
-4*b*c*C + 9*b*B*d + 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(7*
d*f) - (2*(((-28*a^2*C*d^2 + 18*a*b*d*(2*c*C - 7*B*d) - b^2*(8*c^2*C - 18*
B*c*d + 63*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2))/(10*d*f) + ((I/2)*(((
63*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (63*(2*a*b*B - a^2*(A - C)
+ b^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(
c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d)
 + 2*Sqrt[c + d*Tan[e + f*x]])))/f - ((I/2)*(((-63*I)/4)*(a^2*B - b^2*B +
2*a*b*(A - C))*d^2 + (63*(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^2)/4)*((2
*(c + d*Tan[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I*d)^(3/2)*ArcTanh[Sqr
t[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*Sqrt[c + d*Tan[e + f*
x]])))/f))/(7*d)))/(9*d)
```

3.98.3 Rubi [A] (warning: unable to verify)

Time = 2.92 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.02, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.404$, Rules used = {3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4130$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left((4bcC-4adC-9bBd)\tan^2(e+fx)-9(Ab-Cb+aB)d\tan(e\right.}{9d}$$
$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left((4bcC-4adC-9bBd)\tan^2(e+fx)-9(Ab-Cb+aB)d\tan(e+f\right.}{9d}$$

$\Big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left((4bcC-4adC-9bBd)\tan(e+fx)^2-9(Ab-Cb+aB)d\tan(e+f\right.}{9d}$$

$\Big\downarrow$ 4120

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}-\frac{2\int-\frac{1}{2}(c+d\tan(e+fx))^{3/2}\left(-2c(4cC-9Bd)b^2+36acCdb-7a^2(9A-5C)d^2-\left((8Cc^2-\right.\right.}{}}{9d}$$

$\Big\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{\int(c+d\tan(e+fx))^{3/2}\left(-2c(4cC-9Bd)b^2+36acCdb-7a^2(9A-5C)d^2-\left(\left(8Cc^2-18Bdc+63(A-C)d^2\right)b^2-18ad(2cC-7Bd)b+28a^2Cd^2\right)\tan^2(e+f\right.}{7d}}{9d}$$

$\Big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{\int(c+d\tan(e+fx))^{3/2}\left(-2c(4cC-9Bd)b^2+36acCdb-7a^2(9A-5C)d^2-\left(\left(8Cc^2-18Bdc+63(A-C)d^2\right)b^2-18ad(2cC-7Bd)b+28a^2Cd^2\right)\tan(e+fx)\right.}{7d}}{9d}$$

$\Big\downarrow$ 4113

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{\int(c+d\tan(e+fx))^{3/2}\left(63\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-63\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd\right.}{}}{7d}}{9d}$$

$\Big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-\frac{\frac{\int(c+d\tan(e+fx))^{3/2}\left(63\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-63\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd\right.}{}}{7d}}{9d}$$

$\Big\downarrow$ 4011

3.98.

$$\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\int \sqrt{c+d\tan(e+fx)}(-63((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd))d^2-63((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\int \sqrt{c+d\tan(e+fx)}(-63((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd))d^2-63((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)d)$$

$\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\int \frac{63\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+63\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2\right)+2b\left(Cc^2+2B\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\int \frac{63\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+63\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2\right)+2b\left(Cc^2+2B\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4022

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{-\frac{63}{2}d^2(a+ib)^2(c+id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{63}{2}d^2(a-ib)^2(c-id)^2(A-iB}{}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{-\frac{63}{2}d^2(a+ib)^2(c+id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{63}{2}d^2(a-ib)^2(c-id)^2(A-iB}{}$$

$\downarrow$ 4020

3.98.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$
$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{-\frac{63id^2(a-ib)^2(c-id)^2(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{63i\cdots}{}}{}$$

$\downarrow$ 25

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$
$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{\frac{63id^2(a-ib)^2(c-id)^2(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{63id^2(c\cdots}{}}{}$$

$\downarrow$ 73

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$
$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{-\frac{63d(a+ib)^2(c+id)^2(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{63d(a-ib)^2(c-\cdots}{}}{}$$

$\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}{9df}-$$
$$\frac{2b\tan(e+fx)(-4aCd-9bBd+4bcC)(c+d\tan(e+fx))^{5/2}}{7df}+\frac{-\frac{2(c+d\tan(e+fx))^{5/2}\left(28a^2Cd^2-18abd(2cC-7Bd)+b^2\left(63d^2(A-C)-18Bcd+8c^2C\right)\right)}{5df}-\frac{42d\cdots}{}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2))/(9*d*f) - ((2*b*(4
*b*c*C - 9*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(7*d*
f) + ((-63*(a - I*b)^2*(A - I*B - C)*(c - I*d)^(3/2)*d^2*ArcTan[Tan[e + f*
x]/Sqrt[c - I*d]])/f - (63*(a + I*b)^2*(A + I*B - C)*(c + I*d)^(3/2)*d^2*A
rcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f - (126*d^2*(2*a*b*(A*c - c*C - B*d) +
 a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*Sqrt[c + d*Tan[e + f*x]])/
f - (42*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(c + d*Tan[e + f*x])^(3/2))/f
- (2*(28*a^2*C*d^2 - 18*a*b*d*(2*c*C - 7*B*d) + b^2*(8*c^2*C - 18*B*c*d +
63*(A - C)*d^2))*(c + d*Tan[e + f*x])^(5/2))/(5*d*f))/(7*d))/(9*d)
```

3.98.
$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7938 vs. $2(357) = 714$.

Time = 0.21 (sec) , antiderivative size = 7939, normalized size of antiderivative = 20.05

method	result	size
parts	Expression too large to display	7939
derivativedivides	Expression too large to display	8031
default	Expression too large to display	8031

3.98.
$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

input
```
int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58971 vs. $2(347) = 694$.

Time = 69.93 (sec) , antiderivative size = 58971, normalized size of antiderivative = 148.92

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.98.6 Sympy [F]

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*ta
n(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

3.98.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.98.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x)
+ C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.99 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2} (A+B\tan(e+$

3.99.1 Optimal result

Integrand size = 45, antiderivative size = 273

$$\begin{aligned}\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = & -\frac{(ia+b)(A-iB-C)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} \\ & +\frac{(ia-b)(A+iB-C)(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} \\ & +\frac{2(Abc+aBc-bcC+aAd-bBd-aCd)\sqrt{c+d\tan(e+fx)}}{f} \\ & +\frac{2(Ab+aB-bC)(c+d\tan(e+fx))^{3/2}}{3f} \\ & -\frac{2(2bcC-7bBd-7aCd)(c+d\tan(e+fx))^{5/2}}{35d^2f} \\ & +\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}\end{aligned}$$

output
```
-(I*a+b)*(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1
/2))/f+(I*a-b)*(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I
*d)^(1/2))/f+2*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d-C*b*c)*(c+d*tan(f*x+e))^(1/2
)/f+2/3*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(3/2)/f-2/35*(-7*B*b*d-7*C*a*d+2*C*
b*c)*(c+d*tan(f*x+e))^(5/2)/d^2/f+2/7*b*C*tan(f*x+e)*(c+d*tan(f*x+e))^(5/2
)/d/f
```

3.99.
$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$

3.99.2 Mathematica [A] (verified)

Time = 4.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{\frac{2(-2bcC+7bBd+7aCd)(c+d\tan(e+fx))^{5/2}}{d}+10bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}+\frac{35}{3}(ia}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f
*x] + C*Tan[e + f*x]^2),x]
```

output
```
((2*(-2*b*c*C + 7*b*B*d + 7*a*C*d)*(c + d*Tan[e + f*x])^(5/2))/d + 10*b*C*
Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2) + (35*(I*a + b)*(A - I*B - C)*d*(-
3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c
 + d*Tan[e + f*x]]*(4*c - (3*I)*d + d*Tan[e + f*x])))/3 + (35*((-I)*a + b)
*(A + I*B - C)*d*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt
[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3
)/(35*d*f)
```

3.99.3 Rubi [A] (warning: unable to verify)

Time = 1.67 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.356, Rules used = {3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{2\int\frac{1}{2}(c+d\tan(e+fx))^{3/2}\left((2bcC-7adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+fx)+2bcC-7a\right.}{7d}$$

$\downarrow$ 27

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}\left((2bcC-7adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+fx)+2bcC-7aAd\right)}{7d}$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}\left((2bcC-7adC-7bBd)\tan(e+fx)^2-7(Ab-Cb+aB)d\tan(e+fx)+2bcC-7aAd\right)}{7d}$$

$\downarrow$ 4113

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}(7(bB-a(A-C))d-7(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-7aCd-7bBd+2bcC)(c+d\tan(e+}{5df}}{7d}$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}(7(bB-a(A-C))d-7(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-7aCd-7bBd+2bcC)(c+d\tan(e+}{5df}}{7d}$$

$\downarrow$ 4011

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int\sqrt{c+d\tan(e+fx)}(7d(bBc+b(A-C)d-a(Ac-Cc-Bd))-7d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{7d}$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{\int\sqrt{c+d\tan(e+fx)}(7d(bBc+b(A-C)d-a(Ac-Cc-Bd))-7d(Abc+aBc-bCc+aAd-bBd-aCd)\tan}{7d}$$

$\downarrow$ 4011

3.99.

$$\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\int \frac{7d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-7d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+}{\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\int \frac{7d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-7d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+}{\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4022

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$-\frac{7}{2}d(a+ib)(c+id)^2(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{7}{2}d(a-ib)(c-id)^2(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-$$

$\big\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$-\frac{7}{2}d(a+ib)(c+id)^2(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{7}{2}d(a-ib)(c-id)^2(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-$$

$\big\downarrow$ 4020

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$-\frac{7id(a-ib)(c-id)^2(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{7id(a+ib)(c+id)^2(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e}}}{2f}$$

$\big\downarrow$ 25

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$\frac{7id(a-ib)(c-id)^2(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{7id(a+ib)(c+id)^2(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d}{2f}$$

$\big\downarrow$ 73

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-$$
$$-\frac{7(a+ib)(c+id)^2(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{7(a-ib)(c-id)^2(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}$$

3.99.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{221}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{5/2}}{7df}-\frac{-\frac{7d(a-ib)(c-id)^{3/2}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}-\frac{7d(a+ib)(c+id)^{3/2}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f}-\frac{14d(aB+Ab-bC)(c+d\tan(e+fx)}{3f}}{7d}$$

input
```
Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] +
C*Tan[e + f*x]^2),x]
```

output
```
(2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(5/2))/(7*d*f) - ((-7*(a - I*b)*(
A - I*B - C)*(c - I*d)^(3/2)*d*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f - (7*
(a + I*b)*(A + I*B - C)*(c + I*d)^(3/2)*d*ArcTan[Tan[e + f*x]/Sqrt[c + I*d
]])/f - (14*d*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*Sqrt[c + d*T
an[e + f*x]])/f - (14*(A*b + a*B - b*C)*d*(c + d*Tan[e + f*x])^(3/2))/(3*f
) + (2*(2*b*c*C - 7*b*B*d - 7*a*C*d)*(c + d*Tan[e + f*x])^(5/2))/(5*d*f))/
(7*d)
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.99. $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5106 vs. $2(239) = 478$.

Time = 0.19 (sec) , antiderivative size = 5107, normalized size of antiderivative = 18.71

method	result	size
parts	Expression too large to display	5107
derivativedivides	Expression too large to display	5149
default	Expression too large to display	5149

input
```
int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31081 vs. $2(232) = 464$.

Time = 16.16 (sec) , antiderivative size = 31081, normalized size of antiderivative = 113.85

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.99.6 Sympy [F]

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (a+b\tan(e+fx))\,(c+d\tan(e+fx))^{\frac{3}{2}}\,(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

3.99.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.99.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Hanged}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) +
C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.100 $\int (c+d\tan(e+fx))^{3/2} (A+B\tan(e+fx)+C\tan^2($

3.100.1 Optimal result

Integrand size = 35, antiderivative size = 187

$$\begin{aligned}\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx= & -\frac{(iA+B-iC)(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f} \\ & -\frac{(B-i(A-C))(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f} \\ & +\frac{2(Bc+(A-C)d)\sqrt{c+d\tan(e+fx)}}{f} \\ & +\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}\end{aligned}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f
-(B-I*(A-C))*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f
+2*(B*c+(A-C)*d)*(c+d*tan(f*x+e))^(1/2)/f+2/3*B*(c+d*tan(f*x+e))^(3/2)/f+2
/5*C*(c+d*tan(f*x+e))^(5/2)/d/f
```

3.100.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int (c+d\tan(e+fx))^{3/2} (A+B\tan(e+fx) +C\tan^2(e+fx)) \, dx = \frac{\frac{6C(c+d\tan(e+fx))^{5/2}}{d}+5(iA+B-iC)\left(-3(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)+\sqrt{c+\cdots}\right.}{}$$

input
```
Integrate[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2),x]
```

output
```
((6*C*(c + d*Tan[e + f*x])^(5/2))/d + 5*(I*A + B - I*C)*(-3*(c - I*d)^(3/2
)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x
]]*(4*c - (3*I)*d + d*Tan[e + f*x])) + 5*((-I)*A + B + I*C)*(-3*(c + I*d)^
(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e +
 f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/(15*f)
```

3.100.3 Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4113, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \text{4113}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^{3/2}dx+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^{3/2}dx+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{c+d\tan(e+fx)}(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+d\tan(e+fx)}(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B(c^2-d^2)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B(c^2-d^2)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(c+id)^2(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c-id)^2(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(c+id)^2(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c-id)^2(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(c-id)^2(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(c+id)^2(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

3.100. $\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\Big\downarrow \ 25$$

$$-\frac{i(c-id)^2(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{i(c+id)^2(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \ 73$$

$$\frac{(c+id)^2(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(c-id)^2(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

$$\Big\downarrow \ 221$$

$$\frac{(c-id)^{3/2}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{(c+id)^{3/2}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f}+\frac{2(d(A-C)+Bc)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2B(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5df}$$

input
```
Int[(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
((A - I*B - C)*(c - I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f + ((A
 + I*B - C)*(c + I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f + (2*(B*
c + (A - C)*d)*Sqrt[c + d*Tan[e + f*x]])/f + (2*B*(c + d*Tan[e + f*x])^(3/
2))/(3*f) + (2*C*(c + d*Tan[e + f*x])^(5/2))/(5*d*f)
```

3.100. $\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2499 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 2500, normalized size of antiderivative = 13.37

method	result	size
parts	Expression too large to display	2500
derivativedivides	Expression too large to display	2517
default	Expression too large to display	2517

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETUR
NVERBOSE)
```

output
```
A*(2/f*d*(c+d*tan(f*x+e))^(1/2)+1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+
d^2)^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2
*c)^(1/2)*(c^2+d^2)^(1/2)*c-1/4/f/d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)
^(1/2)+2*c)^(1/2)-d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^
(1/2)*c^2+1/4/f*d*ln((c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-
d*tan(f*x+e)-c-(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2/f*d/(2*(c^
2+d^2)^(1/2)-2*c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f
*x+e))^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c+1/f*d/(2*(c^2+d^2)^(1/2)-2*
c)^(1/2)*arctan(((2*(c^2+d^2)^(1/2)+2*c)^(1/2)-2*(c+d*tan(f*x+e))^(1/2))/(
2*(c^2+d^2)^(1/2)-2*c)^(1/2))*(c^2+d^2)^(1/2)-1/4/f/d*ln(d*tan(f*x+e)+c+(c
+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^
2+d^2)^(1/2)+2*c)^(1/2)*(c^2+d^2)^(1/2)*c+1/4/f/d*ln(d*tan(f*x+e)+c+(c+d*t
an(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^
2)^(1/2)+2*c)^(1/2)*c^2-1/4/f*d*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(
2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)+2/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*
(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c-1/f*d/(2*(c^2
+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)+(2*(c^2+d^2)^(1/2)
+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*(c^2+d^2)^(1/2))+B*(2/3/f*(c+d
*tan(f*x+e))^(3/2)+2/f*(c+d*tan(f*x+e))^(1/2)*c-1/4/f*ln((c+d*tan(f*x+e...
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6846 vs. $2(151) = 302$.

Time = 0.95 (sec) , antiderivative size = 6846, normalized size of antiderivative = 36.61

$$\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="fricas")
```

output
```
Too large to include
```

3.100.6 Sympy [F]

$$\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int (c+d\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2), x)
```

3.100.7 Maxima [F]

$$\int (c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int \left(C\tan{(fx+e)}^2+B\tan{(fx+e)}+A\right)(d\tan{(fx+e)}+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/
2), x)
```

3.100.8 Giac [F(-1)]

Timed out.

$$\int (c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="giac")
```

output
```
Timed out
```

3.100.9 Mupad [B] (verification not implemented)

Time = 42.33 (sec) , antiderivative size = 4260, normalized size of antiderivative = 22.78

$$\int (c+d\tan(e+fx))^{3/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \text{Too large to display}$$

input
```
int((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
((2*C*c^2)/(d*f) - (2*C*(d^3*f + c^2*d*f))/(d^2*f^2))*(c + d*tan(e + f*x))
^(1/2) - log((((16*c*d^2*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*
f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*d^2 + f*(((-B^4*d^2*f^4*(3*c^
2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e
 + f*x))^(1/2)))/f - (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6
*c^2*d^2))/f^2)*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B
^2*c*d^2*f^2)/f^4)^(1/2))/2 - (8*B^3*d^2*(c^2 - d^2)*(c^2 + d^2)^2)/f^3)*(
((6*B^4*c^2*d^4*f^4 - B^4*d^6*f^4 - 9*B^4*c^4*d^2*f^4)^(1/2) + B^2*c^3*f^2
 - 3*B^2*c*d^2*f^2)/(4*f^4))^(1/2) - log((((16*c*d^2*(-((-B^4*d^2*f^4*(3*c
^2 - d^2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*
d^2 + f*(-((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^
2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/f - (16*B^2*d^2*(c + d*tan(
e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/f^2)*(-((-B^4*d^2*f^4*(3*c^2 - d^
2)^2)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/f^4)^(1/2))/2 - (8*B^3*d^2*(c
^2 - d^2)*(c^2 + d^2)^2)/f^3)*(-((6*B^4*c^2*d^4*f^4 - B^4*d^6*f^4 - 9*B^4*
c^4*d^2*f^4)^(1/2) - B^2*c^3*f^2 + 3*B^2*c*d^2*f^2)/(4*f^4))^(1/2) + log((
((16*c*d^2*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1/2) + B^2*c^3*f^2 - 3*B^2*c*
d^2*f^2)/f^4)^(1/2)*(B*c^2 + B*d^2 - f*(((-B^4*d^2*f^4*(3*c^2 - d^2)^2)^(1
/2) + B^2*c^3*f^2 - 3*B^2*c*d^2*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)
))/f + (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^4 + d^4 - 6*c^2*d^2))/...
```

3.101 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.101.1 Optimal result

Integrand size = 47, antiderivative size = 271

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = -\frac{(iA+B-iC)(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f} - \frac{(A+iB-C)(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)f} - \frac{2(Ab^2-a(bB-aC))\,(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{5/2}\,(a^2+b^2)\,f} + \frac{2(bcC+bBd-aCd)\sqrt{c+d\tan(e+fx)}}{b^2f} + \frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(
a-I*b)/f-(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(I*a-b)/f-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(c+d
*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(a^2+b^2)/f+2*(B*b*d-C*a*d+C*
b*c)*(c+d*tan(f*x+e))^(1/2)/b^2/f+2/3*C*(c+d*tan(f*x+e))^(3/2)/b/f
```

3.101.2 Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.98

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{3ib\left(-\left((a+ib)(A-iB-C)(c-id)^{3/2}\text{arctanh}\left(\right.\right.\right.}{}$$

input

```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x]),x]
```

output

```
(((3*I)*b*(-((a + I*b)*(A - I*B - C)*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Ta
n[e + f*x]]/Sqrt[c - I*d]]) + (a - I*b)*(A + I*B - C)*(c + I*d)^(3/2)*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]))/(a^2 + b^2) - (6*(A*b^2 + a*
(-(b*B) + a*C))*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]
])/Sqrt[b*c - a*d]])/(b^(3/2)*(a^2 + b^2)) + (6*(b*c*C + b*B*d - a*C*d)*Sq
rt[c + d*Tan[e + f*x]])/b + 2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f)
```

3.101.3 Rubi [A] (warning: unable to verify)

Time = 2.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.404$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \ 4130$$

$$\frac{2\int \frac{3\sqrt{c+d\tan(e+fx)}\left((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{2(a+b\tan(e+fx))}dx}{3b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((bcC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((bcC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 4130

$$\frac{\frac{2\int \frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2(-aCd+bBd+\cdots}{}}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2(-aCd+bBd+bc\cdots}{}}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan(e+fx)^2+ad(aCd-b(2cC+Bd))}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{2(-aCd+bBd+bc\cdots}{}}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 4136

$$\frac{\frac{\frac{\int -\frac{b^2\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{(bc-\cdots}{}}{b}}{b}+\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

3.101. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\int \frac{b^2\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2\left(2aAcd-2acCd-A\right.}{\sqrt{c+d\tan(e+fx)}} \cdots}{a^2+b^2}}{b}}{b}$$

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\int \frac{b^2\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2\left(2aAcd-2acCd-A\right.}{\sqrt{c+d\tan(e+fx)}} \cdots}{a^2+b^2}}{b}}{b}$$

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}$$

$\Big\downarrow$ 4022

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$

$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{-\frac{1}{2}b^2(a+ib)(c-id)^2(A-iB-C)\int \frac{i\tan(e+ \cdots}{\sqrt{c+d\tan \cdots}}}{\cdots}}{b}}{b}$$

$\Big\downarrow$ 3042

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$

$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{-\frac{1}{2}b^2(a+ib)(c-id)^2(A-iB-C)\int \frac{i\tan(e+ \cdots}{\sqrt{c+d\tan \cdots}}}{\cdots}}{b}}{b}$$

$\Big\downarrow$ 4020

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$

$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{ib^2(a-ib)(c+id)^2(A+iB-C)\int -\frac{\cdots}{(i\tan(e+f \cdots}}{2f}}{\cdots}}{\cdots}}{b}$$

$\Big\downarrow$ 25

3.101. $\displaystyle\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{ib^2(a+ib)(c-id)^2(A-iB-C)\int\frac{\cdots}{(1-i\tan(e+\cdots}}{2f}\cdots}{b}}{\cdots}$$

$\downarrow$ 73

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{-\frac{b^2(a+ib)(c-id)^2(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+f\cdots}{d}\cdots}}{df}\cdots}{b}}{b}$$

$\downarrow$ 221

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{-\frac{b^2(a+ib)(c-id)^{3/2}(A-iB-C)\arctan\left(\frac{\tan\cdots}{\sqrt{\cdots}}\right.}{f}\cdots}{b}}{b}$$

$\downarrow$ 4117

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f\left(a^2+b^2\right)}-\frac{-\frac{b^2(a+ib)(c-id)^{3/2}(A-iB-C)\,\mathrm{ar}\cdots}{f}\cdots}{b}}{b}$$

$\downarrow$ 73

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\frac{2(bc-ad)^2\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df\left(a^2+b^2\right)}-\frac{-\frac{b^2(a+ib)(c-id)^{3/2}(A-iB-C)\,\mathrm{ar}\cdots}{f}\cdots}{b}}{b}$$

$\downarrow$ 221

$$\frac{2C(c+d\tan(e+fx))^{3/2}}{3bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)\sqrt{c+d\tan(e+fx)}}{bf}+\frac{-\frac{2(bc-ad)^{3/2}\left(Ab^2-a(bB-aC)\right)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f\left(a^2+b^2\right)}-\frac{-\frac{b^2(a+ib)(c-id)^{3/2}(A-iB-C)\arctan\left(\frac{\mathrm{ta}\cdots}{\cdots}\right.}{f}\cdots}{b}}{b}$$

3.101. $$\int\frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x]),x]
```

output
```
(2*C*(c + d*Tan[e + f*x])^(3/2))/(3*b*f) + ((-((-(((a + I*b)*b^2*(A - I*B
- C)*(c - I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f) - ((a - I*b)*b
^2*(A + I*B - C)*(c + I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f)/(a
^2 + b^2)) - (2*(A*b^2 - a*(b*B - a*C))*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]
*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*f))/b +
(2*(b*c*C + b*B*d - a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(b*f))/b
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.101. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.101.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6054 vs. 2(234) = 468.

Time = 0.15 (sec) , antiderivative size = 6055, normalized size of antiderivative = 22.34

method	result	size
derivativedivides	Expression too large to display	6055
default	Expression too large to display	6055

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.101.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.101.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx=\int \frac{(c+d\tan\left(e+fx\right))^{\frac{3}{2}}\left(A+B\tan\left(\right.\right.}{a+b\tan\left(e\right.}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e)),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x)), x)
```

3.101.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.101.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.101.9 Mupad [B] (verification not implemented)

Time = 55.55 (sec) , antiderivative size = 106783, normalized size of antiderivative = 394.03

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x)),x)
```

output

```
atan(((((((32*(4*B*a*b^8*d^12*f^4 - 4*B*b^9*c*d^11*f^4 + 8*B*a^3*b^6*d^12*
f^4 + 4*B*a^5*b^4*d^12*f^4 - 4*B*b^9*c^3*d^9*f^4 + 8*B*a*b^8*c^2*d^10*f^4
+ 4*B*a*b^8*c^4*d^8*f^4 - 12*B*a^2*b^7*c*d^11*f^4 - 12*B*a^4*b^5*c*d^11*f^
4 - 4*B*a^6*b^3*c*d^11*f^4 - 12*B*a^2*b^7*c^3*d^9*f^4 + 16*B*a^3*b^6*c^2*d
^10*f^4 + 8*B*a^3*b^6*c^4*d^8*f^4 - 12*B*a^4*b^5*c^3*d^9*f^4 + 8*B*a^5*b^4
*c^2*d^10*f^4 + 4*B*a^5*b^4*c^4*d^8*f^4 - 4*B*a^6*b^3*c^3*d^9*f^4))/(b*f^5
) - (32*(c + d*tan(e + f*x))^(1/2)*(-(((8*B^2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*
f^2 - 16*B^2*a*b*d^3*f^2 - 24*B^2*a^2*c*d^2*f^2 + 24*B^2*b^2*c*d^2*f^2 + 4
8*B^2*a*b*c^2*d*f^2)^2/4 - (16*a^4*f^4 + 16*b^4*f^4 + 32*a^2*b^2*f^4)*(B^4
*c^6 + B^4*d^6 + 3*B^4*c^2*d^4 + 3*B^4*c^4*d^2))^(1/2) - 4*B^2*a^2*c^3*f^2
 + 4*B^2*b^2*c^3*f^2 + 8*B^2*a*b*d^3*f^2 + 12*B^2*a^2*c*d^2*f^2 - 12*B^2*b
^2*c*d^2*f^2 - 24*B^2*a*b*c^2*d*f^2)/(16*(a^4*f^4 + b^4*f^4 + 2*a^2*b^2*f^
4)))^(1/2)*(16*b^10*d^10*f^4 + 16*a^2*b^8*d^10*f^4 - 16*a^4*b^6*d^10*f^4 -
 16*a^6*b^4*d^10*f^4 + 24*b^10*c^2*d^8*f^4 + 40*a^2*b^8*c^2*d^8*f^4 + 8*a^
4*b^6*c^2*d^8*f^4 - 8*a^6*b^4*c^2*d^8*f^4 + 8*a*b^9*c*d^9*f^4 + 24*a^3*b^7
*c*d^9*f^4 + 24*a^5*b^5*c*d^9*f^4 + 8*a^7*b^3*c*d^9*f^4))/(b*f^4))*(-(((8*
B^2*a^2*c^3*f^2 - 8*B^2*b^2*c^3*f^2 - 16*B^2*a*b*d^3*f^2 - 24*B^2*a^2*c*d^
2*f^2 + 24*B^2*b^2*c*d^2*f^2 + 48*B^2*a*b*c^2*d*f^2)^2/4 - (16*a^4*f^4 + 1
6*b^4*f^4 + 32*a^2*b^2*f^4)*(B^4*c^6 + B^4*d^6 + 3*B^4*c^2*d^4 + 3*B^4*c^4
*d^2))^(1/2) - 4*B^2*a^2*c^3*f^2 + 4*B^2*b^2*c^3*f^2 + 8*B^2*a*b*d^3*f^...
```

3.101. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.102 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.102.1 Optimal result

Integrand size = 47, antiderivative size = 372

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx =$$
$$-\frac{(iA+B-iC)(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}$$
$$-\frac{(B-i(A-C))(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2f}$$
$$+\frac{\sqrt{bc-ad}(a^3bBd-3a^4Cd-b^4(2Bc+3Ad)-ab^3(4Ac-4cC-5Bd)+a^2b^2(2Bc+(A-7C)d))\operatorname{arctanh}}{b^{5/2}(a^2+b^2)^2f}$$
$$+\frac{(Ab^2-abB+3a^2C+2b^2C)\,d\sqrt{c+d\tan(e+fx)}}{b^2(a^2+b^2)f}$$
$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(
a-I*b)^2/f-(B-I*(A-C))*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d
)^(1/2))/(a+I*b)^2/f+(a^3*b*B*d-3*a^4*C*d-b^4*(3*A*d+2*B*c)-a*b^3*(4*A*c-5
*B*d-4*C*c)+a^2*b^2*(2*B*c+(A-7*C)*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1
/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)/(a^2+b^2)^2/f+(A*b^2-B*a*b+
3*C*a^2+2*C*b^2)*d*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^2)/f-(A*b^2-a*(B*b-C*
a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.102.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2738 vs. $2(372) = 744$.

Time = 6.61 (sec) , antiderivative size = 2738, normalized size of antiderivative = 7.36

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^2,x]
```

output
```
(2*C*(c + d*Tan[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x])) + (2*(-(((3*b*
c*C + b*B*d - 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])
)) - (2*(-((((I*Sqrt[c - I*d]*(b*(b*c - a*d)*((b*(-(A*b^2*c^2) + 3*a^2*C*d
^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*(6*c*C + B*d)))/4 + (a*b^2*(2*c*(A - C)
*d + B*(c^2 - d^2)))/4 - (b*(3*a^2*C*d^2 - a*b*d*(6*c*C + B*d) + b^2*(3*c^
2*C - (A - C)*d^2)))/4) + a*((((b^2*d)/2 - a*(b*c - a*d))*(-(A*b^2*c^2) +
3*a^2*C*d^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*(6*c*C + B*d)))/4 + (-(b*c) +
(a*d)/2)*(-1/4*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))) - (a*(3*a^2*C*d^2 - a
*b*d*(6*c*C + B*d) + b^2*(3*c^2*C - (A - C)*d^2)))/4) - (d*((b^2*(-(A*b^2*
c^2) + 3*a^2*C*d^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*(6*c*C + B*d)))/4 - a*(
-1/4*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))) - (a*(3*a^2*C*d^2 - a*b*d*(6*c*
C + B*d) + b^2*(3*c^2*C - (A - C)*d^2)))/4)))/2) - I*(a*(b*c - a*d)*((b*(-
(A*b^2*c^2) + 3*a^2*C*d^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*(6*c*C + B*d)))/
4 + (a*b^2*(2*c*(A - C)*d + B*(c^2 - d^2)))/4 - (b*(3*a^2*C*d^2 - a*b*d*(6
*c*C + B*d) + b^2*(3*c^2*C - (A - C)*d^2)))/4) - b*((((b^2*d)/2 - a*(b*c -
 a*d))*(-(A*b^2*c^2) + 3*a^2*C*d^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*(6*c*C
+ B*d)))/4 + (-(b*c) + (a*d)/2)*(-1/4*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))
) - (a*(3*a^2*C*d^2 - a*b*d*(6*c*C + B*d) + b^2*(3*c^2*C - (A - C)*d^2)))/
4) - (d*((b^2*(-(A*b^2*c^2) + 3*a^2*C*d^2 + 2*b^2*c*(2*c*C + B*d) - a*b*d*
(6*c*C + B*d)))/4 - a*(-1/4*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))) - (a*...
```

3.102.3 Rubi [A] (warning: unable to verify)

Time = 3.69 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.04, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.404, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3Ca^2-bBa+Ab^2+2b^2C)d\tan^2(e+fx)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(bc-\frac{3ad}{2}\right)+2Ab\left(ac+\frac{3bd}{2}\right)}{2(a+b\tan(e+fx))}}{b\left(a^2+b^2\right)} \frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3Ca^2-bBa+Ab^2+2b^2C)d\tan^2(e+fx)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-3ad)+Ab(2ac+3bd)\right)}{a+b\tan(e+fx)}d}{2b\left(a^2+b^2\right)} \frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3Ca^2-bBa+Ab^2+2b^2C)d\tan(e+fx)^2-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-3ad)+Ab(2ac+3bd)\right)}{a+b\tan(e+fx)}d}{2b\left(a^2+b^2\right)} \frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{\frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-c((bB-aC)(2bc-3ad)+Ab(2ac+3bd))b+a\left(3Ca^2-bBa+Ab^2+2b^2C\right)d^2+}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}}{b}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int \frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-c((bB-aC)(2bc-3a}{}}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int \frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-c((bB-aC)(2bc-3a}{}}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4136

$$\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int \frac{2\left(b^2\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2-2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-\right.\right.\right.\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{2\int \frac{b^2\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2-2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d\right.\right.\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

3.102. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{2d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{2\int\frac{b^2\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2-2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d\right.\right.\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4022

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(}{(a+b\tan(e+f}}{a^2+b^2}}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(}{(a+b\tan(e+f}}{a^2+b^2}}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 4020

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{2d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(}{(a+b\tan(e+f}}{a^2+b^2}}{}$$

$\downarrow$ 25

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{2d(3a^2C-abB+Ab^2+2b^2C)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(}{(a+b\tan(e+f}}{a^2+b^2}}{}$$

$\downarrow$ 73

3.102. $$\int\frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(\cdots}{(a+b\tan(e+f\cdots}}{a^2+b^2}}{}}{}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{\tan(\cdots}{(a+b\tan(e+f\cdots}}{a^2+b^2}}{}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{}{(a+b\tan(e+f\cdots}}{f\left(a^2+b^2\right)}}{}}{2b\left(a^2+b^2\cdots\right.}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\frac{2(bc-ad)\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\int\frac{1}{a+\frac{b(c+d\tan\cdots}{d}}}{df\left(a^2+b^2\right)}}{}}{2b\left(a^2+b^2\cdots\right.}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+$$

$$\frac{\frac{2d\left(3a^2C-abB+Ab^2+2b^2C\right)\sqrt{c+d\tan(e+fx)}}{bf}-\frac{-\frac{2\sqrt{bc-ad}\left(-3a^4Cd+a^3bBd+a^2b^2(d(A-7C)+2Bc)-ab^3(4Ac-5Bd-4cC)-b^4(3Ad+2Bc)\right)\text{arctanh}\left(\cdots\right.}{\sqrt{b}f\left(a^2+b^2\right)}}{}}{2b\left(a^2+b^2\right)}$$

3.102. $\displaystyle\int\frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^2,x]
```

output
```
-(((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*(a
 + b*Tan[e + f*x]))) + (-(((2*(-(((a + I*b)^2*b^2*(A - I*B - C)*(c - I*d)^
(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f) - ((a - I*b)^2*b^2*(A + I*B -
 C)*(c + I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f))/(a^2 + b^2) -
(2*Sqrt[b*c - a*d]*(a^3*b*B*d - 3*a^4*C*d - b^4*(2*B*c + 3*A*d) - a*b^3*(4
*A*c - 4*c*C - 5*B*d) + a^2*b^2*(2*B*c + (A - 7*C)*d))*ArcTanh[(Sqrt[b]*Sq
rt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*f))/b) + (2
*(A*b^2 - a*b*B + 3*a^2*C + 2*b^2*C)*d*Sqrt[c + d*Tan[e + f*x]])/(b*f))/(2
*b*(a^2 + b^2))
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.102. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9864 vs. $2(337) = 674$.

Time = 0.17 (sec) , antiderivative size = 9865, normalized size of antiderivative = 26.52

method	result	size
derivativedivides	Expression too large to display	9865
default	Expression too large to display	9865

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.102.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.102.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\,(A+B\tan{(}\ldots}{(a+b\tan{(e+}\ldots}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**2,x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**2, x)
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.102.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="giac")
```

output
```
Timed out
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

3.103 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.103.1 Optimal result

Integrand size = 47, antiderivative size = 532

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx =$$
$$-\frac{(A-iB-C)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3 f}$$
$$+\frac{(A+iB-C)(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3 f}$$
$$-\frac{(a^5bBd^2+3a^6Cd^2+a^4b^2d(4Bc+3(A+2C)d)-b^6(8Ac^2-8c^2C-12Bcd-3Ad^2)+a^2b^4(24Ac^2-24c\cdots}{\cdots}$$
$$-\frac{(a^3bBd+3a^4Cd+b^4(4Bc+3Ad)+ab^3(8Ac-8cC-7Bd)-a^2b^2(4Bc+5Ad-11Cd))\sqrt{c+d\tan(e\cdots}}{4b^2(a^2+b^2)^2 f(a+b\tan(e+fx))}$$
$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{2b(a^2+b^2)f(a+b\tan(e+fx))^2}$$

output
```
-(A-I*B-C)*(c-I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*
a+b)^3/f+(A+I*B-C)*(c+I*d)^(3/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(I*a-b)^3/f-1/4*(a^5*b*B*d^2+3*a^6*C*d^2+a^4*b^2*d*(4*B*c+3*(A+2*C)*d
)-b^6*(8*A*c^2-3*A*d^2-12*B*c*d-8*C*c^2)+a^2*b^4*(24*A*c^2-26*A*d^2-48*B*c
*d-24*C*c^2+35*C*d^2)-2*a^3*b^3*(12*c*(A-C)*d+B*(4*c^2-9*d^2))+a*b^5*(40*c
*(A-C)*d+3*B*(8*c^2-5*d^2)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)/(a^2+b^2)^3/f/(-a*d+b*c)^(1/2)-1/4*(a^3*b*B*d+3*a^4*C*
d+b^4*(3*A*d+4*B*c)+a*b^3*(8*A*c-7*B*d-8*C*c)-a^2*b^2*(5*A*d+4*B*c-11*C*d)
)*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2-a*(
B*b-C*a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

3.103.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 7678 vs. $2(532) = 1064$.

Time = 7.18 (sec) , antiderivative size = 7678, normalized size of antiderivative = 14.43

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^3,x]
```

output
```
Result too large to show
```

3.103.3 Rubi [A] (warning: unable to verify)

Time = 4.40 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.404$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \ 4128$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(-((-3Ca^2-bBa+Ab^2-4b^2C)d\tan^2(e+fx))-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(2bc-\frac{3ad}{2}\right)+2Ab\left(2a\cdots\right.\right.}{2(a+b\tan(e+fx))^2}}{2b\left(a^2+b^2\right)} \cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}(-((-3Ca^2-bBa+Ab^2-4b^2C)d\tan^2(e+fx))-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-3ad)+Ab(4ac+3b\cdots}{(a+b\tan(e+fx))^2}}{4b\left(a^2+b^2\right)} \cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}(-((-3Ca^2-bBa+Ab^2-4b^2C)d\tan(e+fx)^2)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-3ad)+Ab(4ac+3b\cdots}{(a+b\tan(e+fx))^2}}{4b\left(a^2+b^2\right)} \cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \ 4128$$

$$\frac{\int \frac{-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{bd}{2}\right)((bB-aC)(4bc-3ad)+Ab(4ac+3bd))b+d\left(3Cda^4+\cdots\right.}{2(a+b\tan(e+fx\cdots}}{b(a\cdots} \cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{bd}{2}\right)((bB-aC)(4bc-3ad)+Ab(4ac+3bd))b+d\left(3Cda^4+\cdots\right.}{(a+b\tan(e+fx\cdots}}{2b(\cdots} \cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \ 3042$$

3.103. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

$$\int \frac{-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{bd}{2}\right)((bB-aC)(4bc-3ad)+Ab(4ac+3bd))b+d(3Cda^4+ \cdots}{(a+b\tan(e+fx \cdots}$$

$$2b(\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 4136

$$\int -\frac{8\left(b^2\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2\left(\left(2c(A-C) \cdots\right.\right.\right.}{\sqrt{c+d\tan(e+fx)} \cdots}$$

$$a^2+b^2$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 27

$$\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc)-2a^3b^3\left(12cd(A-C)+B\left(4c^2-9d^2\right)\right)+a^2b^4\left(24Ac^2-26Ad^2-48Bcd-24c^2C+35Cd^2\right)+ab^5\left(40cd(A-C)+3B\left(8c^2-5d^2 \cdots\right.\right.\right.$$

$$a^2+b^2$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc)-2a^3b^3\left(12cd(A-C)+B\left(4c^2-9d^2\right)\right)+a^2b^4\left(24Ac^2-26Ad^2-48Bcd-24c^2C+35Cd^2\right)+ab^5\left(40cd(A-C)+3B\left(8c^2-5d^2 \cdots\right.\right.\right.$$

$$a^2+b^2$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\Big\downarrow$ 4022

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc) \cdots\right.}{\cdots}$$

$\Big\downarrow$ 3042

3.103. $\displaystyle\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}$$

$\downarrow$ 4020

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}$$

$\downarrow$ 25

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}$$

$\downarrow$ 4117

3.103. $\displaystyle\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc)\right.}{}}{}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{2\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4Bc\right.}{}}{}$$

$\downarrow$ 221

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}+$$

$$\frac{-\frac{\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+a^3bBd-a^2b^2(5Ad+4Bc-11Cd)+ab^3(8Ac-7Bd-8cC)+b^4(3Ad+4Bc)\right)}{bf(a^2+b^2)(a+b\tan(e+fx))}+\frac{-\frac{2\left(3a^6Cd^2+a^5bBd^2+a^4b^2d(3d(A+2C)+4\right.}{}}{}}{}$$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f
*(a + b*Tan[e + f*x])^2) + (((-8*(-(((a + I*b)^3*b^2*(A - I*B - C)*(c - I*
d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f) - ((a - I*b)^3*b^2*(A + I*
B - C)*(c + I*d)^(3/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f))/(a^2 + b^2)
 - (2*(a^5*b*B*d^2 + 3*a^6*C*d^2 + a^4*b^2*d*(4*B*c + 3*(A + 2*C)*d) - b^6
*(8*A*c^2 - 8*c^2*C - 12*B*c*d - 3*A*d^2) + a^2*b^4*(24*A*c^2 - 24*c^2*C -
 48*B*c*d - 26*A*d^2 + 35*C*d^2) - 2*a^3*b^3*(12*c*(A - C)*d + B*(4*c^2 -
9*d^2)) + a*b^5*(40*c*(A - C)*d + 3*B*(8*c^2 - 5*d^2)))*ArcTanh[(Sqrt[b]*S
qrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*Sqrt[b*c -
 a*d]*f))/(2*b*(a^2 + b^2)) - ((a^3*b*B*d + 3*a^4*C*d + b^4*(4*B*c + 3*A*d
) + a*b^3*(8*A*c - 8*c*C - 7*B*d) - a^2*b^2*(4*B*c + 5*A*d - 11*C*d))*Sqrt
[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])))/(4*b*(a^2 +
b^2))
```

3.103. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.103. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 14440 vs. $2(492) = 984$.

Time = 0.16 (sec) , antiderivative size = 14441, normalized size of antiderivative = 27.14

method	result	size
derivativedivides	Expression too large to display	14441
default	Expression too large to display	14441

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.103.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.103.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(\ldots}\right.}{(a+b\tan{(e+\ldots}}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**3,x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**3, x)
```

3.103.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.103.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="giac")
```

output
```
Timed out
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^3,x)
```

output
```
\text{Hanged}
```

3.104 $\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(\right.$

3.104.1 Optimal result

Integrand size = 47, antiderivative size = 503

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$

$$-\frac{(a-ib)^2(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$
$$+\frac{(a+ib)^2(iA-B-iC)(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$
$$-\frac{2(2ab(c^2C+2Bcd-Cd^2-A(c^2-d^2))-a^2(2c(A-C)d+B(c^2-d^2))+b^2(2c(A-C)d+B(c^2-d^2))}{f}$$
$$+\frac{2(2ab(Ac-cC-Bd)+a^2(Bc+(A-C)d)-b^2(Bc+(A-C)d))\,(c+d\tan(e+fx))^{3/2}}{3f}$$
$$+\frac{2(a^2B-b^2B+2ab(A-C))\,(c+d\tan(e+fx))^{5/2}}{5f}$$
$$+\frac{2(36a^2Cd^2-22abd(2cC-9Bd)+b^2(8c^2C-22Bcd+99(A-C)d^2))\,(c+d\tan(e+fx))^{7/2}}{693d^3f}$$
$$-\frac{2b(4bcC-11bBd-4aCd)\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{99d^2f}$$
$$+\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}$$

output
```
-(a-I*b)^2*(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d
)^(1/2))/f+(a+I*b)^2*(I*A-B-I*C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1
/2)/(c+I*d)^(1/2))/f-2*(2*a*b*(c^2*C+2*B*c*d-C*d^2-A*(c^2-d^2))-a^2*(2*c*(
A-C)*d+B*(c^2-d^2))+b^2*(2*c*(A-C)*d+B*(c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/
f+2/3*(2*a*b*(A*c-B*d-C*c)+a^2*(B*c+(A-C)*d)-b^2*(B*c+(A-C)*d))*(c+d*tan(f
*x+e))^(3/2)/f+2/5*(B*a^2-B*b^2+2*a*b*(A-C))*(c+d*tan(f*x+e))^(5/2)/f+2/69
3*(36*a^2*C*d^2-22*a*b*d*(-9*B*d+2*C*c)+b^2*(8*c^2*C-22*B*c*d+99*(A-C)*d^2
))*(c+d*tan(f*x+e))^(7/2)/d^3/f-2/99*b*(-11*B*b*d-4*C*a*d+4*C*b*c)*tan(f*x
+e)*(c+d*tan(f*x+e))^(7/2)/d^2/f+2/11*C*(a+b*tan(f*x+e))^2*(c+d*tan(f*x+e)
)^(7/2)/d/f
```

3.104.2 Mathematica [A] (verified)

Time = 6.58 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.12

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}+\frac{2\left(\frac{b(-4bcC+11bBd+4aCd)\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-\frac{2\left(\frac{\left(-36a^2Cd^2+22abd(2cC-9Bd)-b^2\left(8c^2C-22Bcd+99(A-C)d^2\right)\right)(c+d\tan(e+fx))^{7/}}{14df}\right.}{}\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e +
 f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*f) + (2*((b*
(-4*b*c*C + 11*b*B*d + 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(
9*d*f) - (2*(((-36*a^2*C*d^2 + 22*a*b*d*(2*c*C - 9*B*d) - b^2*(8*c^2*C - 2
2*B*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(14*d*f) + ((I/2)*(
((99*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*(2*a*b*B - a^2*(A - C
) + b^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (c - I*d)*((2
*(c + d*Tan[e + f*x])^(3/2))/3 + (c - I*d)*((2*(c - I*d)^(3/2)*ArcTanh[Sqr
t[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/(-c + I*d) + 2*Sqrt[c + d*Tan[e + f*
x]]))))/f - ((I/2)*(((-99*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (99*
(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^2)/4)*((2*(c + d*Tan[e + f*x])^(5/
2))/5 + (c + I*d)*((2*(c + d*Tan[e + f*x])^(3/2))/3 + (c + I*d)*((2*(c + I
*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/(-c - I*d) + 2*
Sqrt[c + d*Tan[e + f*x]]))))/f))/(9*d)))/(11*d)
```

3.104.3 Rubi [A] (warning: unable to verify)

Time = 3.78 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.447, Rules used = {3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left((4bcC-4adC-11bBd)\tan^2(e+fx)-11(Ab-Cb+aB)d\tan\right.}{11d}$$
$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$
$$\frac{\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left((4bcC-4adC-11bBd)\tan^2(e+fx)-11(Ab-Cb+aB)d\tan(e+\right.}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left((4bcC-4adC-11bBd)\tan(e+fx)^2-11(Ab-Cb+aB)d\tan(e+\right.}{11d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}-\frac{2\int-\frac{1}{2}(c+d\tan(e+fx))^{5/2}\left(-2c(4cC-11Bd)b^2+44acCdb-9a^2(11A-7C)d^2-((8C\right.}{}}{11d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\frac{\int(c+d\tan(e+fx))^{5/2}\left(-((8c^2C-22Bcd)b^2)+44acCdb-9a^2(11A-7C)d^2-((8Cc^2-22Bdc+99(A-C)d^2)b^2-22ad(2cC-9Bd)b+36a^2Cd^2)\tan^2(\right.}{9d}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\frac{\int(c+d\tan(e+fx))^{5/2}\left(-((8c^2C-22Bcd)b^2)+44acCdb-9a^2(11A-7C)d^2-((8Cc^2-22Bdc+99(A-C)d^2)b^2-22ad(2cC-9Bd)b+36a^2Cd^2)\tan(e\right.}{9d}}{11d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\frac{\int(c+d\tan(e+fx))^{5/2}\left(99(-((A-C)a^2)+2bBa+b^2(A-C))d^2-99(Ba^2+2b(A-C)a-b^2B)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{7/2}\left(36a^2Cd^2-22abd\right.}{}}{9d}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-\frac{\frac{\int(c+d\tan(e+fx))^{5/2}\left(99(-((A-C)a^2)+2bBa+b^2(A-C))d^2-99(Ba^2+2b(A-C)a-b^2B)d^2\tan(e+fx)\right)dx-\frac{2(c+d\tan(e+fx))^{7/2}\left(36a^2Cd^2-22abd\right.}{}}{9d}}{11d}$$

$$\Big\downarrow \text{4011}$$

3.104.

$$\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int(c+d\tan(e+fx))^{3/2}\left(-99((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd))d^2-99((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)\right.$$

$\big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int(c+d\tan(e+fx))^{3/2}\left(-99((Ac-Cc-Bd)a^2-2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd))d^2-99((Bc+(A-C)d)a^2+2b(Ac-Cc-Bd)a-b^2(Bc+(A-C)\right.$$

$\big\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int\sqrt{c+d\tan(e+fx)}\left(99\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+99\left(-\left(\left(2c(A-C)\right.\right.\right.\right.$$

$\big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int\sqrt{c+d\tan(e+fx)}\left(99\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2+2b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2+99\left(-\left(\left(2c(A-C)\right.\right.\right.\right.$$

$\big\downarrow$ 4011

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int\frac{99d^2\left(-\left(\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^2\right)+2b\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a+b^2\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)\tan(e+fx)-99d^2\left(\left(\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\int\frac{99d^2\left(-\left(\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^2\right)+2b\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a+b^2\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)\tan(e+fx)-99d^2\left(\left(\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4022

3.104.
$$\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;-\frac{99}{2}d^2(a+ib)^2(c+id)^3(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{99}{2}d^2(a-ib)^2(c-id)^3(A-i$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;-\frac{99}{2}d^2(a+ib)^2(c+id)^3(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{99}{2}d^2(a-ib)^2(c-id)^3(A-i$$

$\downarrow$ 4020

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;-\frac{99id^2(a-ib)^2(c-id)^3(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\;99$$

$\downarrow$ 25

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;\frac{99id^2(a-ib)^2(c-id)^3(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\;99id^2$$

$\downarrow$ 73

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;-\frac{99d(a+ib)^2(c+id)^3(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\;99d(a-ib)^2(c$$

$\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{7/2}}{11df}-$$

$$\frac{2b\tan(e+fx)(-4aCd-11bBd+4bcC)(c+d\tan(e+fx))^{7/2}}{9df}+\;-\frac{2(c+d\tan(e+fx))^{7/2}\left(36a^2Cd^2-22abd(2cC-9Bd)+b^2\left(99d^2(A-C)-22Bcd+8c^2C\right)\right)}{7df}+\;19$$

input

```
Int[(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

3.104.
$\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
(2*C*(a + b*Tan[e + f*x])^2*(c + d*Tan[e + f*x])^(7/2))/(11*d*f) - ((2*b*(
4*b*c*C - 11*b*B*d - 4*a*C*d)*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*
d*f) + ((-99*(a - I*b)^2*(A - I*B - C)*(c - I*d)^(5/2)*d^2*ArcTan[Tan[e +
f*x]/Sqrt[c - I*d]])/f - (99*(a + I*b)^2*(A + I*B - C)*(c + I*d)^(5/2)*d^2
*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f + (198*d^2*(2*a*b*(c^2*C + 2*B*c*d
- C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*
(A - C)*d + B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]])/f - (66*d^2*(2*a*b*(
A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d))*(c + d*T
an[e + f*x])^(3/2))/f - (198*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(c + d*Ta
n[e + f*x])^(5/2))/(5*f) - (2*(36*a^2*C*d^2 - 22*a*b*d*(2*c*C - 9*B*d) + b
^2*(8*c^2*C - 22*B*c*d + 99*(A - C)*d^2))*(c + d*Tan[e + f*x])^(7/2))/(7*d
*f))/(9*d))/(11*d)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.104.
$\int(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11279 vs. $2(459) = 918$.

Time = 0.44 (sec) , antiderivative size = 11280, normalized size of antiderivative = 22.43

method	result	size
parts	Expression too large to display	11280
derivativedivides	Expression too large to display	11478
default	Expression too large to display	11478

input
```
int((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91140 vs. $2(449) = 898$.

Time = 169.46 (sec) , antiderivative size = 91140, normalized size of antiderivative = 181.19

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.104.6 Sympy [F]

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*ta
n(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e
+ f*x) + C*tan(e + f*x)**2), x)
```

3.104.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.104.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(
f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^2*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x)
+ C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.105 $\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2} (A+B\tan(e$

3.105.1 Optimal result

Integrand size = 45, antiderivative size = 353

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx =$$
$$-\frac{(ia+b)(A-iB-C)(c-id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}$$
$$+\frac{(ia-b)(A+iB-C)(c+id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}$$
$$+\frac{2(a(Bc^2-2cCd-Bd^2)-b(c^2C+2Bcd-Cd^2)+A(2acd+b(c^2-d^2)))\sqrt{c+d\tan(e+fx)}}{f}$$
$$+\frac{2(Abc+aBc-bcC+aAd-bBd-aCd)(c+d\tan(e+fx))^{3/2}}{3f}$$
$$+\frac{2(Ab+aB-bC)(c+d\tan(e+fx))^{5/2}}{5f}$$
$$-\frac{2(2bcC-9bBd-9aCd)(c+d\tan(e+fx))^{7/2}}{63d^2f}+\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}$$

output
```
-(I*a+b)*(A-I*B-C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1
/2))/f+(I*a-b)*(A+I*B-C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I
*d)^(1/2))/f+2*(a*(B*c^2-B*d^2-2*C*c*d)-b*(2*B*c*d+C*c^2-C*d^2)+A*(2*a*c*d
+b*(c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(A*a*d+A*b*c+B*a*c-B*b*d-C*a*d
-C*b*c)*(c+d*tan(f*x+e))^(3/2)/f+2/5*(A*b+B*a-C*b)*(c+d*tan(f*x+e))^(5/2)/
f-2/63*(-9*B*b*d-9*C*a*d+2*C*b*c)*(c+d*tan(f*x+e))^(7/2)/d^2/f+2/9*b*C*tan
(f*x+e)*(c+d*tan(f*x+e))^(7/2)/d/f
```

3.105.
$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$

3.105.2 Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.92

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{\frac{2(-2bcC+9bBd+9aCd)(c+d\tan(e+fx))^{7/2}}{d}+14bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}+\frac{63}{2}i(a}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f
*x] + C*Tan[e + f*x]^2),x]
```

output
```
((2*(-2*b*c*C + 9*b*B*d + 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/d + 14*b*C*
Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2) + ((63*I)/2)*(a - I*b)*(A - I*B -
C)*d*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*
ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]
*(4*c - (3*I)*d + d*Tan[e + f*x])))/3) - ((63*I)/2)*(a + I*b)*(A + I*B - C
)*d*((2*(c + d*Tan[e + f*x])^(5/2))/5 + (2*(c + I*d)*(-3*(c + I*d)^(3/2)*A
rcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] + Sqrt[c + d*Tan[e + f*x]]*
(4*c + (3*I)*d + d*Tan[e + f*x])))/3))/(63*d*f)
```

3.105.3 Rubi [A] (warning: unable to verify)

Time = 2.23 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{2\int\frac{1}{2}(c+d\tan(e+fx))^{5/2}\left((2bcC-9adC-9bBd)\tan^2(e+fx)-9(Ab-Cb+aB)d\tan(e+fx)+2bcC-9a\right.}{9d}$$

↓ 27

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{5/2}\left((2bcC-9adC-9bBd)\tan^2(e+fx)-9(Ab-Cb+aB)d\tan(e+fx)+2bcC-9aAd\right)}{9d}$$

↓ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{5/2}\left((2bcC-9adC-9bBd)\tan(e+fx)^2-9(Ab-Cb+aB)d\tan(e+fx)+2bcC-9aAd\right)}{9d}$$

↓ 4113

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{5/2}(9(bB-a(A-C))d-9(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-9aCd-9bBd+2bcC)(c+d\tan(e+}{7df}}{9d}$$

↓ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{5/2}(9(bB-a(A-C))d-9(Ab-Cb+aB)d\tan(e+fx))dx+\frac{2(-9aCd-9bBd+2bcC)(c+d\tan(e+}{7df}}{9d}$$

↓ 4011

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}(9d(bBc+b(A-C)d-a(Ac-Cc-Bd))-9d(Abc+aBc-bCc+aAd-bBd-aCd)}{9d}$$

↓ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\frac{\int(c+d\tan(e+fx))^{3/2}(9d(bBc+b(A-C)d-a(Ac-Cc-Bd))-9d(Abc+aBc-bCc+aAd-bBd-aCd)}{9d}$$

↓ 4011

3.105.

$\int(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\int \sqrt{c+d\tan(e+fx)}\left(9d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-9d(2aAcd-2a$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\int \sqrt{c+d\tan(e+fx)}\left(9d\left(a\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-9d(2aAcd-2a$$

$\downarrow$ 4011

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\int \frac{9d\left(a\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)+b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)-9d\left(A\left(bc^3+3adc^2-3bd^2c-ad^3\right)-b(Cc^3+3Bdc^2-3Cd^2c}{\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$\int \frac{9d\left(a\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)+b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\right)-9d\left(A\left(bc^3+3adc^2-3bd^2c-ad^3\right)-b(Cc^3+3Bdc^2-3Cd^2c}{\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4022

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$-\frac{9}{2}d(a+ib)(c+id)^3(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{9}{2}d(a-ib)(c-id)^3(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{}$$

$\downarrow$ 3042

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$-\frac{9}{2}d(a+ib)(c+id)^3(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{9}{2}d(a-ib)(c-id)^3(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{}$$

$\downarrow$ 4020

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df}-$$
$$-\frac{9id(a-ib)(c-id)^3(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{9id(a+ib)(c+id)^3(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+}}}{2f}$$

$\downarrow$ 25

3.105.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df} - \frac{\frac{9id(a-ib)(c-id)^3(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f} - \frac{9id(a+ib)(c+id)^3(A+iB-C)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d}{2f}}{}$$

$\downarrow$ 73

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df} - \frac{-\frac{9(a+ib)(c+id)^3(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f} - \frac{9(a-ib)(c-id)^3(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}}{}$$

$\downarrow$ 221

$$\frac{2bC\tan(e+fx)(c+d\tan(e+fx))^{7/2}}{9df} - \frac{-\frac{9d(a-ib)(c-id)^{5/2}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f} - \frac{9d(a+ib)(c+id)^{5/2}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f} - \frac{18d\sqrt{c+d\tan(e+fx)}(2aAcd+aB}{}}{}$$

input
```
Int[(a + b*Tan[e + f*x])*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] +
C*Tan[e + f*x]^2),x]
```

output
```
(2*b*C*Tan[e + f*x]*(c + d*Tan[e + f*x])^(7/2))/(9*d*f) - ((-9*(a - I*b)*(
A - I*B - C)*(c - I*d)^(5/2)*d*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f - (9*
(a + I*b)*(A + I*B - C)*(c + I*d)^(5/2)*d*ArcTan[Tan[e + f*x]/Sqrt[c + I*d
]])/f - (18*d*(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) -
 b*(c^2*C + 2*B*c*d - C*d^2))*Sqrt[c + d*Tan[e + f*x]])/f - (6*d*(A*b*c +
a*B*c - b*c*C + a*A*d - b*B*d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/f - (18
*(A*b + a*B - b*C)*d*(c + d*Tan[e + f*x])^(5/2))/(5*f) + (2*(2*b*c*C - 9*b
*B*d - 9*a*C*d)*(c + d*Tan[e + f*x])^(7/2))/(7*d*f))/(9*d)
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.105.

$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7293 vs. $2(315) = 630$.

Time = 0.21 (sec) , antiderivative size = 7294, normalized size of antiderivative = 20.66

method	result	size
parts	Expression too large to display	7294
derivativedivides	Expression too large to display	7402
default	Expression too large to display	7402

input
```
int((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48734 vs. $2(308) = 616$.

Time = 37.57 (sec) , antiderivative size = 48734, normalized size of antiderivative = 138.06

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.105.6 Sympy [F]

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (a+b\tan(e+fx))\,(c+d\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f
*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)**2), x)
```

3.105.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.105.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) +
C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.106 $\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

3.106.1 Optimal result

Integrand size = 35, antiderivative size = 229

$$\begin{aligned}\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=&\\ -\frac{(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{f}&\\ -\frac{(B-i(A-C))(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{f}&\\ +\frac{2(2c(A-C)d+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}&\\ +\frac{2(Bc+(A-C)d)(c+d\tan(e+fx))^{3/2}}{3f}&\\ +\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}&\end{aligned}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f
-(B-I*(A-C))*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f
+2*(2*c*(A-C)*d+B*(c^2-d^2))*(c+d*tan(f*x+e))^(1/2)/f+2/3*(B*c+(A-C)*d)*(c
+d*tan(f*x+e))^(3/2)/f+2/5*B*(c+d*tan(f*x+e))^(5/2)/f+2/7*C*(c+d*tan(f*x+e
))^(7/2)/d/f
```

3.106.2 Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.14

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{\frac{4C(c+d\tan(e+fx))^{7/2}}{d}+7i(A-iB-C)\left(\frac{2}{5}(c+d\tan(e+fx))^{5/2}+\frac{2}{3}(c-id)\left(-3(c-i\right.\right.}{}$$

input
```
Integrate[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^
2),x]
```

output
```
((4*C*(c + d*Tan[e + f*x])^(7/2))/d + (7*I)*(A - I*B - C)*((2*(c + d*Tan[e
 + f*x])^(5/2))/5 + (2*(c - I*d)*(-3*(c - I*d)^(3/2)*ArcTanh[Sqrt[c + d*Ta
n[e + f*x]]/Sqrt[c - I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c - (3*I)*d + d*T
an[e + f*x])))/3) - (7*I)*(A + I*B - C)*((2*(c + d*Tan[e + f*x])^(5/2))/5
+ (2*(c + I*d)*(-3*(c + I*d)^(3/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 + I*d]] + Sqrt[c + d*Tan[e + f*x]]*(4*c + (3*I)*d + d*Tan[e + f*x])))/3))
/(14*f)
```

3.106.3 Rubi [A] (warning: unable to verify)

Time = 1.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 4113}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^{5/2}dx+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{ 3042}$$

$$\int (A-C+B\tan(e+fx))(c+d\tan(e+fx))^{5/2}dx+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{4011}$$

$$\int (c+d\tan(e+fx))^{3/2}(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^{3/2}(Ac-Cc-Bd+(Bc+(A-C)d)\tan(e+fx))dx+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{c+d\tan(e+fx)}\left(-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)\right)dx+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+d\tan(e+fx)}\left(-Cc^2-2Bdc+Cd^2+A(c^2-d^2)+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)\right)dx+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Cc^3-3Bdc^2+3Cd^2c+Bd^3+A(c^3-3cd^2)+\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow \text{3042}$$

3.106. $\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

$$\int \frac{-Cc^3-3Bdc^2+3Cd^2c+Bd^3+A(c^3-3cd^2)+((A-C)d(3c^2-d^2)+B(c^3-3cd^2))\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$\downarrow$ 4022

$$\frac{1}{2}(c+id)^3(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c-id)^3(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$\downarrow$ 3042

$$\frac{1}{2}(c+id)^3(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c-id)^3(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$\downarrow$ 4020

$$\frac{i(c-id)^3(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(c+id)^3(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$\downarrow$ 25

$$-\frac{i(c-id)^3(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{i(c+id)^3(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2(2cd(A-C)+B(c^2-d^2))\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

3.106. $\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\Big\downarrow 73$$

$$\frac{(c+id)^3(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(c-id)^3(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

$$\Big\downarrow 221$$

$$\frac{(c-id)^{5/2}(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f}+\frac{(c+id)^{5/2}(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f}+\frac{2\left(2cd(A-C)+B\left(c^2-d^2\right)\right)\sqrt{c+d\tan(e+fx)}}{f}+\frac{2(d(A-C)+Bc)(c+d\tan(e+fx))^{3/2}}{3f}+\frac{2B(c+d\tan(e+fx))^{5/2}}{5f}+\frac{2C(c+d\tan(e+fx))^{7/2}}{7df}$$

input
```
Int[(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
((A - I*B - C)*(c - I*d)^(5/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f + ((A
 + I*B - C)*(c + I*d)^(5/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f + (2*(2*
c*(A - C)*d + B*(c^2 - d^2))*Sqrt[c + d*Tan[e + f*x]])/f + (2*(B*c + (A -
C)*d)*(c + d*Tan[e + f*x])^(3/2))/(3*f) + (2*B*(c + d*Tan[e + f*x])^(5/2))
/(5*f) + (2*C*(c + d*Tan[e + f*x])^(7/2))/(7*d*f)
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.106. $\int (c+d\tan(e+fx))^{5/2} \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) \, dx$

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3561 vs. $2(196) = 392$.

Time = 0.15 (sec) , antiderivative size = 3562, normalized size of antiderivative = 15.55

method	result	size
parts	Expression too large to display	3562
derivativedivides	Expression too large to display	3614
default	Expression too large to display	3614

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x,method=_RETUR
NVERBOSE)
```

output
```
A*(2/3/f*d*(c+d*tan(f*x+e))^(3/2)+4/f*d*(c+d*tan(f*x+e))^(1/2)*c+1/4/f/d*l
n(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2
+d^2)^(1/2))*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-1/4/f*d*ln(
d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d
^2)^(1/2))*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)-1/4/f/d*ln(d*tan(
f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1
/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+3/4/f*d*ln(d*tan(f*x+e)+c-(c+d*tan(
f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^
(1/2)+2*c)^(1/2)*c+3/f*d/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(
f*x+e))^(1/2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)
)*c^2-1/f*d^3/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/
2)-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))-2/f*d/(2*
(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2)^(
1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*(c^2+d^2)^(1/2)*c-1/4/f/d*
ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^
2+d^2)^(1/2))*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+1/4/f*d*ln
(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+
d^2)^(1/2))*(c^2+d^2)^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+1/4/f/d*ln(d*tan
(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(
1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3-3/4/f*d*ln(d*tan(f*x+e)+c+(c+d*...
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10840 vs. $2(189) = 378$.

Time = 1.98 (sec) , antiderivative size = 10840, normalized size of antiderivative = 47.34

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="fricas")
```

output
```
Too large to include
```

3.106.6 Sympy [F]

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int (c+d\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2), x)
```

3.106.7 Maxima [F]

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int \left(C\tan(fx+e)^2+B\tan(fx+e)+A\right)(d\tan(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(5/
2), x)
```

3.106.8 Giac [F(-1)]

Timed out.

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algori
thm="giac")
```

output
```
Timed out
```

3.106.9 Mupad [B] (verification not implemented)

Time = 114.33 (sec) , antiderivative size = 5863, normalized size of antiderivative = 25.60

$$\int (c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Too large to display}$$

input
```
int((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
((2*C*c^2)/(3*d*f) - (2*C*(d^3*f + c^2*d*f))/(3*d^2*f^2))*(c + d*tan(e + f
*x))^(3/2) - log(((((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^
2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(((((-B^4*d^2
*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^
2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*c^4*d^2 - 32*B*d^6 + 32*c*d^2*f*(((-
B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3
*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2)))/(2*f)
- (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4 - 15*c^4*
d^2))/f^2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(((20*B^4*c
^2*d^8*f^4 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f^4 + 100*B^4*c^6*d^4*f^4 - 25
*B^4*c^8*d^2*f^4)^(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f
^2)/(4*f^4))^(1/2) + log(- ((((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^
(1/2) + B^2*c^5*f^2 - 10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(((
((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 - 10*B^2*
c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(32*B*d^6 - 32*B*c^4*d^2 + 32*c*
d^2*f*(((-B^4*d^2*f^4*(5*c^4 + d^4 - 10*c^2*d^2)^2)^(1/2) + B^2*c^5*f^2 -
10*B^2*c^3*d^2*f^2 + 5*B^2*c*d^4*f^2)/f^4)^(1/2)*(c + d*tan(e + f*x))^(1/2
)))/(2*f) - (16*B^2*d^2*(c + d*tan(e + f*x))^(1/2)*(c^6 - d^6 + 15*c^2*d^4
 - 15*c^4*d^2))/f^2))/2 - (8*B^3*c*d^2*(c^2 - 3*d^2)*(c^2 + d^2)^3)/f^3)*(
(20*B^4*c^2*d^8*f^4 - B^4*d^10*f^4 - 110*B^4*c^4*d^6*f^4 + 100*B^4*c^6*...
```

3.107 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

3.107.1 Optimal result

Integrand size = 47, antiderivative size = 336

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = -\frac{(iA+B-iC)(c-id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)f} + \frac{(iA-B-iC)(c+id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)f} - \frac{2(Ab^2-a(bB-aC))\,(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{b^{7/2}\,(a^2+b^2)\,f} + \frac{2(b^2d(Bc+(A-C)d)+(bc-ad)(bcC+bBd-aCd))\,\sqrt{c+d\tan(e+fx)}}{b^3f} + \frac{2(bcC+bBd-aCd)(c+d\tan(e+fx))^{3/2}}{3b^2f} + \frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(
a-I*b)/f+(I*A-B-I*C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^
(1/2))/(a+I*b)/f-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(c
+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(a^2+b^2)/f+2*(b^2*d*(B*c+(
A-C)*d)+(-a*d+b*c)*(B*b*d-C*a*d+C*b*c))*(c+d*tan(f*x+e))^(1/2)/b^3/f+2/3*(
B*b*d-C*a*d+C*b*c)*(c+d*tan(f*x+e))^(3/2)/b^2/f+2/5*C*(c+d*tan(f*x+e))^(5/
2)/b/f
```

3.107.2 Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.96

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \frac{15\Big(b^{7/2}(-ia+b)(A-iB-C)(c-id)^{5/2}\text{arctanh}\Big(}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x]),x]
```

output
```
((15*(b^(7/2)*((-I)*a + b)*(A - I*B - C)*(c - I*d)^(5/2)*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c - I*d]] + b^(7/2)*(I*a + b)*(A + I*B - C)*(c + I*d)
^(5/2)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]] - 2*(A*b^2 + a*(-(b
*B) + a*C))*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/S
qrt[b*c - a*d]]))/(b^(5/2)*(a^2 + b^2)) + (30*(b^2*d*(B*c + (A - C)*d) + (
b*c - a*d)*(b*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*x]])/b^2 + (10*(b
*c*C + b*B*d - a*C*d)*(c + d*Tan[e + f*x])^(3/2))/b + 6*C*(c + d*Tan[e + f
*x])^(5/2))/(15*b*f)
```

3.107.3 Rubi [A] (warning: unable to verify)

Time = 3.71 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.01, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.489$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4136, 25, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \; 4130$$

$$\frac{2\int \frac{5(c+d\tan(e+fx))^{3/2}\left((bC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{2(a+b\tan(e+fx))}dx}{5b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((bC-adC+bBd)\tan^2(e+fx)+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((bC-adC+bBd)\tan(e+fx)^2+b(Bc+(A-C)d)\tan(e+fx)+Abc-aCd\right)}{a+b\tan(e+fx)}dx}{b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 4130$$

$$\frac{\frac{2\int \frac{3\sqrt{c+d\tan(e+fx)}\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))\right)}{2(a+b\tan(e+fx))}dx}{3b}+}{b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan^2(e+fx)+ad(aCd-b(2cC+Bd))\right)}{a+b\tan(e+fx)}dx}{b}+}{b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(Ac^2b^2+\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2+\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\tan(e+fx)^2+ad(aCd-b(2cC+Bd))\right)}{a+b\tan(e+fx)}dx}{b}+}{b}+\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 4130$$

3.107. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$$\frac{\frac{2\int \frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2+\left(d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\right)\tan\cdots}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2+\left(d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\right)\tan\cdots}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+A\left(bc^3-ad^3\right)b^2+\left(d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(d(Bc+(A-C)d)b^2+(bc-ad)(bcC-adC+bBd)\right)\right)\tan\cdots}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\cdots}{b}\cdots}{b}\cdots$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$\downarrow$ 4136

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\int -\frac{b^3\left(a\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)-b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\cdots}{\cdots}}{\cdots}}{b}\cdots$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$\downarrow$ 25

$$\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\int -\frac{\left(b(A-C)d\left(3c^2-d^2\right)+bB\left(c^3-3cd^2\right)+a\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)\right)b^3+\cdots}{\cdots}}{\cdots}}{b}\cdots$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$\downarrow$ 25

3.107. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$$\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\int\frac{\left(b(A-C)d\left(3c^2-d^2\right)+bB\left(c^3-3cd^2\right)-a\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)\right)b^3+(\ldots}{\ldots}$$

$$b$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 3042$$

$$\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\int\frac{\left(b(A-C)d\left(3c^2-d^2\right)+bB\left(c^3-3cd^2\right)-a\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)\right)b^3+(\ldots}{\ldots}$$

$$b$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}$$

$$\Big\downarrow \; 4022$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$

$$\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\ldots}{(a+b\tan(\ldots}}{a^2+b^2}\ldots$$

$$b$$

$$\Big\downarrow \; 3042$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$

$$\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\ldots}{(a+b\tan(\ldots}}{a^2+b^2}\ldots$$

$$b$$

$$\Big\downarrow \; 4020$$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$

$$\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{\ldots}{(a+b\tan(\ldots}}{a^2+b^2}\ldots$$

$$\Big\downarrow \; 25$$

3.107. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(}}{a^2+b^2}}{}}{}}{}$$

$\downarrow$ 73

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(}}{a^2+b^2}}{}}{}}{}$$

$\downarrow$ 221

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(}}{a^2+b^2}}{}}{b}}{}$$

$\downarrow$ 4117

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{\frac{(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(}}{f\left(a^2+b\right.}}{}}{b}}{}$$

$\downarrow$ 73

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+$$
$$\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{\frac{2(bc-ad)^3\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+\frac{b(c+}{}}}{df\left(a^2+\right.}}{}}{b}}{}$$

$\downarrow$ 221

3.107. $\int\frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

$$\frac{2C(c+d\tan(e+fx))^{5/2}}{5bf}+\frac{\frac{2(-aCd+bBd+bcC)(c+d\tan(e+fx))^{3/2}}{3bf}+\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-aCd+bBd+bcC)+b^2d(d(A-C)+Bc)\right)}{bf}+\frac{-\frac{2(bc-ad)^{5/2}\left(Ab^2-a(bB-aC)\right)\text{arcta}\cdots}{\sqrt{b}f\left(a^2+b^2\cdots\right.}}{\cdots}}{b}}{\cdots}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x]),x]
```

output
```
(2*C*(c + d*Tan[e + f*x])^(5/2))/(5*b*f) + ((2*(b*c*C + b*B*d - a*C*d)*(c
+ d*Tan[e + f*x])^(3/2))/(3*b*f) + (((((a + I*b)*b^3*(A - I*B - C)*(c - I*
d)^(5/2)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/f + ((a - I*b)*b^3*(A + I*B -
 C)*(c + I*d)^(5/2)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/f)/(a^2 + b^2) - (
2*(A*b^2 - a*(b*B - a*C))*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Ta
n[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*f))/b + (2*(b^2*d*(B*c
 + (A - C)*d) + (b*c - a*d)*(b*c*C + b*B*d - a*C*d))*Sqrt[c + d*Tan[e + f*
x]])/(b*f))/b)/b
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.107. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{a+b\tan(e+fx)}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8697 vs. 2(294) = 588.

Time = 0.18 (sec) , antiderivative size = 8698, normalized size of antiderivative = 25.89

method	result	size
derivativedivides	Expression too large to display	8698
default	Expression too large to display	8698

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.107.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e)),x)
```

output
```
Timed out
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.107.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{a+b\tan(e+fx)}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x)),x)
```

output
```
\text{Hanged}
```

3.108 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.108.1 Optimal result

Integrand size = 47, antiderivative size = 473

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx =$$

$$-\frac{(iA+B-iC)(c-id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2f}$$

$$-\frac{(B-i(A-C))(c+id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2f}$$

$$+\frac{(bc-ad)^{3/2}\left(3a^3bBd-5a^4Cd-b^4(2Bc+5Ad)-ab^3(4Ac-4cC-7Bd)+a^2b^2(2Bc-(A+9C)d)\right)\text{arc}}{b^{7/2}\left(a^2+b^2\right)^2f}$$

$$-\frac{d(5a^3Cd-Ab^2(bc-ad)-2b^3(2cC+Bd)-a^2b(5cC+3Bd)+ab^2(Bc+4Cd))\sqrt{c+d\tan(e+fx)}}{b^3\left(a^2+b^2\right)f}$$

$$+\frac{\left(3Ab^2-3abB+5a^2C+2b^2C\right)d(c+d\tan(e+fx))^{3/2}}{3b^2\left(a^2+b^2\right)f}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{b\left(a^2+b^2\right)f(a+b\tan(e+fx))}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(
a-I*b)^2/f-(B-I*(A-C))*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d
)^(1/2))/(a+I*b)^2/f+(-a*d+b*c)^(3/2)*(3*a^3*b*B*d-5*a^4*C*d-b^4*(5*A*d+2*
B*c)-a*b^3*(4*A*c-7*B*d-4*C*c)+a^2*b^2*(2*B*c-(A+9*C)*d))*arctanh(b^(1/2)*
(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(a^2+b^2)^2/f-d*(5*a^3*C*
d-A*b^2*(-a*d+b*c)-2*b^3*(B*d+2*C*c)-a^2*b*(3*B*d+5*C*c)+a*b^2*(B*c+4*C*d)
)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)/f+1/3*(3*A*b^2-3*B*a*b+5*C*a^2+2*C*
b^2)*d*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)/f-(A*b^2-a*(B*b-C*a))*(c+d*tan
(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.108.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6112 vs. 2(473) = 946.

Time = 7.02 (sec) , antiderivative size = 6112, normalized size of antiderivative = 12.92

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.108.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-3bBa+3Ab^2+2b^2C)d\tan^2(e+fx)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(bc-\frac{5ad}{2}\right)+2Ab\left(ac+\right.\right.}{2(a+b\tan(e+fx))}}{b(a^2+b^2)}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}((5Ca^2-3bBa+3Ab^2+2b^2C)d\tan^2(e+fx)-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-5ad)+Ab(2ac+5b}{a+b\tan(e+fx)}}{2b(a^2+b^2)}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}((5Ca^2-3bBa+3Ab^2+2b^2C)d\tan(e+fx)^2-2b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(2bc-5ad)+Ab(2ac+5b}{a+b\tan(e+fx)}}{2b(a^2+b^2)}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

$\downarrow$ 4130

$$\frac{\frac{2\int -\frac{3\sqrt{c+d\tan(e+fx)}\left(-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-c((bB-aC)(2bc-5ad)+Ab(2ac+5bd))b+a\left(5Ca^2-3b\right.\right.}{2(a+b\tan(e+fx))}}{3b}}{2b}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\int \frac{\sqrt{c+d\tan(e+fx)}\left(-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx\right.}{}}{2b(}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\int \frac{\sqrt{c+d\tan(e+fx)}\left(-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx\right.}{}}{2b(}$$

$$\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{bf(a^2+b^2)(a+b\tan(e+fx))}$$

3.108. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

$\downarrow$ 4130

$$\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{2\int -\frac{5Cd^3a^4-bd^2(10cC+3Bd)a^3+b^2d\left(5Cc^2+4Bdc+(A+4C)d^2\right)a^2+b^3\left(2Ac^3-2Cc^3-5Bdc^2-4Ad\cdots\right.}{\cdots}}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \frac{\int \frac{5Cd\cdots}{\cdots}}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 3042

$$\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \frac{\int \frac{5Cd\cdots}{\cdots}}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 4136

$$\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \frac{\int \frac{2((\cdots}{\cdots}}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 27

$$\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \frac{2\int -\frac{(\cdots}{\cdots}}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

3.108. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \cdots}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \cdots}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \cdots}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \cdots}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf} - \frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf} - \cdots}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

3.108. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{2\int-\frac{(}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{2\int-}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{2\int-\frac{(}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{2\int-}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{2d\left(5a^2C-3abB+3Ab^2+2b^2C\right)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{2\int-\frac{(}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

3.108. $\displaystyle\int\frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\cdots}{\cdots}}{\cdots}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$\downarrow$ 25

3.108. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{2\int-\frac{(}{}}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\,(a^2+b^2)\,(a+b\tan(e+fx))}$$

$\Big\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{-\frac{2\int-}{}}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\,(a^2+b^2)\,(a+b\tan(e+fx))}$$

$\Big\downarrow$ 25

$$\frac{\frac{2d(5a^2C-3abB+3Ab^2+2b^2C)(c+d\tan(e+fx))^{3/2}}{3bf}-\frac{\frac{2d\sqrt{c+d\tan(e+fx)}\left(5a^3Cd-a^2b(3Bd+5cC)-Ab^2(bc-ad)+ab^2(Bc+4Cd)-2b^3(Bd+2cC)\right)}{bf}-\frac{2\int-\frac{(}{}}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\,(a^2+b^2)\,(a+b\tan(e+fx))}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^2,x]
```

output
```
$Aborted
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.108. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.108. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^2}\,dx$

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 14118 vs. 2(434) = 868.

Time = 0.20 (sec) , antiderivative size = 14119, normalized size of antiderivative = 29.85

method	result	size
derivativedivides	Expression too large to display	14119
default	Expression too large to display	14119

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**2,x)
```

output
```
Timed out
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.108.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^2,x, algorithm="giac")
```

output
```
Timed out
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

3.109 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.109.1 Optimal result

Integrand size = 47, antiderivative size = 643

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{(A-iB-C)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)^3f}$$

$$+\frac{(A+iB-C)(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)^3f}$$

$$+\frac{\sqrt{bc-ad}(3a^5bBd^2-15a^6Cd^2+a^4b^2d(4Bc+(A-46C)d)-3a^2b^4(8Ac^2-8c^2C-16Bcd-6Ad^2+21C}{}$$

$$-\frac{d(3a^3bBd-15a^4Cd-ab^3(8Ac-8cC-11Bd)+a^2b^2(4Bc+(A-31C)d)-b^4(4Bc+7Ad+8Cd))\sqrt{c}}{4b^3\left(a^2+b^2\right)^2f}$$

$$+\frac{(a^3bBd-5a^4Cd-b^4(4Bc+5Ad)-ab^3(8Ac-8cC-9Bd)+a^2b^2(4Bc+3Ad-13Cd))(c+d\tan(e+}{4b^2\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}$$

$$-\frac{(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

output
```
-(A-I*B-C)*(c-I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*
a+b)^3/f+(A+I*B-C)*(c+I*d)^(5/2)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(I*a-b)^3/f+1/4*(3*a^5*b*B*d^2-15*a^6*C*d^2+a^4*b^2*d*(4*B*c+(A-46*C)
*d)-3*a^2*b^4*(8*A*c^2-6*A*d^2-16*B*c*d-8*C*c^2+21*C*d^2)-a*b^5*(56*c*(A-C
)*d+B*(24*c^2-35*d^2))-b^6*(4*c*(5*B*d+2*C*c)-A*(8*c^2-15*d^2))+2*a^3*b^3*
(4*c*(A-C)*d+B*(4*c^2+3*d^2)))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)/(a^2+b^2)^3/f-1/4*d*(3*a^3*b*B*d-15
*a^4*C*d-a*b^3*(8*A*c-11*B*d-8*C*c)+a^2*b^2*(4*B*c+(A-31*C)*d)-b^4*(7*A*d+
4*B*c+8*C*d))*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^2/f+1/4*(a^3*b*B*d-5*a^
4*C*d-b^4*(5*A*d+4*B*c)-a*b^3*(8*A*c-9*B*d-8*C*c)+a^2*b^2*(3*A*d+4*B*c-13*
C*d))*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))-1/2*(A*b^2
-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^2
```

3.109.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 17248 vs. 2(643) = 1286.

Time = 7.95 (sec) , antiderivative size = 17248, normalized size of antiderivative = 26.82

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^3,x]
```

output
```
Result too large to show
```

3.109.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \; 4128$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-bBa+Ab^2+4b^2C)d\tan^2(e+fx)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+2(bB-aC)\left(2bc-\frac{5ad}{2}\right)+2Ab\left(2ac+\cdots\right.\right.}{2(a+b\tan(e+fx))^2}\cdots}{2b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-bBa+Ab^2+4b^2C)d\tan^2(e+fx)-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-5ad)+Ab(4ac+5bd)\cdots\right.}{(a+b\tan(e+fx))^2}\cdots}{4b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-bBa+Ab^2+4b^2C)d\tan(e+fx)^2-4b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(4bc-5ad)+Ab(4ac+5bd)\cdots\right.}{(a+b\tan(e+fx))^2}\cdots}{4b\left(a^2+b^2\right)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 4128$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{3bd}{2}\right)((bB-aC)(4bc-5ad)+Ab(4ac+\cdots\right.}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{3bd}{2}\right)((bB-aC)(4bc-5ad)+Ab(4ac+\cdots\right.}{\cdots}\cdots$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-8((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+2\left(ac+\frac{3bd}{2}\right)((bB-aC)(4bc-5ad)+Ab(4ac+\right.}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 4130

$$2\int -\frac{15Cd^3a^5-3bd^2(5cC+Bd)a^4-b^2d^2(Bc+(A-31C)d)a^3-b^3\left(8Ac^3-8Cc^3-20Bdc^2-17Ad^2c+47Cd^2c+11Bd^3\right)a^2-b^4\left(16Bc^3+40Adc^2-40Cdc^2-31Bd^2c-7Ad\right.}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{15Cd^3a^5-3bd^2(5cC+Bd)a^4-b^2d^2(Bc+(A-31C)d)a^3-b^3\left(8Ac^3-8Cc^3-20Bdc^2-17Ad^2c+47Cd^2c+11Bd^3\right)a^2-b^4\left(16Bc^3+40Adc^2-40Cdc^2-31Bd^2c-7Ad^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 3042

$$-\frac{\int \frac{15Cd^3a^5-3bd^2(5cC+Bd)a^4-b^2d^2(Bc+(A-31C)d)a^3-b^3\left(8Ac^3-8Cc^3-20Bdc^2-17Ad^2c+47Cd^2c+11Bd^3\right)a^2-b^4\left(16Bc^3+40Adc^2-40Cdc^2-31Bd^2c-7Ad^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2bf\left(a^2+b^2\right)(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 4136

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 27

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\left(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad)\right)(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\big\downarrow$ 25

3.109. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))}+-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$$\Big\downarrow \; 25$$

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{(-5Cda^4+bBda^3+b^2(4Bc+3Ad-13Cd)a^2-b^3(8Ac-8Cc-9Bd)a-b^4(4Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{-\frac{2d\sqrt{c+d\tan(e+fx)}\left(-15Cda^4+3bBda^3\right.}{}}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{2b\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^3,x]
```

output
```
$Aborted
```

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.109. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^3}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20662 vs. $2(599) = 1198$.

Time = 0.30 (sec) , antiderivative size = 20663, normalized size of antiderivative = 32.14

method	result	size
derivativedivides	Expression too large to display	20663
default	Expression too large to display	20663

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.109.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.109.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^3,x, algorithm="giac")
```

output
```
Timed out
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^3}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^3,x)
```

output
```
\text{Hanged}
```

3.110 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.110.1 Optimal result

Integrand size = 47, antiderivative size = 407

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$= \frac{(ia+b)^3(A-iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f}$$
$$- \frac{(ia-b)^3(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}f}$$
$$+ \frac{2(72a^3Cd^3-6a^2bd^2(32cC-49Bd)+21ab^2d(8c^2C-10Bcd+15(A-C)d^2)-b^3(48c^3C-56Bc^2d+}{105d^4f}$$
$$+ \frac{2b(35b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-7bBd-6aCd))\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{105d^3f}$$
$$- \frac{2(6bcC-7bBd-6aCd)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{35d^2f}$$
$$+ \frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}$$

output
```
(I*a+b)^3*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d
)^(1/2)-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/
f/(c+I*d)^(1/2)+2/105*(72*a^3*C*d^3-6*a^2*b*d^2*(-49*B*d+32*C*c)+21*a*b^2*
d*(8*c^2*C-10*B*c*d+15*(A-C)*d^2)-b^3*(48*c^3*C-56*B*c^2*d+70*c*(A-C)*d^2+
105*B*d^3))*(c+d*tan(f*x+e))^(1/2)/d^4/f+2/105*b*(35*b*(A*b+B*a-C*b)*d^2+4
*(-a*d+b*c)*(-7*B*b*d-6*C*a*d+6*C*b*c))*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/
d^3/f-2/35*(-7*B*b*d-6*C*a*d+6*C*b*c)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+
e))^2/d^2/f+2/7*C*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3/d/f
```

3.110.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1200 vs. 2(407) = 814.

Time = 6.51 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.95

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$= \frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} + \frac{2\left(\frac{(-6bcC+7bBd+6aCd)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} + \frac{2\left(\frac{b\left(35b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-7bBd-6aCd)\right)\tan(e+fx)\sqrt{c+d\tan(e}{6df}\right.}{}\right.}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])/(7*d*f) + (2*(((-6*b
*c*C + 7*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])
/(5*d*f) + (2*((b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7
*b*B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(6*d*f) - (2*((I
*Sqrt[c - I*d]*((b*c*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C
- 7*b*B*d - 6*a*C*d)))/4 + (3*a*d*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a
*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - (3*a*d*(-5*a*d*(6*b*c*C - a*(7*A -
 C)*d) + (4*b*c + a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - b*((-105*(a^2*B
 - b^2*B + 2*a*b*(A - C))*d^3)/8 + (c*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c
 - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4) + ((3*I)/2)*d*((35*a*(a^2*B - b
^2*B + 2*a*b*(A - C))*d^2)/4 - (b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a
*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4 + (b*(-5*a*d*(6*b*c*C - a*(7*A - C)*
d) + (4*b*c + a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4))*ArcTanh[Sqrt[c + d*
Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*c*(35*
b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d - 6*a*C*d)))/4
+ (3*a*d*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*d -
6*a*C*d)))/8 - (3*a*d*(-5*a*d*(6*b*c*C - a*(7*A - C)*d) + (4*b*c + a*d)*(6
*b*c*C - 7*b*B*d - 6*a*C*d)))/8 - b*((-105*(a^2*B - b^2*B + 2*a*b*(A - C))
*d^3)/8 + (c*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*B*
d - 6*a*C*d)))/4) - ((3*I)/2)*d*((35*a*(a^2*B - b^2*B + 2*a*b*(A - C))*...
```

3.110.3 Rubi [A] (warning: unable to verify)

Time = 2.87 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.383$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{(a+b\tan(e+fx))^2((6bcC-6adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+fx)+6bcC-a(7A-C)d)}{2\sqrt{c+d\tan(e+fx)}}dx}{7d}+\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}$$

$\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-\frac{\int \frac{(a+b\tan(e+fx))^2((6bcC-6adC-7bBd)\tan^2(e+fx)-7(Ab-Cb+aB)d\tan(e+fx)+6bcC-a(7A-C)d)}{\sqrt{c+d\tan(e+fx)}}dx}{7d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-\frac{\int \frac{(a+b\tan(e+fx))^2((6bcC-6adC-7bBd)\tan(e+fx)^2-7(Ab-Cb+aB)d\tan(e+fx)+6bcC-a(7A-C)d)}{\sqrt{c+d\tan(e+fx)}}dx}{7d}$$

$\downarrow$ 4130

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-\frac{\frac{2\int -\frac{(a+b\tan(e+fx))\left(4c(6cC-7Bd)b^2-ad(48cC+7Bd)b+a^2(35A-11C)d^2+\left(35b(Ab-Cb+aB)d^2+4(bc-ad)(6bcC-6adC-7bBd)\right)\tan^2(e+fx)+35\left(Ba^2+2b(A-\right.}{2\sqrt{c+d\tan(e+fx)}}}{5d}}{7d}$$

$\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\int \frac{(a+b\tan(e+fx))\left(4c(6cC-7Bd)b^2-ad(48cC+7Bd)b+a^2(35A-11C)d^2+\left(35b(Ab-Cb+a\right.}{\sqrt{c+}}}{}}{7d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\int \frac{(a+b\tan(e+fx))\left(4c(6cC-7Bd)b^2-ad(48cC+7Bd)b+a^2(35A-11C)d^2+\left(35b(Ab-Cb+a\right.}{\sqrt{c+}}}{}}{7d}$$

$\downarrow$ 4120

3.110. $$\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df} - \frac{2 \cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\int \frac{-2c\left(24Cc^2-28Bdc+35(A-C)d^2\right)b^3+42acd(4cC-5Bd)b^2-3a^2d^2(64cC+7Bd)b+3a^3(35 \cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\int \frac{-2c\left(24Cc^2-28Bdc+35(A-C)d^2\right)b^3+42acd(4cC-5Bd)b^2-3a^2d^2(64cC+7Bd)b+3a^3(35 \cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 4113

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\int \frac{105\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-105\left(-\left((A-C)a^3\right)+3bBa^2+3 \cdots\right.}{\sqrt{c+d\tan(e+fx)}}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\int \frac{105\left(Ba^3+3b(A-C)a^2-3b^2Ba-b^3(A-C)\right)d^3\tan(e+fx)-105\left(-\left((A-C)a^3\right)+3bBa^2+3 \cdots\right.}{\sqrt{c+d\tan(e+fx)}}}{\cdots}}{\cdots}$$

$\downarrow$ 4022

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df} - \frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df} - \frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df} + \frac{1 \cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

3.110. $$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-$$

$$\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df}+}{}}{}$$

$\downarrow$ 4020

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-$$

$$\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df}+}{}}{}$$

$\downarrow$ 25

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-$$

$$\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df}+}{}}{}$$

$\downarrow$ 73

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-$$

$$\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df}+}{}}{}$$

$\downarrow$ 221

$$\frac{2C(a+b\tan(e+fx))^3\sqrt{c+d\tan(e+fx)}}{7df}-$$

$$\frac{\frac{2(-6aCd-7bBd+6bcC)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(35bd^2(aB+Ab-bC)+4(bc-ad)(-6aCd-7bBd+6bcC)\right)}{3df}+}{}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[
c + d*Tan[e + f*x]],x]
```

3.110. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

output
```
(2*C*(a + b*Tan[e + f*x])^3*Sqrt[c + d*Tan[e + f*x]])/(7*d*f) - ((2*(6*b*c
*C - 7*b*B*d - 6*a*C*d)*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(
5*d*f) - ((2*b*(35*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 7*b*
B*d - 6*a*C*d))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) + ((105*(a
- I*b)^3*(A - I*B - C)*d^3*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I
*d]*f) + (105*(a + I*b)^3*(A + I*B - C)*d^3*ArcTan[Tan[e + f*x]/Sqrt[c + I
*d]])/(Sqrt[c + I*d]*f) + (2*(72*a^3*C*d^3 - 6*a^2*b*d^2*(32*c*C - 49*B*d)
 + 21*a*b^2*d*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2) - b^3*(48*c^3*C - 56*B
*c^2*d + 70*c*(A - C)*d^2 + 105*B*d^3))*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(
3*d))/(5*d))/(7*d)
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.110. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5977 vs. 2(371) = 742.

Time = 0.31 (sec) , antiderivative size = 5978, normalized size of antiderivative = 14.69

method	result	size
parts	Expression too large to display	5978
derivativedivides	Expression too large to display	25426
default	Expression too large to display	25426

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37247 vs. 2(363) = 726.

Time = 11.35 (sec) , antiderivative size = 37247, normalized size of antiderivative = 91.52

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.110.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{\left(a+b\tan\left(e+fx\right)\right)^3\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)}{\sqrt{c+d\tan\left(e+fx\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(c + d*tan(e + f*x)), x)
```

3.110.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.110.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.110.9 Mupad [B] (verification not implemented)

Time = 112.14 (sec) , antiderivative size = 28858, normalized size of antiderivative = 70.90

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(1/2),x)
```

output
```
atan(((((8*(4*A*a^3*d^3*f^2 - 12*A*a*b^2*d^3*f^2 + 4*A*b^3*c*d^2*f^2 - 12*
A*a^2*b*c*d^2*f^2))/f^3 - 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^
6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*
A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (
16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^
4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a
^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*
A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*
f^4 + d^2*f^4)))^(1/2))*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b
^5*d*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*
f^2 - 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A
^4*b^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^
4 + 6*A^4*a^10*b^2))^(1/2) - 4*A^2*a^6*c*f^2 + 4*A^2*b^6*c*f^2 - 24*A^2*a*
b^5*d*f^2 - 24*A^2*a^5*b*d*f^2 - 60*A^2*a^2*b^4*c*f^2 + 60*A^2*a^4*b^2*c*f
^2 + 80*A^2*a^3*b^3*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*ta
n(e + f*x))^(1/2)*(A^2*a^6*d^2 - A^2*b^6*d^2 + 15*A^2*a^2*b^4*d^2 - 15*A^2
*a^4*b^2*d^2))/f^2)*((((8*A^2*a^6*c*f^2 - 8*A^2*b^6*c*f^2 + 48*A^2*a*b^5*d
*f^2 + 48*A^2*a^5*b*d*f^2 + 120*A^2*a^2*b^4*c*f^2 - 120*A^2*a^4*b^2*c*f^2
- 160*A^2*a^3*b^3*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^12 + A^4*b
^12 + 6*A^4*a^2*b^10 + 15*A^4*a^4*b^8 + 20*A^4*a^6*b^6 + 15*A^4*a^8*b^4...
```

3.111 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.111.1 Optimal result

Integrand size = 47, antiderivative size = 287

$$\begin{aligned}
&\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx \\
&= -\frac{(a-ib)^2(B+i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} \\
&\quad+\frac{(a+ib)^2(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}f} \\
&\quad+\frac{2(12a^2Cd^2-10abd(2cC-3Bd)+b^2(8c^2C-10Bcd+15(A-C)d^2))\sqrt{c+d\tan(e+fx)}}{15d^3f} \\
&\quad-\frac{2b(4bcC-5bBd-4aCd)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{15d^2f} \\
&\quad+\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}
\end{aligned}$$

output
```
-(a-I*b)^2*(B+I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-
I*d)^(1/2)+(a+I*b)^2*(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/f/(c+I*d)^(1/2)+2/15*(12*a^2*C*d^2-10*a*b*d*(-3*B*d+2*C*c)+b^2*(8*c^2
*C-10*B*c*d+15*(A-C)*d^2))*(c+d*tan(f*x+e))^(1/2)/d^3/f-2/15*b*(-5*B*b*d-4
*C*a*d+4*C*b*c)*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/f+2/5*C*(c+d*tan(f*x
+e))^(1/2)*(a+b*tan(f*x+e))^2/d/f
```

3.111.2 Mathematica [A] (verified)

Time = 6.36 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$= \frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}$$
$$+ \frac{2\left(\frac{b(-4bcC+5bBd+4aCd)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{2\left(\frac{i\sqrt{c-id}\left(\frac{15}{4}i\left(a^2B-b^2B+2ab(A-C)\right)d^2+\frac{15}{4}\left(2abB-a^2(A-C)+b^2(A-C)\right)d^2\right)\text{arct}}{(-c+id)f}\right.}{}\right.}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d*f) + (2*((b*(-4
*b*c*C + 5*b*B*d + 4*a*C*d)*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f)
 - (2*((I*Sqrt[c - I*d]*(((15*I)/4)*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2 +
(15*(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^2)/4)*ArcTanh[Sqrt[c + d*Tan[e
 + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*(((-15*I)/4)*(a
^2*B - b^2*B + 2*a*b*(A - C))*d^2 + (15*(2*a*b*B - a^2*(A - C) + b^2*(A -
C))*d^2)/4)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f
) + ((-12*a^2*C*d^2 + 10*a*b*d*(2*c*C - 3*B*d) - b^2*(8*c^2*C - 10*B*c*d +
 15*(A - C)*d^2))*Sqrt[c + d*Tan[e + f*x]])/(2*d*f)))/(3*d)))/(5*d)
```

3.111.3 Rubi [A] (warning: unable to verify)

Time = 1.82 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.319, Rules used = {3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{2\int -\frac{(a+b\tan(e+fx))\left((4bcC-4adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)+4bcC-a(5A-C)d\right)}{2\sqrt{c+d\tan(e+fx)}}dx}{5d}+\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\int \frac{(a+b\tan(e+fx))\left((4bcC-4adC-5bBd)\tan^2(e+fx)-5(Ab-Cb+aB)d\tan(e+fx)+4bcC-a(5A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\int \frac{(a+b\tan(e+fx))\left((4bcC-4adC-5bBd)\tan(e+fx)^2-5(Ab-Cb+aB)d\tan(e+fx)+4bcC-a(5A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{5d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$
$$\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{2\int -\frac{-2c(4cC-5Bd)b^2+20acCdb-3a^2(5A-C)d^2-\left(\left(8Cc^2-10Bdc+15(A-C)d^2\right)b^2-10ad(2cC-\cdots}{2\sqrt{c+d\tan(e+fx)}\cdots}}{3d}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$
$$\frac{\frac{\int \frac{-2c(4cC-5Bd)b^2+20acCdb-3a^2(5A-C)d^2-\left(\left(8Cc^2-10Bdc+15(A-C)d^2\right)b^2-10ad(2cC-3Bd)b+12a^2Cd^2\right)\tan^2(e+fx)-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e\cdots}{\sqrt{c+d\tan(e+fx)}}}{3d}\cdots}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$
$$\frac{\frac{\int \frac{-2c(4cC-5Bd)b^2+20acCdb-3a^2(5A-C)d^2-\left(\left(8Cc^2-10Bdc+15(A-C)d^2\right)b^2-10ad(2cC-3Bd)b+12a^2Cd^2\right)\tan(e+fx)^2-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e\cdots}{\sqrt{c+d\tan(e+fx)}}}{3d}\cdots}{5d}$$

$$\Big\downarrow \text{4113}$$

3.111. $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{\int\frac{15\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(15d^2(A-C)-10Bcd\cdots\right.\right.}{df}}{3d}}{5d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{\int\frac{15\left(-\left((A-C)a^2\right)+2bBa+b^2(A-C)\right)d^2-15\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(15d^2(A-C)-10Bcd\cdots\right.\right.}{df}}{3d}}{5d}$$

$\downarrow$ 4022

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{-\frac{15}{2}d^2(a+ib)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{15}{2}d^2(a-ib)^2(A-iB-C)\int\frac{i\tan(e+fx\cdots}{\sqrt{c+d\tan(e\cdots}}}{3d}}{5d}$$

$\downarrow$ 3042

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{-\frac{15}{2}d^2(a+ib)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{15}{2}d^2(a-ib)^2(A-iB-C)\int\frac{i\tan(e+fx\cdots}{\sqrt{c+d\tan(e\cdots}}}{3d}}{5d}$$

$\downarrow$ 4020

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{-\frac{15id^2(a-ib)^2(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{15id^2(a+ib)^2(\cdots}{\cdots}}{\cdots}}{5d}$$

$\downarrow$ 25

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-$$

$$\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{\frac{15id^2(a-ib)^2(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{15id^2(a+ib)^2(A+\cdots}{\cdots}}{\cdots}}{5d}$$

$\downarrow$ 73

3.111. $\int\frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{-\frac{15d(a-ib)^2(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{15d(a+ib)^2(A+iB-C)\int -\cdots}{\cdots}}{\cdots}}{5d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2C(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b\tan(e+fx)(-4aCd-5bBd+4bcC)\sqrt{c+d\tan(e+fx)}}{3df}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(12a^2Cd^2-10abd(2cC-3Bd)+b^2\left(15d^2(A-C)-10Bcd+8c^2C\right)\right)}{df}-\frac{15d^2(a-i\cdots}{\cdots}}{3d}}{5d}$$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[
c + d*Tan[e + f*x]],x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d*f) - ((2*b*(4*b
*c*C - 5*b*B*d - 4*a*C*d)*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) +
 ((-15*(a - I*b)^2*(A - I*B - C)*d^2*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(
Sqrt[c - I*d]*f) - (15*(a + I*b)^2*(A + I*B - C)*d^2*ArcTan[Tan[e + f*x]/S
qrt[c + I*d]])/(Sqrt[c + I*d]*f) - (2*(12*a^2*C*d^2 - 10*a*b*d*(2*c*C - 3*
B*d) + b^2*(8*c^2*C - 10*B*c*d + 15*(A - C)*d^2))*Sqrt[c + d*Tan[e + f*x]]
)/(d*f))/(3*d))/(5*d)
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.111. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5512 vs. $2(254) = 508$.

Time = 0.16 (sec) , antiderivative size = 5513, normalized size of antiderivative = 19.21

method	result	size
parts	Expression too large to display	5513
derivativedivides	Expression too large to display	18289
default	Expression too large to display	18289

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25627 vs. $2(244) = 488$.

Time = 4.81 (sec) , antiderivative size = 25627, normalized size of antiderivative = 89.29

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.111.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2\,(A+B\tan{(e+fx)}+C\tan^2{(e+fx)})}{\sqrt{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
sqrt(c + d*tan(e + f*x)), x)
```

3.111.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.111.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.111.9 Mupad [B] (verification not implemented)

Time = 43.42 (sec) , antiderivative size = 21254, normalized size of antiderivative = 74.06

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(1/2),x)
```

output

```
atan(((((16*(2*C*b^2*d^3*f^2 - 2*C*a^2*d^3*f^2 + 4*C*a*b*c*d^2*f^2))/f^3 -
 64*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2
 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (
16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^
4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b
^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f
^4)))^(1/2))*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 +
32*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)
*(C^4*a^8 + C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2
) - 4*C^2*a^4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*
d*f^2 + 24*C^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d
*tan(e + f*x))^(1/2)*(C^2*a^4*d^2 + C^2*b^4*d^2 - 6*C^2*a^2*b^2*d^2))/f^2)
*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 32*C^2*a^3*b
*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(C^4*a^8 +
C^4*b^8 + 4*C^4*a^2*b^6 + 6*C^4*a^4*b^4 + 4*C^4*a^6*b^2))^(1/2) - 4*C^2*a^
4*c*f^2 - 4*C^2*b^4*c*f^2 + 16*C^2*a*b^3*d*f^2 - 16*C^2*a^3*b*d*f^2 + 24*C
^2*a^2*b^2*c*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2)*1i - (((16*(2*C*b^2*d^3*
f^2 - 2*C*a^2*d^3*f^2 + 4*C*a*b*c*d^2*f^2))/f^3 + 64*c*d^2*(c + d*tan(e +
f*x))^(1/2)*((((8*C^2*a^4*c*f^2 + 8*C^2*b^4*c*f^2 - 32*C^2*a*b^3*d*f^2 + 3
2*C^2*a^3*b*d*f^2 - 48*C^2*a^2*b^2*c*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^...
```

3.111. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.112 $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.112.1 Optimal result

Integrand size = 45, antiderivative size = 194

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(ia+b)(A-iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} + \frac{(ia-b)(A+iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}f} - \frac{2(2bcC-3bBd-3aCd)\sqrt{c+d\tan(e+fx)}}{3d^2f} + \frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}$$

output
```
-(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)
^(1/2)+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(
c+I*d)^(1/2)-2/3*(-3*B*b*d-3*C*a*d+2*C*b*c)*(c+d*tan(f*x+e))^(1/2)/d^2/f+2
/3*b*C*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d/f
```

3.112.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$= \frac{2\left(-\frac{3i(a-ib)(A-iB-C)d\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{2\sqrt{c-id}}+\frac{3i(a+ib)(A+iB-C)d\,\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{2\sqrt{c+id}}+\frac{(-2bcC+3bBd+3aCd)\sqrt{\cdots}}{d}\right.}{3df}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/S
qrt[c + d*Tan[e + f*x]],x]
```

output
```
(2*((((-3*I)/2)*(a - I*b)*(A - I*B - C)*d*ArcTanh[Sqrt[c + d*Tan[e + f*x]]
/Sqrt[c - I*d]])/Sqrt[c - I*d] + (((3*I)/2)*(a + I*b)*(A + I*B - C)*d*ArcT
anh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] + ((-2*b*c*C +
3*b*B*d + 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/d + b*C*Tan[e + f*x]*Sqrt[c +
 d*Tan[e + f*x]]))/(3*d*f)
```

3.112.3 Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{4120}$$
$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{2\int \frac{(2bcC-3adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx)+2bcC-3aAd}{2\sqrt{c+d\tan(e+fx)}}dx}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{(2bcC-3adC-3bBd)\tan^2(e+fx)-3(Ab-Cb+aB)d\tan(e+fx)+2bcC-3aAd}{\sqrt{c+d\tan(e+fx)}}dx}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{(2bcC-3adC-3bBd)\tan(e+fx)^2-3(Ab-Cb+aB)d\tan(e+fx)+2bcC-3aAd}{\sqrt{c+d\tan(e+fx)}}dx}{3d}$$

$$\Big\downarrow \; 4113$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{3(bB-a(A-C))d-3(Ab-Cb+aB)d\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(-3aCd-3bBd+2bcC)\sqrt{c+d\tan(e+fx)}}{df}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{3(bB-a(A-C))d-3(Ab-Cb+aB)d\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(-3aCd-3bBd+2bcC)\sqrt{c+d\tan(e+fx)}}{df}}{3d}$$

$$\Big\downarrow \; 4022$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3}{2}d(a+ib)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{3}{2}d(a-ib)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(-3aCd-3bBd+2bcC)}{df}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3}{2}d(a+ib)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{3}{2}d(a-ib)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2(-3aCd-3bBd+2bcC)}{df}}{3d}$$

$$\Big\downarrow \; 4020$$

3.112. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{-\frac{3id(a-ib)(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f} + \frac{3id(a+ib)(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e\cdots}{2f}}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{\frac{3id(a-ib)(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f} - \frac{3id(a+ib)(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}}{3d}$$

$$\Big\downarrow \; 73$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{-\frac{3(a+ib)(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f} - \frac{3(a-ib)(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f} + \frac{2(-3aCd-3\cdots}{}}{3d}$$

$$\Big\downarrow \; 221$$

$$\frac{2bC\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3df} - \frac{-\frac{3d(a-ib)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}} - \frac{3d(a+ib)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}} + \frac{2(-3aCd-3bBd+2bcC)\sqrt{c+d\tan(e+fx)}}{df}}{3d}$$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqrt[c
+ d*Tan[e + f*x]],x]
```

output
```
(2*b*C*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) - ((-3*(a - I*b)*(A
- I*B - C)*d*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f) - (3*(a
 + I*b)*(A + I*B - C)*d*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]
*f) + (2*(2*b*c*C - 3*b*B*d - 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(3
*d)
```

3.112. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.112. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3852 vs. $2(166) = 332$.

Time = 0.14 (sec) , antiderivative size = 3853, normalized size of antiderivative = 19.86

method	result	size
parts	Expression too large to display	3853
derivativedivides	Expression too large to display	4138
default	Expression too large to display	4138

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2
),x,method=_RETURNVERBOSE)
```

output
```
A*a*(-1/4/f/d/(c^2+d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d
^2)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-1/
4/f*d/(c^2+d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2
)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+1/4/f/d/(c^2+d
^2)^(3/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)
^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+1/4/f*d/(c^2+d^2
)^(3/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-1/f/d/(c^2+d^2)^(1/2
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^2-1/f*d/(c^2+d^2)^(
1/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^
2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))+1/f/d/(c^2+d^2)^(3
/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2
+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^4+3/f*d/(c^2+d^2)
^(3/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(
c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^2+2/f*d^3/(c^2
+d^2)^(3/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)
-(2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))+1/4/f/d/(c^
2+d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1
/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+1/4/f*d/(c^2+d^2...
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13473 vs. $2(159) = 318$.

Time = 1.75 (sec) , antiderivative size = 13473, normalized size of antiderivative = 69.45

$$\int \frac{(a + b\tan(e + fx))\,(A + B\tan(e + fx) + C\tan^2(e + fx))}{\sqrt{c + d\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.112.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})\left(A+B\tan{(e+fx)}+C\tan^2{(e+fx)}\right)}{\sqrt{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqr
t(c + d*tan(e + f*x)), x)
```

3.112.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{\left(C\tan{(fx+e)}^2+B\tan{(fx+e)}+A\right)(b\tan{(fx+e)}+a)}{\sqrt{d\tan{(fx+e)}+c}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)/sqr
t(d*tan(f*x + e) + c), x)
```

3.112.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.112.9 Mupad [B] (verification not implemented)

Time = 21.65 (sec) , antiderivative size = 16400, normalized size of antiderivative = 84.54

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x))^(1/2),x)
```

output
```
((2*B*b*d - 6*C*b*c)/(d^2*f) + (4*C*b*c)/(d^2*f))*(c + d*tan(e + f*x))^(1/
2) - atan(((((8*(4*C*a*d^3*f^2 - 4*A*a*d^3*f^2 + 4*B*a*c*d^2*f^2))/f^3 - 6
4*c*d^2*(c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 +
 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)
^2/4 - (16*c^2*f^4 + 16*d^2*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^
4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*
C*a^4))^(1/2) - 4*A^2*a^2*c*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*
B*a^2*d*f^2 + 8*A*C*a^2*c*f^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))
^(1/2))*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a
^2*d*f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2
*f^4)*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2
*a^4 + 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c
*f^2 + 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f
^2 + 8*B*C*a^2*d*f^2)/(16*(c^2*f^4 + d^2*f^4)))^(1/2) - (16*(c + d*tan(e +
 f*x))^(1/2)*(A^2*a^2*d^2 - B^2*a^2*d^2 + C^2*a^2*d^2 - 2*A*C*a^2*d^2))/f^
2)*((((8*A^2*a^2*c*f^2 - 8*B^2*a^2*c*f^2 + 8*C^2*a^2*c*f^2 + 16*A*B*a^2*d*
f^2 - 16*A*C*a^2*c*f^2 - 16*B*C*a^2*d*f^2)^2/4 - (16*c^2*f^4 + 16*d^2*f^4)
*(A^4*a^4 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4
+ 6*A^2*C^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c*f^2
+ 4*B^2*a^2*c*f^2 - 4*C^2*a^2*c*f^2 - 8*A*B*a^2*d*f^2 + 8*A*C*a^2*c*f^2...
```

3.113 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.113.1 Optimal result

Integrand size = 35, antiderivative size = 133

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}f} - \frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}f} + \frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/f/(c-I*d)^(1/2)
-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/f/(c+I*d)^(1/2)
+2*C*(c+d*tan(f*x+e))^(1/2)/d/f
```

3.113.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{-\frac{i(A-iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{i(A+iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} + \frac{2C\sqrt{c+d\tan(e+fx)}}{d}}{f}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]]
,x]
```

output
```
(((-I)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt
[c - I*d] + (I*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d
]])/Sqrt[c + I*d] + (2*C*Sqrt[c + d*Tan[e + f*x]])/d)/f
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4113}$$

$$\int \frac{A-C+B\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-C+B\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$\Big\downarrow$ 4020

$$\frac{i(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$\Big\downarrow$ 25

$$-\frac{i(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{i(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$\Big\downarrow$ 73

$$\frac{(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

$\Big\downarrow$ 221

$$\frac{(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2C\sqrt{c+d\tan(e+fx)}}{df}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
((A - I*B - C)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f) + ((A
 + I*B - C)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*C*S
qrt[c + d*Tan[e + f*x]])/(d*f)
```

3.113. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.113. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3462 vs. $2(112) = 224$.

Time = 0.12 (sec) , antiderivative size = 3463, normalized size of antiderivative = 26.04

method	result	size
parts	Expression too large to display	3463
derivativedivides	Expression too large to display	5570
default	Expression too large to display	5570

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
A*(-1/4/f/d/(c^2+d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2
)^(1/2)+2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2-1/4/
f*d/(c^2+d^2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+
2*c)^(1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)+1/4/f/d/(c^2+d^2
)^(3/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(
1/2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^3+1/4/f*d/(c^2+d^2)^
(3/2)*ln(d*tan(f*x+e)+c-(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/
2)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c-1/f/d/(c^2+d^2)^(1/2)/
(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d^2
)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^2-1/f*d/(c^2+d^2)^(1/
2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+
d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))+1/f/d/(c^2+d^2)^(3/2
)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^2+d
^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^4+3/f*d/(c^2+d^2)^(
3/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(2*(c^
2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))*c^2+2/f*d^3/(c^2+d
^2)^(3/2)/(2*(c^2+d^2)^(1/2)-2*c)^(1/2)*arctan((2*(c+d*tan(f*x+e))^(1/2)-(
2*(c^2+d^2)^(1/2)+2*c)^(1/2))/(2*(c^2+d^2)^(1/2)-2*c)^(1/2))+1/4/f/d/(c^2+
d^2)*ln(d*tan(f*x+e)+c+(c+d*tan(f*x+e))^(1/2)*(2*(c^2+d^2)^(1/2)+2*c)^(1/2
)+(c^2+d^2)^(1/2))*(2*(c^2+d^2)^(1/2)+2*c)^(1/2)*c^2+1/4/f*d/(c^2+d^2)*...
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3194 vs. $2(105) = 210$.

Time = 0.39 (sec) , antiderivative size = 3194, normalized size of antiderivative = 24.02

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algori
thm="fricas")
```

output
```
1/2*(d*f*sqrt(-((c^2 + d^2)*f^2*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c
^2 - 4*(A^3*B - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4
- 2*A^2*B^2 + B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*
C)*d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4)) + (A^2 - B^2 - 2*A*C + C^2)*c + 2*(
A*B - B*C)*d)/((c^2 + d^2)*f^2))*log(-(2*(A^3*B + A*B^3 + 3*A*B*C^2 - B*C^
3 - (3*A^2*B + B^3)*C)*c - (A^4 - B^4 - 4*A^3*C + 6*A^2*C^2 - 4*A*C^3 + C^
4)*d)*sqrt(d*tan(f*x + e) + c) + (((A - C)*c^3 + B*c^2*d + (A - C)*c*d^2 +
 B*d^3)*f^3*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c^2 - 4*(A^3*B - A*B^
3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4 - 2*A^2*B^2 + B^4 -
4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*C)*d^2)/((c^4 + 2*c^
2*d^2 + d^4)*f^4)) + (2*(A*B^2 - B^2*C)*c^2 - (3*A^2*B - B^3 - 6*A*B*C + 3
*B*C^2)*c*d + (A^3 - A*B^2 + 3*A*C^2 - C^3 - (3*A^2 - B^2)*C)*d^2)*f)*sqrt
(-((c^2 + d^2)*f^2*sqrt(-(4*(A^2*B^2 - 2*A*B^2*C + B^2*C^2)*c^2 - 4*(A^3*B
 - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B^3)*C)*c*d + (A^4 - 2*A^2*B^2 +
 B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2 - 4*(A^3 - A*B^2)*C)*d^2)/((c^4
 + 2*c^2*d^2 + d^4)*f^4)) + (A^2 - B^2 - 2*A*C + C^2)*c + 2*(A*B - B*C)*d)
/((c^2 + d^2)*f^2))) - d*f*sqrt(-((c^2 + d^2)*f^2*sqrt(-(4*(A^2*B^2 - 2*A*
B^2*C + B^2*C^2)*c^2 - 4*(A^3*B - A*B^3 + 3*A*B*C^2 - B*C^3 - (3*A^2*B - B
^3)*C)*c*d + (A^4 - 2*A^2*B^2 + B^4 - 4*A*C^3 + C^4 + 2*(3*A^2 - B^2)*C^2
- 4*(A^3 - A*B^2)*C)*d^2)/((c^4 + 2*c^2*d^2 + d^4)*f^4)) + (A^2 - B^2 -...
```

3.113.6 Sympy [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\,dx = \int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/sqrt(c + d*tan(e + f*x))
, x)
```

3.113.7 Maxima [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\, dx = \int \frac{C\tan{(fx + e)}^2 + B\tan{(fx + e)} + A}{\sqrt{d\tan{(fx + e)} + c}}\, dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algori
thm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/sqrt(d*tan(f*x + e) + c)
, x)
```

3.113.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.113.9 Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 4326, normalized size of antiderivative = 32.53

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{\sqrt{c + d\tan(e + fx)}}\, dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(1/2),x)
```

output
```
2*atanh((32*C^2*d^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (C
^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))^(1/2))/(((16*
C^3*c*d^3*f^3)/(c^2*f^4 + d^2*f^4) - (4*C*d^3*f^2*(-16*C^4*d^2*f^4)^(1/2))
/(c^2*f^5 + d^2*f^5)) + (8*c*d^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d
^2*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e + f*x))
^(1/2)*(-16*C^4*d^2*f^4)^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) -
(4*C*d^5*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^
3*f^5)/(c^2*f^4 + d^2*f^4) - (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^
2*f^5 + d^2*f^5)) - (32*C^2*c^2*d^2*f^2*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*
f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*tan(e
+ f*x))^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 + d^2*f^4) - (4*C*d^5*f^4*(-16
*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16*C^3*c^3*d^3*f^5)/(c^2*f^4 +
 d^2*f^4) - (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5))
)*((-16*C^4*d^2*f^4)^(1/2)/(16*(c^2*f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*(c^2*
f^4 + d^2*f^4)))^(1/2) - 2*atanh((8*c*d^2*(- (-16*C^4*d^2*f^4)^(1/2)/(16*(
c^2*f^4 + d^2*f^4)) - (C^2*c*f^2)/(4*(c^2*f^4 + d^2*f^4)))^(1/2)*(c + d*ta
n(e + f*x))^(1/2)*(-16*C^4*d^2*f^4)^(1/2))/((16*C^3*c*d^5*f^5)/(c^2*f^4 +
d^2*f^4) + (4*C*d^5*f^4*(-16*C^4*d^2*f^4)^(1/2))/(c^2*f^5 + d^2*f^5) + (16
*C^3*c^3*d^3*f^5)/(c^2*f^4 + d^2*f^4) + (4*C*c^2*d^3*f^4*(-16*C^4*d^2*f^4)
^(1/2))/(c^2*f^5 + d^2*f^5)) - (32*C^2*d^2*(- (-16*C^4*d^2*f^4)^(1/2)/(...
```

3.113. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.114 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

3.114.1 Optimal result

Integrand size = 47, antiderivative size = 210

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)\sqrt{c-id}f} - \frac{(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(ia-b)\sqrt{c+id}f} - \frac{2(Ab^2-a(bB-aC))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(a^2+b^2)\sqrt{bc-ad}f}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)/f/(c-I*
d)^(1/2)-(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(I*a-b)/f
/(c+I*d)^(1/2)-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2
)/(-a*d+b*c)^(1/2))/(a^2+b^2)/f/b^(1/2)/(-a*d+b*c)^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\frac{(-ia+b)(A-iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} + \frac{(ia+b)(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}} - \frac{2(Ab^2+a(-bB+aC))\operatorname{arctanh}\left(\cdots\right)}{\sqrt{b}\sqrt{bc-ad}}}{(a^2+b^2)\,f}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sq
rt[c + d*Tan[e + f*x]]),x]
```

output
```
((((-I)*a + b)*(A - I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d
]])/Sqrt[c - I*d] + ((I*a + b)*(A + I*B - C)*ArcTanh[Sqrt[c + d*Tan[e + f*
x]]/Sqrt[c + I*d]])/Sqrt[c + I*d] - (2*(A*b^2 + a*(-(b*B) + a*C))*ArcTanh[
(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d]))/((a^2 + b^2)*f)
```

3.114.3 Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.255$, Rules used = {3042, 4136, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4136}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\int \frac{bB+a(A-C)-(Ab-Cb-aB)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{bB+a(A-C)-(Ab-Cb-aB)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{4022}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{1}{2}(a-ib)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(a+ib)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{1}{2}(a-ib)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(a+ib)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 4020}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{i(a+ib)(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(a-ib)(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}}{a^2+b^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{i(a-ib)(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}-\frac{i(a+ib)(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}}{a^2+b^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{(a-ib)(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(a+ib)(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}}{a^2+b^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\frac{\frac{(a+ib)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}}{a^2+b^2}$$

$$\Big\downarrow \text{ 4117}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f\left(a^2+b^2\right)}+\frac{\frac{(a+ib)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}}{a^2+b^2}$$

$$\Big\downarrow \text{ 73}$$

3.114. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df\,(a^2+b^2)}+\frac{\frac{(a+ib)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}}{a^2+b^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f\,(a^2+b^2)\sqrt{bc-ad}}+\frac{\frac{(a+ib)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}}{a^2+b^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*Sqrt[c +
 d*Tan[e + f*x]]),x]
```

output
```
(((a + I*b)*(A - I*B - C)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*
d]*f) + ((a - I*b)*(A + I*B - C)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt
[c + I*d]*f))/(a^2 + b^2) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b]*Sq
rt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*Sqrt[b*c -
a*d]*f)
```

3.114.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13473 vs. $2(179) = 358$.

Time = 0.14 (sec) , antiderivative size = 13474, normalized size of antiderivative = 64.16

method	result	size
derivativedivides	Expression too large to display	13474
default	Expression too large to display	13474

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.114.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.114.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$$
$$=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})\sqrt{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*ta
n(f*x+e)),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*sq
rt(c + d*tan(e + f*x))), x)
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.114.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

3.114.9 Mupad [B] (verification not implemented)

Time = 65.48 (sec) , antiderivative size = 25341, normalized size of antiderivative = 120.67

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx=\text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))^(1/2)),x)
```

output
```
(log(((((((((128*C*b^2*d^8*(a*d + b*c)^2*(a^2 + b^2)^2)/f - 64*b^2*d^8*(a
^2 + b^2)^2*(c + d*tan(e + f*x))^(1/2)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*
b*c)^2)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*
(a^2 + b^2)^2*(c^2 + d^2)))^(1/2)*(3*b^3*c^2 + 2*b^3*d^2 - a^2*b*c^2 - 2*a
^2*b*d^2 + a^3*c*d + a*b^2*c*d))*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2
)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 +
 b^2)^2*(c^2 + d^2)))^(1/2))/4 + (64*C^2*b*d^8*(c + d*tan(e + f*x))^(1/2)*
(5*b^6*c - 4*a^6*c - 2*a^2*b^4*c + 5*a^4*b^2*c - 2*a^3*b^3*d + 7*a*b^5*d +
 7*a^5*b*d))/f^2)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*C^2
*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2 +
d^2)))^(1/2))/4 + (32*C^3*b*d^8*(4*a^5*d - b^5*c - 9*a^2*b^3*c - 15*a^3*b^
2*d + 12*a^4*b*c + a*b^4*d))/f^3)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^
2)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2
+ b^2)^2*(c^2 + d^2)))^(1/2))/4 - (32*C^4*b*d^8*(2*a^4 + b^4)*(c + d*tan(e
 + f*x))^(1/2))/f^4)*((4*(-C^4*f^4*(a^2*d - b^2*d + 2*a*b*c)^2)^(1/2) - 4*
C^2*a^2*c*f^2 + 4*C^2*b^2*c*f^2 + 8*C^2*a*b*d*f^2)/(f^4*(a^2 + b^2)^2*(c^2
 + d^2)))^(1/2))/4 + (32*C^5*a^2*b^2*d^8)/f^5)*(((32*C^4*a^2*b^2*d^2*f^4 -
 16*C^4*b^4*d^2*f^4 - 64*C^4*a^2*b^2*c^2*f^4 - 16*C^4*a^4*d^2*f^4 + 64*C^4
*a*b^3*c*d*f^4 - 64*C^4*a^3*b*c*d*f^4)^(1/2) - 4*C^2*a^2*c*f^2 + 4*C^2*b^2
*c*f^2 + 8*C^2*a*b*d*f^2)/(a^4*c^2*f^4 + a^4*d^2*f^4 + b^4*c^2*f^4 + b^...
```

3.114. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\,dx$

3.115 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

3.115.1 Optimal result

Integrand size = 47, antiderivative size = 327

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2\sqrt{c-id}f} - \frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2\sqrt{c+id}f}$$
$$-\frac{(3a^3bBd-a^4Cd+b^4(2Bc-Ad)+ab^3(4Ac-4cC-Bd)-a^2b^2(2Bc+5Ad-3Cd))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+\cdots}}{\sqrt{\cdots}}\right)}{\sqrt{b}\,(a^2+b^2)^2\,(bc-ad)^{3/2}f}$$
$$-\frac{(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))}$$

output
```
-(3*a^3*b*B*d-a^4*C*d+b^4*(-A*d+2*B*c)+a*b^3*(4*A*c-B*d-4*C*c)-a^2*b^2*(5*
A*d+2*B*c-3*C*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))
/(a^2+b^2)^2/(-a*d+b*c)^(3/2)/f/b^(1/2)-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e
))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c+d
*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)^2/f/(c+I*d)^(1/2)-(A*b^2-a*(B*b-
C*a))*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))
```

3.115.2 Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.59

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx =$$

$$-\frac{\frac{\frac{i\sqrt{c-id}\left(i\left(a^2B-b^2B-2ab(A-C)\right)(bc-ad)-\left(2abB+a^2(A-C)-b^2(A-C)\right)(bc-ad)\right)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(-c+id)f}-\frac{i\sqrt{c+id}\left(-i\left(a^2B-b^2B-2ab(A-C)\right)(\cdots}{\cdots}}{a^2+b^2}}{\cdots}$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
Sqrt[c + d*Tan[e + f*x]]),x]
```

output
```
-((((I*Sqrt[c - I*d]*(I*(a^2*B - b^2*B - 2*a*b*(A - C))*(b*c - a*d) - (2*a
*b*B + a^2*(A - C) - b^2*(A - C))*(b*c - a*d))*ArcTanh[Sqrt[c + d*Tan[e +
f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((-I)*(a^2*B - b^2
*B - 2*a*b*(A - C))*(b*c - a*d) - (2*a*b*B + a^2*(A - C) - b^2*(A - C))*(b
*c - a*d))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/((-c - I*d)*f)
)/(a^2 + b^2) + (2*Sqrt[b*c - a*d]*((a^2*(A*b^2 - a*(b*B - a*C))*d)/2 - a*
b*(A*b - a*B - b*C)*(b*c - a*d) + (b^2*(A*b^2*d - 2*a*A*(b*c - a*d) - 2*(b
*B - a*C)*(b*c - (a*d)/2)))/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/
Sqrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*(-(b*c) + a*d)*f))/((a^2 + b^2)*(b*
c - a*d))) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^
2)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

3.115.3 Rubi [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.340$, Rules used = {3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \; 4132$$

$$-\frac{\int \frac{(2A-C)da^2-b(2Ac-2Cc-Bd)a+\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-b^2(2Bc-Ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(2A-C)da^2-b(2Ac-2Cc-Bd)a+\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-b^2(2Bc-Ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{(2A-C)da^2-b(2Ac-2Cc-Bd)a+\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-b^2(2Bc-Ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 4136$$

$$-\frac{\frac{\int -\frac{2\left(\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-ad)+\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4A\ldots}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{2\int \frac{\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-ad)+\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac\ldots}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$$\Big\downarrow \; 3042$$

3.115. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

$$-\frac{-\frac{2\int \frac{\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-ad)+\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-}{}}{2\left(a^2+b^2\right)(bc-ad)}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}$$

$\downarrow$ 4022

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-$$

$$-\frac{\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a-ib)^2(A+iB-\right.}{}}{2\left(a^2+b^2\right)(bc-ad)}$$

$\downarrow$ 3042

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-$$

$$-\frac{\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a-ib)^2(A+iB-\right.}{}}{2\left(a^2+b^2\right)(bc-ad)}$$

$\downarrow$ 4020

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-$$

$$-\frac{\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{i(a+ib)^2(A-iB-C)}{}\right.}{}}{2\left(a^2+b^2\right)(bc-}$$

$\downarrow$ 25

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-$$

$$-\frac{\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{i(a-ib)^2(A+iB-C)}{}\right.}{}}{2\left(a^2+b^2\right)(bc-}$$

$\downarrow$ 73

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-$$

$$-\frac{\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{(a-ib)^2(A+iB-C)(}{}\right.}{}}{2\left(a^2+b^2\right)(bc-ad)}$$

3.115. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

$$\Big\downarrow \; 221$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{-\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{2\left(\frac{(a-ib)^2(A+iB-C)(\ldots}{\ldots}\right)}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}$$

$$\Big\downarrow \; 4117$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{-\frac{\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f(a^2+b^2)}-\frac{2\left(\frac{(a-ib)^2(\ldots}{\ldots}\right)}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}$$

$$\Big\downarrow \; 73$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{-\frac{2\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df(a^2+b^2)}-\frac{2\left(\frac{(a-ib)^2(\ldots}{\ldots}\right)}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))}-\frac{\frac{2\left(a^4(-C)d+3a^3bBd-a^2b^2(5Ad+2Bc-3Cd)+ab^3(4Ac-Bd-4cC)+b^4(2Bc-Ad)\right)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{\sqrt{b}f(a^2+b^2)\sqrt{bc-ad}}-\frac{2\left(\frac{(a-ib)^2(A+iB-C)(bc-a\ldots}{f\sqrt{c+\ldots}}\right)}{\ldots}}{2\left(a^2+b^2\right)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*Sqrt[c
 + d*Tan[e + f*x]]),x]
```

3.115. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

output
```
-1/2*((-2*(((a + I*b)^2*(A - I*B - C)*(b*c - a*d)*ArcTan[Tan[e + f*x]/Sqrt
[c - I*d]])/(Sqrt[c - I*d]*f) + ((a - I*b)^2*(A + I*B - C)*(b*c - a*d)*Arc
Tan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f)))/(a^2 + b^2) + (2*(3*a
^3*b*B*d - a^4*C*d + b^4*(2*B*c - A*d) + a*b^3*(4*A*c - 4*c*C - B*d) - a^2
*b^2*(2*B*c + 5*A*d - 3*C*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/S
qrt[b*c - a*d]])/(Sqrt[b]*(a^2 + b^2)*Sqrt[b*c - a*d]*f))/((a^2 + b^2)*(b*
c - a*d)) - ((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2
)*(b*c - a*d)*f*(a + b*Tan[e + f*x]))
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.115. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20869 vs. 2(294) = 588.

Time = 0.16 (sec) , antiderivative size = 20870, normalized size of antiderivative = 63.82

method	result	size
derivativedivides	Expression too large to display	20870
default	Expression too large to display	20870

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.115.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.115.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})^2\sqrt{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*ta
n(f*x+e))**2,x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**2
*sqrt(c + d*tan(e + f*x))), x)
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2 \sqrt{c + d\tan(e + fx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.115.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2 \sqrt{c + d\tan(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^2,x, algorithm="giac")
```

output
```
Timed out
```

3.115.9 Mupad [B] (verification not implemented)

Time = 57.47 (sec) , antiderivative size = 225004, normalized size of antiderivative = 688.09

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2 \sqrt{c + d\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))^(1/2)),x)
```

output
```
(atan(((((((16*(8*C^3*a^6*b^7*d^11*f^2 - 78*C^3*a^4*b^9*d^11*f^2 + 60*C^3*
a^8*b^5*d^11*f^2 - 24*C^3*a^10*b^3*d^11*f^2 + 2*C^3*a^12*b*d^11*f^2 - 32*C
^3*a*b^12*c^3*d^8*f^2 + 152*C^3*a^3*b^10*c*d^10*f^2 + 128*C^3*a^5*b^8*c*d^
10*f^2 - 64*C^3*a^7*b^6*c*d^10*f^2 - 32*C^3*a^9*b^4*c*d^10*f^2 + 8*C^3*a^1
1*b^2*c*d^10*f^2 - 40*C^3*a^2*b^11*c^2*d^9*f^2 + 64*C^3*a^3*b^10*c^3*d^8*f
^2 - 216*C^3*a^4*b^9*c^2*d^9*f^2 + 96*C^3*a^5*b^8*c^3*d^8*f^2 - 120*C^3*a^
6*b^7*c^2*d^9*f^2 + 56*C^3*a^8*b^5*c^2*d^9*f^2))/(a^10*d^2*f^5 + b^10*c^2*
f^5 + 4*a^2*b^8*c^2*f^5 + 6*a^4*b^6*c^2*f^5 + 4*a^6*b^4*c^2*f^5 + a^8*b^2*
c^2*f^5 + a^2*b^8*d^2*f^5 + 4*a^4*b^6*d^2*f^5 + 6*a^6*b^4*d^2*f^5 + 4*a^8*
b^2*d^2*f^5 - 2*a*b^9*c*d*f^5 - 2*a^9*b*c*d*f^5 - 8*a^3*b^7*c*d*f^5 - 12*a
^5*b^5*c*d*f^5 - 8*a^7*b^3*c*d*f^5) - (((((16*(40*C*a^3*b^14*d^12*f^4 + 19
2*C*a^5*b^12*d^12*f^4 + 360*C*a^7*b^10*d^12*f^4 + 320*C*a^9*b^8*d^12*f^4 +
 120*C*a^11*b^6*d^12*f^4 - 8*C*a^15*b^2*d^12*f^4 + 8*C*b^17*c^3*d^9*f^4 +
40*C*a*b^16*c^2*d^10*f^4 + 32*C*a*b^16*c^4*d^8*f^4 - 88*C*a^2*b^15*c*d^11*
f^4 - 448*C*a^4*b^13*c*d^11*f^4 - 920*C*a^6*b^11*c*d^11*f^4 - 960*C*a^8*b^
9*c*d^11*f^4 - 520*C*a^10*b^7*c*d^11*f^4 - 128*C*a^12*b^5*c*d^11*f^4 - 8*C
*a^14*b^3*c*d^11*f^4 - 32*C*a^2*b^15*c^3*d^9*f^4 + 256*C*a^3*b^14*c^2*d^10
*f^4 + 160*C*a^3*b^14*c^4*d^8*f^4 - 280*C*a^4*b^13*c^3*d^9*f^4 + 680*C*a^5
*b^12*c^2*d^10*f^4 + 320*C*a^5*b^12*c^4*d^8*f^4 - 640*C*a^6*b^11*c^3*d^9*f
^4 + 960*C*a^7*b^10*c^2*d^10*f^4 + 320*C*a^7*b^10*c^4*d^8*f^4 - 680*C*a...
```

3.115. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}\,dx$

3.116 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.116.1 Optimal result

Integrand size = 47, antiderivative size = 511

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{(a-ib)^3(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f}$$

$$-\frac{(ia-b)^3(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f}$$

$$-\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^3}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$+\frac{2b(6a^2d^2(12c^2C-5Bcd+(5A+7C)d^2)-15abd(8c^3C-6Bc^2d+c(3A+5C)d^2-3Bd^3)+b^2(48c^4C-\cdots}{15d^4\,(c^2+d^2)\,f}$$

$$-\frac{2b^2(4(bc-ad)\,(6c^2C-5Bcd+(5A+C)d^2)-5d^2((A-C)(bc-ad)+B(ac+bd)))\tan(e+fx)\sqrt{c+d\cdots}}{15d^3\,(c^2+d^2)\,f}$$

$$+\frac{2b(6c^2C-5Bcd+(5A+C)d^2)\,(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5d^2\,(c^2+d^2)\,f}$$

output
```
-(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*
d)^(3/2)/f-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(c+I*d)^(3/2)/f+2/15*b*(6*a^2*d^2*(12*c^2*C-5*B*c*d+(5*A+7*C)*d^2)-15*a
*b*d*(8*c^3*C-6*B*c^2*d+c*(3*A+5*C)*d^2-3*B*d^3)+b^2*(48*c^4*C-40*B*c^3*d+
6*c^2*(5*A+3*C)*d^2-25*B*c*d^3+15*(A-C)*d^4))*(c+d*tan(f*x+e))^(1/2)/d^4/(
c^2+d^2)/f-2/15*b^2*(4*(-a*d+b*c)*(6*c^2*C-5*B*c*d+(5*A+C)*d^2)-5*d^2*((A-
C)*(-a*d+b*c)+B*(a*c+b*d)))*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^3/(c^2+d^2
)/f+2/5*b*(6*c^2*C-5*B*c*d+(5*A+C)*d^2)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*
x+e))^2/d^2/(c^2+d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^3/d/(c^2+d^
2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.88 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{2C(a+b\tan(e+fx))^3}{5df\sqrt{c+d\tan(e+fx)}}$$

$$+\frac{2\left(\frac{(-6bcC+5bBd+6aCd)(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{\left(15b(Ab+aB-bC)d^2+4(bc-ad)(6bcC-5bBd-6aCd)\right)(a+b\tan(e+fx))}{2df\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2\left(-48b^3c^3C+40b^3Bc^2d+\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^3)/(5*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-6*b
*c*C + 5*b*B*d + 6*a*C*d)*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e
+ f*x]]) + (2*(((15*b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(6*b*c*C - 5*b
*B*d - 6*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*Sqrt[c + d*Tan[e + f*x]]) +
((-2*(-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b^3*c*d^2
- 110*a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 60*a*A*b^2*d^3
+ 85*a^2*b*B*d^3 - 15*b^3*B*d^3 + 48*a^3*C*d^3 - 60*a*b^2*C*d^3))/(d*Sqrt[
c + d*Tan[e + f*x]]) + (2*(((45*a^2*A*b*d^3 - 15*A*b^3*d^3 + 15*a^3*B*d^3
- 45*a*b^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C*d^3)*(((-I)*ArcTanh[Sqrt[c +
d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[
e + f*x]]/Sqrt[c + I*d]])/Sqrt[c + I*d]))/2 + ((-1/2*(c*d*(45*a^2*A*b*d^3
- 15*A*b^3*d^3 + 15*a^3*B*d^3 - 45*a*b^2*B*d^3 - 45*a^2*b*C*d^3 + 15*b^3*C
*d^3)) + d^2*((-48*b^3*c^3*C + 40*b^3*B*c^2*d + 144*a*b^2*c^2*C*d - 30*A*b
^3*c*d^2 - 110*a*b^2*B*c*d^2 - 144*a^2*b*c*C*d^2 + 30*b^3*c*C*d^2 + 15*a^3
*A*d^3 + 15*a*A*b^2*d^3 + 40*a^2*b*B*d^3 + 33*a^3*C*d^3 - 15*a*b^2*C*d^3)/
2 + (48*b^3*c^3*C - 40*b^3*B*c^2*d - 144*a*b^2*c^2*C*d + 30*A*b^3*c*d^2 +
110*a*b^2*B*c*d^2 + 144*a^2*b*c*C*d^2 - 30*b^3*c*C*d^2 - 60*a*A*b^2*d^3 -
85*a^2*b*B*d^3 + 15*b^3*B*d^3 - 48*a^3*C*d^3 + 60*a*b^2*C*d^3)/2))*(-(Hype
rgeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqr
t[c + d*Tan[e + f*x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e ...
```

3.116.3 Rubi [A] (warning: unable to verify)

Time = 3.84 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.383, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{(a+b\tan(e+fx))^2\left(b(6Cc^2-5Bdc+(5A+C)d^2)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+6bd)+2\left(3bc-\frac{ad}{2}\right)(cC-Bd)}{2\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{\int \frac{(a+b\tan(e+fx))^2(b(6Cc^2-5Bdc+(5A+C)d^2)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+6bd)+(6bc-ad)(cC-Bd))}{\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(e+fx))^2(b(6Cc^2-5Bdc+(5A+C)d^2)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+6bd)+(6bc-ad)(cC-Bd))}{\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4130

$$2\int -\frac{\frac{(a+b\tan(e+fx))\left(-5\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-5a(Ad(ac+6bd)+(6bc-ad)(cC-Bd))d+b\left(4(bc-ad)\left(6Cc^2-5Bc\right.\right.\right.}{2\sqrt{c+d\tan(e+fx)}}}{5d}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{2b(d^2(5A+C)-5Bcd+6c^2C)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\int \frac{(a+b\tan(e+fx))\left(-5\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan\right.}{}}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

3.116. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\int\frac{(a+b\tan(e+fx))\Big(-5\Big((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\Big)\tan(\ldots}{\ldots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 4120$$

$$\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\Big(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(b\ldots}{3df}}{\ldots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\Big(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(b\ldots}{3df}}{\ldots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\Big(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(b\ldots}{3df}}{\ldots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 4113$$

$$\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\Big(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(b\ldots}{3df}}{\ldots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

3.116. $\displaystyle\int\frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4022

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$\downarrow$ 3042

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$\downarrow$ 4020

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$\downarrow$ 25

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$\downarrow$ 73

3.116. $\displaystyle\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

$\downarrow$ 221

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b\left(d^2(5A+C)-5Bcd+6c^2C\right)(a+b\tan(e+fx))^2\sqrt{c+d\tan(e+fx)}}{5df}-\frac{\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(4(bc-ad)\left(d^2(5A+C)-5Bcd+6c^2C\right)-5d^2((A-C)(bc}{3df}}{}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(d*(c^2 + d^2)*f*Sqrt[
c + d*Tan[e + f*x]]) + ((2*b*(6*c^2*C - 5*B*c*d + (5*A + C)*d^2)*(a + b*Ta
n[e + f*x])^2*Sqrt[c + d*Tan[e + f*x]])/(5*d*f) - ((2*b^2*(4*(b*c - a*d)*(
6*c^2*C - 5*B*c*d + (5*A + C)*d^2) - 5*d^2*((A - C)*(b*c - a*d) + B*(a*c +
 b*d)))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) - ((15*(a - I*b)^3*
(A - I*B - C)*(c + I*d)*d^3*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c -
I*d]*f) + (15*(a + I*b)^3*(A + I*B - C)*(c - I*d)*d^3*ArcTan[Tan[e + f*x]/
Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*b*(6*a^2*d^2*(12*c^2*C - 5*B*c*d +
(5*A + 7*C)*d^2) - 15*a*b*d*(8*c^3*C - 6*B*c^2*d + c*(3*A + 5*C)*d^2 - 3*B
*d^3) + b^2*(48*c^4*C - 40*B*c^3*d + 6*c^2*(5*A + 3*C)*d^2 - 25*B*c*d^3 +
15*(A - C)*d^4))*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(3*d))/(5*d))/(d*(c^2 +
d^2))
```

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.116. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.116. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.116. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11254 vs. 2(476) = 952.

Time = 0.44 (sec) , antiderivative size = 11255, normalized size of antiderivative = 22.03

method	result	size
parts	Expression too large to display	11255
derivativedivides	Expression too large to display	49725
default	Expression too large to display	49725

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.116.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.116.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{(a+b\tan{(e+fx)})^3\,(A+B\tan{(e+}$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(3/2), x)
```

3.116.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.116.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(3/2),x)
```

output
```
\text{Hanged}
```

3.117 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.117.1 Optimal result

Integrand size = 47, antiderivative size = 343

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{(a-ib)^2(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} - \frac{(a+ib)^2(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f} - \frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^2}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}} + \frac{2b(6ad(2c^2C-Bcd+(A+C)d^2)-b(8c^3C-6Bc^2d+c(3A+5C)d^2-3Bd^3))\,\sqrt{c+d\tan(e+fx)}}{3d^3\,(c^2+d^2)\,f} + \frac{2b^2(4c^2C-3Bcd+(3A+C)d^2)\tan(e+fx)\sqrt{c+d\tan(e+fx)}}{3d^2\,(c^2+d^2)\,f}$$

output
```
-(a-I*b)^2*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*
d)^(3/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(c+I*d)^(3/2)/f+2/3*b*(6*a*d*(2*c^2*C-B*c*d+(A+C)*d^2)-b*(8*c^3*C-6*B
*c^2*d+c*(3*A+5*C)*d^2-3*B*d^3))*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)/f+2/
3*b^2*(4*c^2*C-3*B*c*d+(3*A+C)*d^2)*(c+d*tan(f*x+e))^(1/2)*tan(f*x+e)/d^2/
(c^2+d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^2)/f/(c+d*ta
n(f*x+e))^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.57 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.39

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\frac{2C(a+b\tan(e+fx))^2}{3df\sqrt{c+d\tan(e+fx)}}$$

$$+\frac{2\left(\frac{(-4bcC+3bBd+4aCd)(a+b\tan(e+fx))}{df\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2\left(8b^2c^2C-6b^2Bcd-16abcCd+3Ab^2d^2+9abBd^2+8a^2Cd^2-3b^2Cd^2\right)}{d\sqrt{c+d\tan(e+fx)}}+\frac{2\left(\frac{3}{2}\left(a^2B-b^2B+2ab(A-C)\right)d^2\left(\cdots\right.\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*C*(a + b*Tan[e + f*x])^2)/(3*d*f*Sqrt[c + d*Tan[e + f*x]]) + (2*(((-4*b
*c*C + 3*b*B*d + 4*a*C*d)*(a + b*Tan[e + f*x]))/(d*f*Sqrt[c + d*Tan[e + f*
x]]) + ((-2*(8*b^2*c^2*C - 6*b^2*B*c*d - 16*a*b*c*C*d + 3*A*b^2*d^2 + 9*a*
b*B*d^2 + 8*a^2*C*d^2 - 3*b^2*C*d^2))/(d*Sqrt[c + d*Tan[e + f*x]]) + (2*((
3*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*
x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sq
rt[c + I*d]])/Sqrt[c + I*d]))/2 + (((-3*c*(a^2*B - b^2*B + 2*a*b*(A - C))*
d^3)/2 - (3*(2*a*b*B - a^2*(A - C) + b^2*(A - C))*d^4)/2)*(-(Hypergeometri
c2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*T
an[e + f*x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c +
 I*d)]/((I*c - d)*Sqrt[c + d*Tan[e + f*x]])))/d))/d)/(2*d*f)))/(3*d)
```

3.117.3 Rubi [A] (warning: unable to verify)

Time = 2.33 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.319, Rules used = {3042, 4128, 27, 3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-3Bdc+(3A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+4bd)+2\left(2bc-\frac{ad}{2}\right)(cC-Bd)\right)}{2\sqrt{c+d\tan(e+fx)}}}{d\left(c^2+d^2\right)}$$
$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-3Bdc+(3A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+4bd)+(4bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{d\left(c^2+d^2\right)} -$$
$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-3Bdc+(3A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+4bd)+(4bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{d\left(c^2+d^2\right)} -$$
$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 4120}$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{2\int \frac{2c\left(4Cc^2-3Bdc+(3A+C)d^2\right)b^2-\left(6ad\left(2Cc^2-Bdc+(A+C)d^2\right)-b\left(8Cc^3-6Bdc^2+(3A+\cdots\right.\right.}{\cdots}}{\cdots}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{2c\left(4Cc^2-3Bdc+(3A+C)d^2\right)b^2-\left(6ad\left(2Cc^2-Bdc+(A+C)d^2\right)-b\left(8Cc^3-6Bdc^2+(3A+5\cdots\right.\right.}{\cdots}}{\cdots}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{2c\left(4Cc^2-3Bdc+(3A+C)d^2\right)b^2-\left(6ad\left(2Cc^2-Bdc+(A+C)d^2\right)-b\left(8Cc^3-6Bdc^2+(3A+5\cdots\right.\right.}{\cdots}}{\cdots}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4113

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{-3\left((Ac-Cc+Bd)a^2-2b(Bc-(A-C)d)a-b^2(Ac-Cc+Bd)\right)d^2-3\left((Bc-(A-C)d)a^2+2b(\cdots\right.}{\sqrt{c+d\tan(e+fx)}}\cdots}{\cdots}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int \frac{-3\left((Ac-Cc+Bd)a^2-2b(Bc-(A-C)d)a-b^2(Ac-Cc+Bd)\right)d^2-3\left((Bc-(A-C)d)a^2+2b(\cdots\right.}{\sqrt{c+d\tan(e+fx)}}\cdots}{\cdots}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4022

3.117. $$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3}{2}d^2(a+ib)^2(c-id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{3}{2}d^2(a-ib)^2(c+id)(A-iB-}{}}{d\left(c^2+d^2\right)}$$

↓ 3042

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3}{2}d^2(a+ib)^2(c-id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{3}{2}d^2(a-ib)^2(c+id)(A-iB-}{}}{d\left(c^2+d^2\right)}$$

↓ 4020

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3id^2(a-ib)^2(c+id)(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{3id}{}}{}}{d}$$

↓ 25

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\frac{3id^2(a-ib)^2(c+id)(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{3id^2(a}{}}{}}{d(}$$

↓ 73

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3d(a-ib)^2(c+id)(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\frac{3d(a+ib)^2(c-id}{}}{}}{d\left(c^2+d^2\right.}$$

↓ 221

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{2b^2\tan(e+fx)\left(d^2(3A+C)-3Bcd+4c^2C\right)\sqrt{c+d\tan(e+fx)}}{3df}-\frac{-\frac{3d^2(a-ib)^2(c+id)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}-\frac{3d^2(a+ib)^2(c-id)(A+iB-C)\arctan}{f\sqrt{c+id}}}{}}{d\left(c^2+d^2\right)}$$

3.117. $\int\frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}dx$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(d*(c^2 + d^2)*f*Sqrt[
c + d*Tan[e + f*x]]) + ((2*b^2*(4*c^2*C - 3*B*c*d + (3*A + C)*d^2)*Tan[e +
 f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) - ((-3*(a - I*b)^2*(A - I*B - C)*(
c + I*d)*d^2*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f) - (3*(a
 + I*b)^2*(A + I*B - C)*(c - I*d)*d^2*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/
(Sqrt[c + I*d]*f) - (2*b*(6*a*d*(2*c^2*C - B*c*d + (A + C)*d^2) - b*(8*c^3
*C - 6*B*c^2*d + c*(3*A + 5*C)*d^2 - 3*B*d^3))*Sqrt[c + d*Tan[e + f*x]])/(
d*f))/(3*d))/(d*(c^2 + d^2))
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.117. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9398 vs. 2(312) = 624.

Time = 0.22 (sec) , antiderivative size = 9399, normalized size of antiderivative = 27.40

method	result	size
parts	Expression too large to display	9399
derivativedivides	Expression too large to display	36710
default	Expression too large to display	36710

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+fx))^2\,(A+B\tan(e}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(3/2), x)
```

3.117.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.117.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.117.9 Mupad [B] (verification not implemented)

Time = 63.78 (sec) , antiderivative size = 54886, normalized size of antiderivative = 160.02

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(3/2),x)
```

output
```
(2*(B*b^2*c^3 + B*a^2*c*d^2 - 2*B*a*b*c^2*d))/(d^2*f*(c^2 + d^2)*(c + d*ta
n(e + f*x))^(1/2)) - atan((((-(((8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 4
8*B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B
^2*a^4*c*d^2*f^2 - 24*B^2*b^4*c*d^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*
a^3*b*c^2*d*f^2 + 144*B^2*a^2*b^2*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^
4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 +
6*B^4*a^4*b^4 + 4*B^4*a^6*b^2))^(1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*
f^2 + 24*B^2*a^2*b^2*c^3*f^2 - 16*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2
 + 12*B^2*a^4*c*d^2*f^2 + 12*B^2*b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 -
48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4
+ 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*((c + d*tan(e + f*x))^(1/2)*(-(((
8*B^2*a^4*c^3*f^2 + 8*B^2*b^4*c^3*f^2 - 48*B^2*a^2*b^2*c^3*f^2 + 32*B^2*a*
b^3*d^3*f^2 - 32*B^2*a^3*b*d^3*f^2 - 24*B^2*a^4*c*d^2*f^2 - 24*B^2*b^4*c*d
^2*f^2 - 96*B^2*a*b^3*c^2*d*f^2 + 96*B^2*a^3*b*c^2*d*f^2 + 144*B^2*a^2*b^2
*c*d^2*f^2)^2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f
^4)*(B^4*a^8 + B^4*b^8 + 4*B^4*a^2*b^6 + 6*B^4*a^4*b^4 + 4*B^4*a^6*b^2))^(
1/2) - 4*B^2*a^4*c^3*f^2 - 4*B^2*b^4*c^3*f^2 + 24*B^2*a^2*b^2*c^3*f^2 - 16
*B^2*a*b^3*d^3*f^2 + 16*B^2*a^3*b*d^3*f^2 + 12*B^2*a^4*c*d^2*f^2 + 12*B^2*
b^4*c*d^2*f^2 + 48*B^2*a*b^3*c^2*d*f^2 - 48*B^2*a^3*b*c^2*d*f^2 - 72*B^2*a
^2*b^2*c*d^2*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^...
```

3.117. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.118 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.118.1 Optimal result

Integrand size = 45, antiderivative size = 201

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{(ia+b)(A-iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} + \frac{(ia-b)(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f} + \frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{d^2\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}} + \frac{2bC\sqrt{c+d\tan(e+fx)}}{d^2f}$$

output
```
-(I*a+b)*(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(
3/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+
I*d)^(3/2)/f+2*(-a*d+b*c)*(A*d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x
+e))^(1/2)+2*b*C*(c+d*tan(f*x+e))^(1/2)/d^2/f
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.78 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{(Ab+aB-bC)\left(-\frac{i\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+}}{\sqrt{c-id}}\right.}{\sqrt{c-id}}\right.}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(3/2),x]
```

output
```
((A*b + a*B - b*C)*(((-I)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]])
/Sqrt[c - I*d] + (I*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqrt[
c + I*d]) - (2*(-2*b*c*C + b*B*d + 2*a*C*d))/(d*Sqrt[c + d*Tan[e + f*x]])
+ ((A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)*(((-I)*c + d)*Hypergeom
etric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hyperge
ometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sq
rt[c + d*Tan[e + f*x]]) + (2*C*(a + b*Tan[e + f*x]))/Sqrt[c + d*Tan[e + f*
x]])/(d*f)
```

3.118.3 Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {3042, 4118, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 4118$$

$$\frac{\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{bC(c^2+d^2)\tan(e+fx)^2+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4113

$$\frac{\int \frac{d(a(Ac-Cc+Bd)-b(Bc-(A-C)d))+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2bC(c^2+d^2)\sqrt{c+d\tan(e+fx)}}{df}}{d(c^2+d^2)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{d(a(Ac-Cc+Bd)-b(Bc-(A-C)d))+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2bC(c^2+d^2)\sqrt{c+d\tan(e+fx)}}{df}}{d(c^2+d^2)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4022

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{1}{2}d(a+ib)(c-id)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}d(a-ib)(c+id)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2bC(c}{}}{d(c^2+d^2)}$$

$\big\downarrow$ 3042

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{1}{2}d(a+ib)(c-id)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}d(a-ib)(c+id)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx+\frac{2bC(c}{}}{d(c^2+d^2)}$$

$\big\downarrow$ 4020

3.118. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{id(a-ib)(c+id)(A-iB-C)\int -\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{id(a+ib)(c-id)(A+iB-C)\int -\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(\cdots}{2f}}{d\left(c^2+d^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{id(a-ib)(c+id)(A-iB-C)\int \frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}+\frac{id(a+ib)(c-id)(A+iB-C)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-\cdots}{2f}}{d\left(c^2+d^2\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{(a+ib)(c-id)(A+iB-C)\int \frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}+\frac{(a-ib)(c+id)(A-iB-C)\int \frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}+\frac{2bC\cdots}{}}{d\left(c^2+d^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{d(a-ib)(c+id)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{d(a+ib)(c-id)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}+\frac{2bC\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}{df}}{d\left(c^2+d^2\right)}$$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*
Tan[e + f*x])^(3/2),x]
```

output
```
(2*(b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(d^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[
e + f*x]]) + (((a - I*b)*(A - I*B - C)*(c + I*d)*d*ArcTan[Tan[e + f*x]/Sqr
t[c - I*d]])/(Sqrt[c - I*d]*f) + ((a + I*b)*(A + I*B - C)*(c - I*d)*d*ArcT
an[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*b*C*(c^2 + d^2)*Sqr
t[c + d*Tan[e + f*x]])/(d*f))/(d*(c^2 + d^2))
```

3.118. $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.118. $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7395 vs. $2(177) = 354$.

Time = 0.16 (sec) , antiderivative size = 7396, normalized size of antiderivative = 36.80

method	result	size
parts	Expression too large to display	7396
derivativedivides	Expression too large to display	23472
default	Expression too large to display	23472

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31879 vs. $2(170) = 340$.

Time = 61.21 (sec) , antiderivative size = 31879, normalized size of antiderivative = 158.60

$$\int \frac{(a + b\tan(e + fx))\,(A + B\tan(e + fx) + C\tan^2(e + fx))}{(c + d\tan(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.118.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{(a+b\tan{(e+fx)})\,(A+B\tan{(e+$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c
+ d*tan(e + f*x))**(3/2), x)
```

3.118.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.118.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.118.9 Mupad [B] (verification not implemented)

Time = 38.60 (sec) , antiderivative size = 40542, normalized size of antiderivative = 201.70

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x))^(3/2),x)
```

output
```
atan((((c + d*tan(e + f*x))^(1/2)*(16*A^2*a^2*d^10*f^3 - 16*B^2*a^2*d^10*f
^3 + 16*C^2*a^2*d^10*f^3 + 32*A^2*a^2*c^2*d^8*f^3 - 32*A^2*a^2*c^6*d^4*f^3
 - 16*A^2*a^2*c^8*d^2*f^3 - 32*B^2*a^2*c^2*d^8*f^3 + 32*B^2*a^2*c^6*d^4*f^
3 + 16*B^2*a^2*c^8*d^2*f^3 + 32*C^2*a^2*c^2*d^8*f^3 - 32*C^2*a^2*c^6*d^4*f
^3 - 16*C^2*a^2*c^8*d^2*f^3 - 32*A*C*a^2*d^10*f^3 - 64*A*B*a^2*c*d^9*f^3 +
 64*B*C*a^2*c*d^9*f^3 - 192*A*B*a^2*c^3*d^7*f^3 - 192*A*B*a^2*c^5*d^5*f^3
- 64*A*B*a^2*c^7*d^3*f^3 - 64*A*C*a^2*c^2*d^8*f^3 + 64*A*C*a^2*c^6*d^4*f^3
 + 32*A*C*a^2*c^8*d^2*f^3 + 192*B*C*a^2*c^3*d^7*f^3 + 192*B*C*a^2*c^5*d^5*
f^3 + 64*B*C*a^2*c^7*d^3*f^3) - ((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2
+ 8*C^2*a^2*c^3*f^2 - 16*A*B*a^2*d^3*f^2 - 16*A*C*a^2*c^3*f^2 + 16*B*C*a^2
*d^3*f^2 - 24*A^2*a^2*c*d^2*f^2 + 24*B^2*a^2*c*d^2*f^2 - 24*C^2*a^2*c*d^2*
f^2 + 48*A*B*a^2*c^2*d*f^2 + 48*A*C*a^2*c*d^2*f^2 - 48*B*C*a^2*c^2*d*f^2)^
2/4 - (16*c^6*f^4 + 16*d^6*f^4 + 48*c^2*d^4*f^4 + 48*c^4*d^2*f^4)*(A^4*a^4
 + B^4*a^4 + C^4*a^4 - 4*A*C^3*a^4 - 4*A^3*C*a^4 + 2*A^2*B^2*a^4 + 6*A^2*C
^2*a^4 + 2*B^2*C^2*a^4 - 4*A*B^2*C*a^4))^(1/2) - 4*A^2*a^2*c^3*f^2 + 4*B^2
*a^2*c^3*f^2 - 4*C^2*a^2*c^3*f^2 + 8*A*B*a^2*d^3*f^2 + 8*A*C*a^2*c^3*f^2 -
 8*B*C*a^2*d^3*f^2 + 12*A^2*a^2*c*d^2*f^2 - 12*B^2*a^2*c*d^2*f^2 + 12*C^2*
a^2*c*d^2*f^2 - 24*A*B*a^2*c^2*d*f^2 - 24*A*C*a^2*c*d^2*f^2 + 24*B*C*a^2*c
^2*d*f^2)/(16*(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4)))^(1/2)*
((c + d*tan(e + f*x))^(1/2)*((((8*A^2*a^2*c^3*f^2 - 8*B^2*a^2*c^3*f^2 +...
```

3.119 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.119.1 Optimal result

Integrand size = 35, antiderivative size = 157

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{3/2}f} - \frac{(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{3/2}f} - \frac{2(c^2C-Bcd+Ad^2)}{d(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(3/2)/f
-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(3/2)/f
-2*(A*d^2-B*c*d+C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.39

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{-iB\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{\sqrt{c+id}}\right) - \cdots}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/
2),x]
```

output
```
((-I)*B*(ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c - I*d]]/Sqrt[c - I*d] - A
rcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]]/Sqrt[c + I*d]) - (2*C)/Sqrt
[c + d*Tan[e + f*x]] + ((B*c + (-A + C)*d)*(((-I)*c + d)*Hypergeometric2F1
[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] + (I*c + d)*Hypergeometric2
F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]))/((c^2 + d^2)*Sqrt[c + d
*Tan[e + f*x]]))/(d*f)
```

3.119.3 Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {3042, 4111, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 4111$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \ 4022$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{1}{2}(c-id)(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c+id)(A-iB-C)\int \frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{1}{2}(c-id)(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c+id)(A-iB-C)\int\frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}$$

$$\Big\downarrow \; 4020$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{i(c+id)(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(c-id)(A+iB-C)\int-\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}}{c^2+d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{i(c-id)(A+iB-C)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}-\frac{i(c+id)(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}}{c^2+d^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{(c-id)(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(c+id)(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}}{c^2+d^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{(c+id)(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(c-id)(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}}{c^2+d^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(((A - I*B - C)*(c + I*d)*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*
d]*f) + ((A + I*B - C)*(c - I*d)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt
[c + I*d]*f))/(c^2 + d^2) - (2*(c^2*C - B*c*d + A*d^2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]])
```

3.119. $\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.119.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.119. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5612 vs. 2(136) = 272.

Time = 0.13 (sec) , antiderivative size = 5613, normalized size of antiderivative = 35.75

method	result	size
parts	Expression too large to display	5613
derivativedivides	Expression too large to display	11427
default	Expression too large to display	11427

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7982 vs. 2(129) = 258.

Time = 1.83 (sec) , antiderivative size = 7982, normalized size of antiderivative = 50.84

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algori
thm="fricas")
```

output
```
Too large to include
```

3.119.6 Sympy [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^{3/2}}\,dx = \int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(3
/2), x)
```

3.119.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algori
thm="maxima")
```

output
```
Timed out
```

3.119.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(3/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.119.9 Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 8588, normalized size of antiderivative = 54.70

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(3/2),x)
```

output
```
(log((((((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2
) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 +
 3*c^4*d^2*f^4))^(1/2)*(64*C*c*d^11*f^4 - ((c + d*tan(e + f*x))^(1/2)*(((9
6*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^
3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f
^4))^(1/2)*(64*c*d^12*f^5 + 320*c^3*d^10*f^5 + 640*c^5*d^8*f^5 + 640*c^7*d
^6*f^5 + 320*c^9*d^4*f^5 + 64*c^11*d^2*f^5))/4 + 256*C*c^3*d^9*f^4 + 384*C
*c^5*d^7*f^4 + 256*C*c^7*d^5*f^4 + 64*C*c^9*d^3*f^4))/4 + (c + d*tan(e + f
*x))^(1/2)*(16*C^2*d^10*f^3 + 32*C^2*c^2*d^8*f^3 - 32*C^2*c^6*d^4*f^3 - 16
*C^2*c^8*d^2*f^3))*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^
2*f^4)^(1/2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^
2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 - 8*C^3*d^9*f^2 - 24*C^3*c^2*d^7*f^2
- 24*C^3*c^4*d^5*f^2 - 8*C^3*c^6*d^3*f^2)*(((96*C^4*c^2*d^4*f^4 - 16*C^4*d
^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) - 4*C^2*c^3*f^2 + 12*C^2*c*d^2*f^2)/(c
^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1/2))/4 + (log(((((-((
96*C^4*c^2*d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c
^3*f^2 - 12*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*
f^4))^(1/2)*(64*C*c*d^11*f^4 - ((c + d*tan(e + f*x))^(1/2)*(-((96*C^4*c^2*
d^4*f^4 - 16*C^4*d^6*f^4 - 144*C^4*c^4*d^2*f^4)^(1/2) + 4*C^2*c^3*f^2 - 12
*C^2*c*d^2*f^2)/(c^6*f^4 + d^6*f^4 + 3*c^2*d^4*f^4 + 3*c^4*d^2*f^4))^(1...
```

3.119. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.120 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

3.120.1 Optimal result

Integrand size = 47, antiderivative size = 262

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \frac{(A-iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{3/2}f} + \frac{(iA-B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)(c+id)^{3/2}f} - \frac{2\sqrt{b}(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)\,(bc-ad)^{3/2}f} + \frac{2(c^2C-Bcd+Ad^2)}{(bc-ad)\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

output
```
(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(3
/2)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)/(c
+I*d)^(3/2)/f-2*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(1/2)
/(-a*d+b*c)^(1/2))*b^(1/2)/(a^2+b^2)/(-a*d+b*c)^(3/2)/f+2*(A*d^2-B*c*d+C*c
^2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.13

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \frac{-\frac{i\left(\frac{(a+ib)(A-iB-C)(c+id)(-bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{\sqrt{c-id}}+\frac{(a-ib)(A+\cdots}{\cdots}\right.}{a^2+b^2}\cdots}{\cdots}$$

input

```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
 + d*Tan[e + f*x])^(3/2)),x]
```

output

```
(((-I)*(((a + I*b)*(A - I*B - C)*(c + I*d)*(-(b*c) + a*d)*ArcTanh[Sqrt[c +
 d*Tan[e + f*x]]/Sqrt[c - I*d]])/Sqrt[c - I*d] + ((a - I*b)*(A + I*B - C)*
(c - I*d)*(b*c - a*d)*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c + I*d]])/Sqr
t[c + I*d]))/(a^2 + b^2) + (2*Sqrt[b]*(A*b^2 + a*(-(b*B) + a*C))*(c^2 + d^
2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^
2)*Sqrt[b*c - a*d]) - (2*(c^2*C - B*c*d + A*d^2))/Sqrt[c + d*Tan[e + f*x]]
)/((-(b*c) + a*d)*(c^2 + d^2)*f)
```

3.120.3 Rubi [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.340, Rules used = {3042, 4132, 27, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{2\int -\frac{-b(Cc^2-Bdc+Ad^2)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab(c^2+d^2)}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+$$
$$\frac{2(Ad^2-Bcd+c^2C)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan(e+fx)^2-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 4136$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{\int -\frac{(bc-ad)(bBc-b(A-C)d+a(Ac-Cc+Bd))+(bc-ad)(aBc+bCc-bBd+aCd-A(bc+ad))\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{b\left(c^2+d^2\right)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+}{(a+b\tan(e+fx))\sqrt{}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b\left(c^2+d^2\right)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan^2(e+fx)+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\int \frac{(bc-ad)(bBc-b(A-C)d+a(Ac-Cc+Bd))+(bc-ad)(aBc+bCc-bBd+aCd-A(bc+}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b\left(c^2+d^2\right)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\int \frac{(bc-ad)(bBc-b(A-C)d+a(Ac-Cc+Bd))+(bc-ad)(aBc+bCc-bBd+aCd-A(bc+}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 4022$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b\left(c^2+d^2\right)\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{1}{2}(a-ib)(c-id)(A+iB-C)(bc-ad)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(a+ib)(c+id)(A}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

3.120. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{1}{2}(a-ib)(c-id)(A+iB-C)(bc-ad)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(a+ib)(c+id)(A}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 4020

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{i(a+ib)(c+id)(A-iB-C)(bc-ad)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+f}{2f}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 25

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{i(a-ib)(c-id)(A+iB-C)(bc-ad)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+f}{2f}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 73

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{(a-ib)(c-id)(A+iB-C)(bc-ad)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}+\frac{(a+}{}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 221

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}-\frac{\frac{(a+ib)(c+id)(A-iB-C)(bc-ad)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(c-id)(A+iB-C)(b}{f\sqrt{c}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 4117

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{b(c^2+d^2)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{f(a^2+b^2)}-\frac{\frac{(a+ib)(c+id)(A-iB-C)(bc-ad)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(c-id)(A}{}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 73

3.120. $\displaystyle\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{2(Ad^2-Bcd+c^2C)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{-\frac{2b(c^2+d^2)(Ab^2-a(bB-aC))\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e+fx)}}{df(a^2+b^2)}-\frac{\frac{(a+ib)(c+id)(A-iB-C)(bc-ad)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(c-id)(\cdots}{\cdots}}{a^2+b^2}}{(c^2+d^2)(bc-ad)}$$

$\downarrow$ 221

$$\frac{2(Ad^2-Bcd+c^2C)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2\sqrt{b}(c^2+d^2)(Ab^2-a(bB-aC))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f(a^2+b^2)\sqrt{bc-ad}}-\frac{\frac{(a+ib)(c+id)(A-iB-C)(bc-ad)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+\frac{(a-ib)(c-id)(A+iB-C)(bc-\cdots}{f\sqrt{c+i\cdots}}}{a^2+b^2}}{(c^2+d^2)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*T
an[e + f*x])^(3/2)),x]
```

output
```
-((-((((a + I*b)*(A - I*B - C)*(c + I*d)*(b*c - a*d)*ArcTan[Tan[e + f*x]/S
qrt[c - I*d]])/(Sqrt[c - I*d]*f) + ((a - I*b)*(A + I*B - C)*(c - I*d)*(b*c
 - a*d)*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f))/(a^2 + b^2)
) + (2*Sqrt[b]*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)*Sqrt[b*c - a*d]*f))/((b
*c - a*d)*(c^2 + d^2))) + (2*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)*(c^2 +
d^2)*f*Sqrt[c + d*Tan[e + f*x]])
```

3.120.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.120. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26342 vs. $2(229) = 458$.

Time = 0.15 (sec) , antiderivative size = 26343, normalized size of antiderivative = 100.55

method	result	size
derivativedivides	Expression too large to display	26343
default	Expression too large to display	26343

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(3/2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})\,(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*(c
 + d*tan(e + f*x))**(3/2)), x)
```

3.120.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.120.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))(c + d\tan(e + fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))^(3/2)),x)
```

output
```
\text{Hanged}
```

3.121 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

3.121.1 Optimal result

Integrand size = 47, antiderivative size = 447

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{3/2}f}-\frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{3/2}f}$$
$$-\frac{\sqrt{b}(5a^3bBd-3a^4Cd+b^4(2Bc-3Ad)+ab^3(4Ac-4cC+Bd)-a^2b^2(2Bc+(7A-C)d))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{(a^2+b^2)^2\,(bc-ad)^{5/2}f}$$
$$-\frac{d(2b^2c(cC-Bd)-abB(c^2+d^2)+a^2(3c^2C-2Bcd+Cd^2)+A(2a^2d^2+b^2(c^2+3d^2)))}{(a^2+b^2)\,(bc-ad)^2\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$
$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)\,(bc-ad)f(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*
d)^(3/2)/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*
b)^2/(c+I*d)^(3/2)/f-(5*a^3*b*B*d-3*a^4*C*d+b^4*(-3*A*d+2*B*c)+a*b^3*(4*A*
c+B*d-4*C*c)-a^2*b^2*(2*B*c+(7*A-C)*d))*arctanh(b^(1/2)*(c+d*tan(f*x+e))^(
1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(a^2+b^2)^2/(-a*d+b*c)^(5/2)/f-d*(2*b^2*c*(
-B*d+C*c)-a*b*B*(c^2+d^2)+a^2*(-2*B*c*d+3*C*c^2+C*d^2)+A*(2*a^2*d^2+b^2*(c
^2+3*d^2)))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)+(-A*
b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*
x+e))
```

3.121.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2078 vs. $2(447) = 894$.

Time = 6.43 (sec) , antiderivative size = 2078, normalized size of antiderivative = 4.65

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
(c + d*Tan[e + f*x])^(3/2)),x]
```

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
Sqrt[c + d*Tan[e + f*x]])) - ((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((
-3*(A*b^2 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(
3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-1/
2*(a*d*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*
d))) + (((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d)
- (b*B - a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d
)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a
*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2) - I*((a*(-(b*c) + a*d)*((-3*(A*b^
2 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*
d - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 - b*(-1/2*(a*d*(
(-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) + (
((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b*B -
 a*C)*(2*b*c + a*d)))/2 - (b*(-(c*((-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A
*b - a*B - b*C)*d*(b*c - a*d))) + (d^2*(3*A*b^2*d - 2*a*A*(b*c - a*d) - (b
*B - a*C)*(2*b*c + a*d)))/2))/2)))*ArcTanh[Sqrt[c + d*Tan[e + f*x]]/Sqrt[c
 - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c) + a*d)*((-3*(A*b^2
 - a*(b*B - a*C))*d^2)/2 - c*(A*b - a*B - b*C)*(b*c - a*d) + (d*(3*A*b^2*d
 - 2*a*A*(b*c - a*d) - (b*B - a*C)*(2*b*c + a*d)))/2))/2 + a*(-1/2*(a*d*((
-3*c*(A*b^2 - a*(b*B - a*C))*d)/2 + (A*b - a*B - b*C)*d*(b*c - a*d))) +...
```

3.121.3 Rubi [A] (warning: unable to verify)

Time = 4.08 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.404, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\int \frac{3Adb^2+3\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-2aA(bc-ad)-(bB-aC)(2bc+ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3Adb^2+3\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-2aA(bc-ad)-(bB-aC)(2bc+ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3Adb^2+3\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-2aA(bc-ad)-(bB-aC)(2bc+ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{2\int -\frac{2d^2(Ac-Cc+Bd)a^3-b(4A-C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3+Bdc^2+4Cd^2c-Bd^3-2A\left(c^3+2d^2c\right)\right)a-bd\left(2Ad^2a^2+\left(3Cc^2-2Bdc+Cd^2\right)a^2-bB\left(c^2+d^2\right)a+2b^2c\cdots}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx}\cdots}\cdots}{\left(c^2+d^2\right)(bc-ad)}\cdots}{\cdots}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2d(2a^2Ad^2+a^2(-2Bcd+3c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+3d^2)+2b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\int\frac{2d^2(Ac-Cc+Bd)a^3-b(4A-C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3+B\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2d(2a^2Ad^2+a^2(-2Bcd+3c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+3d^2)+2b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\int\frac{2d^2(Ac-Cc+Bd)a^3-b(4A-C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3+B\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4136

$$-\frac{\frac{2d(2a^2Ad^2+a^2(-2Bcd+3c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+3d^2)+2b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\int\frac{2\left((bc-ad)^2\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac-\ldots\right.\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2d(2a^2Ad^2+a^2(-2Bcd+3c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+3d^2)+2b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{2\int\frac{(bc-ad)^2\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac-\ldots\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2d(2a^2Ad^2+a^2(-2Bcd+3c^2C+Cd^2)-abB(c^2+d^2)+Ab^2(c^2+3d^2)+2b^2c(cC-Bd))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{2\int\frac{(bc-ad)^2\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac-\ldots\right.}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4022

3.121. $\displaystyle\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}}{}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}}{}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}}{}$$

$$\Big\downarrow \text{25}$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}}{}$$

$$\Big\downarrow \text{73}$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$
$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}}{}$$

$$\Big\downarrow \text{221}$$

3.121. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{a}$$

$\downarrow$ 4117

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4Ac\right.}{}$$

$\downarrow$ 73

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{2b\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3(4A\right.}{}$$

$\downarrow$ 221

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+3c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+3d^2\right)+2b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2\sqrt{b}\left(c^2+d^2\right)\left(-3a^4Cd+5a^3bBd-a^2b^2(d(7A-C)+2Bc)+ab^3\right.}{f\left(a^2\right.}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d
*Tan[e + f*x])^(3/2)),x]
```

3.121. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
Sqrt[c + d*Tan[e + f*x]])) - (-(((2*(((a + I*b)^2*(A - I*B - C)*(c + I*d)*
(b*c - a*d)^2*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f) + ((a
- I*b)^2*(A + I*B - C)*(c - I*d)*(b*c - a*d)^2*ArcTan[Tan[e + f*x]/Sqrt[c
+ I*d]])/(Sqrt[c + I*d]*f)))/(a^2 + b^2) - (2*Sqrt[b]*(c^2 + d^2)*(5*a^3*b
*B*d - 3*a^4*C*d + b^4*(2*B*c - 3*A*d) + a*b^3*(4*A*c - 4*c*C + B*d) - a^2
*b^2*(2*B*c + (7*A - C)*d))*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqr
t[b*c - a*d]])/((a^2 + b^2)*Sqrt[b*c - a*d]*f))/((b*c - a*d)*(c^2 + d^2)))
 + (2*d*(2*a^2*A*d^2 + 2*b^2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^
2 + 3*d^2) + a^2*(3*c^2*C - 2*B*c*d + C*d^2)))/((b*c - a*d)*(c^2 + d^2)*f*
Sqrt[c + d*Tan[e + f*x]]))/(2*(a^2 + b^2)*(b*c - a*d))
```

3.121.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.121. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.121. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{3/2}}\,dx$

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40618 vs. $2(411) = 822$.

Time = 0.24 (sec) , antiderivative size = 40619, normalized size of antiderivative = 90.87

method	result	size
derivativedivides	Expression too large to display	40619
default	Expression too large to display	40619

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(3
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\, dx = \int \frac{A + B\tan\left(e + fx\right) + C\tan^2\left(e + fx\right)}{\left(a + b\tan\left(e + fx\right)\right)^2\left(c + d\tan\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*
x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**2
*(c + d*tan(e + f*x))**(3/2)), x)
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.121.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{3/2}}\, dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))^(3/2)),x)
```

output
```
\text{Hanged}
```

3.122 $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.122.1 Optimal result

Integrand size = 47, antiderivative size = 585

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{(a-ib)^3(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}$$

$$-\frac{(ia-b)^3(A+iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}$$

$$-\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^3}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{2(b(2c^4C-Bc^3d+4c^2Cd^2-3Bcd^3+2Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\,(a+b\tan(e+fx))^2}{d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}}$$

$$+\frac{2b(3abd(8c^4C-2Bc^3d-c^2(A-17C)d^2-8Bcd^3+(5A+3C)d^4)-b^2(16c^5C-8Bc^4d+2c^3(A+15C)d}{3d^4\,(c^2+d^2)^2}$$

$$+\frac{2b^2(b(8c^4C-4Bc^3d+c^2(A+15C)d^2-10Bcd^3+(7A+C)d^4)+3ad^2(2c(A-C)d-B(c^2-d^2)))\tan(e}{3d^3\,(c^2+d^2)^2\,f}$$

output
```
-(a-I*b)^3*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*
d)^(5/2)/f-(I*a-b)^3*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2
))/(c+I*d)^(5/2)/f+2/3*b*(3*a*b*d*(8*c^4*C-2*B*c^3*d-c^2*(A-17*C)*d^2-8*B*
c*d^3+(5*A+3*C)*d^4)-b^2*(16*c^5*C-8*B*c^4*d+2*c^3*(A+15*C)*d^2-17*B*c^2*d
^3+8*c*(A+C)*d^4-3*B*d^5)+6*a^2*d^3*(2*c*(A-C)*d-B*(c^2-d^2)))*(c+d*tan(f*
x+e))^(1/2)/d^4/(c^2+d^2)^2/f+2/3*b^2*(b*(8*c^4*C-4*B*c^3*d+c^2*(A+15*C)*d
^2-10*B*c*d^3+(7*A+C)*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(c+d*tan(f*x
+e))^(1/2)*tan(f*x+e)/d^3/(c^2+d^2)^2/f-2*(b*(2*A*d^4-B*c^3*d-3*B*c*d^3+2*
C*c^4+4*C*c^2*d^2)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^2/d^2
/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x
+e))^3/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.93 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.15

$$\int \frac{(a+b\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\frac{2C(a+b\tan(e+fx))^3}{3df(c+d\tan(e+fx))^{3/2}}$$

$$+\frac{2\left(-\frac{3(2bcC-bBd-2aCd)(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))^{3/2}}+\frac{2\left(-\frac{3\left(b(Ab+aB-bC)d^2+4(bc-ad)(2bcC-bBd-2aCd)\right)(a+b\tan(e+fx))}{2df(c+d\tan(e+fx))^{3/2}}-\frac{3\left(-\frac{2\left(-16b^3c^3C+8b^3Bc^2 \cdots\right.}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^(5/2),x]
```

3.122. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

output
```
(2*C*(a + b*Tan[e + f*x])^3)/(3*d*f*(c + d*Tan[e + f*x])^(3/2)) + (2*((-3*
(2*b*c*C - b*B*d - 2*a*C*d)*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*
x])^(3/2)) + (2*((-3*(b*(A*b + a*B - b*C)*d^2 + 4*(b*c - a*d)*(2*b*c*C - b
*B*d - 2*a*C*d))*(a + b*Tan[e + f*x]))/(2*d*f*(c + d*Tan[e + f*x])^(3/2))
- (3*((-2*(-16*b^3*c^3*C + 8*b^3*B*c^2*d + 48*a*b^2*c^2*C*d - 2*A*b^3*c*d^
2 - 18*a*b^2*B*c*d^2 - 48*a^2*b*c*C*d^2 + 2*b^3*c*C*d^2 + 9*a^2*b*B*d^3 +
b^3*B*d^3 + 16*a^3*C*d^3))/(3*d*(c + d*Tan[e + f*x])^(3/2)) + (2*((((3*c*(
a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C))*d^4)/2 + (3*(3*a^2*b*B
- b^3*B - a^3*(A - C) + 3*a*b^2*(A - C))*d^5)/2)*(-1/3*Hypergeometric2F1[-
3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*(c + d*Tan[e + f*
x])^(3/2)) + Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*
d)]/(3*(I*c - d)*(c + d*Tan[e + f*x])^(3/2))))/d - (3*(a^3*B - 3*a*b^2*B +
 3*a^2*b*(A - C) - b^3*(A - C))*d^3*(-(Hypergeometric2F1[-1/2, 1, 1/2, (c
+ d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c + d*Tan[e + f*x]])) + Hyper
geometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)]/((I*c - d)*Sqrt
[c + d*Tan[e + f*x]])))/2))/(3*d)))/(4*d*f)))/d))/(3*d)
```

3.122.3 Rubi [A] (warning: unable to verify)

Time = 4.58 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.383, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4120, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{3(a+b\tan(e+fx))^2\left(b\left(2Cc^2-Bdc+(A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+2bd)+\frac{2}{3}\left(3bc-\frac{3ad}{2}\right)(cC-Bd)}{2(c+d\tan(e+fx))^{3/2}}}{3d\left(c^2+d^2\right)}$$
$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(2Cc^2-Bdc+(A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+2bd)+(2bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}-$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(a+b\tan(e+fx))^2\left(b\left(2Cc^2-Bdc+(A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+2bd)+(2bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}-$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4128$$

$$2\int \frac{(a+b\tan(e+fx))\left(((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+2\left(\frac{ac}{2}+2bd\right)(Ad(ac+2bd)+(2bc-ad)(cC-Bc\ldots\right.}{2\sqrt{c+\ldots}}$$

$$d(c\ldots$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(a+b\tan(e+fx))\left(((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+2\left(\frac{ac}{2}+2bd\right)(Ad(ac+2bd)+(2bc-ad)(cC-Bd)\ldots\right.}{\sqrt{c+d\,\mathrm{ta}\ldots}}$$

$$d(c^2\ldots$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))\left(((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+2\left(\frac{ac}{2}+2bd\right)(Ad(ac+2bd)+(2bc-ad)(cC-Bd)\ldots\right.}{\sqrt{c+d\,\mathrm{ta}\ldots}}$$

$$d(c^2\ldots$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4120$$

3.122. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(c^2d^2(A+15C)+d^4(7A+C)-4Bc^3d-10Bcd^3+8c^4C\right)\right)}{3df} - \frac{2\int -\frac{-2c\left(8Cc^4-4Bdc^3+(A+15C)d^2c \right.}{}}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{-2c\left(8Cc^4-4Bdc^3+(A+15C)d^2c^2-10Bd^3c+(7A+C)d^4\right)b^3+6ad\left(4Cc^4-Bdc^3-2(A-5C)d^2c^2-7Bd^3c+4Ad^4\right)b^2+\left(6a^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^3+3ab\left(8Cc^4\right.\right.}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{-2c\left(8Cc^4-4Bdc^3+(A+15C)d^2c^2-10Bd^3c+(7A+C)d^4\right)b^3+6ad\left(4Cc^4-Bdc^3-2(A-5C)d^2c^2-7Bd^3c+4Ad^4\right)b^2+\left(6a^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^3+3ab\left(8Cc^4\right.\right.}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4113$$

$$\int \frac{-3\left(\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)d^3-3\left(\left(2c(A-C)d-\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{-3\left(\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)d^3-3\left(\left(2c(A-C)d-\right.\right.}{\sqrt{c+d\tan(e+fx)}}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4022$$

3.122. $\displaystyle\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\,(3ad^2(2cd(A-C)-B}{}$$

$$\Big\downarrow \ 3042$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\,(3ad^2(2cd(A-C)-B}{}$$

$$\Big\downarrow \ 4020$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\,(3ad^2(2cd(A-C)-B}{}$$

$$\Big\downarrow \ 25$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\,(3ad^2(2cd(A-C)-B}{}$$

$$\Big\downarrow \ 73$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\,(3ad^2(2cd(A-C)-B}{}$$

$$\Big\downarrow \ 221$$

3.122. $\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^3}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2(a+b\tan(e+fx))^2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(2Ad^4-Bc^3d-3Bcd^3+2c^4C+4c^2Cd^2\right)\right)}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{2b^2\tan(e+fx)\sqrt{c+d\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\right.\right.}{}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^3)/(3*d*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])^(3/2)) + ((-2*(b*(2*c^4*C - B*c^3*d + 4*c^2*C*d^2 - 3*B*
c*d^3 + 2*A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(a + b*Tan[e + f
*x])^2)/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + ((2*b^2*(b*(8*c^4*C -
 4*B*c^3*d + c^2*(A + 15*C)*d^2 - 10*B*c*d^3 + (7*A + C)*d^4) + 3*a*d^2*(2
*c*(A - C)*d - B*(c^2 - d^2)))*Tan[e + f*x]*Sqrt[c + d*Tan[e + f*x]])/(3*d
*f) + ((3*(a - I*b)^3*(A - I*B - C)*(c + I*d)^2*d^3*ArcTan[Tan[e + f*x]/Sq
rt[c - I*d]])/(Sqrt[c - I*d]*f) + (3*(a + I*b)^3*(A + I*B - C)*(c - I*d)^2
*d^3*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*b*(3*a*b*d
*(8*c^4*C - 2*B*c^3*d - c^2*(A - 17*C)*d^2 - 8*B*c*d^3 + (5*A + 3*C)*d^4)
- b^2*(16*c^5*C - 8*B*c^4*d + 2*c^3*(A + 15*C)*d^2 - 17*B*c^2*d^3 + 8*c*(A
 + C)*d^4 - 3*B*d^5) + 6*a^2*d^3*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[c +
 d*Tan[e + f*x]])/(d*f))/(3*d))/(d*(c^2 + d^2)))/(d*(c^2 + d^2))
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.122. $\int \frac{(a+b\tan(e+fx))^3(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13585 vs. 2(550) = 1100.

Time = 0.48 (sec) , antiderivative size = 13586, normalized size of antiderivative = 23.22

method	result	size
parts	Expression too large to display	13586
derivativedivides	Expression too large to display	85156
default	Expression too large to display	85156

input
```
int((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.122.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.122.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(5/2), x)
```

3.122.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.122.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(((a + b*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.123 $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.123.1 Optimal result

Integrand size = 47, antiderivative size = 358

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{(a-ib)^2(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} - \frac{(a+ib)^2(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f} - \frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^2}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}} + \frac{2(bc-ad)\,(b(4c^4C-Bc^3d-2c^2(A-5C)d^2-7Bcd^3+4Ad^4)+3ad^2(2c(A-C)d-B(c^2-d^2)))}{3d^3\,(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}} + \frac{2b^2(4c^2C-Bcd+(A+3C)d^2)\,\sqrt{c+d\tan(e+fx)}}{3d^3\,(c^2+d^2)\,f}$$

output
```
-(a-I*b)^2*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*
d)^(5/2)/f-(a+I*b)^2*(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1
/2))/(c+I*d)^(5/2)/f+2/3*(-a*d+b*c)*(b*(4*c^4*C-B*c^3*d-2*c^2*(A-5*C)*d^2-
7*B*c*d^3+4*A*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/d^3/(c^2+d^2)^2/f/(c
+d*tan(f*x+e))^(1/2)+2/3*b^2*(4*c^2*C-B*c*d+(A+3*C)*d^2)*(c+d*tan(f*x+e))^
(1/2)/d^3/(c^2+d^2)/f-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^2/d/(c^2+d^
2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.63 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \frac{2C(a+b\tan(e+fx))^2}{df(c+d\tan(e+fx))^{3/2}}$$

$$+ \frac{2\left(-\frac{(-4bcC+bBd+4aCd)(a+b\tan(e+fx))}{df(c+d\tan(e+fx))^{3/2}} - \frac{-\frac{2\left(8b^2c^2C-2b^2Bcd-16abcCd-Ab^2d^2+abBd^2+8a^2Cd^2+b^2Cd^2\right)}{3d(c+d\tan(e+fx))^{3/2}} + \frac{2\left(\frac{\left(\frac{3}{2}c\left(a^2B-b^2B+2ab(A-C)\right)d^3 \right.}{}\right.}{}}{}\right.}{}$$

```
input
Integrate[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^(5/2),x]
```

```
output
(2*C*(a + b*Tan[e + f*x])^2)/(d*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(-(((-4
*b*c*C + b*B*d + 4*a*C*d)*(a + b*Tan[e + f*x]))/(d*f*(c + d*Tan[e + f*x])^
(3/2))) - ((-2*(8*b^2*c^2*C - 2*b^2*B*c*d - 16*a*b*c*C*d - A*b^2*d^2 + a*b
*B*d^2 + 8*a^2*C*d^2 + b^2*C*d^2))/(3*d*(c + d*Tan[e + f*x])^(3/2)) + (2*(
(((3*c*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3)/2 + (3*(2*a*b*B - a^2*(A - C)
+ b^2*(A - C))*d^4)/2)*(-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (c + d*Tan[e
 + f*x])/(c - I*d)]/((I*c + d)*(c + d*Tan[e + f*x])^(3/2)) + Hypergeometri
c2F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]/(3*(I*c - d)*(c + d*Ta
n[e + f*x])^(3/2))))/d - (3*(a^2*B - b^2*B + 2*a*b*(A - C))*d^2*(-(Hyperge
ometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)]/((I*c + d)*Sqrt[c
 + d*Tan[e + f*x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x]
)/(c + I*d)]/((I*c - d)*Sqrt[c + d*Tan[e + f*x]])))/2))/(3*d))/(2*d*f)))/d
```

3.123.3 Rubi [A] (warning: unable to verify)

Time = 2.56 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.298, Rules used = {3042, 4128, 27, 3042, 4118, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 4128$$

$$\frac{2\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-Bdc+(A+3C)d^2\right)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+3ad(Ac-Cc+Bd)+4b\left(Cc^2-Bdc+Ac\cdots\right)\right)}{2(c+d\tan(e+fx))^{3/2}}\cdots}{3d\left(c^2+d^2\right)} \cdots \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-Bdc+(A+3C)d^2\right)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+3ad(Ac-Cc+Bd)+4b\left(Cc^2-Bdc+Ad^2\right)\cdots\right)}{(c+d\tan(e+fx))^{3/2}}\cdots}{3d\left(c^2+d^2\right)} \cdots \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(a+b\tan(e+fx))\left(b\left(4Cc^2-Bdc+(A+3C)d^2\right)\tan(e+fx)^2+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+3ad(Ac-Cc+Bd)+4b\left(Cc^2-Bdc+Ad^2\right)\cdots\right)}{(c+d\tan(e+fx))^{3/2}}\cdots}{3d\left(c^2+d^2\right)} \cdots \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4118$$

$$\frac{\int \frac{\left(c^2+d^2\right)\left(4Cc^2-Bdc+(A+3C)d^2\right)\tan^2(e+fx)b^2+\left(4Cc^4-Bdc^3-2(A-5C)d^2c^2-7Bd^3c+4Ad^4\right)b^2+6ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)b-3a^2d^2\left(Cc^2-2Bdc-Cd^2-\right.}{\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{\left(c^2+d^2\right)\left(4Cc^2-Bdc+(A+3C)d^2\right)\tan(e+fx)^2b^2+\left(4Cc^4-Bdc^3-2(A-5C)d^2c^2-7Bd^3c+4Ad^4\right)b^2+6ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)b-3a^2d^2\left(Cc^2-2Bdc-Cd^2-\right.}{\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4113

$$\frac{\int \frac{-3\left(\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2-2b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2-3\left(\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a^2+2b\left(Cc^2-2Bdc-\right.\right.}{\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{-3\left(\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^2-2b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a-b^2\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)\right)d^2-3\left(\left(2c(A-C)d-B\left(c^2-d^2\right)\right)a^2+2b\left(Cc^2-2Bdc-\right.\right.}{\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4022

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{3}{2}d^2(a+ib)^2(c-id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}}{}}{3d\left(c^2+d^2\right)}$$

$\downarrow$ 3042

3.123. $\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{3}{2}d^2(a+ib)^2(c-id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}$$

$$3d\left(c^2+d^2\right)$$

$\downarrow$ 4020

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{3id^2(a-ib)^2(c+id)^2(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{}}}{2f}$$

$\downarrow$ 25

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{3id^2(a-ib)^2(c+id)^2(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{}}}{2f}$$

$\downarrow$ 73

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{3d(a+ib)^2(c-id)^2(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}}{f}$$

$$3d\left(c^2+d\right.$$

$\downarrow$ 221

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^2}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2(bc-ad)\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-2c^2d^2(A-5C)+4Ad^4-Bc^3d-7Bcd^3+4c^4C\right)\right)}{d^2f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{3d^2(a-ib)^2(c+id)^2(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}+}{}$$

$$3d\left(c^2+d^2\right)$$

input
```
Int[((a + b*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^(5/2),x]
```

3.123. $\int\frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^2)/(3*d*(c^2 + d^2)*f*(c
+ d*Tan[e + f*x])^(3/2)) + ((2*(b*c - a*d)*(b*(4*c^4*C - B*c^3*d - 2*c^2*(
A - 5*C)*d^2 - 7*B*c*d^3 + 4*A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2))))/(d^2*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + ((3*(a - I*b)^2*(A -
I*B - C)*(c + I*d)^2*d^2*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d
]*f) + (3*(a + I*b)^2*(A + I*B - C)*(c - I*d)^2*d^2*ArcTan[Tan[e + f*x]/Sq
rt[c + I*d]])/(Sqrt[c + I*d]*f) + (2*b^2*(c^2 + d^2)*(4*c^2*C - B*c*d + (A
 + 3*C)*d^2)*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(d*(c^2 + d^2)))/(3*d*(c^2 +
 d^2))
```

3.123.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.123. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.123. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11359 vs. $2(325) = 650$.

Time = 0.22 (sec) , antiderivative size = 11360, normalized size of antiderivative = 31.73

method	result	size
parts	Expression too large to display	11360
derivativedivides	Expression too large to display	61833
default	Expression too large to display	61833

input
```
int((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.123.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.123.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+fx))^2 (A+B\tan(e}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**(5/2), x)
```

3.123.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.123.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.123.9 Mupad [B] (verification not implemented)

Time = 109.69 (sec) , antiderivative size = 88684, normalized size of antiderivative = 247.72

$$\int \frac{(a+b\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^(5/2),x)
```

output
```
atan((((c + d*tan(e + f*x))^(1/2)*(96*A^2*a^2*b^2*d^18*f^3 - 16*A^2*b^4*d^
18*f^3 - 16*A^2*a^4*d^18*f^3 + 320*A^2*a^4*c^4*d^14*f^3 + 1024*A^2*a^4*c^6
*d^12*f^3 + 1440*A^2*a^4*c^8*d^10*f^3 + 1024*A^2*a^4*c^10*d^8*f^3 + 320*A^
2*a^4*c^12*d^6*f^3 - 16*A^2*a^4*c^16*d^2*f^3 + 320*A^2*b^4*c^4*d^14*f^3 +
1024*A^2*b^4*c^6*d^12*f^3 + 1440*A^2*b^4*c^8*d^10*f^3 + 1024*A^2*b^4*c^10*
d^8*f^3 + 320*A^2*b^4*c^12*d^6*f^3 - 16*A^2*b^4*c^16*d^2*f^3 - 256*A^2*a*b
^3*c*d^17*f^3 + 256*A^2*a^3*b*c*d^17*f^3 - 1280*A^2*a*b^3*c^3*d^15*f^3 - 2
304*A^2*a*b^3*c^5*d^13*f^3 - 1280*A^2*a*b^3*c^7*d^11*f^3 + 1280*A^2*a*b^3*
c^9*d^9*f^3 + 2304*A^2*a*b^3*c^11*d^7*f^3 + 1280*A^2*a*b^3*c^13*d^5*f^3 +
256*A^2*a*b^3*c^15*d^3*f^3 + 1280*A^2*a^3*b*c^3*d^15*f^3 + 2304*A^2*a^3*b*
c^5*d^13*f^3 + 1280*A^2*a^3*b*c^7*d^11*f^3 - 1280*A^2*a^3*b*c^9*d^9*f^3 -
2304*A^2*a^3*b*c^11*d^7*f^3 - 1280*A^2*a^3*b*c^13*d^5*f^3 - 256*A^2*a^3*b*
c^15*d^3*f^3 - 1920*A^2*a^2*b^2*c^4*d^14*f^3 - 6144*A^2*a^2*b^2*c^6*d^12*f
^3 - 8640*A^2*a^2*b^2*c^8*d^10*f^3 - 6144*A^2*a^2*b^2*c^10*d^8*f^3 - 1920*
A^2*a^2*b^2*c^12*d^6*f^3 + 96*A^2*a^2*b^2*c^16*d^2*f^3) + ((((8*A^2*a^4*c^
5*f^2 + 8*A^2*b^4*c^5*f^2 - 48*A^2*a^2*b^2*c^5*f^2 - 80*A^2*a^4*c^3*d^2*f^
2 - 80*A^2*b^4*c^3*d^2*f^2 - 32*A^2*a*b^3*d^5*f^2 + 32*A^2*a^3*b*d^5*f^2 +
 40*A^2*a^4*c*d^4*f^2 + 40*A^2*b^4*c*d^4*f^2 - 160*A^2*a*b^3*c^4*d*f^2 + 1
60*A^2*a^3*b*c^4*d*f^2 + 320*A^2*a*b^3*c^2*d^3*f^2 - 240*A^2*a^2*b^2*c*d^4
*f^2 - 320*A^2*a^3*b*c^2*d^3*f^2 + 480*A^2*a^2*b^2*c^3*d^2*f^2)^2/4 - (...
```

3.123. $\int \frac{(a+b\tan(e+fx))^2(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.124 $\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.124.1 Optimal result

Integrand size = 45, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx =\\ -\frac{(a-ib)(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f}\\ +\frac{(ia-b)(A+iB-C)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f}\\ +\frac{2(bc-ad)\left(c^2C-Bcd+Ad^2\right)}{3d^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}\\ -\frac{2(b(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))}{d^2\left(c^2+d^2\right)^2f\sqrt{c+d\tan(e+fx)}}\end{aligned}$$

output
```
-(a-I*b)*(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)
^(5/2)/f+(I*a-b)*(A+I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(
c+I*d)^(5/2)/f-2*(b*(c^4*C-c^2*(A-3*C)*d^2-2*B*c*d^3+A*d^4)+a*d^2*(2*c*(A-
C)*d-B*(c^2-d^2)))/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(-a*d+b*c)
*(A*d^2-B*c*d+C*c^2)/d^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{2(c-id)(c+id)(2bcC+bBd-2aCd)+d(Abc+aBc-bcC-aAd+bBd+aCd)\left(i(c+id)\,\text{Hypergeo}\right.}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(5/2),x]
```

output
```
-1/3*(2*(c - I*d)*(c + I*d)*(2*b*c*C + b*B*d - 2*a*C*d) + d*(A*b*c + a*B*c
 - b*c*C - a*A*d + b*B*d + a*C*d)*(I*(c + I*d)*Hypergeometric2F1[-3/2, 1,
-1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[-3/2,
1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) + 6*C*(c - I*d)*(c + I*d)*d*(a +
 b*Tan[e + f*x]) - 3*(A*b + a*B - b*C)*d*(I*(c + I*d)*Hypergeometric2F1[-1
/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2F1[
-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[e + f*x]))/(d^2*
(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

3.124.3 Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4118, 3042, 4111, 25, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{ 4118}$$

$$\frac{\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{bC(c^2+d^2)\tan(e+fx)^2+d(Abc+aBc-bCc-aAd+bBd+aCd)\tan(e+fx)+ad(Ac-Cc+Bd)+b(Cc^2-Bdc+Ad^2)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4111

$$\frac{\frac{\int -\frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\ldots}{d\left(c^2+d^2\right)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\ldots}{d\left(c^2+d^2\right)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{d\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\ldots}{d\left(c^2+d^2\right)}+\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4022

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{1}{2}d(a+ib)(c-id)^2(A+iB-C)\int \frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{2}d(a-ib)(c+\ldots}{c^2+d^2}}{d\left(c^2+d^2\right)}$$

$\downarrow$ 3042

3.124. $\displaystyle \int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{1}{2}d(a+ib)(c-id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{2}d(a-ib)(c+\cdots}{c^2+d^2}}{d\left(c^2+d^2\right)}$$

$\downarrow$ 4020

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{id(a+ib)(c-id)^2(A+iB-C)\int-\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan\cdots}{2f}}{\cdots}}{d\left(c^2+d^2\right)}$$

$\downarrow$ 25

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{id(a-ib)(c+id)^2(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+\cdots}{2f}}{\cdots}}{d\left(c^2+d^2\right)}$$

$\downarrow$ 73

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{(a+ib)(c-id)^2(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{f}-\cdots}{c^2+d^2}}{d\left(c^2+d^2\right)}$$

$\downarrow$ 221

$$\frac{2(bc-ad)\left(Ad^2-Bcd+c^2C\right)}{3d^2f\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{d(a+ib)(c-id)^2(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{d(a-ib)(c+id)^2(A-i\cdots}{f\cdots}}{c^2+d^2}}{d\left(c^2+d^2\right)}$$

input
```
Int[((a + b*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c + d*
Tan[e + f*x])^(5/2),x]
```

3.124. $\int\frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$

output
```
(2*(b*c - a*d)*(c^2*C - B*c*d + A*d^2))/(3*d^2*(c^2 + d^2)*f*(c + d*Tan[e
+ f*x])^(3/2)) + (-((-(((a - I*b)*(A - I*B - C)*(c + I*d)^2*d*ArcTan[Tan[e
 + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f)) - ((a + I*b)*(A + I*B - C)*(c -
 I*d)^2*d*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f))/(c^2 + d^
2)) - (2*(b*(c^4*C - c^2*(A - 3*C)*d^2 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(
A - C)*d - B*(c^2 - d^2))))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/(d
*(c^2 + d^2))
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && !IntegerQ[m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8962 vs. $2(246) = 492$.

Time = 0.25 (sec) , antiderivative size = 8963, normalized size of antiderivative = 32.83

method	result	size
parts	Expression too large to display	8963
derivativedivides	Expression too large to display	40201
default	Expression too large to display	40201

input
```
int((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50755 vs. 2(237) = 474.

Time = 297.23 (sec) , antiderivative size = 50755, normalized size of antiderivative = 185.92

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.124.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+\right.}{(c+d\tan(e+fx}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e
))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c
+ d*tan(e + f*x))**(5/2), x)
```

3.124.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.124.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.124.9 Mupad [B] (verification not implemented)

Time = 85.53 (sec) , antiderivative size = 64641, normalized size of antiderivative = 236.78

$$\int \frac{(a+b\tan(e+fx))\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c + d*
tan(e + f*x))^(5/2),x)
```

output
```
((2*(C*b*c^3 + A*b*c*d^2 - B*b*c^2*d))/(3*(c^2 + d^2)) - (2*(c + d*tan(e +
 f*x))*(A*b*d^4 + C*b*c^4 - 2*B*b*c*d^3 - A*b*c^2*d^2 + 3*C*b*c^2*d^2))/(c
^2 + d^2)^2)/(d^2*f*(c + d*tan(e + f*x))^(3/2)) - atan(-(((c + d*tan(e + f
*x))^(1/2)*(16*A^2*b^2*d^18*f^3 - 16*B^2*b^2*d^18*f^3 + 16*C^2*b^2*d^18*f^
3 - 320*A^2*b^2*c^4*d^14*f^3 - 1024*A^2*b^2*c^6*d^12*f^3 - 1440*A^2*b^2*c^
8*d^10*f^3 - 1024*A^2*b^2*c^10*d^8*f^3 - 320*A^2*b^2*c^12*d^6*f^3 + 16*A^2
*b^2*c^16*d^2*f^3 + 320*B^2*b^2*c^4*d^14*f^3 + 1024*B^2*b^2*c^6*d^12*f^3 +
 1440*B^2*b^2*c^8*d^10*f^3 + 1024*B^2*b^2*c^10*d^8*f^3 + 320*B^2*b^2*c^12*
d^6*f^3 - 16*B^2*b^2*c^16*d^2*f^3 - 320*C^2*b^2*c^4*d^14*f^3 - 1024*C^2*b^
2*c^6*d^12*f^3 - 1440*C^2*b^2*c^8*d^10*f^3 - 1024*C^2*b^2*c^10*d^8*f^3 - 3
20*C^2*b^2*c^12*d^6*f^3 + 16*C^2*b^2*c^16*d^2*f^3 - 32*A*C*b^2*d^18*f^3 -
128*A*B*b^2*c*d^17*f^3 + 128*B*C*b^2*c*d^17*f^3 - 640*A*B*b^2*c^3*d^15*f^3
 - 1152*A*B*b^2*c^5*d^13*f^3 - 640*A*B*b^2*c^7*d^11*f^3 + 640*A*B*b^2*c^9*
d^9*f^3 + 1152*A*B*b^2*c^11*d^7*f^3 + 640*A*B*b^2*c^13*d^5*f^3 + 128*A*B*b
^2*c^15*d^3*f^3 + 640*A*C*b^2*c^4*d^14*f^3 + 2048*A*C*b^2*c^6*d^12*f^3 + 2
880*A*C*b^2*c^8*d^10*f^3 + 2048*A*C*b^2*c^10*d^8*f^3 + 640*A*C*b^2*c^12*d^
6*f^3 - 32*A*C*b^2*c^16*d^2*f^3 + 640*B*C*b^2*c^3*d^15*f^3 + 1152*B*C*b^2*
c^5*d^13*f^3 + 640*B*C*b^2*c^7*d^11*f^3 - 640*B*C*b^2*c^9*d^9*f^3 - 1152*B
*C*b^2*c^11*d^7*f^3 - 640*B*C*b^2*c^13*d^5*f^3 - 128*B*C*b^2*c^15*d^3*f^3)
 + ((((8*A^2*b^2*c^5*f^2 - 8*B^2*b^2*c^5*f^2 + 8*C^2*b^2*c^5*f^2 - 80*A...
```

3.124. $\int \frac{(a+b\tan(e+fx))(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.125 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.125.1 Optimal result

Integrand size = 35, antiderivative size = 209

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(c-id)^{5/2}f} - \frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(c+id)^{5/2}f} - \frac{2(c^2C-Bcd+Ad^2)}{3d(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} - \frac{2(2c(A-C)d-B(c^2-d^2))}{(c^2+d^2)^2 f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(c-I*d)^(5/2)/f
-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(c+I*d)^(5/2)/f
-2*(2*c*(A-C)*d-B*(c^2-d^2))/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d
^2-B*c*d+C*c^2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2C(c^2+d^2)+(Bc+(-A+C)d)\left(i(c+id)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{c+d\tan(e+fx)}{c-id}\right)-(ic+d)\,\text{Hyp}\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/
2),x]
```

output
```
-1/3*(2*C*(c^2 + d^2) + (B*c + (-A + C)*d)*(I*(c + I*d)*Hypergeometric2F1[
-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hypergeometric2
F1[-3/2, 1, -1/2, (c + d*Tan[e + f*x])/(c + I*d)]) - 3*B*(I*(c + I*d)*Hype
rgeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c - I*d)] - (I*c + d)*Hy
pergeometric2F1[-1/2, 1, 1/2, (c + d*Tan[e + f*x])/(c + I*d)])*(c + d*Tan[
e + f*x]))/(d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2))
```

3.125.3 Rubi [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4111, 3042, 4012, 25, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4111}$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}dx}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx)}{(c+d\tan(e+fx))^{3/2}}dx}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}}{c^2+d^2}-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4022$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{1}{2}(c-id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{2}(c+id)^2(A-iB-C)\int\frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}}{c^2+d^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{1}{2}(c-id)^2(A+iB-C)\int\frac{1-i\tan(e+fx)}{\sqrt{c+d\tan(e+fx)}}dx-\frac{1}{2}(c+id)^2(A-iB-C)\int\frac{i\tan(e+fx)+1}{\sqrt{c+d\tan(e+fx)}}dx}{c^2+d^2}}{c^2+d^2}$$

$$\Big\downarrow \; 4020$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{i(c-id)^2(A+iB-C)\int-\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e+fx)}}d(-i\tan(e+fx))}{2f}-\frac{i(c+id)^2(A-iB-C)\int-\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\ldots}}}{2f}}{c^2+d^2}}{c^2+d^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\,(c^2+d^2)\,(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f(c^2+d^2)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{i(c+id)^2(A-iB-C)\int\frac{1}{(1-i\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d(i\tan(e+fx))}{2f}-\frac{i(c-id)^2(A+iB-C)\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c+d\tan(e\ldots}}}{2f}}{c^2+d^2}}{c^2+d^2}$$

$$\Big\downarrow \; 73$$

3.125. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{(c-id)^2(A+iB-C)\int\frac{1}{-\frac{i\tan^2(e+fx)}{d}-\frac{ic}{d}+1}d\sqrt{c+d\tan(e+fx)}}{df}-\frac{(c+id)^2(A-iB-C)\int\frac{1}{\frac{i\tan^2(e+fx)}{d}+\frac{ic}{d}+1}d\sqrt{c+d\tan(e}{df}}{c^2+d^2}}{c^2+d^2}$$

$\Big\downarrow$ 221

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+\frac{-\frac{2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)}{f\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{(c-id)^2(A+iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c+id}}\right)}{f\sqrt{c+id}}-\frac{(c+id)^2(A-iB-C)\arctan\left(\frac{\tan(e+fx)}{\sqrt{c-id}}\right)}{f\sqrt{c-id}}}{c^2+d^2}}{c^2+d^2}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(c + d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2))/(3*d*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)
) + (-((-(((A - I*B - C)*(c + I*d)^2*ArcTan[Tan[e + f*x]/Sqrt[c - I*d]])/(
Sqrt[c - I*d]*f)) - ((A + I*B - C)*(c - I*d)^2*ArcTan[Tan[e + f*x]/Sqrt[c
+ I*d]])/(Sqrt[c + I*d]*f))/(c^2 + d^2)) - (2*(2*c*(A - C)*d - B*(c^2 - d^
2)))/((c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/(c^2 + d^2)
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6787 vs. $2(184) = 368$.

Time = 0.18 (sec) , antiderivative size = 6788, normalized size of antiderivative = 32.48

method	result	size
parts	Expression too large to display	6788
derivativedivides	Expression too large to display	20647
default	Expression too large to display	20647

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13143 vs. $2(177) = 354$.

Time = 7.15 (sec) , antiderivative size = 13143, normalized size of antiderivative = 62.89

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algori
thm="fricas")
```

output
```
Too large to include
```

3.125.6 Sympy [F]

$$\int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{A + B\tan(e+fx) + C\tan^2(e+fx)}{(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(c + d*tan(e + f*x))**(5
/2), x)
```

3.125.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algori
thm="maxima")
```

output
```
Timed out
```

3.125.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(5/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.125.9 Mupad [B] (verification not implemented)

Time = 35.92 (sec) , antiderivative size = 14163, normalized size of antiderivative = 67.77

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(c + d\tan(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/(c + d*tan(e + f*x))^(5/2),x)
```

output
```
(log(96*A^3*c^3*d^13*f^2 - ((((((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1
760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) -
4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^
4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2
)*(((((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 - 1760*A^4*c^4*d^6*f^4 + 1600
*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2) - 4*A^2*c^5*f^2 + 40*A^2*c^3
*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*f^4 + 5*c^2*d^8*f^4 + 10*c^4
*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))^(1/2)*(c + d*tan(e + f*x))^(1/
2)*(64*c*d^22*f^5 + 640*c^3*d^20*f^5 + 2880*c^5*d^18*f^5 + 7680*c^7*d^16*f
^5 + 13440*c^9*d^14*f^5 + 16128*c^11*d^12*f^5 + 13440*c^13*d^10*f^5 + 7680
*c^15*d^8*f^5 + 2880*c^17*d^6*f^5 + 640*c^19*d^4*f^5 + 64*c^21*d^2*f^5))/4
 - 32*A*d^21*f^4 - 160*A*c^2*d^19*f^4 - 128*A*c^4*d^17*f^4 + 896*A*c^6*d^1
5*f^4 + 3136*A*c^8*d^13*f^4 + 4928*A*c^10*d^11*f^4 + 4480*A*c^12*d^9*f^4 +
 2432*A*c^14*d^7*f^4 + 736*A*c^16*d^5*f^4 + 96*A*c^18*d^3*f^4))/4 - (c + d
*tan(e + f*x))^(1/2)*(320*A^2*c^4*d^14*f^3 - 16*A^2*d^18*f^3 + 1024*A^2*c^
6*d^12*f^3 + 1440*A^2*c^8*d^10*f^3 + 1024*A^2*c^10*d^8*f^3 + 320*A^2*c^12*
d^6*f^3 - 16*A^2*c^16*d^2*f^3))*(((320*A^4*c^2*d^8*f^4 - 16*A^4*d^10*f^4 -
 1760*A^4*c^4*d^6*f^4 + 1600*A^4*c^6*d^4*f^4 - 400*A^4*c^8*d^2*f^4)^(1/2)
- 4*A^2*c^5*f^2 + 40*A^2*c^3*d^2*f^2 - 20*A^2*c*d^4*f^2)/(c^10*f^4 + d^10*
f^4 + 5*c^2*d^8*f^4 + 10*c^4*d^6*f^4 + 10*c^6*d^4*f^4 + 5*c^8*d^2*f^4))...
```

3.125. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.126 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

3.126.1 Optimal result

Integrand size = 47, antiderivative size = 365

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \frac{(A-iB-C)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(ia+b)(c-id)^{5/2}f} + \frac{(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)(c+id)^{5/2}f} - \frac{2b^{3/2}(Ab^2-a(bB-aC))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{(a^2+b^2)(bc-ad)^{5/2}f} + \frac{2(c^2C-Bcd+Ad^2)}{3(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(c^4C-2Bc^3d+c^2(3A-C)d^2+Ad^4)-ad^2(2c(A-C)d-B(c^2-d^2)))}{(bc-ad)^2(c^2+d^2)^2f\sqrt{c+d\tan(e+fx)}}$$

output
```
(A-I*B-C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(I*a+b)/(c-I*d)^(5
/2)/f+(I*A-B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*b)/(c
+I*d)^(5/2)/f-2*b^(3/2)*(A*b^2-a*(B*b-C*a))*arctanh(b^(1/2)*(c+d*tan(f*x+e
))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)/(-a*d+b*c)^(5/2)/f+2*(b*(c^4*C-2*B*c^
3*d+c^2*(3*A-C)*d^2+A*d^4)-a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))/(-a*d+b*c)^2/(
c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(A*d^2-B*c*d+C*c^2)/(-a*d+b*c)/(c^
2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.126.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1948 vs. 2(365) = 730.

Time = 6.39 (sec) , antiderivative size = 1948, normalized size of antiderivative = 5.34

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c
 + d*Tan[e + f*x])^(5/2)),x]
```

output
```
(-2*(A*d^2 - c*(-(c*C) + B*d)))/(3*(-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan
[e + f*x])^(3/2)) - (2*((-2*(((I*Sqrt[c - I*d]*((b*(-(b*c) + a*d)*((-3*c*(
b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d
*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a*((-3*((b*d^2)/2
- (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 -
 (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d
^2))/2))/2 - (b*((-3*d^2*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2
- c*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2
))/2)))/2) - I*((a*(-(b*c) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2
- (3*b*d*(c^2*C - B*c*d + A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*
b*(c^2 + d^2)))/2))/2 - b*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d
 - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (
A - C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2))/2 - (b*((-3*d^2*(a*A*c*
d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2 - c*((3*d*(b*c - a*d)*(B*c - (A
- C)*d))/2 - (3*b*c*(c^2*C - B*c*d + A*d^2))/2)))/2)))*ArcTanh[Sqrt[c + d*
Tan[e + f*x]]/Sqrt[c - I*d]])/((-c + I*d)*f) - (I*Sqrt[c + I*d]*((b*(-(b*c
) + a*d)*((-3*c*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*d*(c^2*C - B*c*d +
 A*d^2))/2 - (3*d*(a*A*c*d - a*d*(c*C - B*d) - A*b*(c^2 + d^2)))/2))/2 + a
*((-3*((b*d^2)/2 - (c*(-(b*c) + a*d))/2)*(a*A*c*d - a*d*(c*C - B*d) - A*b*
(c^2 + d^2)))/2 - (a*d*((3*d*(b*c - a*d)*(B*c - (A - C)*d))/2 - (3*b*c*...
```

3.126.3 Rubi [A] (warning: unable to verify)

Time = 3.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.383, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{2\int -\frac{3\left(-b\left(Cc^2-Bdc+Ad^2\right)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)\right)}{2(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{3\left(c^2+d^2\right)(bc-ad)}+\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan^2(e+fx)-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{\int \frac{-b\left(Cc^2-Bdc+Ad^2\right)\tan(e+fx)^2-(bc-ad)(Bc-(A-C)d)\tan(e+fx)+aAcd-ad(cC-Bd)-Ab\left(c^2+d^2\right)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}dx}{\left(c^2+d^2\right)(bc-ad)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2\int \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-b\left(b\left(Cc^4-2Bdc^3+(3A-C)d^2c^2+Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\tan^2(e+fx)+A\left(2abdc^3-b^2\left(c^2+d^2\right)^2 \cdots\right.}{2(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}\cdots}{\left(c^2+d^2\right)(bc-ad)}\cdots}{\left(c^2+d^2\right)(b\cdots}$$

$\downarrow$ 27

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\int \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-b\left(b\left(Cc^4-2Bdc^3+(3A-C)d^2c^2+Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\tan^2(e+fx)+A\left(2abdc^3-b^2\left(c^2+d^2\right)^2-c\right.}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\left(c^2+d^2\right)(bc$$

$\downarrow$ 3042

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\int \frac{\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-b\left(b\left(Cc^4-2Bdc^3+(3A-C)d^2c^2+Ad^4\right)-ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)\tan(e+fx)^2+A\left(2abdc^3-b^2\left(c^2+d^2\right)^2-c\right.}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\left(c^2+d^2\right)(bc$$

$\downarrow$ 4136

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{\int \frac{\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)(bc-ad)^2+\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)-b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)(bc-a}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\left(c^2+d^2\right)$$

$\downarrow$ 3042

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{\int \frac{\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)(bc-ad)^2+\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)-b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)(bc-a}{\sqrt{c+d\tan(e+fx)}}}{a^2+b^2}}{\left(c^2+d^2\right)(bc-ad)}$$

$$\left(c^2+d^2\right)$$

$\downarrow$ 4022

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int \frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+-}{}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 3042

3.126. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\cdots}{\cdots}}{\left(c^2+d^2\right)(bc-ad)\cdots}$$

$\downarrow$ 4020

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\cdots}{\cdots}}{(c^2\cdots}$$

$\downarrow$ 25

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\cdots}{\cdots}}{(c^2\cdots}$$

$\downarrow$ 73

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\cdots}{\cdots}}{\left(c^2+d^2\right)\cdots}$$

$\downarrow$ 221

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{\tan(e+fx)^2+1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}dx}{a^2+b^2}+\cdots}{\cdots}}{\left(c^2+d^2\right)(bc-ad)\cdots}$$

$\downarrow$ 4117

3.126. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$
$$\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt{c+d\tan(e+fx)}}d\tan(e}{f\left(a^2+b^2\right)}}{}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 73

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$
$$\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{2b^2\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\int\frac{1}{a+\frac{b(c+d\tan(e+fx))}{d}-\frac{bc}{d}}d\sqrt{c+d\tan(e}}{df\left(a^2+b^2\right)}}{}}{\left(c^2+d^2\right)(bc-ad)}$$

$\downarrow$ 221

$$\frac{2\left(Ad^2-Bcd+c^2C\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-$$
$$\frac{-\frac{2\left(b\left(c^2d^2(3A-C)+Ad^4-2Bc^3d+c^4C\right)-ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{2b^{3/2}\left(c^2+d^2\right)^2\left(Ab^2-a(bB-aC)\right)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+d\tan(e+fx)}}{\sqrt{bc-ad}}\right)}{f\left(a^2+b^2\right)\sqrt{bc-ad}}+\frac{-\frac{(c}{}}{}}{}}{\left(c^2+d^2\right)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])*(c + d*T
an[e + f*x])^(5/2)),x]
```

output
```
(2*(c^2*C - B*c*d + A*d^2))/(3*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*
x])^(3/2)) - (((-(((a + I*b)*(A - I*B - C)*(c + I*d)^2*(b*c - a*d)^2*ArcTa
n[Tan[e + f*x]/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f)) - ((a - I*b)*(A + I*B -
C)*(c - I*d)^2*(b*c - a*d)^2*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c +
 I*d]*f))/(a^2 + b^2) + (2*b^(3/2)*(A*b^2 - a*(b*B - a*C))*(c^2 + d^2)^2*A
rcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2)*S
qrt[b*c - a*d]*f))/((b*c - a*d)*(c^2 + d^2)) - (2*(b*(c^4*C - 2*B*c^3*d +
c^2*(3*A - C)*d^2 + A*d^4) - a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))))/((b*c
 - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/((b*c - a*d)*(c^2 + d^2))
```

3.126. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.126. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45118 vs. $2(328) = 656$.

Time = 0.23 (sec) , antiderivative size = 45119, normalized size of antiderivative = 123.61

method	result	size
derivativedivides	Expression too large to display	45119
default	Expression too large to display	45119

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e))^(5/2
),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.126.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.126.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)}{\left(a+b\tan\left(e+fx\right)\right)\left(c+d\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e
))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))*(c
 + d*tan(e + f*x))**(5/2)), x)
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.126.8 Giac [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A}{(b\tan\left(fx+e\right)+a)(d\tan\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))/(c+d*tan(f*x+e)
)^(5/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))*(c + d*t
an(e + f*x))^(5/2)),x)
```

output
```
\text{Hanged}
```

3.127 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

3.127.1 Optimal result

Integrand size = 47, antiderivative size = 679

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c-id}}\right)}{(a-ib)^2(c-id)^{5/2}f}-\frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+d\tan(e+fx)}}{\sqrt{c+id}}\right)}{(a+ib)^2(c+id)^{5/2}f}$$

$$-\frac{b^{3/2}(7a^3bBd-5a^4Cd+b^4(2Bc-5Ad)+ab^3(4Ac-4cC+3Bd)-a^2b^2(2Bc+(9A+C)d))\,\text{arctanh}\Big(\frac{\sqrt{b}}{}}{\left(a^2+b^2\right)^2(bc-ad)^{7/2}f}$$

$$-\frac{d(2b^2c(cC-Bd)-3abB(c^2+d^2)+a^2(5c^2C-2Bcd+3Cd^2)+A(2a^2d^2+b^2(3c^2+5d^2)))}{3\left(a^2+b^2\right)(bc-ad)^2\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{d(2a^3d^2(Bc^2+2cCd-Bd^2)+2b^3c(2c^3C-3Bc^2d-Bd^3)-ab^2(Bc^4-4cCd^3+3Bd^4)+a^2b(5c^4C-6B}{\left(a^2+b^2\right)(bc-ad)^3\left(c^2+d^2\right)}$$

output
```
-(I*A+B-I*C)*arctanh((c+d*tan(f*x+e))^(1/2)/(c-I*d)^(1/2))/(a-I*b)^2/(c-I*
d)^(5/2)/f-(B-I*(A-C))*arctanh((c+d*tan(f*x+e))^(1/2)/(c+I*d)^(1/2))/(a+I*
b)^2/(c+I*d)^(5/2)/f-b^(3/2)*(7*a^3*b*B*d-5*a^4*C*d+b^4*(-5*A*d+2*B*c)+a*b
^3*(4*A*c+3*B*d-4*C*c)-a^2*b^2*(2*B*c+(9*A+C)*d))*arctanh(b^(1/2)*(c+d*tan
(f*x+e))^(1/2)/(-a*d+b*c)^(1/2))/(a^2+b^2)^2/(-a*d+b*c)^(7/2)/f-d*(2*a^3*d
^2*(B*c^2-B*d^2+2*C*c*d)+2*b^3*c*(-3*B*c^2*d-B*d^3+2*C*c^3)-a*b^2*(B*c^4+3
*B*d^4-4*C*c*d^3)+a^2*b*(-6*B*c^3*d-2*B*c*d^3+5*C*c^4+2*C*c^2*d^2+C*d^4)-A
*(4*a^3*c*d^3+4*a*b^2*c*d^3-4*a^2*b*d^2*(2*c^2+d^2)-b^3*(c^4+10*c^2*d^2+5*
d^4)))/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-1/3*d*(
2*b^2*c*(-B*d+C*c)-3*a*b*B*(c^2+d^2)+a^2*(-2*B*c*d+5*C*c^2+3*C*d^2)+A*(2*a
^2*d^2+b^2*(3*c^2+5*d^2)))/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x
+e))^(3/2)+(-A*b^2+a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))/(c
+d*tan(f*x+e))^(3/2)
```

3.127.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6052 vs. 2(679) = 1358.

Time = 6.84 (sec) , antiderivative size = 6052, normalized size of antiderivative = 8.91

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*
(c + d*Tan[e + f*x])^(5/2)),x]
```

output
```
Result too large to show
```

3.127.3 Rubi [A] (warning: unable to verify)

Time = 7.03 (sec) , antiderivative size = 767, normalized size of antiderivative = 1.13, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.468$, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 4132$$

$$-\frac{\int \frac{5Adb^2+5\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-2aA(bc-ad)-(bB-aC)(2bc+3ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{5Adb^2+5\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-2aA(bc-ad)-(bB-aC)(2bc+3ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{5Adb^2+5\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-2aA(bc-ad)-(bB-aC)(2bc+3ad)+2(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))(c+d\tan(e+fx))^{5/2}}dx}{2\left(a^2+b^2\right)(bc-ad)}-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{2\int -\frac{3\left(2d^2(Ac-Cc+Bd)a^3-b(4A+C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3-Bdc^2+4Cd^2c-3Bd^3-2A\left(c^3+2d^2c\right)\right)a-bd\left(2Ad^2a^2+\left(5Cc^2-2Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a-\right.\right.}{2(a+b\tan(e+fx))(c+d\tan(e+f}}{3(c^2+d^2)(bc-ad)}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{\int \frac{2d^2(Ac-Cc+Bd)a^3-b(4A+C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3\right.}{}}{}}{}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.127. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{\int\frac{2d^2(Ac-Cc+Bd)a^3-b(4A+C)d\left(c^2+d^2\right)a^2-b^2\left(2Cc^3\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4132

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2Ad^2a^2+\left(5Cc^2-2Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+2b^2c(cC-Bd)+Ab^2\left(3c^2+5d^2\right)\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{2\int\frac{2d^3\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^4-2bd^2\left(3Cc^3-4Bd\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2Ad^2a^2+\left(5Cc^2-2Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+2b^2c(cC-Bd)+Ab^2\left(3c^2+5d^2\right)\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{\int\frac{2d^3\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^4-2bd^2\left(3Cc^3-4Bdc^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2Ad^2a^2+\left(5Cc^2-2Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+2b^2c(cC-Bd)+Ab^2\left(3c^2+5d^2\right)\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{\int\frac{2d^3\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^4-2bd^2\left(3Cc^3-4Bdc^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 4136

$$-\frac{Ab^2-a(bB-aC)}{\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2Ad^2a^2+\left(5Cc^2-2Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+2b^2c(cC-Bd)+Ab^2\left(3c^2+5d^2\right)\right)}{3(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}-\frac{\frac{b^2\left(-5Cda^4+7bBda^3-b^2(2Bc+(9A+C)d)a^2+b^3(4Ac-4Cc+\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 27

3.127. $\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{2d(2Ad^2a^2+(5Cc^2-2Bdc+3Cd^2)a^2-3bB(c^2+d^2)a+2b^2c(cC-Bd)+Ab^2(3c^2+5d^2))}{3(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{b^2(c^2+d^2)^2(-5Cda^4+7bBda^3-b^2(2Bc+(9A+C)d)a^2+b^3(4$$

$\downarrow$ 3042

$$-\frac{Ab^2-a(bB-aC)}{(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{2d(2Ad^2a^2+(5Cc^2-2Bdc+3Cd^2)a^2-3bB(c^2+d^2)a+2b^2c(cC-Bd)+Ab^2(3c^2+5d^2))}{3(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{b^2(c^2+d^2)^2(-5Cda^4+7bBda^3-b^2(2Bc+(9A+C)d)a^2+b^3(4$$

$\downarrow$ 4022

$$-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{2d(2a^2Ad^2+a^2(-2Bcd+5c^2C+3Cd^2)-3abB(c^2+d^2)+Ab^2(3c^2+5d^2)+2b^2c(cC-Bd))}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d(2a^3d^2(Bc^2-Bd^2+2cCd)+a^2b(-6Bc^3d-2Bcd^3+5c^4$$

$\downarrow$ 3042

$$-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{2d(2a^2Ad^2+a^2(-2Bcd+5c^2C+3Cd^2)-3abB(c^2+d^2)+Ab^2(3c^2+5d^2)+2b^2c(cC-Bd))}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d(2a^3d^2(Bc^2-Bd^2+2cCd)+a^2b(-6Bc^3d-2Bcd^3+5c^4$$

$\downarrow$ 4020

$$-\frac{Ab^2-a(bB-aC)}{f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{2d(2a^2Ad^2+a^2(-2Bcd+5c^2C+3Cd^2)-3abB(c^2+d^2)+Ab^2(3c^2+5d^2)+2b^2c(cC-Bd))}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d(2a^3d^2(Bc^2-Bd^2+2cCd)+a^2b(-6Bc^3d-2Bcd^3+5c^4$$

$\downarrow$ 25

3.127. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 73$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 221$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 4117$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

$$\Big\downarrow \ 73$$

3.127. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{Ab^2-a(bB-aC)}{f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))(c+d\tan(e+fx))^{3/2}}-$$

$$\frac{\frac{2d\left(2a^2Ad^2+a^2\left(-2Bcd+5c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+5d^2\right)+2b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}-\frac{-\frac{2d\left(2a^3d^2\left(Bc^2-Bd^2+2cCd\right)+a^2b\left(-6Bc^3d-2Bcd^3+5c^4\right.\right.}{}}{}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^2*(c + d
*Tan[e + f*x])^(5/2)),x]
```

output
```
-((A*b^2 - a*(b*B - a*C))/((a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])*
(c + d*Tan[e + f*x])^(3/2))) - ((2*d*(2*a^2*A*d^2 + 2*b^2*c*(c*C - B*d) -
3*a*b*B*(c^2 + d^2) + A*b^2*(3*c^2 + 5*d^2) + a^2*(5*c^2*C - 2*B*c*d + 3*C
*d^2)))/(3*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (((-2*(
-(((a + I*b)^2*(A - I*B - C)*(c + I*d)^2*(b*c - a*d)^3*ArcTan[Tan[e + f*x]
/Sqrt[c - I*d]])/(Sqrt[c - I*d]*f)) - ((a - I*b)^2*(A + I*B - C)*(c - I*d)
^2*(b*c - a*d)^3*ArcTan[Tan[e + f*x]/Sqrt[c + I*d]])/(Sqrt[c + I*d]*f)))/(
a^2 + b^2) - (2*b^(3/2)*(c^2 + d^2)^2*(7*a^3*b*B*d - 5*a^4*C*d + b^4*(2*B*
c - 5*A*d) + a*b^3*(4*A*c - 4*c*C + 3*B*d) - a^2*b^2*(2*B*c + (9*A + C)*d)
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])/Sqrt[b*c - a*d]])/((a^2 + b^2
)*Sqrt[b*c - a*d]*f))/((b*c - a*d)*(c^2 + d^2)) - (2*d*(2*a^3*d^2*(B*c^2 +
 2*c*C*d - B*d^2) + 2*b^3*c*(2*c^3*C - 3*B*c^2*d - B*d^3) - a*b^2*(B*c^4 -
 4*c*C*d^3 + 3*B*d^4) + a^2*b*(5*c^4*C - 6*B*c^3*d + 2*c^2*C*d^2 - 2*B*c*d
^3 + C*d^4) - A*(4*a^3*c*d^3 + 4*a*b^2*c*d^3 - 4*a^2*b*d^2*(2*c^2 + d^2) -
 b^3*(c^4 + 10*c^2*d^2 + 5*d^4))))/((b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*T
an[e + f*x]]))/((b*c - a*d)*(c^2 + d^2)))/(2*(a^2 + b^2)*(b*c - a*d))
```

3.127. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

3.127. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 67569 vs. $2(639) = 1278$.

Time = 0.35 (sec) , antiderivative size = 67570, normalized size of antiderivative = 99.51

method	result	size
derivativedivides	Expression too large to display	67570
default	Expression too large to display	67570

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+e))^(5
/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.127.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.127.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})^2\,(c+d\tan{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**2/(c+d*tan(f*
x+e))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**2
*(c + d*tan(e + f*x))**(5/2)), x)
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^2(c+d\tan(e+fx))^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.127.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^2/(c+d*tan(f*x+
e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^2(c + d\tan(e + fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^2*(c + d
*tan(e + f*x))^(5/2)),x)
```

output
```
\text{Hanged}
```

3.128 $\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan$

3.128.1 Optimal result

Integrand size = 49, antiderivative size = 679

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$
$$-\frac{(a-ib)^{5/2}(iA+B-iC)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$-\frac{(a+ib)^{5/2}(B-i(A-C))\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$-\frac{(5a^4Cd^4-20a^3bd^3(cC+2Bd)+30a^2b^2d^2(c^2C-4Bcd-8(A-C)d^2)-20ab^3d(c^3C-2Bc^2d+8c(A-}{64b}$$
$$+\frac{(64b(a^2B-b^2B+2ab(A-C))\,d^3-(bc-ad)\,(16b(Ab+aB-bC)d^2+(bc-ad)(5bcC-8bBd-5aCd)}{64bd^3f}$$
$$+\frac{(16b(Ab+aB-bC)d^2+(bc-ad)(5bcC-8bBd-5aCd))\,\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{32d^3f}$$
$$-\frac{(5bcC-8bBd-5aCd)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{24d^2f}$$
$$+\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}$$

output
```
-1/64*(5*a^4*C*d^4-20*a^3*b*d^3*(2*B*d+C*c)+30*a^2*b^2*d^2*(c^2*C-4*B*c*d-
8*(A-C)*d^2)-20*a*b^3*d*(c^3*C-2*B*c^2*d+8*c*(A-C)*d^2-16*B*d^3)+b^4*(5*c^
4*C-8*B*c^3*d+16*c^2*(A-C)*d^2+64*B*c*d^3+128*(A-C)*d^4))*arctanh(d^(1/2)*
(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/d^(7/2)/f-(
a-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I
*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f-(a+I*b)^(5/2)*(B-I*(A-C)
)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+
e))^(1/2))*(c+I*d)^(1/2)/f+1/64*(64*b*(B*a^2-B*b^2+2*a*b*(A-C))*d^3-(-a*d+
b*c)*(16*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-8*B*b*d-5*C*a*d+5*C*b*c)))*(a+b*
tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d^3/f+1/32*(16*b*(A*b+B*a-C*b)*
d^2+(-a*d+b*c)*(-8*B*b*d-5*C*a*d+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan
(f*x+e))^(3/2)/d^3/f-1/24*(-8*B*b*d-5*C*a*d+5*C*b*c)*(a+b*tan(f*x+e))^(3/2
)*(c+d*tan(f*x+e))^(3/2)/d^2/f+1/4*C*(a+b*tan(f*x+e))^(5/2)*(c+d*tan(f*x+e
))^(3/2)/d/f
```

3.128.2 Mathematica [A] (verified)

Time = 10.25 (sec) , antiderivative size = 1202, normalized size of antiderivative = 1.77

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\big(A+B\tan(e+fx)+C\tan^2(e+fx)\big)\,dx=\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}$$

$$+\frac{\frac{(-5bcC+8bBd+5aCd)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{6df}+\frac{3\left(16b(Ab+aB-bC)d^2+(bc-ad)(5bcC-8bBd-5aCd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx}{8df}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
 + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/(4*d*f) + (((-5*
b*c*C + 8*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])
^(3/2))/(6*d*f) + ((3*(16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C -
 8*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/
(8*d*f) + (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(b*c - a*d)*(16
*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)))/8)*
Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*b*d^3*(b*(
3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*C - B*d) + a^3*(B*c + (A - C)*d)
- 3*a*b^2*(B*c + (A - C)*d)) + Sqrt[-b^2]*(a^3*(A*c - c*C - B*d) - 3*a*b^2
*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A - C)*d) + b^3*(B*c + (A - C)*d)))*A
rcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + S
qrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (S
qrt[-b^2]*d)/b]) - (24*b*d^3*(b*(3*a^2*b*(A*c - c*C - B*d) - b^3*(A*c - c*
C - B*d) + a^3*(B*c + (A - C)*d) - 3*a*b^2*(B*c + (A - C)*d)) - Sqrt[-b^2]
*(a^3*(A*c - c*C - B*d) - 3*a*b^2*(A*c - c*C - B*d) - 3*a^2*b*(B*c + (A -
C)*d) + b^3*(B*c + (A - C)*d)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqr
t[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (3*Sqrt[b]*Sqrt[c - (a*d)/
b]*Sqrt[(c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*Sqrt[c/(c - (a*d
)/b) - (a*d)/(b*(c - (a*d)/b))]*(5*a^4*C*d^4 - 20*a^3*b*d^3*(c*C + 2*B*...
```

3.128.3 Rubi [A] (verified)

Time = 5.73 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.327$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{1}{2}(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left((5bcC-5adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e+\right.}{4d}$$
$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}$$

$\Big\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{\int(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left((5bcC-5adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e+f\right.}{8d}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{\int(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left((5bcC-5adC-8bBd)\tan(e+fx)^2-8(Ab-Cb+aB)d\tan(e+f\right.}{8d}$$

$\Big\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{\frac{\int-\frac{3}{2}\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(c(5cC-8Bd)b^2-2ad(5cC+4Bd)b+a^2(16A-11C)d^2+(16b(Ab-Cb+aB)d^2+(bc-ad)(5bcC-5adC-8bB\right.}{3d}}{8d}$$

$\Big\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\int\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(c(5cC-8Bd)b^2-2ad(5cC+4Bd)b+a^2(\right.}{}}{8d}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\int\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(c(5cC-8Bd)b^2-2ad(5cC+4Bd)b+a^2(\right.}{}}{8d}$$

$\Big\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$
$$\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\int\frac{\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2-8Bdc+16(A-C)d^2\right)b^3+ad\left(15Cc^2-32Bdc-48(A-C)d\right.\right.}{}}{}$$

$\Big\downarrow$ 27

3.128.

$$\int(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\int\frac{\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2-8Bdc+16(A-C)d^2\right)b^3+ad\left(15Cc^2-32Bdc-48(A-C)d\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\int\frac{\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2-8Bdc+16(A-C)d^2\right)b^3+ad\left(15Cc^2-32Bdc-48(A-C)d\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\frac{\int-\frac{c\left(5Cc^3-8Bdc^2+16(A-C)d^2c-64Bd^3\right)b^4-4ad\left(5Cc^3-10Bdc^2-56(A-C)d^2c+16\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64bd^3\left(a^2B+2ab(A-C)-b^2B\right)-(bc-ad)\left(16bd^2(a\cdots\right.\right.}{bf}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64bd^3\left(a^2B+2ab(A-C)-b^2B\right)-(bc-ad)\left(16bd^2(a\cdots\right.\right.}{bf}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 4138

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64bd^3\left(a^2B+2ab(A-C)-b^2B\right)-(bc-ad)\left(16bd^2(a\cdots\right.\right.}{bf}}{\cdots}}{\cdots}$$

3.128.

$$\int(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{2348}$$

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$

$$\frac{\frac{(5bcC-5adC-8bBd)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\left(16b(Ab-Cb+aB)d^2+(bc-ad)(5bcC-5adC-8bBd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/}}{2df}}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{C(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}{4df}-$$

$$\frac{\frac{(-5aCd-8bBd+5bcC)(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(16bd^2(aB+Ab-bC)+(bc-ad)(-5aCd-8bBd+5b\right.}{2df}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2)*(c + d*Tan[e + f*x])^(3/2))/(4*d*f) - (((5*b
*c*C - 8*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^
(3/2))/(3*d*f) - (((16*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 8*
b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*
d*f) + (-1/2*(128*(a - I*b)^(5/2)*b*(B + I*(A - C))*Sqrt[c - I*d]*d^3*ArcT
anh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan
[e + f*x]])] - 128*(a + I*b)^(5/2)*b*(I*A - B - I*C)*Sqrt[c + I*d]*d^3*Arc
Tanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Ta
n[e + f*x]])] + (2*(5*a^4*C*d^4 - 20*a^3*b*d^3*(c*C + 2*B*d) + 30*a^2*b^2*
d^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2) - 20*a*b^3*d*(c^3*C - 2*B*c^2*d + 8*
c*(A - C)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 8*B*c^3*d + 16*c^2*(A - C)*d^2
+ 64*B*c*d^3 + 128*(A - C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(b*f) + ((64*b*(
a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (b*c - a*d)*(16*b*(A*b + a*B - b*C)*d
^2 + (b*c - a*d)*(5*b*c*C - 8*b*B*d - 5*a*C*d)))*Sqrt[a + b*Tan[e + f*x]]*
Sqrt[c + d*Tan[e + f*x]])/(b*f))/(4*d))/(2*d))/(8*d)
```

3.128.

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$$

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.128.
$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

3.128.4 Maple [F(-1)]

Timed out.

$$\int \sqrt{c+d\tan(fx+e)}\,(a+b\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.128.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.128.6 Sympy [F]

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx = \int (a+b\tan(e+fx))^{\frac{5}{2}}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(5/2)*sqrt(c + d*tan(e + f*x))*(A + B*tan(e
 + f*x) + C*tan(e + f*x)**2), x)
```

3.128.7 Maxima [F]

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^{\frac{5}{2}}\sqrt{d\tan(fx+e)+c}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(5/
2)*sqrt(d*tan(f*x + e) + c), x)
```

3.128.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\int (a+b\tan(e+f\,x))^{5/2}\,\sqrt{c+d\tan(e+f\,x)}\,(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A)$$

input
```
int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^(5/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2), x)
```

3.129 $\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan$

3.129.1 Optimal result

Integrand size = 49, antiderivative size = 505

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$
$$-\frac{(a-ib)^{3/2}(iA+B-iC)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$+\frac{(a+ib)^{3/2}(iA-B-iC)\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$-\frac{(a^3Cd^3-3a^2bd^2(cC+2Bd)+3ab^2d(c^2C-4Bcd-8(A-C)d^2)-b^3(c^3C-2Bc^2d+8c(A-C)d^2-16}{8b^{3/2}d^{5/2}f}$$
$$+\frac{(8b(Ab+aB-bC)d^2+(bc-ad)(bcC-2bBd-aCd))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{8bd^2f}$$
$$-\frac{(bcC-2bBd-aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{4d^2f}$$
$$+\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}$$

output
```
-1/8*(a^3*C*d^3-3*a^2*b*d^2*(2*B*d+C*c)+3*a*b^2*d*(c^2*C-4*B*c*d-8*(A-C)*d
^2)-b^3*(c^3*C-2*B*c^2*d+8*c*(A-C)*d^2-16*B*d^3))*arctanh(d^(1/2)*(a+b*tan
(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/d^(5/2)/f-(a-I*b)^(
3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2
)/(c+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f+(a+I*b)^(3/2)*(I*A-B-I*C)*arctan
h((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2
))*(c+I*d)^(1/2)/f+1/8*(8*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-2*B*b*d-C*a*d+C
*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d^2/f-1/4*(-2*B*b*d
-C*a*d+C*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/d^2/f+1/3*C*(a
+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)/d/f
```

3.129.2 Mathematica [A] (verified)

Time = 9.10 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.65

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}$$

$$+\frac{-\frac{3(bcC-2bBd-aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{4df}+\frac{\frac{3\left(8b(Ab+aB-bC)d^2+(bc-ad)(bcC-2bBd-aCd)\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4bf}+-\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e
 + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) + ((-3*(
b*c*C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/
2))/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(b*c*C - 2*b*B*
d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + (
(6*b*d^2*(Sqrt[-b^2]*(a^2*(A*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*
b*(B*c + (A - C)*d)) + b*(2*a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d)
- b^2*(B*c + (A - C)*d)))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a
 + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) + (6*b*d^2*(Sqrt[-b^2]*(a^2*(A
*c - c*C - B*d) - b^2*(A*c - c*C - B*d) - 2*a*b*(B*c + (A - C)*d)) - b*(2*
a*b*(A*c - c*C - B*d) + a^2*(B*c + (A - C)*d) - b^2*(B*c + (A - C)*d)))*Ar
cTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt
[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-
b^2]*d)/b]) - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*(a^3*C*d^3 - 3*a^2*b*d^2*(c*C +
 2*B*d) + 3*a*b^2*d*(c^2*C - 4*B*c*d - 8*(A - C)*d^2) - b^3*(c^3*C - 2*B*c
^2*d + 8*c*(A - C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d
)])/(4*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*d))/(3*d)
```

3.129.3 Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{3}{2}\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}((bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx}{3d}$$
$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{\int\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left((bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx)+\right.}{2d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{\int\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left((bcC-adC-2bBd)\tan(e+fx)^2-2(Ab-Cb+aB)d\tan(e+fx)+\right.}{2d}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{\frac{\int-\frac{\sqrt{c+d\tan(e+fx)}\left(c(cC-2Bd)b^2-2ad(cC+3Bd)b+a^2(8A-7C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-2bBd)\right)\tan^2(e+fx)+8\left(Ba^2+2b(A-C)a-b^2B\right)d^2\right.}{2\sqrt{a+b\tan(e+fx)}}}{2d}}{2d}$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\int\frac{\sqrt{c+d\tan(e+fx)}\left(c(cC-2Bd)b^2-2ad(cC+3Bd)b+a^2(8A-7C)d^2+\left(8b(Ab-Cb+aB)d^2+(b\right.\right.}{\sqrt{a+b\tan(e+}}}{4d}}{2d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\int\frac{\sqrt{c+d\tan(e+fx)}\left(c(cC-2Bd)b^2-2ad(cC+3Bd)b+a^2(8A-7C)d^2+\left(8b(Ab-Cb+aB)d^2+(b\right.\right.}{\sqrt{a+b\tan(e+}}}{4d}}{2d}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$
$$\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\int-\frac{-c\left(Cc^2-2Bdc-8(A-C)d^2\right)b^3+ad\left(3Cc^2+20Bdc+8(A-C)d^2\right)b^2-a^2d^2(16Ac-13Cc-\right.}{}}{}$$

$\downarrow$ 27

3.129.

$$\int(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC)\right)}{bf}-\frac{\int-}{}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC)\right)}{bf}-\frac{\int-}{}$$

$\downarrow$ 4138

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC)\right)}{bf}-\frac{\int-}{}$$

$\downarrow$ 2348

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$\frac{(bcC-adC-2bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\left(8b(Ab-Cb+aB)d^2+(bc-ad)(bcC-adC-2bBd)\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int\left(\frac{16B}{}\right.}{}$$

$\downarrow$ 2009

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}{3df}-$$

$$\frac{(-aCd-2bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-aCd-2bBd+bcC)\right)}{bf}-\frac{2(a}{}$$

input
```
Int[(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

3.129.
$$\int(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$$

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2))/(3*d*f) - (((b*c
*C - 2*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))
/(2*d*f) - (-1/2*(16*(a - I*b)^(3/2)*b*(B + I*(A - C))*Sqrt[c - I*d]*d^2*A
rcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*
Tan[e + f*x]])] - 16*(a + I*b)^(3/2)*b*(I*A - B - I*C)*Sqrt[c + I*d]*d^2*A
rcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*
Tan[e + f*x]])] + (2*(a^3*C*d^3 - 3*a^2*b*d^2*(c*C + 2*B*d) + 3*a*b^2*d*(c
^2*C - 4*B*c*d - 8*(A - C)*d^2) - b^3*(c^3*C - 2*B*c^2*d + 8*c*(A - C)*d^2
 - 16*B*d^3))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(b*f) + ((8*b*(A*b + a*B - b*C)*d^2
 + (b*c - a*d)*(b*c*C - 2*b*B*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c
+ d*Tan[e + f*x]])/(b*f))/(4*d))/(2*d)
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.129.
$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.129.4 Maple [F(-1)]

Timed out.

$$\int \sqrt{c+d\tan\left(fx+e\right)}\,\left(a+b\tan\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(fx+e\right)+C\tan\left(fx+e\right)^{2}\right)dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.129.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.129.6 Sympy [F]

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\int (a+b\tan(e+fx))^{\frac{3}{2}}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(3/2)*sqrt(c + d*tan(e + f*x))*(A + B*tan(e
 + f*x) + C*tan(e + f*x)**2), x)
```

3.129.7 Maxima [F]

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx=\int \left(C\tan(fx+e)^2+B\tan(fx+e)+A\right)(b\tan(fx+e)+a)^{\frac{3}{2}}\sqrt{d\tan(fx+e)+c}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/
2)*sqrt(d*tan(f*x + e) + c), x)
```

3.129.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx = \int (a+b\tan(e+f\,x))^{3/2}\,\sqrt{c+d\tan\left(e+f\,x\right)}\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)$$

input
```
int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2), x)
```

3.130 $\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(A+B\tan$

3.130.1 Optimal result

Integrand size = 49, antiderivative size = 381

$$\begin{aligned}
&\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx \\
&= -\frac{\sqrt{a-ib}(iA+B-iC)\sqrt{c-id}\,\mathrm{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f} \\
&\quad - \frac{\sqrt{a+ib}(B-i(A-C))\sqrt{c+id}\,\mathrm{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f} \\
&\quad - \frac{(a^2Cd^2-2abd(cC+2Bd)+b^2(c^2C-4Bcd-8(A-C)d^2))\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{3/2}d^{3/2}f} \\
&\quad - \frac{(bcC-4bBd-aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4bdf} \\
&\quad + \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}
\end{aligned}$$

output
```
-1/4*(a^2*C*d^2-2*a*b*d*(2*B*d+C*c)+b^2*(c^2*C-4*B*c*d-8*(A-C)*d^2))*arcta
nh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/
d^(3/2)/f-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)
^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)*(c-I*d)^(1/2)/f-(B-I*(A-C))*a
rctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))
^(1/2))*(a+I*b)^(1/2)*(c+I*d)^(1/2)/f-1/4*(-4*B*b*d-C*a*d+C*b*c)*(a+b*tan(
f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/d/f+1/2*C*(a+b*tan(f*x+e))^(1/2)*(c
+d*tan(f*x+e))^(3/2)/d/f
```

3.130.
$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))\,dx$

3.130.2 Mathematica [A] (verified)

Time = 7.94 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.62

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}$$
$$+\frac{\frac{(-bcC+4bBd+aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{2bf}+\frac{\frac{2bd\left(b(Abc+aBc-bcC+aAd-bBd-aCd)-\sqrt{-b^2}(bBc+b(A-C)d-a(Ac-cC-Bd))\right)\text{arct}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}}{}}{}$$

input
```
Integrate[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e +
 f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*d*f) + (((-(b*c
*C) + 4*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/
(2*b*f) + ((2*b*d*(b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - Sqr
t[-b^2]*(b*B*c + b*(A - C)*d - a*(A*c - c*C - B*d)))*ArcTanh[(Sqrt[-c + (S
qrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c +
d*Tan[e + f*x]])]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (
2*b*d*(b*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) + Sqrt[-b^2]*(b*B
*c + b*(A - C)*d - a*(A*c - c*C - B*d)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/
b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]
])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (Sqrt[b]*Sqrt[c -
 (a*d)/b]*(a^2*C*d^2 - 2*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A
 - C)*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (
a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c +
d*Tan[e + f*x]]))/(b^2*f))/(2*d)
```

3.130.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.130.
$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \; 4130$$

$$\frac{\int -\frac{\sqrt{c+d\tan(e+fx)}\left((bcC-adC-4bBd)\tan^2(e+fx)-4(Ab-Cb+aB)d\tan(e+fx)+bcC-a(4A-3C)d\right)}{2\sqrt{a+b\tan(e+fx)}}dx}{2d}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}$$

$$\Big\downarrow \; 27$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((bcC-adC-4bBd)\tan^2(e+fx)-4(Ab-Cb+aB)d\tan(e+fx)+bcC-a(4A-3C)d\right)}{\sqrt{a+b\tan(e+fx)}}dx}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((bcC-adC-4bBd)\tan(e+fx)^2-4(Ab-Cb+aB)d\tan(e+fx)+bcC-a(4A-3C)d\right)}{\sqrt{a+b\tan(e+fx)}}dx}{4d}$$

$$\Big\downarrow \; 4130$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\frac{\int \frac{c(cC+4Bd)b^2-2ad(4Ac-3Cc-2Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-4bBd)\right)\tan^2(e+fx)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b}}{4d}$$

$$\Big\downarrow \; 27$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df}-\frac{\frac{\int \frac{c(cC+4Bd)b^2-2ad(4Ac-3Cc-2Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-4bBd)\right)\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{2b}}{4d}$$

$$\Big\downarrow \; 3042$$

3.130.

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df} - \frac{\frac{\int \frac{c(cC+4Bd)b^2-2ad(4Ac-3Cc-2Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-4bBd)\right)\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{2b}}{4d}$$

$\downarrow$ 4138

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df} - \frac{\frac{\int \frac{c(cC+4Bd)b^2-2ad(4Ac-3Cc-2Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-4bBd)\right)\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}}{2bf}}{4d}$$

$\downarrow$ 2348

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df} - \frac{(-aCd-4bBd+bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\int \left(\frac{-8Ad^2b^2+8Cd^2b^2+c^2Cb^2-4Bcdb^2-4aBd^2b-2acCdb+a^2Cd^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} + \frac{-8Bd^2b^2+8Acdb^2-8cC\ldots}{\ldots}\right.}{\ldots}$$

$\downarrow$ 2009

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2df} - \frac{\frac{(-aCd-4bBd+bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf} + \frac{\frac{2\left(a^2Cd^2-2abd(2Bd+cC)+b^2\left(-8d^2(A-C)-4Bcd+c^2C\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}}}{\ldots}}{4d}$$

input
```
Int[Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*d*f) - ((8*Sqrt
[a - I*b]*b*(B + I*(A - C))*Sqrt[c - I*d]*d*ArcTanh[(Sqrt[c - I*d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])] - 8*Sqrt[a +
I*b]*b*(I*A - B - I*C)*Sqrt[c + I*d]*d*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])] + (2*(a^2*C*d^2 -
2*a*b*d*(c*C + 2*B*d) + b^2*(c^2*C - 4*B*c*d - 8*(A - C)*d^2))*ArcTanh[(Sq
rt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt
[b]*Sqrt[d]))/(2*b*f) + ((b*c*C - 4*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]])/(b*f))/(4*d)
```

3.130.

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.130.4 Maple [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(fx+e)}\,\sqrt{c+d\tan(fx+e)}\,\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34031 vs. $2(308) = 616$.

Time = 161.87 (sec) , antiderivative size = 68078, normalized size of antiderivative = 178.68

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.130.6 Sympy [F]

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*sqrt(c + d*tan(e + f*x))*(A + B*tan(e +
f*x) + C*tan(e + f*x)**2), x)
```

3.130.7 Maxima [F]

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{b\tan\left(fx+e\right)+a}\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)
*sqrt(d*tan(f*x + e) + c), x)
```

3.130.8 Giac [F]

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{b\tan\left(fx+e\right)+a}\sqrt{d\tan\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)
*sqrt(d*tan(f*x + e) + c), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Hanged}$$

input
```
int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
\text{Hanged}
```

3.131 $\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

3.131.1 Optimal result

Integrand size = 49, antiderivative size = 287

$$\begin{aligned}\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx &= -\frac{(iA+B-iC)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\,f} \\ &\quad -\frac{(B-i(A-C))\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\,f} \\ &\quad +\frac{(bcC+2bBd-aCd)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}\sqrt{d}\,f} \\ &\quad +\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}\end{aligned}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/f/(a-I*b)^(1/2)-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
c+I*d)^(1/2)/f/(a+I*b)^(1/2)+(2*B*b*d-C*a*d+C*b*c)*arctanh(d^(1/2)*(a+b*ta
n(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(3/2)/f/d^(1/2)+C*(a+b*t
an(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b/f
```

3.131.2 Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$$
$$= \frac{\frac{b\left(bBc+b(A-C)d+\sqrt{-b^2}(Ac-cC-Bd)\right)\operatorname{arctanh}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}+\frac{b\left(\sqrt{-b^2}(Ac-cC-Bd)-b(Bc+(A-C)d)\right)\operatorname{arctanh}\left(\cdots\right.}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+\frac{\sqrt{-b^2}d}{b}}}}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/Sqrt[a + b*Tan[e + f*x]],x]
```

output
```
((b*(b*B*c + b*(A - C)*d + Sqrt[-b^2]*(A*c - c*C - B*d))*ArcTanh[(Sqrt[-c
+ (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b])
 + (b*(Sqrt[-b^2]*(A*c - c*C - B*d) - b*(B*c + (A - C)*d))*ArcTanh[(Sqrt[c
 + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) +
 b*C*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]] + (Sqrt[b]*Sqrt[c -
 (a*d)/b]*(b*c*C + 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d
)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f)
```

3.131.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan(e+fx)^2)}{\sqrt{a+b\tan(e+fx)}}dx$$

$$\Big\downarrow \ 4130$$

$$\frac{\int \frac{(bcC-adC+2bBd)\tan^2(e+fx)+2b(Bc+(A-C)d)\tan(e+fx)+2Abc-C(bc+ad)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{b}+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(bcC-adC+2bBd)\tan^2(e+fx)+2b(Bc+(A-C)d)\tan(e+fx)+2Abc-C(bc+ad)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{2b}+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{(bcC-adC+2bBd)\tan(e+fx)^2+2b(Bc+(A-C)d)\tan(e+fx)+2Abc-C(bc+ad)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{2b}+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}$$

$$\Big\downarrow \ 4138$$

$$\frac{\int \frac{(bcC-adC+2bBd)\tan^2(e+fx)+2b(Bc+(A-C)d)\tan(e+fx)+2Abc-C(bc+ad)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(\tan^2(e+fx)+1)}d\tan(e+fx)}{2bf}+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}$$

$$\Big\downarrow \ 2348$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\int \left(\frac{bcC-adC+2bBd}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-2bBc-2Abd+2bCd+i(2Abc-2bCc-2bBd)}{2(i-\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{2bBc+2Abd-2bCd+i(2Abc-2bCc-2bBd)}{2(\tan(e+fx)+i)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\right)}{2bf}$$

$$\Big\downarrow \ 2009$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}+\frac{-\frac{2b\sqrt{c-id}(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}}-\frac{2b\sqrt{c+id}(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}}+\frac{2(-aCd+2bBd+bC}{}}{2bf}$$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqr
t[a + b*Tan[e + f*x]],x]
```

3.131. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

output
```
((-2*b*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I*b] - (2*
b*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] + (2*(b*c*
C + 2*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*S
qrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(2*b*f) + (C*Sqrt[a + b*Tan[
e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f)
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.131. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.131.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(fx+e)}\,(A+B\tan(fx+e)+C\tan(fx+e)^2)}{\sqrt{a+b\tan(fx+e)}}dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39018 vs. $2(225) = 450$.

Time = 105.64 (sec) , antiderivative size = 78051, normalized size of antiderivative = 271.95

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.131.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$$
$$=\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/sqrt(a + b*tan(e + f*x)), x)
```

3.131.7 Maxima [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$$
$$=\int \frac{(C\tan(fx+e)^2+B\tan(fx+e)+A)\sqrt{d\tan(fx+e)+c}}{\sqrt{b\tan(fx+e)+a}}\,dx$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(d*tan(f*x + e) + c)
/sqrt(b*tan(f*x + e) + a), x)
```

3.131.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(1/2),x)
```

output
```
\text{Hanged}
```

3.132 $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

3.132.1 Optimal result

Integrand size = 49, antiderivative size = 300

$$\begin{aligned}\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx =& -\frac{(iA+B-iC)\sqrt{c-id}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}\\ &-\frac{(B-i(A-C))\sqrt{c+id}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}\\ &+\frac{2C\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{3/2}f}-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{b(a^2+b^2)f\sqrt{a+b\tan(e+fx)}}\end{aligned}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(3/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
c+I*d)^(1/2)/(a+I*b)^(3/2)/f+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^
(1/2)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(3/2)/f-2*(A*b^2-a*(B*b-C*a))*(c+d
*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \frac{\frac{(iA+B-iC)\sqrt{-c+id}\,\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+}}\right.}{(-a+ib)^{3/2}}}{}$$

input

```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x])^(3/2),x]
```

output

```
(((I*A + B - I*C)*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])]])/(-a + I*b)^(3/2) + (I
*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(3/2) + ((B + I*(
A - C))*Sqrt[c + d*Tan[e + f*x]])/((a - I*b)*Sqrt[a + b*Tan[e + f*x]]) + (
((-I)*A + B + I*C)*Sqrt[c + d*Tan[e + f*x]])/((a + I*b)*Sqrt[a + b*Tan[e +
 f*x]]) + (2*C*(-((b*(c + d*Tan[e + f*x]))/Sqrt[a + b*Tan[e + f*x]]) + Sqr
t[d]*Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/Sqrt[b*c -
 a*d]]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]))/(b^2*Sqrt[c + d*Tan[e
+ f*x]]))/f
```

3.132.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4128, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan(e+fx)^2)}{(a+b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2\int \frac{(a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(a^2+b^2)Cd\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}$$

$\Big\downarrow$ 4138

$$\frac{\int \frac{(a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(\tan^2(e+fx)+1)}d\tan(e+fx)}{bf(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}$$

$\Big\downarrow$ 2348

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{\int \left(\frac{(a^2+b^2)Cd}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{Acb^2-cCb^2-Bdb^2-aBcb-aAdb+aCdb+i(Bcb^2+Adb^2-Cdb^2+aAcb-acCb-aBdb)}{2(i-\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-Acb^2+cCb}{}\right.}{bf(a^2+b^2)}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{\frac{2C\sqrt{d}(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}}-\frac{b(a+ib)\sqrt{c-id}(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}}+\frac{b(b+ia)\sqrt{c+id}(A+iB-C)a}{}}{bf(a^2+b^2)}$$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x])^(3/2),x]
```

3.132. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

output
```
(-(((a + I*b)*b*(I*A + B - I*C)*Sqrt[c - I*d]*ArcTanh[(Sqrt[c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I
*b]) + (b*(I*a + b)*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a +
 I*b] + (2*(a^2 + b^2)*C*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[b])/(b*(a^2 + b^2)*f) - (2*(A*
b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(b*(a^2 + b^2)*f*Sqrt[a + b
*Tan[e + f*x]])
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.132. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.132.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(fx+e)}\,(A+B\tan(fx+e)+C\tan(fx+e)^2)}{(a+b\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69754 vs. 2(237) = 474.

Time = 198.89 (sec) , antiderivative size = 139535, normalized size of antiderivative = 465.12

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.132.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{c+d\tan\left(e+fx\right)}(A+B\tan\left(e+\right.}{(a+b\tan\left(e+f\right.}$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x))**(3/2), x)
```

3.132.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.132.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(3/2),x)
```

output
```
\text{Hanged}
```

3.133 $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

3.133.1 Optimal result

Integrand size = 49, antiderivative size = 370

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{(iA+B-iC)\sqrt{c-id}\,\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f} - \frac{(B-i(A-C))\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f} - \frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3b(a^2+b^2)f(a+b\tan(e+fx))^{3/2}} - \frac{2(2a^3bBd+a^4Cd+b^4(3Bc+Ad)+2ab^3(3Ac-3cC-2Bd)-a^2b^2(3Bc+5Ad-7Cd))\sqrt{c+d\tan(e+fx)}}{3b(a^2+b^2)^2(bc-ad)f\sqrt{a+b\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(5/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
c+I*d)^(1/2)/(a+I*b)^(5/2)/f-2/3*(2*a^3*b*B*d+a^4*C*d+b^4*(A*d+3*B*c)+2*a*
b^3*(3*A*c-2*B*d-3*C*c)-a^2*b^2*(5*A*d+3*B*c-7*C*d))*(c+d*tan(f*x+e))^(1/2
)/b/(a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a)
)*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

3.133.2 Mathematica [A] (verified)

Time = 7.10 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{C\sqrt{c+d\tan(e+fx)}}{bf(a+b\tan(e+fx))^{3/2}} - \frac{-\frac{2\left(\frac{1}{2}b^2(-2Abc+3bcC-aCd)-a\left(-b^2(Bc+(A-C)d)-\frac{1}{2}a(bcC-2bBd-aCd)\right)\right)\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}} - \frac{2\left(-\frac{3b(bc-ad)\left(\frac{(a+ib)^2(iA+B-iC)\sqrt{-c+id}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x])^(5/2),x]
```

output
```
-((C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])^(3/2))) - ((-2*((
b^2*(-2*A*b*c + 3*b*c*C - a*C*d))/2 - a*(-(b^2*(B*c + (A - C)*d)) - (a*(b*
c*C - 2*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c
- a*d)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*((-3*b*(b*c - a*d)*(((a + I*b)^2
*(I*A + B - I*C)*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] + ((a -
 I*b)^2*(B - I*(A - C))*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b]))/(2
*(a^2 + b^2)*f) - (2*((b^2*(b*c - a*d)*(a^2*C*d + b^2*(3*B*c + A*d) + a*b*
(3*A*c - 3*c*C - B*d)))/2 - a*((a*(2*A*b^2 - 2*a*b*B - a^2*C - 3*b^2*C)*d*
(b*c - a*d))/2 - (3*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d
 + a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a
 + b*Tan[e + f*x]])))/(3*(a^2 + b^2)*(b*c - a*d)))/b
```

3.133.3 Rubi [A] (verified)

Time = 2.59 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.245$, Rules used = {3042, 4128, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.133. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2\int \frac{-\left(\left(-Ca^2-2bBa+2Ab^2-3b^2C\right)d\tan^2(e+fx)\right)-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-ad)+Ab(3ac+bd)}{2(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-2bBa+2Ab^2-3b^2C\right)d\tan^2(e+fx)\right)-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-ad)+Ab(3ac+bd)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-2bBa+2Ab^2-3b^2C\right)d\tan(e+fx)^2\right)-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-ad)+Ab(3ac+bd)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{-\frac{2\int -\frac{3\left(b(bc-ad)\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)\right)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{\left(a^2+b^2\right)(bc-ad)}}{3b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

3.133. $$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\frac{\frac{3\int \frac{b(bc-ad)\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(a^2+b^2)(bc-ad)} - 2}{3b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\int \frac{b(bc-ad)\left((Ac-Cc-Bd)a^2+2b(Bc+(A-C)d)a-b^2(Ac-Cc-Bd)\right)-b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc+(A-C)d)\right)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(a^2+b^2)(bc-ad)} - 2}{3b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 4099

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}+\frac{3\left(\frac{1}{2}b(a-ib)^2(c+id)(A+iB-C)(bc-ad)\right.}{}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}+\frac{3\left(\frac{1}{2}b(a-ib)^2(c+id)(A+iB-C)(bc-ad)\right.}{}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 4098

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}+\frac{3\left(\frac{b(a-ib)^2(c+id)(A+iB-C)(bc-ad)\int\frac{}{(i\,t}}{}\right.}{}}{3b\,(a}$$

$\downarrow$ 104

3.133. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}+\frac{3\left(\frac{b(a-ib)^2(c+id)(A+iB-C)(bc-ad)\int \frac{\ldots}{-i\ldots}}{\ldots}\right.}{\ldots}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+2a^3bBd-a^2b^2(5Ad+3Bc-7Cd)+2ab^3(3Ac-2Bd-3cC)+b^4(Ad+3Bc)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}+\frac{3\left(\frac{ib(a-ib)^2\sqrt{c+id}(A+iB-C)(bc-ad)\text{arc}\ldots}{f\sqrt{a+i\ldots}}\right.}{\ldots}}{3b\left(a^2+b^2\right)}$$

input

```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x])^(5/2),x]
```

output

```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)*f*(
a + b*Tan[e + f*x])^(3/2)) + ((3*(((-I)*(a + I*b)^2*b*(A - I*B - C)*Sqrt[c
 - I*d]*(b*c - a*d)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) + (I*(a - I*b)^2*b
*(A + I*B - C)*Sqrt[c + I*d]*(b*c - a*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*
f)))/((a^2 + b^2)*(b*c - a*d)) - (2*(2*a^3*b*B*d + a^4*C*d + b^4*(3*B*c +
A*d) + 2*a*b^3*(3*A*c - 3*c*C - 2*B*d) - a^2*b^2*(3*B*c + 5*A*d - 7*C*d))*
Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*
x]]))/(3*b*(a^2 + b^2))
```

3.133. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

3.133. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.133.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(fx+e)}\,\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(a+b\tan(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

3.133.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.133.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{c+d\tan{(e+fx)}}(A+B\tan{(e+}$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x))**(5/2), x)
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assum
e?` for mo
```

3.133.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.134 $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

3.134.1 Optimal result

Integrand size = 49, antiderivative size = 597

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{(iA+B-iC)\sqrt{c-id}\,\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}$$
$$-\frac{(B-i(A-C))\sqrt{c+id}\,\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{7/2}f}$$
$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{5b(a^2+b^2)f(a+b\tan(e+fx))^{5/2}}$$
$$-\frac{2(4a^3bBd+a^4Cd+b^4(5Bc+Ad)+2ab^3(5Ac-5cC-3Bd)-a^2b^2(5Bc+9Ad-11Cd))\sqrt{c+d\tan(e}{15b(a^2+b^2)^2(bc-ad)f(a+b\tan(e+fx))^{3/2}}$$
$$+\frac{2(8a^5bBd^2+2a^6Cd^2-a^4b^2d(25Bc+33Ad-39Cd)-a^2b^4(45Ac^2-45c^2C-90Bcd-29Ad^2+23Cd^2)}{15b(a^2}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(c-I*d)^(1/2)/(a-I*b)^(7/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
c+I*d)^(1/2)/(a+I*b)^(7/2)/f+2/15*(8*a^5*b*B*d^2+2*a^6*C*d^2-a^4*b^2*d*(33
*A*d+25*B*c-39*C*d)-a^2*b^4*(45*A*c^2-29*A*d^2-90*B*c*d-45*C*c^2+23*C*d^2)
+a^3*b^3*(80*c*(A-C)*d+B*(15*c^2-49*d^2))-a*b^5*(40*c*(A-C)*d+B*(45*c^2-3*
d^2))-b^6*(5*c*(B*d+3*C*c)-A*(15*c^2+2*d^2)))*(c+d*tan(f*x+e))^(1/2)/b/(a^
2+b^2)^3/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))^(1/2)-2/5*(A*b^2-a*(B*b-C*a))*(c+
d*tan(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)-2/15*(4*a^3*b*B*d
+a^4*C*d+b^4*(A*d+5*B*c)+2*a*b^3*(5*A*c-3*B*d-5*C*c)-a^2*b^2*(9*A*d+5*B*c-
11*C*d))*(c+d*tan(f*x+e))^(1/2)/b/(a^2+b^2)^2/(-a*d+b*c)/f/(a+b*tan(f*x+e)
)^(3/2)
```

3.134.2 Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 1109, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx =$$

$$-\frac{C\sqrt{c+d\tan(e+fx)}}{2bf(a+b\tan(e+fx))^{5/2}}$$

$$-\frac{-\frac{2\left(\frac{1}{2}b^2(-4Abc+5bcC-aCd)-a\left(-2b^2(Bc+(A-C)d)-\frac{1}{2}a(bcC-4bBd-aCd)\right)\right)\sqrt{c+d\tan(e+fx)}}{5(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{5/2}}-\frac{2\left(-\frac{2\left(b^2(bc-ad)\left(a^2Cd+b^2(5Bc+Ad)+ab(5\cdots\right.\right.}{\cdots}\cdots\right)}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(a + b*Tan[e + f*x])^(7/2),x]
```

output
```
-1/2*(C*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[e + f*x])^(5/2)) - ((-2*
((b^2*(-4*A*b*c + 5*b*c*C - a*C*d))/2 - a*(-2*b^2*(B*c + (A - C)*d) - (a*(
b*c*C - 4*b*B*d - a*C*d))/2))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*
c - a*d)*f*(a + b*Tan[e + f*x])^(5/2)) - (2*((-2*(b^2*(b*c - a*d)*(a^2*C*d
 + b^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a*(4*A*b^2 - 4*a*b*
B - a^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c - a*d)*(A*b*c - a*B*c - b*
c*C - a*A*d - b*B*d + a*C*d)))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b
*c - a*d)*f*(a + b*Tan[e + f*x])^(3/2)) - (2*((-15*b*(b*c - a*d)^2*(((I*a
- b)^3*(A - I*B - C)*Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan
[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-a + I*b] - (
(I*a + b)^3*(A + I*B - C)*Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*
Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b]))/
(2*(a^2 + b^2)*f) - (2*(b^2*((b*c - a*d)*(b^2*d - (3*a*(b*c - a*d))/2)*(a^
2*C*d + b^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) + ((-3*b*c)/2 + (a*
d)/2)*(a*(4*A*b^2 - 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) - 5*b^2*(b*c
- a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d))) - a*((3*b*(b*c -
a*d)*(b*(4*A*b^2 - 4*a*b*B - a^2*C - 5*b^2*C)*d*(b*c - a*d) + 5*a*b*(b*c -
 a*d)*(A*b*c - a*B*c - b*c*C - a*A*d - b*B*d + a*C*d) + b*(b*c - a*d)*(a^2
*C*d + b^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d))))/2 - a*d*(b^2*(b*c
- a*d)*(a^2*C*d + b^2*(5*B*c + A*d) + a*b*(5*A*c - 5*c*C - B*d)) - a*(a...
```

3.134.3 Rubi [A] (verified)

Time = 4.50 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{-\left(\left(-Ca^2-4bBa+4Ab^2-5b^2C\right)d\tan^2(e+fx)\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-ad)+Ab(5ac+bd)}{2(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}dx}{5b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-4bBa+4Ab^2-5b^2C\right)d\tan^2(e+fx)\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-ad)+Ab(5ac+bd)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}dx}{5b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-\left(\left(-Ca^2-4bBa+4Ab^2-5b^2C\right)d\tan(e+fx)^2\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-ad)+Ab(5ac+bd)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}dx}{5b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{2\int \frac{2d\left(Cda^4+4bBda^3-b^2(5Bc+9Ad-11Cd)a^2+2b^3(5Ac-5Cc-3Bd)a+b^4(5Bc+Ad)\right)\tan^2(e+fx)+15b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(\cdots\right.}{2(a+b\tan(e+fx))^{3\cdots}}}{3(a^2+b^2\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2d\left(Cda^4+4bBda^3-b^2(5Bc+9Ad-11Cd)a^2+2b^3(5Ac-5Cc-3Bd)a+b^4(5Bc+Ad)\right)\tan^2(e+fx)+15b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc\cdots\right.}{(a+b\tan(e+fx))^{3/2\cdots}}}{3(a^2+b^2)\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

3.134. $\displaystyle\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$-\frac{\int \frac{2d\left(Cda^4+4bBda^3-b^2(5Bc+9Ad-11Cd)a^2+2b^3(5Ac-5Cc-3Bd)a+b^4(5Bc+Ad)\right)\tan(e+fx)^2+15b(bc-ad)\left(-\left((Bc+(A-C)d)a^2\right)+2b(Ac-Cc-Bd)a+b^2(Bc\cdots}{(a+b\tan(e+fx))^{3/2}\cdots}}{3(a^2+b^2)\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{-\frac{2\int \frac{15\left(b(bc-ad)^2\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)^2\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(\cdots}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{\left(a^2+b^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{15\int \frac{b(bc-ad)^2\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)^2\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(B\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{\left(a^2+b^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{15\int \frac{b(bc-ad)^2\left((Ac-Cc-Bd)a^3+3b(Bc+(A-C)d)a^2-3b^2(Ac-Cc-Bd)a-b^3(Bc+(A-C)d)\right)-b(bc-ad)^2\left(-\left((Bc+(A-C)d)a^3\right)+3b(Ac-Cc-Bd)a^2+3b^2(B\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{\left(a^2+b^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 4099}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc)\right)}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(2a^6Cd^2+8a^5bBd^2-a\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 3042}$$

3.134. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc))}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(2a^6Cd^2+8a^5bBd^2-a\right.}{}}{}$$

$\downarrow$ 4098

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc))}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(2a^6Cd^2+8a^5bBd^2-a\right.}{}}{}$$

$\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc))}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(2a^6Cd^2+8a^5bBd^2-a\right.}{}}{}$$

$\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(a^4Cd+4a^3bBd-a^2b^2(9Ad+5Bc-11Cd)+2ab^3(5Ac-3Bd-5cC)+b^4(Ad+5Bc))}{3f(a^2+b^2)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(2a^6Cd^2+8a^5bBd^2-a\right.}{}}{}$$

input
```
Int[(Sqrt[c + d*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(a
+ b*Tan[e + f*x])^(7/2),x]
```

3.134. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(5*b*(a^2 + b^2)*f*(
a + b*Tan[e + f*x])^(5/2)) + ((-2*(4*a^3*b*B*d + a^4*C*d + b^4*(5*B*c + A*
d) + 2*a*b^3*(5*A*c - 5*c*C - 3*B*d) - a^2*b^2*(5*B*c + 9*A*d - 11*C*d))*S
qrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f*x])
^(3/2)) - ((-15*(((-I)*(a + I*b)^3*b*(A - I*B - C)*Sqrt[c - I*d]*(b*c - a*
d)^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) + (I*(a - I*b)^3*b*(A + I*B - C)*
Sqrt[c + I*d]*(b*c - a*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]
])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f)))/((a^2 +
b^2)*(b*c - a*d)) - (2*(8*a^5*b*B*d^2 + 2*a^6*C*d^2 - a^4*b^2*d*(25*B*c +
33*A*d - 39*C*d) - a^2*b^4*(45*A*c^2 - 45*c^2*C - 90*B*c*d - 29*A*d^2 + 23
*C*d^2) + a^3*b^3*(80*c*(A - C)*d + B*(15*c^2 - 49*d^2)) - a*b^5*(40*c*(A
- C)*d + B*(45*c^2 - 3*d^2)) - b^6*(5*c*(3*c*C + B*d) - A*(15*c^2 + 2*d^2)
))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]]))/(3*(a^2 + b^2)*(b*c - a*d)))/(5*b*(a^2 + b^2))
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.134. $\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.134. $\int \frac{\sqrt{c+d\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

3.134.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(fx+e)}\,\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(a+b\tan(fx+e))^{\frac{7}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

output
```
int((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

3.134.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.134.6 Sympy [F]

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+$$

input
```
integrate((c+d*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(7/2),x)
```

output
```
Integral(sqrt(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(a + b*tan(e + f*x))**(7/2), x)
```

3.134.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.134.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+d\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

3.135 $\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\,(A+B\tan$

3.135.1 Optimal result

Integrand size = 49, antiderivative size = 682

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$

$$-\frac{(a-ib)^{3/2}(B+i(A-C))(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$-\frac{(a+ib)^{3/2}(B-i(A-C))(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$+\frac{(3a^4Cd^4-4a^3bd^3(3cC+2Bd)+6a^2b^2d^2(3c^2C+12Bcd+8(A-C)d^2)-12ab^3d(c^3C-6Bc^2d-24c(A}{64}$$

$$+\frac{(64b(a^2B-b^2B+2ab(A-C))\,d^3+(bc-ad)\,(48b(Ab+aB-bC)d^2+(bc-ad)(3bcC-8bBd-3aCd)}{64b^2d^2f}$$

$$+\frac{(48b(Ab+aB-bC)d^2+(bc-ad)(3bcC-8bBd-3aCd))\,\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{96bd^2f}$$

$$-\frac{(3bcC-8bBd-3aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{24d^2f}$$

$$+\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}$$

output
```
-(a-I*b)^(3/2)*(B+I*(A-C))*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*
x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f-(a+I*b)^(3/2)*(B-I*(A-
C))*(c+I*d)^(3/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/
2)/(c+d*tan(f*x+e))^(1/2))/f+1/64*(3*a^4*C*d^4-4*a^3*b*d^3*(2*B*d+3*C*c)+6
*a^2*b^2*d^2*(3*c^2*C+12*B*c*d+8*(A-C)*d^2)-12*a*b^3*d*(c^3*C-6*B*c^2*d-24
*c*(A-C)*d^2+16*B*d^3)+b^4*(3*c^4*C-8*B*c^3*d+48*c^2*(A-C)*d^2-192*B*c*d^3
-128*(A-C)*d^4))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f
*x+e))^(1/2))/b^(5/2)/d^(5/2)/f+1/64*(64*b*(B*a^2-B*b^2+2*a*b*(A-C))*d^3+(
-a*d+b*c)*(48*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-8*B*b*d-3*C*a*d+3*C*b*c))*
(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/d^2/f+1/96*(48*b*(A*b+B*
a-C*b)*d^2+(-a*d+b*c)*(-8*B*b*d-3*C*a*d+3*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(
c+d*tan(f*x+e))^(3/2)/b/d^2/f-1/24*(-8*B*b*d-3*C*a*d+3*C*b*c)*(a+b*tan(f*x
+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/d^2/f+1/4*C*(a+b*tan(f*x+e))^(3/2)*(c+d*
tan(f*x+e))^(5/2)/d/f
```

3.135.2 Mathematica [A] (verified)

Time = 9.56 (sec) , antiderivative size = 1304, normalized size of antiderivative = 1.91

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}$$

$$+\frac{\frac{(-3bcC+8bBd+3aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{6df}+\frac{\left(48b(Ab+aB-bC)d^2+(bc-ad)(3bcC-8bBd-3aCd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/}}{8bf}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan
[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2))/(4*d*f) + (((-3*
b*c*C + 8*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(
5/2))/(6*d*f) + (((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b
*B*d - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b
*f) + (((24*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 - (3*(-(b*c) + a*d)*(48*
b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d - 3*a*C*d)))/8)*S
qrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((24*(-(b^4*Sqrt
[-b^2]*d^2*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - b^2*(c^2*C + 2
*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A - C)*d + B*(c^2 - d^2))))
- b^5*d^2*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) - a^2*(2*c*(A -
 C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sq
rt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]
*Sqrt[c + d*Tan[e + f*x]])])/(b^2*Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b
^2]*d)/b]) - (24*b^2*d^2*(Sqrt[-b^2]*(a^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^
2 - d^2)) - b^2*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + 2*a*b*(2*c*(A
- C)*d + B*(c^2 - d^2))) - b*(2*a*b*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^
2)) - a^2*(2*c*(A - C)*d + B*(c^2 - d^2)) + b^2*(2*c*(A - C)*d + B*(c^2 -
d^2))))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqr
t[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c
 + (Sqrt[-b^2]*d)/b]) + (3*Sqrt[b]*Sqrt[c - (a*d)/b]*Sqrt[(c/(c - (a*d)...
```

3.135.3 Rubi [A] (verified)

Time = 5.86 (sec) , antiderivative size = 711, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.327$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{1}{2}\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left((3bcC-3adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e \cdots\right.}{4d}$$

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left((3bcC-3adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e+ \cdots\right.}{8d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left((3bcC-3adC-8bBd)\tan(e+fx)^2-8(Ab-Cb+aB)d\tan(e+ \cdots\right.}{8d}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{\frac{\int -\frac{(c+d\tan(e+fx))^{3/2}\left(c(3cC-8Bd)b^2-2ad(3cC+20Bd)b+3a^2(16A-15C)d^2+\left(48b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3adC-8bBd)\right)\tan^2(e+fx)+48\left(Ba^2+2b(A \cdots\right.\right.}{2\sqrt{a+b\tan(e+fx)}}}{3d}}{8d}$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(c(3cC-8Bd)b^2-2ad(3cC+20Bd)b+3a^2(16A-15C)d^2+\left(48b(Ab- \cdots\right.\right.}{\sqrt{a \cdots}}}{\cdots}}{8d}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(c(3cC-8Bd)b^2-2ad(3cC+20Bd)b+3a^2(16A-15C)d^2+\left(48b(Ab- \cdots\right.\right.}{\sqrt{a \cdots}}}{\cdots}}{8d}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\int -\frac{3\sqrt{c+d\tan(e+fx)}\left(-c\left(3Cc^2-8Bdc-16(A-C)d^2\right)b^3+ad\left(9Cc^2+64Bdc+48(A-C)d^2 \cdots\right.\right.}{\cdots}}{\cdots}$$

3.135.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}$$

$\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}$$

$\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}$$

$\downarrow$ 4138

3.135.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}}{}$$

$\downarrow$ 2348

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(3bcC-3adC-8bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{\left(48b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3adC-8bBd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}}{}$$

$\downarrow$ 2009

$$\frac{C(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}{4df}-$$

$$\frac{(-3aCd-8bBd+3bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)+(bc-ad)(-3aCd-8bBd+3bcC\right.}{2bf}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f
*x] + C*Tan[e + f*x]^2),x]
```

3.135.
$\int(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*(c + d*Tan[e + f*x])^(5/2))/(4*d*f) - (((3*b
*c*C - 8*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5
/2))/(3*d*f) - (((48*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*
B*d - 3*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*
f) - (3*(-1/2*(-128*(a - I*b)^(3/2)*b^2*(I*A + B - I*C)*(c - I*d)^(3/2)*d^
2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c +
 d*Tan[e + f*x]])] - 128*(a + I*b)^(3/2)*b^2*(B - I*(A - C))*(c + I*d)^(3/
2)*d^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqr
t[c + d*Tan[e + f*x]])] + (2*(3*a^4*C*d^4 - 4*a^3*b*d^3*(3*c*C + 2*B*d) +
6*a^2*b^2*d^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2) - 12*a*b^3*d*(c^3*C - 6
*B*c^2*d - 24*c*(A - C)*d^2 + 16*B*d^3) + b^4*(3*c^4*C - 8*B*c^3*d + 48*c^
2*(A - C)*d^2 - 192*B*c*d^3 - 128*(A - C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(
b*f) - ((64*b*(a^2*B - b^2*B + 2*a*b*(A - C))*d^3 + (b*c - a*d)*(48*b*(A*b
 + a*B - b*C)*d^2 + (b*c - a*d)*(3*b*c*C - 8*b*B*d - 3*a*C*d)))*Sqrt[a + b
*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f)))/(4*b))/(6*d))/(8*d)
```

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.135.4 Maple [F(-1)]

Timed out.

$$\int (a+b\tan(fx+e))^{\frac{3}{2}}(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)dx$$

input
```
int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.135.5 Fricas [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.135.6 Sympy [F]

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (a+b\tan(e+fx))^{\frac{3}{2}}(c+d\tan(e+fx))^{\frac{3}{2}}(A$$

input
```
integrate((a+b*tan(f*x+e))**(3/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(3/2)*(c + d*tan(e + f*x))**(3/2)*(A + B*ta
n(e + f*x) + C*tan(e + f*x)**2), x)
```

3.135.7 Maxima [F]

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/
2)*(d*tan(f*x + e) + c)^(3/2), x)
```

3.135.8 Giac [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int (a+b\tan(e+f\,x))^{3/2}\,(c+d\tan(e+f\,x))^{3/$$

input
```
int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^(3/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2), x)
```

3.136 $\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\,\text{ta}\right.$

3.136.1 Optimal result

Integrand size = 49, antiderivative size = 508

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx =$$

$$-\frac{\sqrt{a-ib}(iA+B-iC)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$-\frac{\sqrt{a+ib}(B-i(A-C))(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$+\frac{(a^3Cd^3-a^2bd^2(3cC+2Bd)+ab^2d(3c^2C+12Bcd+8(A-C)d^2)-b^3(c^3C-6Bc^2d-24c(A-C)d^2+}{8b^{5/2}d^{3/2}f}$$

$$+\frac{(8b(Ab+aB-bC)d^2-(bc-ad)(bcC-6bBd-aCd))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{8b^2df}$$

$$-\frac{(bcC-6bBd-aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{12bdf}$$

$$+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}$$

output
```
1/8*(a^3*C*d^3-a^2*b*d^2*(2*B*d+3*C*c)+a*b^2*d*(3*c^2*C+12*B*c*d+8*(A-C)*d
^2)-b^3*(c^3*C-6*B*c^2*d-24*c*(A-C)*d^2+16*B*d^3))*arctanh(d^(1/2)*(a+b*ta
n(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(5/2)/d^(3/2)/f-(I*A+B-I
*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/
2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f-(B-I*(A-C))*(c+I*d)^(3/2)*arcta
nh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/
2))*(a+I*b)^(1/2)/f+1/8*(8*b*(A*b+B*a-C*b)*d^2-(-a*d+b*c)*(-6*B*b*d-C*a*d+
C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/d/f-1/12*(-6*B*b
*d-C*a*d+C*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b/d/f+1/3*C*
(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/d/f
```

3.136.2 Mathematica [A] (verified)

Time = 9.23 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.71

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}+\frac{\frac{(-bcC+6bBd+aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{4bf}+\frac{\frac{3\left(8b(Ab+aB-bC)d^2-(bc-ad)(bcC-6bBd-aCd)\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4bf}+\frac{6b^2 \cdots}{\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e
 + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*f) + (((-(b*c
*C) + 6*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)
)/(4*b*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d
- a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f) + ((6
*b^2*d*(b*(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(
c^2*C + 2*B*c*d - C*d^2)) - Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^
2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sqrt[-c + (Sqrt[-
b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan
[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (6*b^2
*d*(b*(2*a*A*c*d - 2*a*c*C*d + A*b*(c^2 - d^2) + a*B*(c^2 - d^2) - b*(c^2*
C + 2*B*c*d - C*d^2)) + Sqrt[-b^2]*(a*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 -
d^2)) + b*(2*c*(A - C)*d + B*(c^2 - d^2))))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*
d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f
*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) + (3*Sqrt[b]*Sqr
t[c - (a*d)/b]*(a^3*C*d^3 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3*c^2*C +
 12*B*c*d + 8*(A - C)*d^2) - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)*d^2 + 1
6*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*
d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4*Sqrt[d]*Sqrt[c + d*
Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*d)
```

3.136.3 Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{(c+d\tan(e+fx))^{3/2}\left((bcC-adC-6bBd)\tan^2(e+fx)-6(Ab-Cb+aB)d\tan(e+fx)+bcC-a(6A-5C)d\right)}{2\sqrt{a+b\tan(e+fx)}}dx}{3d}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\int\frac{(c+d\tan(e+fx))^{3/2}\left((bcC-adC-6bBd)\tan^2(e+fx)-6(Ab-Cb+aB)d\tan(e+fx)+bcC-a(6A-5C)d\right)}{\sqrt{a+b\tan(e+fx)}}dx}{6d}$$

$\downarrow$ 3042

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\int\frac{(c+d\tan(e+fx))^{3/2}\left((bcC-adC-6bBd)\tan(e+fx)^2-6(Ab-Cb+aB)d\tan(e+fx)+bcC-a(6A-5C)d\right)}{\sqrt{a+b\tan(e+fx)}}dx}{6d}$$

$\downarrow$ 4130

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{\int\frac{3\sqrt{c+d\tan(e+fx)}\left(c(cC+2Bd)b^2-2ad(4Ac-3Cc-3Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-\right.}{2\sqrt{a+b\tan(e+fx)}}}{2b}}{6d}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{3\int\frac{\sqrt{c+d\tan(e+fx)}\left(c(cC+2Bd)b^2-2ad(4Ac-3Cc-3Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-\right.}{\sqrt{a+b\tan(e+fx)}}}{4b}}{6d}$$

$\downarrow$ 3042

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-\frac{\frac{3\int\frac{\sqrt{c+d\tan(e+fx)}\left(c(cC+2Bd)b^2-2ad(4Ac-3Cc-3Bd)b-8d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+a^2Cd^2-\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-\right.}{\sqrt{a+b\tan(e+fx)}}}{4b}}{6d}$$

$\downarrow$ 4130

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-3\left(\int-\frac{-c\left(Cc^2+10Bdc+8(A-C)d^2\right)b^3-ad\left(13Cc^2+20Bdc-8Cd^2-8A\left(2c^2-d^2\right)\right)b^2+16d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(\right.}{2\sqrt{a+b\tan(e+}}\right.$$

$\downarrow$ 27

3.136.

$$\int\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-$$

$$3\left(-\frac{\int \frac{-c\left(Cc^2+10Bdc+8(A-C)d^2\right)b^3+ad\left(16Ac^2-13Cc^2-20Bdc-8Ad^2+8Cd^2\right)b^2+16d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(}{\sqrt{a+b\tan(e+}}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-$$

$$3\left(-\frac{\int \frac{-c\left(Cc^2+10Bdc+8(A-C)d^2\right)b^3+ad\left(16Ac^2-13Cc^2-20Bdc-8Ad^2+8Cd^2\right)b^2+16d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(}{\sqrt{a+b\tan(e+}}}{}\right.$$

$$\Big\downarrow \text{4138}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-$$

$$3\left(-\frac{\int \frac{-c\left(Cc^2+10Bdc+8(A-C)d^2\right)b^3+ad\left(16Ac^2-13Cc^2-20Bdc-8Ad^2+8Cd^2\right)b^2+16d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(}{\sqrt{a+b\tan(e+fx)}\sqrt{c+c}}}{}\right.$$

$$\Big\downarrow \text{2348}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-$$

$$\frac{(bcC-adC-6bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+\frac{3\left(-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8b(Ab-Cb+aB)d^2-(bc-ad)(bcC-adC-6bBd)\right)}{bf}-\frac{\int}{}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3df}-$$

$$\frac{(-aCd-6bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+\frac{3\left(-\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)-(bc-ad)(-aCd-6bBd+bcC)\right)}{bf}\right.}{}$$

3.136.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*d*f) - (((b*c*C
 - 6*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(
2*b*f) + (3*(-1/2*(-16*Sqrt[a - I*b]*b^2*(I*A + B - I*C)*(c - I*d)^(3/2)*d
*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c +
d*Tan[e + f*x]])] - 16*Sqrt[a + I*b]*b^2*(B - I*(A - C))*(c + I*d)^(3/2)*d
*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c +
d*Tan[e + f*x]])] + (2*(a^3*C*d^3 - a^2*b*d^2*(3*c*C + 2*B*d) + a*b^2*d*(3
*c^2*C + 12*B*c*d + 8*(A - C)*d^2) - b^3*(c^3*C - 6*B*c^2*d - 24*c*(A - C)
*d^2 + 16*B*d^3))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt
[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(b*f) - ((8*b*(A*b + a*B - b*C)
*d^2 - (b*c - a*d)*(b*c*C - 6*b*B*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqr
t[c + d*Tan[e + f*x]])/(b*f)))/(4*b))/(6*d)
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.136.4 Maple [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.136.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.136.6 Sympy [F]

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int \sqrt{a+b\tan{\left(e+fx\right)}}\left(c+d\tan{\left(e+fx\right)}\right)^{\frac{3}{2}}\left(A+B\tan{\left(e+fx\right)}+C\tan^{2}{\left(e+fx\right)}\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(3/2)*(A + B*tan(e
 + f*x) + C*tan(e + f*x)**2), x)
```

3.136.7 Maxima [F]

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\sqrt{b\tan\left(fx+e\right)+a}\left(d\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)
*(d*tan(f*x + e) + c)^(3/2), x)
```

3.136.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int \sqrt{a+b\tan\left(e+f\,x\right)}\,\left(c+d\tan\left(e+f\,x\right)\right)^{3/2}\,\left(C\,{\tan\left(e+f\,x\right)}^2+B\tan\left(e+f\,x\right)+A\right)$$

input
```
int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2), x)
```

3.137 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

3.137.1 Optimal result

Integrand size = 49, antiderivative size = 384

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = -\frac{(iA+B-iC)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}f} + \frac{(iA-B-iC)(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}f} + \frac{(3a^2Cd^2-2abd(3cC+2Bd)+b^2(3c^2C+12Bcd+8(A-C)d^2))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{5/2}\sqrt{d}f} + \frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4b^2f} + \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)+(I*A-B-I*C)*(c+I*d)^(3
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/f/(a+I*b)^(1/2)+1/4*(3*a^2*C*d^2-2*a*b*d*(2*B*d+3*C*c)+b^2*(3
*c^2*C+12*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/
2)/(c+d*tan(f*x+e))^(1/2))/b^(5/2)/f/d^(1/2)+1/4*(4*B*b*d-3*C*a*d+3*C*b*c)
*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^2/f+1/2*C*(a+b*tan(f*x+e)
)^(1/2)*(c+d*tan(f*x+e))^(3/2)/b/f
```

3.137.2 Mathematica [A] (verified)

Time = 7.86 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.60

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+}{2bf}$$
$$+\frac{\frac{(3bcC+4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{2bf}+\frac{-\frac{2b^2\left(\sqrt{-b^2}\left(c^2C+2Bcd-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{-}}{\sqrt{-}}\right.}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}}{}}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/Sqrt[a + b*Tan[e + f*x]],x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f) + (((3*b*c
*C + 4*b*B*d - 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])
/(2*b*f) + ((-2*b^2*(Sqrt[-b^2]*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2))
- b*(2*c*(A - C)*d + B*(c^2 - d^2)))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*
Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])
])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (2*b^2*(Sqrt[-b^2
]*(c^2*C + 2*B*c*d - C*d^2 - A*(c^2 - d^2)) + b*(2*c*(A - C)*d + B*(c^2 -
d^2)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c
+ (Sqrt[-b^2]*d)/b]) + (Sqrt[b]*Sqrt[c - (a*d)/b]*(3*a^2*C*d^2 - 2*a*b*d*(
3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*T
an[e + f*x])/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/
(2*b)
```

3.137.3 Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.224, Rules used = {3042, 4130, 27, 3042, 4130, 27, 25, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{a+b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3bcC-3adC+4bBd)\tan^2(e+fx)+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-C(bc+3ad)\right)}{2\sqrt{a+b\tan(e+fx)}}dx}{2b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3bcC-3adC+4bBd)\tan^2(e+fx)+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-bcC-3aCd\right)}{\sqrt{a+b\tan(e+fx)}}dx}{4b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((3bcC-3adC+4bBd)\tan(e+fx)^2+4b(Bc+(A-C)d)\tan(e+fx)+4Abc-bcC-3aCd\right)}{\sqrt{a+b\tan(e+fx)}}dx}{4b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{4130}$$

$$\frac{\frac{\int -\frac{-8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2c(4Abc-C(bc+3ad))b-\left(8d(Bc+(A-C)d)b^2+(bc-ad)(3bcC-3adC+4bBd)\right)\tan^2(e+fx)+(bc+ad)(3bcC-3adC+4}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b}}{4b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-3aCd+4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int -\frac{8Ac^2b^2-c(5cC+4Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(3cC+2Bd)b+3a}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan}}}{2b}}{4b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{8Ac^2b^2-c(5cC+4Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(3cC+2Bd)b+3a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(3bcC-3adC+4bBd)\right)\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{2b}}{4b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{8Ac^2b^2-c(5cC+4Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(3cC+2Bd)b+3a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(3bcC-3adC+4bBd)\right)\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{2b}}{4b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$\downarrow$ 4138

$$\frac{\frac{\int \frac{8Ac^2b^2-c(5cC+4Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(3cC+2Bd)b+3a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(3bcC-3adC+4bBd)\right)\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}}{2bf}}{4b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}$$

$\downarrow$ 2348

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+$$

$$\frac{(-3aCd+4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}+\frac{\int\left(\frac{8Ad^2b^2-8Cd^2b^2+3c^2Cb^2+12Bcdb^2-4aBd^2b-6acCdb+3a^2Cd^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-8Bc^2b^2+8Bd^2b^2-}{}\right.}{}$$

$\downarrow$ 2009

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+$$

$$\frac{\frac{(-3aCd+4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}+\frac{2\left(3a^2Cd^2-2abd(2Bd+3cC)+b^2\left(8d^2(A-C)+12Bcd+3c^2C\right)\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+}}{\sqrt{b}\sqrt{c+d\tan(e+}}\right.}{\sqrt{b}\sqrt{d}}}{4b}$$

3.137. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/S
qrt[a + b*Tan[e + f*x]],x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f) + (((-8*b^
2*(I*A + B - I*C)*(c - I*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I*b] - (8*b^2
*(B - I*(A - C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] + (2*(3*a
^2*C*d^2 - 2*a*b*d*(3*c*C + 2*B*d) + b^2*(3*c^2*C + 12*B*c*d + 8*(A - C)*d
^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e
+ f*x]])])/(Sqrt[b]*Sqrt[d]))/(2*b*f) + ((3*b*c*C + 4*b*B*d - 3*a*C*d)*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f))/(4*b)
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.137.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c + d\tan(fx + e))^{\frac{3}{2}} (A + B\tan(fx + e) + C\tan(fx + e)^2)}{\sqrt{a + b\tan(fx + e)}} dx$$

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

output
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57709 vs. 2(311) = 622.

Time = 196.39 (sec) , antiderivative size = 115434, normalized size of antiderivative = 300.61

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.137.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\,(A+B\tan{(}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/sqrt(a + b*tan(e + f*x)), x)
```

3.137.7 Maxima [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\int \frac{(C\tan{(fx+e)}^2+B\tan{(fx+e)}}{\sqrt{b\tan{(fx+}}}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^(3/
2)/sqrt(b*tan(f*x + e) + a), x)
```

3.137.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(1/2),x)
```

output
```
\text{Hanged}
```

3.138 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

3.138.1 Optimal result

Integrand size = 49, antiderivative size = 382

$$\begin{aligned}
\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx &= \\
&-\frac{(iA+B-iC)(c-id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f} \\
&-\frac{(B-i(A-C))(c+id)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f} \\
&+\frac{\sqrt{d}(3bcC+2bBd-3aCd)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f} \\
&+\frac{(2Ab^2-2abB+3a^2C+b^2C)\,d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{b^2(a^2+b^2)f} \\
&-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f\sqrt{a+b\tan(e+fx)}}
\end{aligned}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f-(B-I*(A-C))*(c+I*d)^(3
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(3/2)/f+(2*B*b*d-3*C*a*d+3*C*b*c)*arctanh(d^(1/2)*(a+
b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(5/2)/f+(2*A
*b^2-2*B*a*b+3*C*a^2+C*b^2)*d*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2
)/b^2/(a^2+b^2)/f-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)
/f/(a+b*tan(f*x+e))^(1/2)
```

3.138.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1664 vs. $2(382) = 764$.

Time = 7.33 (sec) , antiderivative size = 1664, normalized size of antiderivative = 4.36

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \frac{C(c+d\tan(e+fx))^{3/2}}{bf\sqrt{a+b\tan(e+fx)}}$$

$$-\frac{2b(iA+B-iC)(-c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}f}+\frac{2b(iA-B-iC)(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}-\frac{2b(A+iB}{(a}$$

$$+$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(3/2),x]
```

output
```
(C*(c + d*Tan[e + f*x])^(3/2))/(b*f*Sqrt[a + b*Tan[e + f*x]]) + ((-2*b*(I*
A + B - I*C)*(-c + I*d)^(3/2)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f
*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((-a + I*b)^(3/2)*f) + (
2*b*(I*A - B - I*C)*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((a + I*b)^(3/2)*f)
- (2*b*(A + I*B - C)*(I*c - d)*Sqrt[c + d*Tan[e + f*x]])/((a + I*b)*f*Sqrt
[a + b*Tan[e + f*x]]) + (2*b*(A - I*B - C)*(I*c + d)*Sqrt[c + d*Tan[e + f*
x]])/((a - I*b)*f*Sqrt[a + b*Tan[e + f*x]]) + (6*c*C*Sqrt[c + d*Tan[e + f*
x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a
*b*d)/(b*c - a*d))))^(3/2)*(1 - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*
Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b
*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)
/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((
b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(Sqrt[b/((b^2*
c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))]*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[(b*
(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c
- a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))) + (4*B*d*Sqrt[c + d*
Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c -
 a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*(1 - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b
]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c ...
```

3.138.3 Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2\int \frac{\sqrt{c+d\tan(e+fx)}\left(\left(3Ca^2-2bBa+2Ab^2+b^2C\right)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-3ad)+Ab(ac+3bd)\right)}{2\sqrt{a+b\tan(e+fx)}}dx}{b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(\left(3Ca^2-2bBa+2Ab^2+b^2C\right)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-3ad)+Ab(ac+3bd)\right)}{\sqrt{a+b\tan(e+fx)}}dx}{b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(\left(3Ca^2-2bBa+2Ab^2+b^2C\right)d\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-3ad)+Ab(ac+3bd)\right)}{\sqrt{a+b\tan(e+fx)}}dx}{b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 4130}$$

$$\frac{\frac{\int -\frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-2c((bB-aC)(bc-3ad)+Ab(ac+3bd))b-\left(a^2+b^2\right)d(3bcC-3adC+2bBd)\tan\cdots}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b}\cdots}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int\frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\cdots}{\cdots}}{\cdots}}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int\frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\cdots}{\cdots}}{\cdots}}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 4138

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int\frac{-2\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2\cdots}{\sqrt{a+b\cdots}}}{\cdots}}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 2348

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+$$

$$\frac{\frac{d\left(3a^2C-2abB+2Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{\int\left(\frac{\left(a^2+b^2\right)d(-3bcC+3adC-2bBd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-2Ac^2b^3+2Ad^2b^3-2Cd^2b^3+2c^2Cb^3+4Bcdb\cdots}{\cdots}\right)\cdots}{\cdots}}{\cdots}$$

$\downarrow$ 2009

3.138. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+$$

$$\frac{\frac{d(3a^2C-2abB+2Ab^2+b^2C)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{bf}-\frac{-\frac{2\sqrt{d}\left(a^2+b^2\right)(-3aCd+2bBd+3bcC)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}}+\frac{2b^2(-b+i}{}}{}}{b\left(a^2+b^2\right)}$$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*S
qrt[a + b*Tan[e + f*x]]) + (-1/2*((2*(I*a - b)*b^2*(A - I*B - C)*(c - I*d)
^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqr
t[c + d*Tan[e + f*x]])])/Sqrt[a - I*b] - (2*(a - I*b)*b^2*(I*A - B - I*C)*
(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] - (2*(a^2 + b^2)*Sqrt[d]*(
3*b*c*C + 2*b*B*d - 3*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[b])/(b*f) + ((2*A*b^2 - 2*a*b*B +
3*a^2*C + b^2*C)*d*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f
))/(b*(a^2 + b^2))
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.138.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{(a+b\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

output
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103394 vs. $2(315) = 630$.

Time = 289.27 (sec) , antiderivative size = 206814, normalized size of antiderivative = 541.40

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.138.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(}\right.}{(a+b\tan{(e+}}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(3/2), x)
```

3.138.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c + d\tan(e + fx))^{3/2} (A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.138.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c + d\tan(e + fx))^{3/2} (A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c + d\tan(e + fx))^{3/2} (A + B\tan(e + fx) + C\tan^2(e + fx))}{(a + b\tan(e + fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(3/2),x)
```

output
```
\text{Hanged}
```

3.139 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

3.139.1 Optimal result

Integrand size = 49, antiderivative size = 402

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = -\frac{(iA+B-iC)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f} - \frac{(B-i(A-C))(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f} + \frac{2Cd^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{5/2}f} - \frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3C)d))\sqrt{c+d\tan(e+fx)}}{b^2(a^2+b^2)^2f\sqrt{a+b\tan(e+fx)}} - \frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{3b(a^2+b^2)f(a+b\tan(e+fx))^{3/2}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f-(B-I*(A-C))*(c+I*d)^(3
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(5/2)/f+2*C*d^(3/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^
(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(5/2)/f-2*(a^4*C*d+b^4*(A*d+B*c)+2
*a*b^3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*d))*(c+d*tan(f*x+e))^(1/2)/b^2/(
a^2+b^2)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e
))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.76 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.29

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = \frac{(B+i(A-C))(c+d\tan(e+fx))^{3/2}}{3(a-ib)f(a+b\tan(e+fx))^{3/2}} - \frac{(iA-B-iC)(c+d\tan(e+fx))^{3/2}}{3(a+ib)f(a+b\tan(e+fx))^{3/2}} - \frac{2C(bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)\sqrt{c+d\tan(e+fx)}}{3b^2f(a+b\tan(e+fx))^{3/2}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}} + \frac{(A-iB-C)(ic+d)\left(\frac{\sqrt{-c+id}\,\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}}+\frac{\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)}{(a-ib)f} + \frac{(A+iB-C)(ic-d)\left(\frac{\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}}-\frac{\sqrt{c+d\tan(e+fx)}}{(a+ib)\sqrt{a+b\tan(e+fx)}}\right)}{(a+ib)f}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(5/2),x]
```

output
```
((B + I*(A - C))*(c + d*Tan[e + f*x])^(3/2))/(3*(a - I*b)*f*(a + b*Tan[e +
 f*x])^(3/2)) - ((I*A - B - I*C)*(c + d*Tan[e + f*x])^(3/2))/(3*(a + I*b)*
f*(a + b*Tan[e + f*x])^(3/2)) - (2*C*(b*c - a*d)*Hypergeometric2F1[-3/2, -
3/2, -1/2, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*Sqrt[c + d*Tan[e + f*x
]])/(3*b^2*f*(a + b*Tan[e + f*x])^(3/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c
 - a*d)]) + ((A - I*B - C)*(I*c + d)*((Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I
*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/
(-a + I*b)^(3/2) + Sqrt[c + d*Tan[e + f*x]]/((a - I*b)*Sqrt[a + b*Tan[e +
f*x]])))/((a - I*b)*f) + ((A + I*B - C)*(I*c - d)*((Sqrt[c + I*d]*ArcTanh[
(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e +
 f*x]])])/(a + I*b)^(3/2) - Sqrt[c + d*Tan[e + f*x]]/((a + I*b)*Sqrt[a + b
*Tan[e + f*x]])))/((a + I*b)*f)
```

3.139.3 Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.204$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{3\sqrt{c+d\tan(e+fx)}\left((a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{2(a+b\tan(e+fx))^{3/2}}dx}{3b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^{3/2}}dx}{b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left((a^2+b^2)Cd\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^{3/2}}dx}{b(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{-\left(\left(\left((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd))\right)\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b+\left(a^2+b^2\right)^2Cd^2}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b(a^2+b^2)} \qquad b\,(}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\int \frac{-\left(\left(\left((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd))\right)\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b+\left(a^2+b^2\right)^2Cd^2\tan}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b(a^2+b^2)} \qquad b\,(a}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{-\left(\left(\left((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd))\right)\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b+\left(a^2+b^2\right)^2Cd^2\tan}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{b(a^2+b^2)} \qquad b\,(a}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4138

$$\frac{\int \frac{-\left(\left(\left((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd))\right)\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b+\left(a^2+b^2\right)^2Cd^2\tan}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)}}{bf(a^2+b^2)}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 2348

$$\frac{\int\left(\frac{\left(a^2+b^2\right)^2Cd^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{Bc^2b^4-Bd^2b^4+2Acdb^4-2cCdb^4+2aAc^2b^3-2aAd^2b^3+2aCd^2b^3-2ac^2Cb^3-4aBcdb^3-a^2Bc^2b^2+a^2Bd^2b^2-2a^2Acdb^2+2}{2(i-\tan(}\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{3b\,(a^2+b^2)\,f(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 2009

3.139. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\left(c+d\tan(e+fx)\right)^{3/2}}{3bf\left(a^2+b^2\right)\left(a+b\tan(e+fx)\right)^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{\frac{2Cd^{3/2}\left(a^2+b^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}}-\frac{b^2(a+\cdots}{\cdots}}{\cdots}}{b\left(a^2+b^2\right)}$$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(3*b*(a^2 + b^2)*f
*(a + b*Tan[e + f*x])^(3/2)) + ((-(((a + I*b)^2*b^2*(I*A + B - I*C)*(c - I
*d)^(3/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*
Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I*b]) - ((a - I*b)^2*b^2*(B - I*(A -
C))*(c + I*d)^(3/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] + (2*(a^2 + b^2)^2*C*d
^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[
e + f*x]])])/Sqrt[b])/(b*(a^2 + b^2)*f) - (2*(a^4*C*d + b^4*(B*c + A*d) +
2*a*b^3*(A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*Sqrt[c + d*Tan[e
+ f*x]])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]]))/(b*(a^2 + b^2))
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.139. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.139.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c + d\tan(fx + e))^{\frac{3}{2}} \left(A + B\tan(fx + e) + C\tan(fx + e)^2\right)}{(a + b\tan(fx + e))^{\frac{5}{2}}}\,dx$$

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

output
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

3.139.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.139.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\int \frac{(c+d\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(\right.}{(a+b\tan{(e+}}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(5/2), x)
```

3.139.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.139.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.140 $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

3.140.1 Optimal result

Integrand size = 49, antiderivative size = 586

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{(iA+B-iC)(c-id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}$$
$$-\frac{(B-i(A-C))(c+id)^{3/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{7/2}f}$$
$$-\frac{2(2a^3bBd+3a^4Cd+b^4(5Bc+3Ad)+2ab^3(5Ac-5cC-4Bd)-a^2b^2(5Bc+7Ad-13Cd))\sqrt{c+d\tan}}{15b^2(a^2+b^2)^2 f(a+b\tan(e+fx))^{3/2}}$$
$$-\frac{2(2a^5bBd^2+3a^6Cd^2+a^4b^2d(10Bc+(8A+C)d)+a^2b^4(45Ac^2-45c^2C-90Bcd-49Ad^2+58Cd^2)-}{15b^2(a^2}$$
$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{3/2}}{5b(a^2+b^2)f(a+b\tan(e+fx))^{5/2}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(3/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f-(B-I*(A-C))*(c+I*d)^(3
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(7/2)/f-2/15*(2*a^5*b*B*d^2+3*a^6*C*d^2+a^4*b^2*d*(10
*B*c+(8*A+C)*d)+a^2*b^4*(45*A*c^2-49*A*d^2-90*B*c*d-45*C*c^2+58*C*d^2)-a^3
*b^3*(50*c*(A-C)*d+B*(15*c^2-39*d^2))+a*b^5*(70*c*(A-C)*d+B*(45*c^2-23*d^2
))+b^6*(5*c*(4*B*d+3*C*c)-3*A*(5*c^2-d^2)))*(c+d*tan(f*x+e))^(1/2)/b^2/(a^
2+b^2)^3/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/15*(2*a^3*b*B*d+3*a^4*C*d+b
^4*(3*A*d+5*B*c)+2*a*b^3*(5*A*c-4*B*d-5*C*c)-a^2*b^2*(7*A*d+5*B*c-13*C*d))
*(c+d*tan(f*x+e))^(1/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))^(3/2)-2/5*(A*b^
2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(3/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)
```

3.140.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3134 vs. $2(586) = 1172$.

Time = 9.50 (sec) , antiderivative size = 3134, normalized size of antiderivative = 5.35

$$\int \frac{(c+d\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(7/2),x]
```

output
```
-((C*(c + d*Tan[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x])^(5/2))) - (-1/4
*((3*b*c*C - 2*b*B*d - 3*a*C*d)*Sqrt[c + d*Tan[e + f*x]])/(b*f*(a + b*Tan[
e + f*x])^(5/2)) - ((-2*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C
- B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d)
 + (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*
(c^2 - d^2))))*Sqrt[c + d*Tan[e + f*x]])/(5*(a^2 + b^2)*(b*c - a*d)*f*(a +
 b*Tan[e + f*x])^(5/2)) - (2*((-2*(b^2*(((2*b^2*d - (5*a*(b*c - a*d))/2)*(
8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C - B*d) - 5*b^2*c*(c*C + 2*B*d))
)/4 + ((-5*b*c)/2 + (a*d)/2)*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) + (b*c -
a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c^2 - d^2
)))) - a*((5*b*(b*c - a*d)*((b*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C
 - B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - (b*(8*b^2*d*(B*c + (A - C)*d) + (b*c
 - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d)))/4 - 2*a*b^2*(2*c*(A - C)*d + B*(c^
2 - d^2))))/2 - 2*a*d*((b^2*(8*A*b^2*c^2 + 3*a^2*C*d^2 - 2*a*b*d*(3*c*C -
B*d) - 5*b^2*c*(c*C + 2*B*d)))/4 - a*(-1/4*(a*(8*b^2*d*(B*c + (A - C)*d) +
 (b*c - a*d)*(3*b*c*C - 2*b*B*d - 3*a*C*d))) + 2*b^3*(2*c*(A - C)*d + B*(c
^2 - d^2))))))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c - a*d)*f*(a +
 b*Tan[e + f*x])^(3/2)) - (2*((-15*b^2*(b*c - a*d)^2*(((3*a^2*A*b*c^2 - A*
b^3*c^2 - a^3*B*c^2 + 3*a*b^2*B*c^2 - 3*a^2*b*c^2*C + b^3*c^2*C - 2*a^3*A*
c*d + 6*a*A*b^2*c*d - 6*a^2*b*B*c*d + 2*b^3*B*c*d + 2*a^3*c*C*d - 6*a*b...
```

3.140.3 Rubi [A] (verified)

Time = 4.67 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$2\int \frac{\sqrt{c+d\tan(e+fx)}\left(-\left(\left(-3Ca^2-2bBa+2Ab^2-5b^2C\right)d\tan^2(e+fx)\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-3ad)+Ab(5a\right.}{2(a+b\tan(e+fx))^{5/2}}$$

$$5b\left(a^2+b^2\right)$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 27

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-\left(\left(-3Ca^2-2bBa+2Ab^2-5b^2C\right)d\tan^2(e+fx)\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-3ad)+Ab(5ac+\right.}{(a+b\tan(e+fx))^{5/2}}$$

$$5b\left(a^2+b^2\right)$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-\left(\left(-3Ca^2-2bBa+2Ab^2-5b^2C\right)d\tan(e+fx)^2\right)-5b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(5bc-3ad)+Ab(5ac+\right.}{(a+b\tan(e+fx))^{5/2}}$$

$$5b\left(a^2+b^2\right)$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4128

$$2\int \frac{-15((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(3ac+bd)((bB-aC)(5bc-3ad)+Ab(5ac+3bd))b+d\left(3Cda^4\right.}{2(a+b\tan(}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 27

$$\int \frac{-15((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(3ac+bd)((bB-aC)(5bc-3ad)+Ab(5ac+3bd))b+d\left(3Cda^4+2\right.}{(a+b\tan(e+}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

3.140. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$\int \frac{-15((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(3ac+bd)((bB-aC)(5bc-3ad)+Ab(5ac+3bd))b+d\Big(3Cda^4+2 \ldots}{(a+b\tan(e+ \ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4132

$$-\frac{2\int \frac{15\left(b^2(bc-ad)\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2(\ldots}{2\sqrt{a+b\tan(e+fx \ldots}}}{\left(a^2+b^2\right) \ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 27

$$-\frac{15\int \frac{b^2(bc-ad)\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2(bc \ldots}{\sqrt{a+b\tan(e+fx)} \ldots}}{\left(a^2+b^2\right)(\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{15\int \frac{b^2(bc-ad)\left(\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a^3-3b\left(2c(A-C)d+B\left(c^2-d^2\right)\right)a^2-3b^2\left(Cc^2+2Bdc-Cd^2-A\left(c^2-d^2\right)\right)a+b^3\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\right)-b^2(bc \ldots}{\sqrt{a+b\tan(e+fx)} \ldots}}{\left(a^2+b^2\right)(\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4099

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc)\right)}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^6Cd^2+2a^5bBd^2 \ldots}{\ldots}}{\ldots}$$

$\downarrow$ 3042

3.140. $\displaystyle\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc))}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^6Cd^2+2a^5bBd^2\right.}{}}{}$$

$\downarrow$ 4098

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc))}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^6Cd^2+2a^5bBd^2\right.}{}}{}$$

$\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc))}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^6Cd^2+2a^5bBd^2\right.}{}}{}$$

$\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{3/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}(3a^4Cd+2a^3bBd-a^2b^2(7Ad+5Bc-13Cd)+2ab^3(5Ac-4Bd-5cC)+b^4(3Ad+5Bc))}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(3a^6Cd^2+2a^5bBd^2\right.}{}}{}$$

input
```
Int[((c + d*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(7/2),x]
```

3.140. $\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(3/2))/(5*b*(a^2 + b^2)*f
*(a + b*Tan[e + f*x])^(5/2)) + ((-2*(2*a^3*b*B*d + 3*a^4*C*d + b^4*(5*B*c
+ 3*A*d) + 2*a*b^3*(5*A*c - 5*c*C - 4*B*d) - a^2*b^2*(5*B*c + 7*A*d - 13*C
*d))*Sqrt[c + d*Tan[e + f*x]])/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/
2)) + ((-15*((I*(a + I*b)^3*b^2*(A - I*B - C)*(c - I*d)^(3/2)*(b*c - a*d)*
ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d
*Tan[e + f*x]])])/(Sqrt[a - I*b]*f) - (I*(a - I*b)^3*b^2*(A + I*B - C)*(c
+ I*d)^(3/2)*(b*c - a*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/
(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*f)))/((a^2 + b^2
)*(b*c - a*d)) - (2*(2*a^5*b*B*d^2 + 3*a^6*C*d^2 + a^4*b^2*d*(10*B*c + (8*
A + C)*d) + a^2*b^4*(45*A*c^2 - 45*c^2*C - 90*B*c*d - 49*A*d^2 + 58*C*d^2)
 - a^3*b^3*(50*c*(A - C)*d + B*(15*c^2 - 39*d^2)) + a*b^5*(70*c*(A - C)*d
+ B*(45*c^2 - 23*d^2)) + b^6*(5*c*(3*c*C + 4*B*d) - 3*A*(5*c^2 - d^2)))*Sq
rt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]
]))/(3*b*(a^2 + b^2)))/(5*b*(a^2 + b^2))
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.140. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.140. $\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

3.140.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan (fx+e))^{\frac{3}{2}} \left(A+B\tan (fx+e)+C\tan (fx+e)^{2}\right)}{(a+b\tan (fx+e))^{\frac{7}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

output
```
int((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

3.140.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.140.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \int \frac{(c+d\tan (e+fx))^{\frac{3}{2}} (A+B\tan (}{(a+b\tan (e+}$$

input
```
integrate((c+d*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(7/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(7/2), x)
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assum
e?` for mo
```

3.140.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

3.141 $\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

3.141.1 Optimal result

Integrand size = 49, antiderivative size = 697

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx =$$
$$-\frac{\sqrt{a-ib}(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$+\frac{\sqrt{a+ib}(iA-B-iC)(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$
$$-\frac{(5a^4Cd^4-4a^3bd^3(5cC+2Bd)+2a^2b^2d^2(15c^2C+20Bcd+8(A-C)d^2)-4ab^3d(5c^3C+30Bc^2d+40c(\ldots}{\ldots}$$
$$+\frac{(64b^2d^2(Abc+aBc-bcC+aAd-bBd-aCd)+(bc-ad)\left(48b(Ab+aB-bC)d^2-5(bc-ad)(bcC-8\ldots\right.}{64b^3df}$$
$$+\frac{(48b(Ab+aB-bC)d^2-5(bc-ad)(bcC-8bBd-aCd))\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{96b^2df}$$
$$-\frac{(bcC-8bBd-aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{24bdf}$$
$$+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}$$

output
```
-1/64*(5*a^4*C*d^4-4*a^3*b*d^3*(2*B*d+5*C*c)+2*a^2*b^2*d^2*(15*c^2*C+20*B*
c*d+8*(A-C)*d^2)-4*a*b^3*d*(5*c^3*C+30*B*c^2*d+40*c*(A-C)*d^2-16*B*d^3)+b^
4*(5*c^4*C-40*B*c^3*d-240*c^2*(A-C)*d^2+320*B*c*d^3+128*(A-C)*d^4))*arctan
h(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/d
^(3/2)/f-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^
(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/f+(I*A-B-I*C)*(c
+I*d)^(5/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+
d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/f+1/64*(64*b^2*d^2*(A*a*d+A*b*c+B*a*c-B
*b*d-C*a*d-C*b*c)+(-a*d+b*c)*(48*b*(A*b+B*a-C*b)*d^2-5*(-a*d+b*c)*(-8*B*b*
d-C*a*d+C*b*c)))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/d/f+1/9
6*(48*b*(A*b+B*a-C*b)*d^2-5*(-a*d+b*c)*(-8*B*b*d-C*a*d+C*b*c))*(a+b*tan(f*
x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b^2/d/f-1/24*(-8*B*b*d-C*a*d+C*b*c)*(a+
b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/b/d/f+1/4*C*(a+b*tan(f*x+e))^(1
/2)*(c+d*tan(f*x+e))^(7/2)/d/f
```

3.141.2 Mathematica [A] (verified)

Time = 9.88 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.81

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}$$

$$+\frac{\frac{(-bcC+8bBd+aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{6bf}+\frac{\left(48b(Ab+aB-bC)d^2-5(bc-ad)(bcC-8bBd-aCd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{8bf}+}{}$$

input
```
Integrate[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e
 + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(7/2))/(4*d*f) + (((-(b*c
*C) + 8*b*B*d + a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)
)/(6*b*f) + (((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d
 - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(8*b*f) +
(((24*b^2*d^2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) - (3*(-(b*c)
 + a*d)*(48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d - a*C
*d)))/8)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f) + ((-24*
b^3*d*(Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*
c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) - b*(A*(b*c^3 +
3*a*c^2*d - 3*b*c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3)
 + a*(B*c^3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3)))*ArcTanh[(Sqrt[-c + (Sqrt[-b
^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[
e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (24*b^3
*d*(Sqrt[-b^2]*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - a*(A*c^3
 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3)) + b*(A*(b*c^3 + 3*a
*c^2*d - 3*b*c*d^2 - a*d^3) - b*(c^3*C + 3*B*c^2*d - 3*c*C*d^2 - B*d^3) +
a*(B*c^3 - 3*c^2*C*d - 3*B*c*d^2 + C*d^3)))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*
d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f
*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (3*Sqrt[b]*Sqr
t[c - (a*d)/b]*Sqrt[(c/(c - (a*d)/b) - (a*d)/(b*(c - (a*d)/b)))^(-1)]*S...
```

3.141.3 Rubi [A] (verified)

Time = 5.85 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.327$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{(c+d\tan(e+fx))^{5/2}((bcC-adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e+fx)+bcC-a(8A-7C)d)}{2\sqrt{a+b\tan(e+fx)}}dx}{4d}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-\frac{\int \frac{(c+d\tan(e+fx))^{5/2}((bcC-adC-8bBd)\tan^2(e+fx)-8(Ab-Cb+aB)d\tan(e+fx)+bcC-a(8A-7C)d)}{\sqrt{a+b\tan(e+fx)}}dx}{8d}$$

$\downarrow$ 3042

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-\frac{\int \frac{(c+d\tan(e+fx))^{5/2}((bcC-adC-8bBd)\tan(e+fx)^2-8(Ab-Cb+aB)d\tan(e+fx)+bcC-a(8A-7C)d)}{\sqrt{a+b\tan(e+fx)}}dx}{8d}$$

$\downarrow$ 4130

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-\frac{\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(c(5cC+8Bd)b^2-2ad(24Ac-19Cc-20Bd)b-48d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+5a^2Cd^2-\left(48b(Ab-Cb+aB)d^2-5(bc-ad)(b\right.}{2\sqrt{a+b\tan(e+fx)}}}{3b}}{8d}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-\frac{\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(c(5cC+8Bd)b^2-2ad(24Ac-19Cc-20Bd)b-48d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+5a^2Cd^2-\left(48b(Ab-Cb+aB)d^2-5(bc-ad)(b\right.}{\sqrt{a+b\tan(e+fx)}}}{6b}}{8d}$$

$\downarrow$ 3042

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-\frac{\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(c(5cC+8Bd)b^2-2ad(24Ac-19Cc-20Bd)b-48d(Abc+aBc-bCc+aAd-bBd-aCd)\tan(e+fx)b+5a^2Cd^2-\left(48b(Ab-Cb+aB)d^2-5(bc-ad)(b\right.}{\sqrt{a+b\tan(e+fx)}}}{6b}}{8d}$$

$\downarrow$ 4130

3.141.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$\int -\frac{3\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2+24Bdc+16(A-C)d^2\right)b^3+ad\left(64Ac^2-49Cc^2-96Bdc-48Ad^2+48Cd^2\right)b^2+64d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2\right.\right.\right.}{}$

$$\Big\downarrow \textbf{27}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$-\frac{3\int \frac{\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2+24Bdc+16(A-C)d^2\right)b^3+ad\left(64Ac^2-49Cc^2-96Bdc-48Ad^2+48Cd^2\right)b^2+64d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2\right.\right.\right.}{}}{}$

$$\Big\downarrow \textbf{3042}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$-\frac{3\int \frac{\sqrt{c+d\tan(e+fx)}\left(-c\left(5Cc^2+24Bdc+16(A-C)d^2\right)b^3+ad\left(64Ac^2-49Cc^2-96Bdc-48Ad^2+48Cd^2\right)b^2+64d\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)-b\left(Cc^2\right.\right.\right.}{}}{}$

$$\Big\downarrow \textbf{4130}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$-\frac{3\left(\int -\frac{c\left(5Cc^3+88Bdc^2+144(A-C)d^2c-64Bd^3\right)b^4+4ad\left(27Cc^3+66Bdc^2-56Cd^2c-16Bd^3-8A\left(4c^3-7cd^2\right)\right)b^3-128d\left(A\left(bc^3+3adc^2-3bd^2c-ad^3\right)-b\left(Cc^3+3B\right.\right.\right.}{}\right.}{}$

$$\Big\downarrow \textbf{27}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$-\frac{3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64b^2d^2(aAd+aBc-aCd+Abc-bBd-bcC)+(bc-ad)\left(48bd^2(aB+Ab-bC)-5(bc-ad)(-aCd-8bBd+bcC)\right)\right)}{bf}-\frac{\int \frac{c\left(5Cc^3+8\right.}{}}{}\right.}{}$

$$\Big\downarrow \textbf{3042}$$

3.141.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$$-\frac{3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64b^2d^2(aAd+aBc-aCd+Abc-bBd-bcC)+(bc-ad)\left(48bd^2(aB+Ab-bC)-5(bc-ad)(-aCd-8bBd+bcC)\right)\right)}{bf}-\frac{\int\frac{c\left(5Cc^3+8\right.}{}}{}\right.}{}$$

$$\Big\downarrow \; 4138$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$$-\frac{3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(64b^2d^2(aAd+aBc-aCd+Abc-bBd-bcC)+(bc-ad)\left(48bd^2(aB+Ab-bC)-5(bc-ad)(-aCd-8bBd+bcC)\right)\right)}{bf}-\frac{\int\frac{c\left(5Cc^3+8\right.}{}}{}\right.}{}$$

$$\Big\downarrow \; 2348$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$$\frac{(bcC-adC-8bBd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}+\frac{-\frac{\left(48b(Ab-Cb+aB)d^2-5(bc-ad)(bcC-adC-8bBd)\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-}{}$$

$$\Big\downarrow \; 2009$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{7/2}}{4df}-$$

$$\frac{(-aCd-8bBd+bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}+\frac{-\frac{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}\left(48bd^2(aB+Ab-bC)-5(bc-ad)(-aCd-8bBd+bcC)\right)}{2bf}}{}$$

input
```
Int[Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

3.141.

$$\int\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(7/2))/(4*d*f) - (((b*c*C
 - 8*b*B*d - a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(
3*b*f) + (-1/2*((48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B
*d - a*C*d))*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(b*f) -
(3*(-1/2*(128*Sqrt[a - I*b]*b^3*(B + I*(A - C))*(c - I*d)^(5/2)*d*ArcTanh[
(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e +
 f*x]])] - 128*Sqrt[a + I*b]*b^3*(I*A - B - I*C)*(c + I*d)^(5/2)*d*ArcTanh
[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e
+ f*x]])] + (2*(5*a^4*C*d^4 - 4*a^3*b*d^3*(5*c*C + 2*B*d) + 2*a^2*b^2*d^2*
(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - 4*a*b^3*d*(5*c^3*C + 30*B*c^2*d +
40*c*(A - C)*d^2 - 16*B*d^3) + b^4*(5*c^4*C - 40*B*c^3*d - 240*c^2*(A - C)
*d^2 + 320*B*c*d^3 + 128*(A - C)*d^4))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(b*f) + ((
64*b^2*d^2*(A*b*c + a*B*c - b*c*C + a*A*d - b*B*d - a*C*d) + (b*c - a*d)*(
48*b*(A*b + a*B - b*C)*d^2 - 5*(b*c - a*d)*(b*c*C - 8*b*B*d - a*C*d)))*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f)))/(4*b))/(6*b))/(8*d
)
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.141.4 Maple [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*
x+e)^2),x)
```

3.141.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="fricas")
```

output
```
Timed out
```

3.141.6 Sympy [F]

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+
C*tan(f*x+e)**2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*(c + d*tan(e + f*x))**(5/2)*(A + B*tan(e
 + f*x) + C*tan(e + f*x)**2), x)
```

3.141.7 Maxima [F]

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\int \left(C\tan(fx+e)^2+B\tan(fx+e)+A\right)\sqrt{b\tan(fx+e)+a}(d\tan(fx+e)+c)^{\frac{5}{2}}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)
*(d*tan(f*x + e) + c)^(5/2), x)
```

3.141.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*
tan(f*x+e)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx = \int \sqrt{a+b\tan\left(e+f\,x\right)}\,\left(c+d\tan\left(e+f\,x\right)\right)^{5/2}\,\left(C\tan\left(e+f\,x\right)^2+B\tan\left(e+f\,x\right)+A\right)$$

input
```
int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^(1/2)*(c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f
*x) + C*tan(e + f*x)^2), x)
```

3.142 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

3.142.1 Optimal result

Integrand size = 49, antiderivative size = 505

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx =$$

$$-\frac{(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}f}$$
$$-\frac{(B-i(A-C))(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}f}$$
$$-\frac{(5a^3Cd^3-3a^2bd^2(5cC+2Bd)+ab^2d(15c^2C+20Bcd+8(A-C)d^2)-b^3(5c^3C+30Bc^2d+40c(A-C}{8b^{7/2}\sqrt{d}f}$$
$$+\frac{(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd-5aCd))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{8b^3f}$$
$$+\frac{(5bcC+6bBd-5aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{12b^2f}$$
$$+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)-(B-I*(A-C))*(c+I*d)^(5
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/f/(a+I*b)^(1/2)-1/8*(5*a^3*C*d^3-3*a^2*b*d^2*(2*B*d+5*C*c)+a*
b^2*d*(15*c^2*C+20*B*c*d+8*(A-C)*d^2)-b^3*(5*c^3*C+30*B*c^2*d+40*c*(A-C)*d
^2-16*B*d^3))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+
e))^(1/2))/b^(7/2)/f/d^(1/2)+1/8*(8*b^2*d*(B*c+(A-C)*d)+(-a*d+b*c)*(6*B*b*
d-5*C*a*d+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/f+1/
12*(6*B*b*d-5*C*a*d+5*C*b*c)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)
/b^2/f+1/3*C*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(5/2)/b/f
```

3.142.2 Mathematica [A] (verified)

Time = 9.04 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.54

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx=\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+ }{3bf}$$

$$+\frac{\frac{(5bcC+6bBd-5aCd)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{4bf}+\frac{\frac{3\left(8b^2d(Bc+(A-C)d)+(bc-ad)(5bcC+6bBd-5aCd)\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4bf}+}{}}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/Sqrt[a + b*Tan[e + f*x]],x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*b*f) + (((5*b*c
*C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2
))/(4*b*f) + ((3*(8*b^2*d*(B*c + (A - C)*d) + (b*c - a*d)*(5*b*c*C + 6*b*B
*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*b*f)
+ ((6*b^3*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) + Sqrt[-b^2]*(A
*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTanh[(Sqrt[-
c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqr
t[c + d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b
]) - (6*b^3*(b*(A - C)*d*(3*c^2 - d^2) + b*B*(c^3 - 3*c*d^2) - Sqrt[-b^2]*
(A*c^3 - c^3*C - 3*B*c^2*d - 3*A*c*d^2 + 3*c*C*d^2 + B*d^3))*ArcTanh[(Sqrt
[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqr
t[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b])
 - (3*Sqrt[b]*Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d)
 + a*b^2*d*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C + 30*B*c^2
*d + 40*c*(A - C)*d^2 - 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)
])/(4*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b^2*f))/(2*b))/(3*b)
```

3.142.3 Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{a+b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5bcC-5adC+6bBd)\tan^2(e+fx)+6b(Bc+(A-C)d)\tan(e+fx)+6Abc-C(bc+5ad)\right)}{2\sqrt{a+b\tan(e+fx)}}dx}{3b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5bcC-5adC+6bBd)\tan^2(e+fx)+6b(Bc+(A-C)d)\tan(e+fx)+6Abc-bcC-5aCd\right)}{\sqrt{a+b\tan(e+fx)}}dx}{6b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5bcC-5adC+6bBd)\tan(e+fx)^2+6b(Bc+(A-C)d)\tan(e+fx)+6Abc-bcC-5aCd\right)}{\sqrt{a+b\tan(e+fx)}}dx}{6b}+\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$\Big\downarrow$ 4130

$$\frac{\frac{\int \frac{3\sqrt{c+d\tan(e+fx)}\left(8Ac^2b^2-c(3cC+2Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(5cC+3Bd)b+5a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(5bcC-5adC+6\right.\right.}{2\sqrt{a+b\tan(e+fx)}}}{2b}}{6b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{\sqrt{c+d\tan(e+fx)}\left(8Ac^2b^2-c(3cC+2Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(5cC+3Bd)b+5a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(5bcC-5adC+\right.\right.}{\sqrt{a+b\tan(e+fx)}}}{4b}}{6b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3\int \frac{\sqrt{c+d\tan(e+fx)}\left(8Ac^2b^2-c(3cC+2Bd)b^2+8\left(2c(A-C)d+B\left(c^2-d^2\right)\right)\tan(e+fx)b^2-2ad(5cC+3Bd)b+5a^2Cd^2+\left(8d(Bc+(A-C)d)b^2+(bc-ad)(5bcC-5adC+\right.\right.}{\sqrt{a+b\tan(e+fx)}}}{4b}}{6b}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$\Big\downarrow$ 4130

$$3\left(\frac{\int \frac{16\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+2c\left(8Ac^2b^2-c(3cC+2Bd)b^2-2ad(5cC+3Bd)b+5a^2Cd^2\right)b+\left(16d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(\right.\right.}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+f}}}{b}\right.$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

3.142. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

$$\Big\downarrow \; 27$$

$$3\left(\frac{\int \frac{16\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+2c\left(8Ac^2b^2-c(3cC+2Bd)b^2-2ad(5cC+3Bd)b+5a^2Cd^2\right)b+\left(16d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(\ldots\right.\right.}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+f\ldots}}}{2b}\ldots\right.$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$$\Big\downarrow \; 3042$$

$$3\left(\frac{\int \frac{16\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+2c\left(8Ac^2b^2-c(3cC+2Bd)b^2-2ad(5cC+3Bd)b+5a^2Cd^2\right)b+\left(16d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(\ldots\right.\right.}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+f\ldots}}}{2b}\ldots\right.$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$$\Big\downarrow \; 4138$$

$$3\left(\frac{\int \frac{16\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)\tan(e+fx)b^3+2c\left(8Ac^2b^2-c(3cC+2Bd)b^2-2ad(5cC+3Bd)b+5a^2Cd^2\right)b+\left(16d\left(2c(A-C)d+B\left(c^2-d^2\right)\right)b^3+(bc-ad)\left(\ldots\right.\right.}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan^2(\ldots\right.}}{2bf}\ldots\right.$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}$$

$$\Big\downarrow \; 2348$$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}+$$

$$\frac{(-5aCd+6bBd+5bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+\frac{3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-5aCd+6bBd+5bcC)+8b^2d(d(A-C)+Bc\ldots\right.}{bf}\ldots\right.}{\ldots}$$

$$\Big\downarrow \; 2009$$

3.142. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx$

$$\frac{C\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}{3bf}+$$

$$\frac{(-5aCd+6bBd+5bcC)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}+\frac{3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left((bc-ad)(-5aCd+6bBd+5bcC)+8b^2d(d(A-C)+Bc\right.}{bf}\right.}{}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/S
qrt[a + b*Tan[e + f*x]],x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(5/2))/(3*b*f) + (((5*b*c
*C + 6*b*B*d - 5*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2
))/(2*b*f) + (3*(((-16*b^3*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c
 - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])
])/Sqrt[a - I*b] - (16*b^3*(B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c
 + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])
])/Sqrt[a + I*b] - (2*(5*a^3*C*d^3 - 3*a^2*b*d^2*(5*c*C + 2*B*d) + a*b^2*d
*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C + 30*B*c^2*d + 40*c*
(A - C)*d^2 - 16*B*d^3))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(2*b*f) + ((8*b^2*d*(B*c
 + (A - C)*d) + (b*c - a*d)*(5*b*c*C + 6*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[
e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f)))/(4*b))/(6*b)
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

3.142. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.142.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

output
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(1/2),x)
```

3.142.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.142.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \int \frac{(c+d\tan{(e+fx)})^{\frac{5}{2}}\left(A+B\tan{(\ldots}\right.}{\sqrt{a+b\tan{(e\ldots}}}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/sqrt(a + b*tan(e + f*x)), x)
```

3.142.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{a+b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.142.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{a+b\tan(e+fx)}}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(1/2),x)
```

output
```
\text{Hanged}
```

3.143 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

3.143.1 Optimal result

Integrand size = 49, antiderivative size = 535

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}f}$$
$$-\frac{(B-i(A-C))(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}f}$$
$$+\frac{\sqrt{d}(15a^2Cd^2-6abd(5cC+2Bd)+b^2(15c^2C+20Bcd+8(A-C)d^2))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4b^{7/2}f}$$
$$-\frac{d(15a^3Cd-8Ab^2(bc-ad)-3a^2b(5cC+4Bd)-b^3(7cC+4Bd)+ab^2(8Bc+7Cd))\sqrt{a+b\tan(e+fx)}}{4b^3(a^2+b^2)f}$$
$$+\frac{(4Ab^2-4abB+5a^2C+b^2C)\,d\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2b^2(a^2+b^2)f}$$
$$-\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{b(a^2+b^2)f\sqrt{a+b\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f-(B-I*(A-C))*(c+I*d)^(5
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(3/2)/f+1/4*(15*a^2*C*d^2-6*a*b*d*(2*B*d+5*C*c)+b^2*(
15*c^2*C+20*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(
1/2)/(c+d*tan(f*x+e))^(1/2))*d^(1/2)/b^(7/2)/f-1/4*d*(15*a^3*C*d-8*A*b^2*(
-a*d+b*c)-3*a^2*b*(4*B*d+5*C*c)-b^3*(4*B*d+7*C*c)+a*b^2*(8*B*c+7*C*d))*(a+
b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)/f+1/2*(4*A*b^2-4*
B*a*b+5*C*a^2+C*b^2)*d*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(3/2)/b^2/(
a^2+b^2)/f-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b
*tan(f*x+e))^(1/2)
```

3.143.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1774 vs. 2(535) = 1070.

Time = 8.66 (sec) , antiderivative size = 1774, normalized size of antiderivative = 3.32

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(3/2),x]
```

output
```
(C*(c + d*Tan[e + f*x])^(5/2))/(2*b*f*Sqrt[a + b*Tan[e + f*x]]) + (((5*b*c
*C + 4*b*B*d - 5*a*C*d)*(c + d*Tan[e + f*x])^(3/2))/(2*b*f*Sqrt[a + b*Tan[
e + f*x]]) + ((8*b^2*(I*A + B - I*C)*(-c + I*d)^(5/2)*ArcTanh[(Sqrt[-c + I
*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/
((-a + I*b)^(3/2)*f) - (8*b^2*(B - I*(A - C))*(c + I*d)^(5/2)*ArcTanh[(Sqr
t[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x
]])])/((a + I*b)^(3/2)*f) + (8*b^2*(I*A + B - I*C)*(c - I*d)^2*Sqrt[c + d*
Tan[e + f*x]])/((a - I*b)*f*Sqrt[a + b*Tan[e + f*x]]) + (8*b^2*(A + I*B -
C)*(c + I*d)^2*Sqrt[c + d*Tan[e + f*x]])/((I*a - b)*f*Sqrt[a + b*Tan[e + f
*x]]) + (30*a^2*C*d^2*Sqrt[c + d*Tan[e + f*x]]*(1 + (b*d*(a + b*Tan[e + f*
x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^(3/2)*(1 -
 (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a
*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d)
 - (a*b*d)/(b*c - a*d)))]))))/(b*Sqrt[b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
 - a*d))]*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c -
a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) -
 (a*b*d)/(b*c - a*d))))) - (12*a*d*(5*c*C + 2*B*d)*Sqrt[c + d*Tan[e + f*x]
]*(1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (...
```

3.143.3 Rubi [A] (verified)

Time = 4.48 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-4bBa+4Ab^2+b^2C)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-5ad)+Ab(ac+5bd)\right)}{2\sqrt{a+b\tan(e+fx)}}}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-4bBa+4Ab^2+b^2C)d\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-5ad)+Ab(ac+5bd)\right)}{\sqrt{a+b\tan(e+fx)}}d}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((5Ca^2-4bBa+4Ab^2+b^2C)d\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-5ad)+Ab(ac+5bd)\right)}{\sqrt{a+b\tan(e+fx)}}d}{b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\big\downarrow$ 4130

$$\frac{\int -\frac{\sqrt{c+d\tan(e+fx)}\left(-4\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-Cd^2\right)\right)\tan(e+fx)b^2-4c((bB-aC)(bc-5ad)+Ab(ac+5bd))b+d\left(15Cda^3-3b(5\right.\right.}{2\sqrt{a+b\tan(e+fx)}}}{2b}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\big\downarrow$ 27

$$\frac{d(5a^2C-4abB+4Ab^2+b^2C)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(-4\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-\right.\right.\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\big\downarrow$ 3042

3.143. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

$$\frac{d\left(5a^2C-4abB+4Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\int\frac{\sqrt{c+d\tan(e+fx)}\left(-4\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)+aB\left(c^2-d^2\right)+b\left(Cc^2+2Bdc-\right.\right.\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 4130

$$\frac{d\left(5a^2C-4abB+4Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\int-\frac{15Cd^3a^4-6bd^2(5cC+2Bd)a^3+b^2d\left(15Cc^2+20Bdc+(8A+7C)d^2\right)a^2-2b^3\left(4Cc^3\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{d\left(5a^2C-4abB+4Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(15a^3Cd-3a^2b(4Bd+5cC)-8Ab^2(bc-ad)+ab^2\right.}{bf}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{d\left(5a^2C-4abB+4Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(15a^3Cd-3a^2b(4Bd+5cC)-8Ab^2(bc-ad)+ab^2\right.}{bf}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 4138

$$\frac{d\left(5a^2C-4abB+4Ab^2+b^2C\right)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(15a^3Cd-3a^2b(4Bd+5cC)-8Ab^2(bc-ad)+ab^2\right.}{bf}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}$$

$\downarrow$ 2348

3.143. $\int\frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

$$\frac{\frac{(5Ca^2-4bBa+4Ab^2+b^2C)d\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{d\left(15Cda^3-3b(5cC+4Bd)a^2+b^2(8Bc+7Cd)a-8Ab^2(bc-ad)-b^3(7cC+4Bd)\right)\sqrt{a+b}}{bf}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{b\left(a^2+b^2\right)f\sqrt{a+b\tan(e+fx)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{bf\left(a^2+b^2\right)\sqrt{a+b\tan(e+fx)}}+$$

$$\frac{\frac{d(5a^2C-4abB+4Ab^2+b^2C)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}{2bf}-\frac{d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(15a^3Cd-3a^2b(4Bd+5cC)-8Ab^2(bc-ad)+ab^2(\right.}{bf}}{}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(b*(a^2 + b^2)*f*S
qrt[a + b*Tan[e + f*x]]) + (((4*A*b^2 - 4*a*b*B + 5*a^2*C + b^2*C)*d*Sqrt[
a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])^(3/2))/(2*b*f) - (-1/2*((-8*(a +
I*b)*b^3*(I*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I*b] +
 (8*b^3*(I*a + b)*(A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a +
 I*b] + (2*(a^2 + b^2)*Sqrt[d]*(15*a^2*C*d^2 - 6*a*b*d*(5*c*C + 2*B*d) + b
^2*(15*c^2*C + 20*B*c*d + 8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[b])/(b*f) + (d*(15*a^
3*C*d - 8*A*b^2*(b*c - a*d) - 3*a^2*b*(5*c*C + 4*B*d) - b^3*(7*c*C + 4*B*d
) + a*b^2*(8*B*c + 7*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x
]])/(b*f))/(4*b))/(b*(a^2 + b^2))
```

3.143. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx$

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.143. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.143.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{(a+b\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

output
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

3.143.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.143.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \int \frac{(c+d\tan\left(e+fx\right))^{\frac{5}{2}}\left(A+B\tan\left(\right.\right.}{(a+b\tan\left(e+\right.}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(3/2), x)
```

3.143.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.143.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(3/2),x)
```

output
```
\text{Hanged}
```

3.144 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

3.144.1 Optimal result

Integrand size = 49, antiderivative size = 545

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{(iA+B-iC)(c-id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}f}$$
$$-\frac{(B-i(A-C))(c+id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}f}$$
$$+\frac{d^{3/2}(5bcC+2bBd-5aCd)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{7/2}f}$$
$$-\frac{d(2a^3bBd-5a^4Cd-2ab^3(2Ac-2cC-3Bd)+2a^2b^2(Bc-5Cd)-b^4(2Bc+(4A+C)d))\sqrt{a+b\tan(e+fx)}\ldots}{b^3(a^2+b^2)^2f}$$
$$+\frac{2(2a^3bBd-5a^4Cd-b^4(3Bc+5Ad)-2ab^3(3Ac-3cC-4Bd)+a^2b^2(3Bc+(A-11C)d))(c+d\tan(e\ldots}{3b^2(a^2+b^2)^2f\sqrt{a+b\tan(e+fx)}}$$
$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{3b(a^2+b^2)f(a+b\tan(e+fx))^{3/2}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f-(B-I*(A-C))*(c+I*d)^(5
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(5/2)/f+d^(3/2)*(2*B*b*d-5*C*a*d+5*C*b*c)*arctanh(d^(
1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/f-d*(2
*a^3*b*B*d-5*a^4*C*d-2*a*b^3*(2*A*c-3*B*d-2*C*c)+2*a^2*b^2*(B*c-5*C*d)-b^4
*(2*B*c+(4*A+C)*d))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2
+b^2)^2/f+2/3*(2*a^3*b*B*d-5*a^4*C*d-b^4*(5*A*d+3*B*c)-2*a*b^3*(3*A*c-4*B*
d-3*C*c)+a^2*b^2*(3*B*c+(A-11*C)*d))*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^2)^
2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/2)/
b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(3/2)
```

3.144.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.63 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.47

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\frac{C(c+d\tan(e+fx))^{5/2}}{bf(a+b\tan(e+fx))^{3/2}}$$

$$+\frac{-\frac{2b(A-iB-C)(c-id)(c+d\tan(e+fx))^{3/2}}{3(ia+b)f(a+b\tan(e+fx))^{3/2}}+\frac{2b(A+iB-C)(c+id)(c+d\tan(e+fx))^{3/2}}{3(ia-b)f(a+b\tan(e+fx))^{3/2}}-\frac{10cC(bc-ad)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{d(\cdots}{}\right.}{3bf(a+b\tan(e+fx))^{3/2}\sqrt{\frac{b(c+\cdots}{}}}}{}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(5/2),x]
```

output
```
(C*(c + d*Tan[e + f*x])^(5/2))/(b*f*(a + b*Tan[e + f*x])^(3/2)) + ((-2*b*(
A - I*B - C)*(c - I*d)*(c + d*Tan[e + f*x])^(3/2))/(3*(I*a + b)*f*(a + b*T
an[e + f*x])^(3/2)) + (2*b*(A + I*B - C)*(c + I*d)*(c + d*Tan[e + f*x])^(3
/2))/(3*(I*a - b)*f*(a + b*Tan[e + f*x])^(3/2)) - (10*c*C*(b*c - a*d)*Hype
rgeometric2F1[-3/2, -3/2, -1/2, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*S
qrt[c + d*Tan[e + f*x]])/(3*b*f*(a + b*Tan[e + f*x])^(3/2)*Sqrt[(b*(c + d*
Tan[e + f*x]))/(b*c - a*d)]) - (4*B*d*(b*c - a*d)*Hypergeometric2F1[-3/2,
-3/2, -1/2, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*Sqrt[c + d*Tan[e + f*
x]])/(3*b*f*(a + b*Tan[e + f*x])^(3/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c
- a*d)]) + (10*a*C*d*(b*c - a*d)*Hypergeometric2F1[-3/2, -3/2, -1/2, -((d*
(a + b*Tan[e + f*x]))/(b*c - a*d))]*Sqrt[c + d*Tan[e + f*x]])/(3*b^2*f*(a
+ b*Tan[e + f*x])^(3/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]) + (2*b
*(I*A + B - I*C)*(c - I*d)^2*((Sqrt[-c + I*d]*ArcTanh[(Sqrt[-c + I*d]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-a + I*
b)^(3/2) + Sqrt[c + d*Tan[e + f*x]]/((a - I*b)*Sqrt[a + b*Tan[e + f*x]])))
/((a - I*b)*f) - (2*b*(A + I*B - C)*(c + I*d)^2*((Sqrt[c + I*d]*ArcTanh[(S
qrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f
*x]])])/(a + I*b)^(3/2) - Sqrt[c + d*Tan[e + f*x]]/((a + I*b)*Sqrt[a + b*T
an[e + f*x]])))/((I*a - b)*f))/(2*b)
```

3.144.3 Rubi [A] (verified)

Time = 5.18 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{(c+d\tan(e+fx))^{3/2}\left(\left(5Ca^2-2bBa+2Ab^2+3b^2C\right)d\tan^2(e+fx)-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-5ad)+Ab(3ac+5}{2(a+b\tan(e+fx))^{3/2}}}{3b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(\left(5Ca^2-2bBa+2Ab^2+3b^2C\right)d\tan^2(e+fx)-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-5ad)+Ab(3ac+5b}{(a+b\tan(e+fx))^{3/2}}}{3b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left(\left(5Ca^2-2bBa+2Ab^2+3b^2C\right)d\tan(e+fx)^2-3b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(3bc-5ad)+Ab(3ac+5b}{(a+b\tan(e+fx))^{3/2}}}{3b\left(a^2+b^2\right)}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 4128

$$2\int \frac{\sqrt{c+d\tan(e+fx)}\left(-3((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(ac+3bd)((bB-aC)(3bc-5ad)+Ab(3ac+}{}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$\int \frac{\sqrt{c+d\tan(e+fx)}\left(-3((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(ac+3bd)((bB-aC)(3bc-5ad)+Ab(3ac+5b}{}$$
$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

3.144. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(-3((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(ac+3bd)((bB-aC)(3bc-5ad)+Ab(3ac+5b}{}$$

$$\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4130

$$\int -\frac{3(5Cd^3a^5-bd^2(5cC+2Bd)a^4+10b^2Cd^3a^3-2b^3(Ac^3-Cc^3-3Bdc^2-3Ad^2c+8Cd^2c+3Bd^3)a^2-b^4(4Ad(3c^2-d^2)-Cd(12c^2+d^2)+4B(c^3-3cd^2))a-(a^2+b^2}{}$$

$$\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{3\int \frac{5Cd^3a^5-bd^2(5cC+2Bd)a^4+10b^2Cd^3a^3-2b^3(Ac^3-Cc^3-3Bdc^2-3Ad^2c+8Cd^2c+3Bd^3)a^2-b^4(4Ad(3c^2-d^2)-Cd(12c^2+d^2)+4B(c^3-3cd^2))a-(a^2+b^2}{}}{}$$

$$\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{3\int \frac{5Cd^3a^5-bd^2(5cC+2Bd)a^4+10b^2Cd^3a^3-2b^3(Ac^3-Cc^3-3Bdc^2-3Ad^2c+8Cd^2c+3Bd^3)a^2-b^4(4Ad(3c^2-d^2)-Cd(12c^2+d^2)+4B(c^3-3cd^2))a-(a^2+b^2}{}}{}$$

$$\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4138

$$-\frac{3\int \frac{5Cd^3a^5-bd^2(5cC+2Bd)a^4+10b^2Cd^3a^3-2b^3(Ac^3-Cc^3-3Bdc^2-3Ad^2c+8Cd^2c+3Bd^3)a^2-b^4(4Ad(3c^2-d^2)-Cd(12c^2+d^2)+4B(c^3-3cd^2))a-(a^2+b^2}{}}{}$$

$$\frac{2(Ab^2-a(bB-aC))\,(c+d\tan(e+fx))^{5/2}}{3bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 2348

3.144. $$\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$$

$$\frac{\frac{2(-5Cda^4+2bBda^3+b^2(3Bc+(A-11C)d)a^2-2b^3(3Ac-3Cc-4Bd)a-b^4(3Bc+5Ad))(c+d\tan(e+fx))^{3/2}}{b(a^2+b^2)f\sqrt{a+b\tan(e+fx)}}+\frac{-\frac{3d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{3bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2(c+d\tan(e+fx))^{3/2}(-5a^4Cd+2a^3bBd+a^2b^2(d(A-11C)+3Bc)-2ab^3(3Ac-4Bd-3cC)-b^4(5Ad+3Bc))}{bf(a^2+b^2)\sqrt{a+b\tan(e+fx)}}+\frac{-\frac{3d\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{}}{}}{}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(3*b*(a^2 + b^2)*f
*(a + b*Tan[e + f*x])^(3/2)) + ((2*(2*a^3*b*B*d - 5*a^4*C*d - b^4*(3*B*c +
 5*A*d) - 2*a*b^3*(3*A*c - 3*c*C - 4*B*d) + a^2*b^2*(3*B*c + (A - 11*C)*d)
)*(c + d*Tan[e + f*x])^(3/2))/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]]) +
 ((-3*((2*(a + I*b)^2*b^3*(B + I*(A - C))*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c
- I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])]
)/Sqrt[a - I*b] - (2*(a - I*b)^2*b^3*(I*A - B - I*C)*(c + I*d)^(5/2)*ArcTa
nh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[
e + f*x]])])/Sqrt[a + I*b] - (2*(a^2 + b^2)^2*d^(3/2)*(5*b*c*C + 2*b*B*d -
 5*a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*T
an[e + f*x]])])/Sqrt[b]))/(2*b*f) - (3*d*(2*a^3*b*B*d - 5*a^4*C*d - 2*a*b^
3*(2*A*c - 2*c*C - 3*B*d) + 2*a^2*b^2*(B*c - 5*C*d) - b^4*(2*B*c + (4*A +
C)*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(b*f))/(b*(a^2 +
 b^2)))/(3*b*(a^2 + b^2))
```

3.144. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

3.144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.144. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{5/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.144.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{(a+b\tan(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

output
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

3.144.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.144.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = \int \frac{(c+d\tan\left(e+fx\right))^{\frac{5}{2}}\left(A+B\tan\left(\right.\right.}{(a+b\tan\left(e+\right.}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(5/2), x)
```

3.144.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.144.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(5/2),x)
```

output
```
\text{Hanged}
```

3.145 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx$

3.145.1 Optimal result

Integrand size = 49, antiderivative size = 590

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx =$$

$$-\frac{(iA+B-iC)(c-id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{7/2}f}$$

$$-\frac{(B-i(A-C))(c+id)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{7/2}f}$$

$$+\frac{2Cd^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{b^{7/2}f}$$

$$-\frac{2(a^6Cd^2+3a^4b^2Cd^2-3a^2b^4(c^2C+2Bcd-2Cd^2-A(c^2-d^2))+b^6(c(cC+2Bd)-A(c^2-d^2))-a^3b^3}{b^3\left(a^2+b^2\right)^3 f\sqrt{a+b\tan(e}}$$

$$-\frac{2(a^4Cd+b^4(Bc+Ad)+2ab^3(Ac-cC-Bd)-a^2b^2(Bc+(A-3C)d))\,(c+d\tan(e+fx))^{3/2}}{3b^2\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))^{3/2}}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(7/2)/f-(B-I*(A-C))*(c+I*d)^(5
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(7/2)/f+2*C*d^(5/2)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^
(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/b^(7/2)/f-2*(a^6*C*d^2+3*a^4*b^2*C*d
^2-3*a^2*b^4*(c^2*C+2*B*c*d-2*C*d^2-A*(c^2-d^2))+b^6*(c*(2*B*d+C*c)-A*(c^2
-d^2))-a^3*b^3*(2*c*(A-C)*d+B*(c^2-d^2))+3*a*b^5*(2*c*(A-C)*d+B*(c^2-d^2))
)*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^3/f/(a+b*tan(f*x+e))^(1/2)-2/3*(a^4
*C*d+b^4*(A*d+B*c)+2*a*b^3*(A*c-B*d-C*c)-a^2*b^2*(B*c+(A-3*C)*d))*(c+d*tan
(f*x+e))^(3/2)/b^2/(a^2+b^2)^2/f/(a+b*tan(f*x+e))^(3/2)-2/5*(A*b^2-a*(B*b-
C*a))*(c+d*tan(f*x+e))^(5/2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(5/2)
```

3.145.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.14 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.09

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx = \frac{(B+i(A-C))(c+d\tan(e+fx))^{5/2}}{5(a-ib)f(a+b\tan(e+fx))^{5/2}}$$
$$-\frac{(iA-B-iC)(c+d\tan(e+fx))^{5/2}}{5(a+ib)f(a+b\tan(e+fx))^{5/2}}$$
$$-\frac{2C(bc-ad)^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{2},-\frac{3}{2},-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)\sqrt{c+d\tan(e+fx)}}{5b^3f(a+b\tan(e+fx))^{5/2}\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}}$$
$$+\frac{(A-iB-C)(ic+d)\left(\frac{(c+d\tan(e+fx))^{3/2}}{(a-ib)(a+b\tan(e+fx))^{3/2}}+\frac{3(c-id)\left(\frac{\sqrt{-c+id}\,\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-a+ib)^{3/2}}+\frac{\sqrt{c+d\tan(e+fx)}}{(a-ib)\sqrt{a+b\tan(e+fx)}}\right)}{a-ib}\right)}{3(a-ib)f}$$
$$-\frac{(A+iB-C)(ic-d)\left(\frac{(c+d\tan(e+fx))^{3/2}}{(a+ib)(a+b\tan(e+fx))^{3/2}}-\frac{3(c+id)\left(\frac{\sqrt{c+id}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}}-\frac{\sqrt{c+d\tan(e+fx)}}{(a+ib)\sqrt{a+b\tan(e+fx)}}\right)}{a+ib}\right)}{3(a+ib)f}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(7/2),x]
```

output
```
((B + I*(A - C))*(c + d*Tan[e + f*x])^(5/2))/(5*(a - I*b)*f*(a + b*Tan[e +
 f*x])^(5/2)) - ((I*A - B - I*C)*(c + d*Tan[e + f*x])^(5/2))/(5*(a + I*b)*
f*(a + b*Tan[e + f*x])^(5/2)) - (2*C*(b*c - a*d)^2*Hypergeometric2F1[-5/2,
 -5/2, -3/2, -((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*Sqrt[c + d*Tan[e + f
*x]])/(5*b^3*f*(a + b*Tan[e + f*x])^(5/2)*Sqrt[(b*(c + d*Tan[e + f*x]))/(b
*c - a*d)]) + ((A - I*B - C)*(I*c + d)*((c + d*Tan[e + f*x])^(3/2)/((a - I
*b)*(a + b*Tan[e + f*x])^(3/2)) + (3*(c - I*d)*((Sqrt[-c + I*d]*ArcTanh[(S
qrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e +
 f*x]])])/(-a + I*b)^(3/2) + Sqrt[c + d*Tan[e + f*x]]/((a - I*b)*Sqrt[a +
b*Tan[e + f*x]])))/(a - I*b)))/(3*(a - I*b)*f) - ((A + I*B - C)*(I*c - d)*
((c + d*Tan[e + f*x])^(3/2)/((a + I*b)*(a + b*Tan[e + f*x])^(3/2)) - (3*(c
 + I*d)*((Sqrt[c + I*d]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(a + I*b)^(3/2) - Sqrt[c + d*Tan
[e + f*x]]/((a + I*b)*Sqrt[a + b*Tan[e + f*x]])))/(a + I*b)))/(3*(a + I*b)
*f)
```

3.145.3 Rubi [A] (verified)

Time = 5.26 (sec) , antiderivative size = 658, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4128, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{5(c+d\tan(e+fx))^{3/2}\left(\left(a^2+b^2\right)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{2(a+b\tan(e+fx))^{5/2}}dx}{5b\left(a^2+b^2\right)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((a^2+b^2)Cd\tan^2(e+fx)-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^{5/2}}dx}{b\,(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}\left((a^2+b^2)Cd\tan(e+fx)^2-b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(bc-ad)+Ab(ac+bd)\right)}{(a+b\tan(e+fx))^{5/2}}dx}{b\,(a^2+b^2)}-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4128

$$\frac{2\int \frac{3\sqrt{c+d\tan(e+fx)}\left(-\left(((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd\right.}{2(a+b\tan(e+fx))^{3/2}}}{3b(a^2+b^2)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(-\left(((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b\right.}{(a+b\tan(e+fx))^{3/2}}}{b(a^2+b^2)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{\sqrt{c+d\tan(e+fx)}\left(-\left(((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2\right)+(ac+bd)((bB-aC)(bc-ad)+Ab(ac+bd))b\right.}{(a+b\tan(e+fx))^{3/2}}}{b(a^2+b^2)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\,(a^2+b^2)\,(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4128

3.145. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$2\int \frac{Cd^3a^6+3b^2Cd^3a^4-b^3\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^3+3b^4\left(Bc^3+3Adc^2-3Cdc^2-3Bd^2c-Ad^3+2Cd^3\right)a^2-3b^5\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3C\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 27

$$\int \frac{Cd^3a^6+3b^2Cd^3a^4-b^3\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^3+3b^4\left(Bc^3+3Adc^2-3Cdc^2-3Bd^2c-Ad^3+2Cd^3\right)a^2-3b^5\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 3042

$$\int \frac{Cd^3a^6+3b^2Cd^3a^4-b^3\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^3+3b^4\left(Bc^3+3Adc^2-3Cdc^2-3Bd^2c-Ad^3+2Cd^3\right)a^2-3b^5\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 4138

$$\int \frac{Cd^3a^6+3b^2Cd^3a^4-b^3\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^3+3b^4\left(Bc^3+3Adc^2-3Cdc^2-3Bd^2c-Ad^3+2Cd^3\right)a^2-3b^5\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

$\downarrow$ 2348

$$\int \left(\frac{\left(a^2+b^2\right)^3Cd^3}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-Ac^3b^6-Bd^3b^6+3Acd^2b^6-3cCd^2b^6+c^3Cb^6+3Bc^2db^6+3aBc^3b^5-3aAd^3b^5+3aCd^3b^5-9aBcd^2b^5+9aAc^2db^5-9ac^2Cdb^5\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{5/2}}$$

3.145. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{5bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{5/2}}+$$

$$\frac{-\frac{2(c+d\tan(e+fx))^{3/2}\left(a^4Cd-a^2b^2(d(A-3C)+Bc)+2ab^3(Ac-Bd-cC)+b^4(Ad+Bc)\right)}{3bf(a^2+b^2)(a+b\tan(e+fx))^{3/2}}+\frac{-\frac{2\sqrt{c+d\tan(e+fx)}\left(a^6Cd^2+3a^4b^2Cd^2-a^3b^3\left(2cd(A-C)\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(7/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*Tan[e + f*x])^(5/2))/(5*b*(a^2 + b^2)*f
*(a + b*Tan[e + f*x])^(5/2)) + ((-2*(a^4*C*d + b^4*(B*c + A*d) + 2*a*b^3*(
A*c - c*C - B*d) - a^2*b^2*(B*c + (A - 3*C)*d))*(c + d*Tan[e + f*x])^(3/2)
)/(3*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^(3/2)) + ((-(((a + I*b)^3*b^3*(I
*A + B - I*C)*(c - I*d)^(5/2)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*
x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a - I*b]) - (b^3*(I*a
 + b)^3*(A + I*B - C)*(c + I*d)^(5/2)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[a + I*b] + (2
*(a^2 + b^2)^3*C*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[b])/(b*(a^2 + b^2)*f) - (2*(a^6*C*d^2
+ 3*a^4*b^2*C*d^2 - 3*a^2*b^4*(c^2*C + 2*B*c*d - 2*C*d^2 - A*(c^2 - d^2))
+ b^6*(c*(c*C + 2*B*d) - A*(c^2 - d^2)) - a^3*b^3*(2*c*(A - C)*d + B*(c^2
- d^2)) + 3*a*b^5*(2*c*(A - C)*d + B*(c^2 - d^2)))*Sqrt[c + d*Tan[e + f*x]
])/(b*(a^2 + b^2)*f*Sqrt[a + b*Tan[e + f*x]]))/(b*(a^2 + b^2)))/(b*(a^2 +
b^2))
```

3.145.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.145. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx$

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.145.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(a+b\tan(fx+e))^{\frac{7}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

output
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(7/2),x)
```

3.145.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.145.6 Sympy [F]

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \int \frac{(c+d\tan{(e+fx)})^{\frac{5}{2}}\,(A+B\tan{(}$$
$$(a+b\tan{(e+}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(7/2),x)
```

output
```
Integral((c + d*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(a + b*tan(e + f*x))**(7/2), x)
```

3.145.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.145.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{7/2}}\,dx=\text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(7/2),x)
```

output
```
\text{Hanged}
```

3.146 $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{9/2}}\,dx$

3.146.1 Optimal result

Integrand size = 49, antiderivative size = 946

$$\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{9/2}}\,dx =$$

$$-\frac{(iA+B-iC)(c-id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{9/2}f}$$

$$-\frac{(B-i(A-C))(c+id)^{5/2}\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{9/2}f}$$

$$-\frac{2(6a^5bBd^2+15a^6Cd^2+a^4b^2d(14Bc+8Ad+37Cd)+3a^2b^4(35Ac^2-35c^2C-70Bcd-39Ad^2+54Cd^2}{}$$

$$-\frac{2(6a^7bBd^3+15a^8Cd^3+2a^6b^2d^2(7Bc+4Ad+26Cd)-2ab^7(210Ac^3-210c^3C-525Bc^2d-406Acd^2+}{}$$

$$-\frac{2(2a^3bBd+5a^4Cd+b^4(7Bc+5Ad)+2ab^3(7Ac-7cC-6Bd)-a^2b^2(7Bc+9Ad-19Cd))(c+d\tan(}{35b^2(a^2+b^2)^2f(a+b\tan(e+fx))^{5/2}}$$

$$-\frac{2(Ab^2-a(bB-aC))(c+d\tan(e+fx))^{5/2}}{7b(a^2+b^2)f(a+b\tan(e+fx))^{7/2}}$$

output
```
-(I*A+B-I*C)*(c-I*d)^(5/2)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(a-I*b)^(9/2)/f-(B-I*(A-C))*(c+I*d)^(5
/2)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(a+I*b)^(9/2)/f-2/105*(6*a^7*b*B*d^3+15*a^8*C*d^3+2*a^6*b^2*d
^2*(4*A*d+7*B*c+26*C*d)-2*a*b^7*(210*A*c^3-406*A*c*d^2-525*B*c^2*d+88*B*d^
3-210*C*c^3+406*C*c*d^2)-a^4*b^4*(525*A*c^2*d-311*A*d^3+105*B*c^3-749*B*c*
d^2-525*C*c^2*d+221*C*d^3)+2*a^2*b^6*(875*A*c^2*d-261*A*d^3+315*B*c^3-812*
B*c*d^2-875*C*c^2*d+291*C*d^3)+2*a^5*b^3*d*(56*c*(A-C)*d+B*(35*c^2-12*d^2)
)-b^8*(5*d*(49*A*c^2-3*A*d^2-49*C*c^2)+7*B*(15*c^3-23*c*d^2))-2*a^3*b^5*(2
10*c^3*C+700*B*c^2*d-798*C*c*d^2-317*B*d^3-42*A*(5*c^3-19*c*d^2)))*(c+d*ta
n(f*x+e))^(1/2)/b^3/(a^2+b^2)^4/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)-2/105*
(6*a^5*b*B*d^2+15*a^6*C*d^2+a^4*b^2*d*(8*A*d+14*B*c+37*C*d)+3*a^2*b^4*(35*
A*c^2-39*A*d^2-70*B*c*d-35*C*c^2+54*C*d^2)-a^3*b^3*(98*c*(A-C)*d+B*(35*c^2
-75*d^2))+a*b^5*(182*c*(A-C)*d+B*(105*c^2-71*d^2))+b^6*(7*c*(8*B*d+5*C*c)-
5*A*(7*c^2-3*d^2)))*(c+d*tan(f*x+e))^(1/2)/b^3/(a^2+b^2)^3/f/(a+b*tan(f*x+
e))^(3/2)-2/35*(2*a^3*b*B*d+5*a^4*C*d+b^4*(5*A*d+7*B*c)+2*a*b^3*(7*A*c-6*B
*d-7*C*c)-a^2*b^2*(9*A*d+7*B*c-19*C*d))*(c+d*tan(f*x+e))^(3/2)/b^2/(a^2+b^
2)^2/f/(a+b*tan(f*x+e))^(5/2)-2/7*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(5/
2)/b/(a^2+b^2)/f/(a+b*tan(f*x+e))^(7/2)
```

3.146.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 10121 vs. 2(946) = 1892.

Time = 56.13 (sec) , antiderivative size = 10121, normalized size of antiderivative = 10.70

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(a + b*Tan[e + f*x])^(9/2),x]
```

output
```
Result too large to show
```

3.146.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(a+b\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{(c+d\tan(e+fx))^{3/2}(-((-5Ca^2-2bBa+2Ab^2-7b^2C)d\tan^2(e+fx))-7b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(7bc-5ad)+Ab(}{2(a+b\tan(e+fx))^{7/2}}}{7b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}(-((-5Ca^2-2bBa+2Ab^2-7b^2C)d\tan^2(e+fx))-7b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(7bc-5ad)+Ab(7c}{(a+b\tan(e+fx))^{7/2}}}{7b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\tan(e+fx))^{3/2}(-((-5Ca^2-2bBa+2Ab^2-7b^2C)d\tan(e+fx)^2)-7b((A-C)(bc-ad)-B(ac+bd))\tan(e+fx)+(bB-aC)(7bc-5ad)+Ab(7c}{(a+b\tan(e+fx))^{7/2}}}{7b\left(a^2+b^2\right)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{\sqrt{c+d\tan(e+fx)}\left(-35((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(5ac+3bd)((bB-aC)(7bc-5ad)+Ab(7ac\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(-35((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(5ac+3bd)((bB-aC)(7bc-5ad)+Ab(7ac+ \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 3042

$$\int \frac{\sqrt{c+d\tan(e+fx)}\,(-35((ac+bd)((A-C)(bc-ad)-B(ac+bd))+(bc-ad)(bBc+b(A-C)d+a(Ac-Cc-Bd)))\tan(e+fx)b^2+(5ac+3bd)((bB-aC)(7bc-5ad)+Ab(7ac+ \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7bf\left(a^2+b^2\right)(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 4128

$$2\int \frac{15Cd^3a^6+6bBd^3a^5+b^2d^2(14Bc+8Ad+37Cd)a^4-b^3(105Cc^3+245Bdc^2-203Cd^2c-75Bd^3-7A(15c^3-29cd^2))a^3+3b^4(35B(3c^3-5cd^2)+d(245Ac^2-245Cc^2- \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 27

$$\int \frac{15Cd^3a^6+6bBd^3a^5+b^2d^2(14Bc+8Ad+37Cd)a^4-b^3(105Cc^3+245Bdc^2-203Cd^2c-75Bd^3-7A(15c^3-29cd^2))a^3+3b^4(35B(3c^3-5cd^2)+d(245Ac^2-245Cc^2- \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 3042

$$\int \frac{15Cd^3a^6+6bBd^3a^5+b^2d^2(14Bc+8Ad+37Cd)a^4-b^3(105Cc^3+245Bdc^2-203Cd^2c-75Bd^3-7A(15c^3-29cd^2))a^3+3b^4(35B(3c^3-5cd^2)+d(245Ac^2-245Cc^2- \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 4132

3.146. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\Big(56c(A-C)d+B\Big(35c^2-12d^2\Big)\Big)a^5-b^4\Big(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\Big(56c(A-C)d+B\Big(35c^2-12d^2\Big)\Big)a^5-b^4\Big(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$105\int -\frac{b^3(bc-ad)\Big(\Big(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\Big(c^3-3cd^2\Big)\Big)a^4-4b\Big((A-C)d\Big(3c^2-d^2\Big)+B\Big(c^3-3cd^2\Big)\Big)a^3+6b^2\Big(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\Big)a^2+4 \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\Big(56c(A-C)d+B\Big(35c^2-12d^2\Big)\Big)a^5-b^4\Big(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad \cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

3.146. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

$$105 \int -\frac{b^3(bc-ad)\left(\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^4-4b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\left(56c(A-C)d+B\left(35c^2-12d^2\right)\right)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\cdots\right.\right.}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$105 \int -\frac{b^3(bc-ad)\left(\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^4-4b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\left(56c(A-C)d+B\left(35c^2-12d^2\right)\right)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\cdots\right.\right.}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

3.146. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

$$105\int -\frac{b^3(bc-ad)\Big(\big(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\big)a^4-4b\big((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\big)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\big(56c(A-C)d+B\left(35c^2-12d^2\right)\big)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$105\int -\frac{b^3(bc-ad)\Big(\big(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\big)a^4-4b\big((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\big)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\big(56c(A-C)d+B\left(35c^2-12d^2\right)\big)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

3.146. $\displaystyle\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

$$105\int -\frac{b^3(bc-ad)\left(\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^4-4b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\left(56c(A-C)d+B\left(35c^2-12d^2\right)\right)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\right.\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$105\int -\frac{b^3(bc-ad)\left(\left(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\left(c^3-3cd^2\right)\right)a^4-4b\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)a^3+6b^2\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+d\tan(e+fx)}\left(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\left(56c(A-C)d+B\left(35c^2-12d^2\right)\right)a^5-b^4\left(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad\right.\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$$\Big\downarrow \; 25$$

3.146. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

$$105\int -\frac{b^3(bc-ad)\Big(\big(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\big(c^3-3cd^2\big)\big)a^4-4b\big((A-C)d\big(3c^2-d^2\big)+B\big(c^3-3cd^2\big)\big)a^3+6b^2\big(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\big)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\big(56c(A-C)d+B\big(35c^2-12d^2\big)\big)a^5-b^4\big(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$105\int -\frac{b^3(bc-ad)\Big(\big(Cc^3+3Bdc^2-3Cd^2c-Bd^3-A\big(c^3-3cd^2\big)\big)a^4-4b\big((A-C)d\big(3c^2-d^2\big)+B\big(c^3-3cd^2\big)\big)a^3+6b^2\big(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\big)a^2+4}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

$\downarrow$ 25

$$-\frac{2\sqrt{c+d\tan(e+fx)}\Big(15Cd^3a^8+6bBd^3a^7+2b^2d^2(7Bc+4Ad+26Cd)a^6+2b^3d\big(56c(A-C)d+B\big(35c^2-12d^2\big)\big)a^5-b^4\big(105Bc^3+525Adc^2-525Cdc^2-749Bd^2c-311Ad}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+d\tan(e+fx))^{5/2}}{7b\left(a^2+b^2\right)f(a+b\tan(e+fx))^{7/2}}$$

input
```
Int[((c + d*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
a + b*Tan[e + f*x])^(9/2),x]
```

output
```
$Aborted
```

3.146. $\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx$

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.146. $\int \frac{(c+d\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(a+b\tan(e+fx))^{9/2}}\,dx$

3.146.4 Maple [F(-1)]

Timed out.

$$\int \frac{(c+d\tan (fx+e))^{\frac{5}{2}} \left(A+B\tan (fx+e)+C\tan (fx+e)^{2}\right)}{(a+b\tan (fx+e))^{\frac{9}{2}}}dx$$

input
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(9/2),x)
```

output
```
int((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e)
)^(9/2),x)
```

3.146.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.146.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx=\text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*ta
n(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assum
e?` for mo
```

3.146.8 Giac [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((c+d*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(
f*x+e))^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+d\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(a+b\tan(e+fx))^{9/2}}\,dx = \text{Hanged}$$

input
```
int(((c + d*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
a + b*tan(e + f*x))^(9/2),x)
```

output
```
\text{Hanged}
```

3.147 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.147.1 Optimal result

Integrand size = 49, antiderivative size = 505

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx =$$

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}f}$$

$$-\frac{(a+ib)^{5/2}(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}f}$$

$$+\frac{(5a^3Cd^3-15a^2bd^2(cC-2Bd)+5ab^2d(3c^2C-4Bcd+8(A-C)d^2)-b^3(5c^3C-6Bc^2d+8c(A-C)d^2}{8\sqrt{b}d^{7/2}f}$$

$$+\frac{(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd-5aCd))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{8d^3f}$$

$$-\frac{(5bcC-6bBd-5aCd)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{12d^2f}$$

$$+\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}$$

output
```
1/8*(5*a^3*C*d^3-15*a^2*b*d^2*(-2*B*d+C*c)+5*a*b^2*d*(3*c^2*C-4*B*c*d+8*(A
-C)*d^2)-b^3*(5*c^3*C-6*B*c^2*d+8*c*(A-C)*d^2+16*B*d^3))*arctanh(d^(1/2)*(
a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(7/2)/f/b^(1/2)-(a
-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*
b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)-(a+I*b)^(5/2)*(B-I*(A-C))
*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e
))^(1/2))/f/(c+I*d)^(1/2)+1/8*(8*b*(A*b+B*a-C*b)*d^2+(-a*d+b*c)*(-6*B*b*d-
5*C*a*d+5*C*b*c))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/f-1/12
*(-6*B*b*d-5*C*a*d+5*C*b*c)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)/
d^2/f+1/3*C*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(5/2)/d/f
```

3.147.2 Mathematica [A] (verified)

Time = 8.84 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e}{3df}$$

$$+\frac{\frac{(-5bcC+6bBd+5aCd)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{4df}+\frac{3\left(8b(Ab+aB-bC)d^2+(bc-ad)(5bcC-6bBd-5aCd)\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4df}+}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) + (((-5*b*
c*C + 6*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x
]])/(4*d*f) + ((3*(8*b*(A*b + a*B - b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6*b*
B*d - 5*a*C*d))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(4*d*f)
 + ((-6*(Sqrt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) -
b*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTanh[(Sqrt[-
c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqr
t[c + d*Tan[e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b
]) - (6*(Sqrt[-b^2]*(3*a^2*b*B - b^3*B - a^3*(A - C) + 3*a*b^2*(A - C)) +
b*(a^3*B - 3*a*b^2*B + 3*a^2*b*(A - C) - b^3*(A - C)))*d^3*ArcTanh[(Sqrt[c
 + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[
c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) +
 (3*Sqrt[b]*Sqrt[c - (a*d)/b]*(5*a^3*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) +
5*a*b^2*d*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d +
 8*c*(A - C)*d^2 + 16*B*d^3))*ArcSinh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(
Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(b*c - a*d)])/(4
*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f))/(2*d))/(3*d)
```

3.147.3 Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{(a+b\tan(e+fx))^{3/2}\left((5bcC-5adC-6bBd)\tan^2(e+fx)-6(Ab-Cb+aB)d\tan(e+fx)+5bcC-a(6A-C)d\right)}{2\sqrt{c+d\tan(e+fx)}}dx}{3d}+\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int\frac{(a+b\tan(e+fx))^{3/2}\left((5bcC-5adC-6bBd)\tan^2(e+fx)-6(Ab-Cb+aB)d\tan(e+fx)+5bcC-a(6A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{6d}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\int\frac{(a+b\tan(e+fx))^{3/2}\left((5bcC-5adC-6bBd)\tan(e+fx)^2-6(Ab-Cb+aB)d\tan(e+fx)+5bcC-a(6A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{6d}$$

$\Big\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\frac{\int-\frac{3\sqrt{a+b\tan(e+fx)}\left(c(5cC-6Bd)b^2-2ad(5cC+Bd)b+a^2(8A-3C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(5bcC-5adC-6bBd)\right)\tan^2(e+fx)+8\left(Ba^2+2b(A-C)a-b^2\right.}{2\sqrt{c+d\tan(e+fx)}}}{2d}}{6d}$$

$\Big\downarrow$ 27

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{3\int\frac{\sqrt{a+b\tan(e+fx)}\left(c(5cC-6Bd)b^2-2ad(5cC+Bd)b+a^2(8A-3C)d^2+\left(8b(Ab-Cb+aB)\right.\right.}{\sqrt{c+d}}}{}}{6d}$$

$\Big\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{3\int\frac{\sqrt{a+b\tan(e+fx)}\left(c(5cC-6Bd)b^2-2ad(5cC+Bd)b+a^2(8A-3C)d^2+\left(8b(Ab-Cb+aB)\right.\right.}{\sqrt{c+d}}}{}}{6d}$$

$\Big\downarrow$ 4130

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{3\left(\int\frac{-c\left(5Cc^2-6Bdc+8(A-C)d^2\right)b^3+ad\left(15Cc^2-20Bdc-8(A-C)d^2\right)b^2-3a^2d^2(5cC+\right.}{}\right.}{}$$

$\Big\downarrow$ 27

3.147. $\displaystyle\int\frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx$

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-$$
$$\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} - 3\left(\int \frac{-c\left(5Cc^2-6Bdc+8(A-C)d^2\right)b^3+ad\left(15Cc^2-20Bdc-8(A-C)d^2\right)b^2-3a^2d^2(5cC+6 \ldots}{\ldots}\right.$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-$$
$$\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} - 3\left(\int \frac{-c\left(5Cc^2-6Bdc+8(A-C)d^2\right)b^3+ad\left(15Cc^2-20Bdc-8(A-C)d^2\right)b^2-3a^2d^2(5cC+6 \ldots}{\ldots}\right.$$

$\downarrow$ 4138

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-$$
$$\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} - 3\left(\int \frac{-c\left(5Cc^2-6Bdc+8(A-C)d^2\right)b^3+ad\left(15Cc^2-20Bdc-8(A-C)d^2\right)b^2-3a^2d^2(5cC+6 \ldots}{\ldots}\right.$$

$\downarrow$ 2348

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-$$
$$\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} - 3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-5aCd-6bBd+5bcC)\ldots\right.}{df}\right.$$

$\downarrow$ 2009

$$\frac{C(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}{3df}-$$
$$\frac{(-5aCd-6bBd+5bcC)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df} - 3\left(\frac{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(8bd^2(aB+Ab-bC)+(bc-ad)(-5aCd-6bBd+5bcC)\ldots\right.}{df}\right.$$

3.147. $\displaystyle\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

input
```
Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/S
qrt[c + d*Tan[e + f*x]],x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2)*Sqrt[c + d*Tan[e + f*x]])/(3*d*f) - (((5*b*c
*C - 6*b*B*d - 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]
])/(2*d*f) - (3*(((-16*(a - I*b)^(5/2)*(I*A + B - I*C)*d^3*ArcTanh[(Sqrt[c
 - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])
])/Sqrt[c - I*d] - (16*(a + I*b)^(5/2)*(B - I*(A - C))*d^3*ArcTanh[(Sqrt[c
 + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])
])/Sqrt[c + I*d] + (2*(5*a^3*C*d^3 - 15*a^2*b*d^2*(c*C - 2*B*d) + 5*a*b^2*
d*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2) - b^3*(5*c^3*C - 6*B*c^2*d + 8*c*(A
- C)*d^2 + 16*B*d^3))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*
Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(2*d*f) + ((8*b*(A*b + a*B
- b*C)*d^2 + (b*c - a*d)*(5*b*c*C - 6*b*B*d - 5*a*C*d))*Sqrt[a + b*Tan[e +
 f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f)))/(4*d))/(6*d)
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.147.4 Maple [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{\sqrt{c+d\tan(fx+e)}}dx$$

input
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

output
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

3.147.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.147.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx=\int \frac{(a+b\tan{(e+fx)})^{\frac{5}{2}}\left(A+B\tan{(\right.}}{\sqrt{c+d\tan{(e}}}$$

input
```
integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/sqrt(c + d*tan(e + f*x)), x)
```

3.147.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.147.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx=\int \frac{(a+b\tan(e+f\,x))^{5/2}\,\left(C\tan(e+\right.}{\sqrt{c+d\tan\left(\right.}}$$

input
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(1/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(1/2), x)
```

3.148 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.148.1 Optimal result

Integrand size = 49, antiderivative size = 383

$$\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(a-ib)^{3/2}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}f} + \frac{(a+ib)^{3/2}(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}f} + \frac{(3a^2Cd^2-6abd(cC-2Bd)+b^2(3c^2C-4Bcd+8(A-C)d^2))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4\sqrt{b}d^{5/2}f} - \frac{(3bcC-4bBd-3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{4d^2f} + \frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}$$

output
```
1/4*(3*a^2*C*d^2-6*a*b*d*(-2*B*d+C*c)+b^2*(3*c^2*C-4*B*c*d+8*(A-C)*d^2))*a
rctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(5
/2)/f/b^(1/2)-(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x
+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f/(c-I*d)^(1/2)+(a+I*b)^(
3/2)*(I*A-B-I*C)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2
)/(c+d*tan(f*x+e))^(1/2))/f/(c+I*d)^(1/2)-1/4*(-4*B*b*d-3*C*a*d+3*C*b*c)*(
a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^2/f+1/2*C*(c+d*tan(f*x+e))^
(1/2)*(a+b*tan(f*x+e))^(3/2)/d/f
```

3.148.2 Mathematica [A] (verified)

Time = 7.76 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.58

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e\cdots}}{2df}$$
$$+\frac{\frac{(-3bcC+4bBd+3aCd)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{2df}+\frac{2\left(b\left(a^2B-b^2B+2ab(A-C)\right)-\sqrt{-b^2}\left(2abB-a^2(A-C)+b^2(A-C)\right)\right)d^2\text{arctanh}\left(\frac{\sqrt{-c+\cdots}}{\sqrt{-a-\cdots}}\right.}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}}{\cdots}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f) + (((-3*b*
c*C + 4*b*B*d + 3*a*C*d)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]
)/(2*d*f) + ((2*(b*(a^2*B - b^2*B + 2*a*b*(A - C)) - Sqrt[-b^2]*(2*a*b*B -
 a^2*(A - C) + b^2*(A - C)))*d^2*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt
[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(
Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - (2*(b*(a^2*B - b^2*B
+ 2*a*b*(A - C)) + Sqrt[-b^2]*(2*a*b*B - a^2*(A - C) + b^2*(A - C)))*d^2*A
rcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqr
t[-b^2]]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[
-b^2]*d)/b]) + (Sqrt[b]*Sqrt[c - (a*d)/b]*(3*a^2*C*d^2 - 6*a*b*d*(c*C - 2*
B*d) + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)*d^2))*ArcSinh[(Sqrt[d]*Sqrt[a +
b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x]
)/(b*c - a*d)])/(2*Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f))/(2*d)
```

3.148.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.204, Rules used = {3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{\sqrt{a+b\tan(e+fx)}\left((3bcC-3adC-4bBd)\tan^2(e+fx)-4(Ab-Cb+aB)d\tan(e+fx)+3bcC-a(4A-C)d\right)}{2\sqrt{c+d\tan(e+fx)}}dx}{2d}+\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left((3bcC-3adC-4bBd)\tan^2(e+fx)-4(Ab-Cb+aB)d\tan(e+fx)+3bcC-a(4A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left((3bcC-3adC-4bBd)\tan(e+fx)^2-4(Ab-Cb+aB)d\tan(e+fx)+3bcC-a(4A-C)d\right)}{\sqrt{c+d\tan(e+fx)}}dx}{4d}$$

$$\Big\downarrow \text{4130}$$

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{\int -\frac{c(3cC-4Bd)b^2-2ad(3cC+2Bd)b+a^2(8A-5C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3adC-4bBd)\right)\tan^2(e+fx)+8\left(Ba^2+2b(A-C)a-b^2B\right)d^2\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{d}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int\frac{c(3cC-4Bd)b^2-2ad(3cC+2Bd)b+a^2(8A-5C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3a\ldots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e\ldots}}{2d}}{4d}}{}$$

$\downarrow$ 3042

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int\frac{c(3cC-4Bd)b^2-2ad(3cC+2Bd)b+a^2(8A-5C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3a\ldots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e\ldots}}{2d}}{4d}$$

$\downarrow$ 4138

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int\frac{c(3cC-4Bd)b^2-2ad(3cC+2Bd)b+a^2(8A-5C)d^2+\left(8b(Ab-Cb+aB)d^2+(bc-ad)(3bcC-3a\ldots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan\ldots\right.}}{2df}}{4d}$$

$\downarrow$ 2348

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int\left(\frac{8Ad^2b^2-8Cd^2b^2+3c^2Cb^2-4Bcdb^2+12aBd^2b-6acCdb+3a^2Cd^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-16aAbd^2-8a^2Bd^2\ldots}{\ldots}\right.}{\ldots}}{\ldots}$$

$\downarrow$ 2009

$$\frac{C(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{(-3aCd-4bBd+3bcC)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\frac{2\left(3a^2Cd^2-6abd(cC-2Bd)+b^2\left(8d^2(A-C)-4Bcd+3c^2C\right)\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx\ldots}}{\sqrt{b}\sqrt{c+d\tan(e+fx\ldots}}\right.}{\sqrt{b}\sqrt{d}}}{\ldots}}{4d}$$

input
```
Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/S
qrt[c + d*Tan[e + f*x]],x]
```

3.148. $\int\frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx$

output
```
(C*(a + b*Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f) - (-1/2*((
-8*(a - I*b)^(3/2)*(I*A + B - I*C)*d^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c - I*d] - (
8*(a + I*b)^(3/2)*(B - I*(A - C))*d^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I*d] + (2
*(3*a^2*C*d^2 - 6*a*b*d*(c*C - 2*B*d) + b^2*(3*c^2*C - 4*B*c*d + 8*(A - C)
*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[
e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(d*f) + ((3*b*c*C - 4*b*B*d - 3*a*C*d)*Sqr
t[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(4*d)
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.148. $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.148.4 Maple [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{\sqrt{c+d\tan(fx+e)}}dx$$

input
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

output
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57789 vs. $2(309) = 618$.

Time = 271.99 (sec) , antiderivative size = 115594, normalized size of antiderivative = 301.81

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.148.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^{\frac{3}{2}}\left(A+B\tan{(\ldots}\right.}{\sqrt{c+d\tan{(e\ldots}}}$$

input
```
integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/sqrt(c + d*tan(e + f*x)), x)
```

3.148.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{\left(C\tan{(fx+e)}^2+B\tan{(fx+e)}\ldots\right.}{\sqrt{d\tan{(fx+\ldots}}}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^(3/
2)/sqrt(d*tan(f*x + e) + c), x)
```

3.148.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^{3/2}\,(C\tan(e+}{\sqrt{c+d\tan(}$$

input
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(1/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(1/2), x)
```

3.149 $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

3.149.1 Optimal result

Integrand size = 49, antiderivative size = 290

$$\begin{aligned}
&\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx\\
&= -\frac{\sqrt{a-ib}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}f}\\
&\quad+\frac{\sqrt{a+ib}(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}f}\\
&\quad-\frac{(bcC-2bBd-aCd)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}d^{3/2}f}\\
&\quad+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}
\end{aligned}$$

output
```
-(-2*B*b*d-C*a*d+C*b*c)*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+
d*tan(f*x+e))^(1/2))/d^(3/2)/f/b^(1/2)-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(
a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/
f/(c-I*d)^(1/2)+(I*A-B-I*C)*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(
a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(a+I*b)^(1/2)/f/(c+I*d)^(1/2)+C*(a+b*
tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d/f
```

3.149.2 Mathematica [A] (verified)

Time = 7.04 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$
$$=\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}$$
$$+\frac{-\frac{\left(\sqrt{-b^2}(bB-a(A-C))-b(Ab+aB-bC)\right)d\,\text{arctanh}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}-\frac{\left(\sqrt{-b^2}(bB-a(A-C))+b(Ab+aB-bC)\right)d\,\text{arct}\cdots}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+\cdots}}\cdots}{bdf}$$

input
```
Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/Sqrt[c + d*Tan[e + f*x]],x]
```

output
```
(C*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f) + (-(((Sqrt[-b
^2]*(b*B - a*(A - C)) - b*(A*b + a*B - b*C))*d*ArcTanh[(Sqrt[-c + (Sqrt[-b
^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[
e + f*x]])])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b])) - ((Sqrt
[-b^2]*(b*B - a*(A - C)) + b*(A*b + a*B - b*C))*d*ArcTanh[(Sqrt[c + (Sqrt[
-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + d*Tan
[e + f*x]])])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (Sqrt[-b^2]*d)/b]) - (Sqrt[b]
*Sqrt[c - (a*d)/b]*(b*c*C - 2*b*B*d - a*C*d)*ArcSinh[(Sqrt[d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*c + b*d*Tan[e + f*x])/(
b*c - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*d*f)
```

3.149.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4130}$$

$$\frac{\int -\frac{(bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx)+bcC-2aAd+aCd}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{d}+\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int \frac{(bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx)+bcC-a(2A-C)d}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int \frac{(bcC-adC-2bBd)\tan(e+fx)^2-2(Ab-Cb+aB)d\tan(e+fx)+bcC-a(2A-C)d}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{2d}$$

$$\Big\downarrow \text{4138}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int \frac{(bcC-adC-2bBd)\tan^2(e+fx)-2(Ab-Cb+aB)d\tan(e+fx)+bcC-a(2A-C)d}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(\tan^2(e+fx)+1)}d\tan(e+fx)}{2df}$$

$$\Big\downarrow \text{2348}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\int \left(\frac{bcC-adC-2bBd}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{2Abd+2aBd-2bCd+i(-2aAd+2bBd+2aCd)}{2(i-\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-2Abd-2aBd+2bCd+i(-2aAd+2bBd+2a}{2(\tan(e+fx)+i)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(}}\right.}{2df}$$

$$\Big\downarrow \text{2009}$$

$$\frac{C\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\frac{2d\sqrt{a-ib}(B+i(A-C))\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}}-\frac{2d\sqrt{a+ib}(iA-B-iC)\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}}+\frac{2(-aCd-2bBd+bC}{}}{2df}$$

3.149. $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}dx$

input
```
Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/Sqr
t[c + d*Tan[e + f*x]],x]
```

output
```
-1/2*((2*Sqrt[a - I*b]*(B + I*(A - C))*d*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c - I*d] -
 (2*Sqrt[a + I*b]*(I*A - B - I*C)*d*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I*d] + (2*(
b*c*C - 2*b*B*d - a*C*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[b]*Sqrt[d]))/(d*f) + (C*Sqrt[a + b*Ta
n[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f)
```

3.149.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.149. $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.149.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(fx+e)}\,\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{\sqrt{c+d\tan(fx+e)}}dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38950 vs. $2(226) = 452$.

Time = 129.20 (sec) , antiderivative size = 77916, normalized size of antiderivative = 268.68

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.149.6 Sympy [F]

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/sqrt(c + d*tan(e + f*x)), x)
```

3.149.7 Maxima [F]

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{\left(C\tan(fx+e)^2+B\tan(fx+e)+A\right)\sqrt{b\tan(fx+e)+a}}{\sqrt{d\tan(fx+e)+c}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*sqrt(b*tan(f*x + e) + a)
/sqrt(d*tan(f*x + e) + c), x)
```

3.149.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{\sqrt{c+d\tan(e+fx)}}\,dx=\text{Hanged}$$

input
```
int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(1/2),x)
```

output
```
\text{Hanged}
```

3.150 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$

3.150.1 Optimal result

Integrand size = 49, antiderivative size = 239

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(B+i(A-C))\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\sqrt{c-id}f} + \frac{(iA-B-iC)\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}f} + \frac{2C\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}f}$$

output
```
-(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/f/(a-I*b)^(1/2)/(c-I*d)^(1/2)+(I*A-B-I*C)*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/f
/(a+I*b)^(1/2)/(c+I*d)^(1/2)+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^
(1/2)/(c+d*tan(f*x+e))^(1/2))/f/b^(1/2)/d^(1/2)
```

3.150.2 Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.51

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\frac{\left(bB+\sqrt{-b^2}(A-C)\right)\operatorname{arctanh}\left(\frac{\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+\sqrt{-b^2}}\sqrt{-c+\frac{\sqrt{-b^2}d}{b}}}-\frac{\left(bB+\sqrt{-b^2}(-A+C)\right)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{\sqrt{-b^2}d}{b}}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+\sqrt{-b^2}}\sqrt{c+\frac{\sqrt{-b^2}d}{b}}}+\frac{2\sqrt{b}C}{}}{bf}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]
]*Sqrt[c + d*Tan[e + f*x]]),x]
```

output
```
(((b*B + Sqrt[-b^2]*(A - C))*ArcTanh[(Sqrt[-c + (Sqrt[-b^2]*d)/b]*Sqrt[a +
 b*Tan[e + f*x]])/(Sqrt[-a + Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt
[-a + Sqrt[-b^2]]*Sqrt[-c + (Sqrt[-b^2]*d)/b]) - ((b*B + Sqrt[-b^2]*(-A +
C))*ArcTanh[(Sqrt[c + (Sqrt[-b^2]*d)/b]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
+ Sqrt[-b^2]]*Sqrt[c + d*Tan[e + f*x]])]])/(Sqrt[a + Sqrt[-b^2]]*Sqrt[c + (
Sqrt[-b^2]*d)/b]) + (2*Sqrt[b]*C*Sqrt[c - (a*d)/b]*ArcSinh[(Sqrt[d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c - (a*d)/b])]*Sqrt[(b*(c + d*Tan[e + f*
x]))/(b*c - a*d)])/(Sqrt[d]*Sqrt[c + d*Tan[e + f*x]]))/(b*f)
```

3.150.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.082$, Rules used = {3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 4138}$$

$$\frac{\int \frac{C\tan^2(e+fx)+B\tan(e+fx)+A}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(\tan^2(e+fx)+1)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(\frac{i(A-C)-B}{2(i-\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{C}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{B+i(A-C)}{2(\tan(e+fx)+i)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(B+i(A-C))\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}\sqrt{c-id}}-\frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}+\frac{2C\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{b}\sqrt{d}}}{f}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*Sqrt
[c + d*Tan[e + f*x]]),x]
```

output
```
(-(((B + I*(A - C))*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d])) - ((B
 - I*(A - C))*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I
*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c + I*d]) + (2*C*ArcTa
nh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])
/(Sqrt[b]*Sqrt[d]))/f
```

3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.150.4 Maple [F(-1)]

Timed out.

$$\int \frac{A+B\tan(fx+e)+C\tan(fx+e)^2}{\sqrt{a+b\tan(fx+e)}\,\sqrt{c+d\tan(fx+e)}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(1/2),x)
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48154 vs. $2(181) = 362$.

Time = 141.31 (sec) , antiderivative size = 96324, normalized size of antiderivative = 403.03

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.150.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*ta
n(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x)
)*sqrt(c + d*tan(e + f*x))), x)
```

3.150.7 Maxima [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \int \frac{C\tan(fx+e)^2+B\tan(fx+e)+A}{\sqrt{b\tan(fx+e)+a}\sqrt{d\tan(fx+e)+c}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(sqrt(b*tan(f*x + e) + a
)*sqrt(d*tan(f*x + e) + c)), x)
```

3.150.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
 + d*tan(e + f*x))^(1/2)),x)
```

output
```
\text{Hanged}
```

3.151 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

3.151.1 Optimal result

Integrand size = 49, antiderivative size = 251

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx = -\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}\sqrt{c-id}f} - \frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}\sqrt{c+id}f} - \frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
a+I*b)^(3/2)/f/(c+I*d)^(1/2)-2*(A*b^2-a*(B*b-C*a))*(c+d*tan(f*x+e))^(1/2)/
(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.05

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\frac{(a+ib)(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}} + \frac{(ia+b)(A+iB-}{}}{(a^2}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3
/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

output
```
(((a + I*b)*(I*A + B - I*C)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c +
I*d]) + ((I*a + b)*(A + I*B - C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c +
 I*d]) + (2*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*Tan[e + f*x]])/((-(b*c)
+ a*d)*Sqrt[a + b*Tan[e + f*x]]))/((a^2 + b^2)*f)
```

3.151.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \; 4132$$

$$-\frac{2\int -\frac{(bB+a(A-C))(bc-ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(a^2+b^2)(bc-ad)} - \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}$$

$$\downarrow 27$$

$$\frac{\int \frac{(bB+a(A-C))(bc-ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(a^2+b^2)(bc-ad)} - \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\int \frac{(bB+a(A-C))(bc-ad)-(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(a^2+b^2)(bc-ad)} - \frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}$$

$$\downarrow 4099$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}} + \frac{\frac{1}{2}(a-ib)(A+iB-C)(bc-ad)\int \frac{1-i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx + \frac{1}{2}(a+ib)(A-iB-C)(bc-ad)\int \frac{i\tan\ldots}{\sqrt{a+b\tan(e+\ldots}}}{(a^2+b^2)(bc-ad)}$$

$$\downarrow 3042$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}} + \frac{\frac{1}{2}(a-ib)(A+iB-C)(bc-ad)\int \frac{1-i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx + \frac{1}{2}(a+ib)(A-iB-C)(bc-ad)\int \frac{i\tan\ldots}{\sqrt{a+b\tan(e+\ldots}}}{(a^2+b^2)(bc-ad)}$$

$$\downarrow 4098$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}} + \frac{\frac{(a+ib)(A-iB-C)(bc-ad)\int \frac{1}{(1-i\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{2f} + \frac{(a-ib)(A+iB-C)(bc-ad)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{a+b\tan(\ldots}}}{2f}}{(a^2+b^2)(bc-ad)}$$

$$\downarrow 104$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}} + \frac{\frac{(a-ib)(A+iB-C)(bc-ad)\int \frac{1}{-ia+b+\frac{(ic-d)(a+b\tan(e+fx))}{c+d\tan(e+fx)}}d\frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{f} + \frac{(a+ib)(A-iB-C)(bc-ad)\int \frac{1}{ia+b-\frac{(ic+d)(a+b\tan(e+fx))}{c+d\tan(e+fx)}}d\frac{\sqrt{a+b\tan\ldots}}{\sqrt{c+d\tan\ldots}}}{f}}{(a^2+b^2)(bc-ad)}$$

$$\downarrow 221$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}} + \frac{\frac{i(a-ib)(A+iB-C)(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}} - \frac{i(a+ib)(A-iB-C)(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}}{(a^2+b^2)(bc-ad)}$$

3.151. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}\,dx$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*Sq
rt[c + d*Tan[e + f*x]]),x]
```

output
```
(((-I)*(a + I*b)*(A - I*B - C)*(b*c - a*d)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a +
 b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b
]*Sqrt[c - I*d]*f) + (I*(a - I*b)*(A + I*B - C)*(b*c - a*d)*ArcTanh[(Sqrt[
c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/(Sqrt[a + I*b]*Sqrt[c + I*d]*f))/((a^2 + b^2)*(b*c - a*d)) - (2*(A*b^2
 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqr
t[a + b*Tan[e + f*x]])
```

3.151.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.151.4 Maple [F(-1)]

Timed out.

$$\int \frac{A + B\tan(fx+e) + C\tan(fx+e)^2}{\sqrt{c + d\tan(fx+e)}\,(a + b\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
)^(3/2),x)
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83974 vs. $2(200) = 400$.

Time = 274.81 (sec) , antiderivative size = 83974, normalized size of antiderivative = 334.56

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}\sqrt{c + d\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.151.6 Sympy [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}\sqrt{c + d\tan(e + fx)}}\,dx = \int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{\frac{3}{2}}\sqrt{c + d\tan(e + fx)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(
3/2)*sqrt(c + d*tan(e + f*x))), x)
```

3.151.7 Maxima [F]

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}\sqrt{c + d\tan(e + fx)}}\,dx = \int \frac{C\tan(fx + e)^2 + B\tan(fx + e) + A}{(b\tan(fx + e) + a)^{\frac{3}{2}}\sqrt{d\tan(fx + e) + c}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/((b*tan(f*x + e) + a)^(3
/2)*sqrt(d*tan(f*x + e) + c)), x)
```

3.151.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}\sqrt{c + d\tan(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}\sqrt{c + d\tan(e + fx)}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
 + d*tan(e + f*x))^(1/2)),x)
```

output
```
\text{Hanged}
```

3.152 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

3.152.1 Optimal result

Integrand size = 49, antiderivative size = 375

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx =$$
$$-\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}\sqrt{c-id}f}$$
$$-\frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}\sqrt{c+id}f}$$
$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}}$$
$$-\frac{2(5a^3bBd-2a^4Cd+b^4(3Bc-2Ad)+ab^3(6Ac-6cC-Bd)-a^2b^2(3Bc+8Ad-4Cd))\sqrt{c+d\tan(e+fx)}}{3(a^2+b^2)^2(bc-ad)^2f\sqrt{a+b\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/f/(c-I*d)^(1/2)-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
a+I*b)^(5/2)/f/(c+I*d)^(1/2)-2/3*(5*a^3*b*B*d-2*a^4*C*d+b^4*(-2*A*d+3*B*c)
+a*b^3*(6*A*c-B*d-6*C*c)-a^2*b^2*(8*A*d+3*B*c-4*C*d))*(c+d*tan(f*x+e))^(1/
2)/(a^2+b^2)^2/(-a*d+b*c)^2/f/(a+b*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(B*b-C*a
))*(c+d*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(3/2)
```

3.152.2 Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.03

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx = \frac{\frac{3(a+ib)^2(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{3i(a-ib)^2(A-}{}}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5
/2)*Sqrt[c + d*Tan[e + f*x]]),x]
```

output
```
((3*(a + I*b)^2*(I*A + B - I*C)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-
c + I*d]) + ((3*I)*(a - I*b)^2*(A + I*B - C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I
*b]*Sqrt[c + I*d]) + (2*(a^2 + b^2)*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*
Tan[e + f*x]])/((-(b*c) + a*d)*(a + b*Tan[e + f*x])^(3/2)) + (2*(-5*a^3*b*
B*d + 2*a^4*C*d + b^4*(-3*B*c + 2*A*d) + a*b^3*(-6*A*c + 6*c*C + B*d) + a^
2*b^2*(3*B*c + 8*A*d - 4*C*d))*Sqrt[c + d*Tan[e + f*x]])/((b*c - a*d)^2*Sq
rt[a + b*Tan[e + f*x]]))/(3*(a^2 + b^2)^2*f)
```

3.152.3 Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.245, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{2\int \frac{2Adb^2+2\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-3aA(bc-ad)-(bB-aC)(3bc-ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\int \frac{2Adb^2+2\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-3aA(bc-ad)-(bB-aC)(3bc-ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\int \frac{2Adb^2+2\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-3aA(bc-ad)-(bB-aC)(3bc-ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 4132

$$-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{2\int\frac{3\left(\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-\ldots\right.}{2\sqrt{a+b\tan\ldots}}}{(a\ldots}}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\int\frac{\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-ad)\ldots}{\sqrt{a+b\tan(e\ldots}}}{(a^2\ldots}}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\int\frac{\left((A-C)a^2+2bBa-b^2(A-C)\right)(bc-ad)\ldots}{\sqrt{a+b\tan(e\ldots}}}{(a^2\ldots}}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}$$

$\downarrow$ 4099

3.152. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\left(\frac{1}{2}(a-ib)^2(A+iB-C)(bc-ad)^2\int\frac{}{\sqrt{a+}}\right.}{}}{3\left(a^2+b^2\right)(bc-ad)}$$

$\downarrow$ 3042

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\left(\frac{1}{2}(a-ib)^2(A+iB-C)(bc-ad)^2\int\frac{}{\sqrt{a+}}\right.}{}}{3\left(a^2+b^2\right)(bc-ad)}$$

$\downarrow$ 4098

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\left(\frac{(a+ib)^2(A-iB-C)(bc-ad)^2\int\frac{}{(1-i\tan(e}}{}\right.}{}}{3\left(a^2+b^2\right)(bc}$$

$\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\left(\frac{(a-ib)^2(A+iB-C)(bc-ad)^2\int\frac{}{-ia+b+}}{}\right.}{}}{3\left(a^2+b^2\right)(bc-ad)}$$

$\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+d\tan(e+fx)}}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}}-\frac{\frac{2\sqrt{c+d\tan(e+fx)}\left(-2a^4Cd+5a^3bBd-a^2b^2(8Ad+3Bc-4Cd)+ab^3(6Ac-Bd-6cC)+b^4(3Bc-2Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}}-\frac{3\left(\frac{i(a-ib)^2(A+iB-C)(bc-ad)^2\text{arctanh}}{f\sqrt{a+ib}\sqrt{c+ic}}\right.}{}}{3\left(a^2+b^2\right)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*Sq
rt[c + d*Tan[e + f*x]]),x]
```

3.152. $\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*Tan[e + f*x]])/(3*(a^2 + b^2)*(b*c
- a*d)*f*(a + b*Tan[e + f*x])^(3/2)) - ((-3*(((-I)*(a + I*b)^2*(A - I*B -
C)*(b*c - a*d)^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
- I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) + (I*(a
 - I*b)^2*(A + I*B - C)*(b*c - a*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Ta
n[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqr
t[c + I*d]*f)))/((a^2 + b^2)*(b*c - a*d)) + (2*(5*a^3*b*B*d - 2*a^4*C*d +
b^4*(3*B*c - 2*A*d) + a*b^3*(6*A*c - 6*c*C - B*d) - a^2*b^2*(3*B*c + 8*A*d
 - 4*C*d))*Sqrt[c + d*Tan[e + f*x]])/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b
*Tan[e + f*x]]))/(3*(a^2 + b^2)*(b*c - a*d))
```

3.152.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

3.152. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx$

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.152.4 Maple [F(-1)]

Timed out.

$$\int \frac{A + B\tan(fx+e) + C\tan(fx+e)^2}{\sqrt{c + d\tan(fx+e)}\,(a + b\tan(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e)
)^(5/2),x)
```

3.152.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.152.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})^{\frac{5}{2}}\sqrt{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*x+e))**(1/2)/(a+b*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(
5/2)*sqrt(c + d*tan(e + f*x))), x)
```

3.152.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.152.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}\sqrt{c+d\tan(e+fx)}}\,dx=\text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(5/2)*(c
 + d*tan(e + f*x))^(1/2)),x)
```

output
```
\text{Hanged}
```

3.153 $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 49, antiderivative size = 528

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}$$

$$-\frac{(a+ib)^{5/2}(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}$$

$$+\frac{\sqrt{b}(15a^2Cd^2-10abd(3cC-2Bd)+b^2(15c^2C-12Bcd+8(A-C)d^2))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{4d^{7/2}f}$$

$$-\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^{5/2}}{d\,(c^2+d^2)\,f\sqrt{c+d\tan(e+fx)}}$$

$$-\frac{b(3(bc-ad)\,(5c^2C-4Bcd+(4A+C)d^2)-4d^2((A-C)(bc-ad)+B(ac+bd)))\,\sqrt{a+b\tan(e+fx)}\sqrt{}}{4d^3\,(c^2+d^2)\,f}$$

$$+\frac{b(5c^2C-4Bcd+(4A+C)d^2)\,(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2d^2\,(c^2+d^2)\,f}$$

output
```
-(a-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f-(a+I*b)^(5/2)*(B-I*(A-
C))*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(c+I*d)^(3/2)/f+1/4*(15*a^2*C*d^2-10*a*b*d*(-2*B*d+3*C*c)+b^2
*(15*c^2*C-12*B*c*d+8*(A-C)*d^2))*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b
^(1/2)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(7/2)/f-1/4*b*(3*(-a*d+b*c)*(5*c^
2*C-4*B*c*d+(4*A+C)*d^2)-4*d^2*((A-C)*(-a*d+b*c)+B*(a*c+b*d)))*(a+b*tan(f*
x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d^2)/f+1/2*b*(5*c^2*C-4*B*c*d+
(4*A+C)*d^2)*(c+d*tan(f*x+e))^(1/2)*(a+b*tan(f*x+e))^(3/2)/d^2/(c^2+d^2)/f
-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(5/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e
))^(1/2)
```

3.153.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2245 vs. $2(528) = 1056$.

Time = 9.61 (sec) , antiderivative size = 2245, normalized size of antiderivative = 4.25

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2))/(2*d*f*Sqrt[c + d*Tan[e + f*x]]) + (((-5*b*
c*C + 4*b*B*d + 5*a*C*d)*(a + b*Tan[e + f*x])^(3/2))/(2*d*f*Sqrt[c + d*Tan
[e + f*x]]) + ((8*(-a + I*b)^(5/2)*(I*A + B - I*C)*d^2*ArcTanh[(Sqrt[-c +
I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])
/((-c + I*d)^(3/2)*f) - (8*(a + I*b)^(5/2)*(B - I*(A - C))*d^2*ArcTanh[(Sq
rt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*
x]])])/((c + I*d)^(3/2)*f) + (8*(a - I*b)^2*(I*A + B - I*C)*d^2*Sqrt[a + b
*Tan[e + f*x]])/((c - I*d)*f*Sqrt[c + d*Tan[e + f*x]]) + (8*(a + I*b)^2*(B
 - I*(A - C))*d^2*Sqrt[a + b*Tan[e + f*x]])/((c + I*d)*f*Sqrt[c + d*Tan[e
+ f*x]]) + (30*a^2*C*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c -
a*d)))^(3/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*Sqrt[(b*(c + d*
Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*
((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))*(-((b*d*(a + b*Tan[e + f*x])
)/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a +
 b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))
)))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b
*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c
- a*d) - (a*b*d)/(b*c - a*d)))])))/(b^2*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt...
```

3.153.3 Rubi [A] (verified)

Time = 4.42 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 4128$$

$$\frac{2\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-4Bdc+(4A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+5bd)+(5bc-ad)(cC-Bd)\right)}{2\sqrt{c+d\tan(e+fx)}}}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-4Bdc+(4A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+5bd)+(5bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}d}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-4Bdc+(4A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+5bd)+(5bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}d}{d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow 4130$$

$$\frac{\int -\frac{\sqrt{a+b\tan(e+fx)}\left(-4\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-4a(Ad(ac+5bd)+(5bc-ad)(cC-Bd))d+b\left(3(bc-ad)\left(5Cc^2-4Bdc\right.\right.\right.}{2\sqrt{c+d\tan(e+fx)}}}{2d}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(-4\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan\right.}{}}{}}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow 3042$$

3.153. $\displaystyle\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(-4\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4130

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\int -\frac{c\left(15Cc^3-12Bdc^2+(8A+7C)d^2c-4Bd^3\right)b^3-2ad\left(15Cc^3-10Bdc^2+3(4A+C)d^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 27

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(3(bc-ad)\left(d^2(4A+C)-4Bcd+5c^2C\right)-4d^2((A-\cdots\right.}{df}\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(3(bc-ad)\left(d^2(4A+C)-4Bcd+5c^2C\right)-4d^2((A-\cdots\right.}{df}\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4138

$$\frac{\frac{b\left(d^2(4A+C)-4Bcd+5c^2C\right)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(3(bc-ad)\left(d^2(4A+C)-4Bcd+5c^2C\right)-4d^2((A-\cdots\right.}{df}\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 2348

3.153. $\displaystyle\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$\frac{\frac{b(5Cc^2-4Bdc+(4A+C)d^2)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{b\left(3(bc-ad)\left(5Cc^2-4Bdc+(4A+C)d^2\right)-4d^2((A-C)(bc-ad)+B(ac+bd))\right)\sqrt{a+b\tan}}{df}}{}$$

$$\frac{2\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{d\left(c^2+d^2\right)f\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{\frac{b(d^2(4A+C)-4Bcd+5c^2C)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}{2df}-\frac{b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(3(bc-ad)\left(d^2(4A+C)-4Bcd+5c^2C\right)-4d^2((A-}{df}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(5/2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]]) + ((b*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2)*(a + b*
Tan[e + f*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]])/(2*d*f) - (-1/2*((-8*(a - I*
b)^(5/2)*(I*A + B - I*C)*(c + I*d)*d^3*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c - I*d] + (
8*(a + I*b)^(5/2)*(A + I*B - C)*d^3*(I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I
*d] + (2*Sqrt[b]*(c^2 + d^2)*(15*a^2*C*d^2 - 10*a*b*d*(3*c*C - 2*B*d) + b^
2*(15*c^2*C - 12*B*c*d + 8*(A - C)*d^2))*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e
 + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[d])/(d*f) + (b*(3*(b*c
 - a*d)*(5*c^2*C - 4*B*c*d + (4*A + C)*d^2) - 4*d^2*((A - C)*(b*c - a*d) +
 B*(a*c + b*d)))*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f))
/(4*d))/(d*(c^2 + d^2))
```

3.153. $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.153. $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.153.4 Maple [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{(c+d\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.153.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^{2}(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.153.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^{\frac{5}{2}}\,(A+B\tan{(}$$

$$(c+d\tan{(e+}$$

input
```
integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(c + d*tan(e + f*x))**(3/2), x)
```

3.153.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.153.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^{5/2}\,(C\tan(e+\ldots}{(c+d\tan(e+\ldots}$$

input
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2), x)
```

3.154 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 49, antiderivative size = 380

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx =\\ -\frac{(a-ib)^{3/2}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f}\\ -\frac{(a+ib)^{3/2}(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f}\\ -\frac{\sqrt{b}(3bcC-2bBd-3aCd)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f}\\ -\frac{2(c^2C-Bcd+Ad^2)(a+b\tan(e+fx))^{3/2}}{d(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}\\ +\frac{b(3c^2C-2Bcd+(2A+C)d^2)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{d^2(c^2+d^2)f}\end{aligned}$$

output
```
-(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f-(a+I*b)^(3/2)*(B-I*(A-
C))*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(c+I*d)^(3/2)/f-(-2*B*b*d-3*C*a*d+3*C*b*c)*arctanh(d^(1/2)*(a
+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(5/2)/f+b*(
3*c^2*C-2*B*c*d+(2*A+C)*d^2)*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)
/d^2/(c^2+d^2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(3/2)/d/(c^2+d^2)/
f/(c+d*tan(f*x+e))^(1/2)
```

3.154.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2141 vs. $2(380) = 760$.

Time = 7.68 (sec) , antiderivative size = 2141, normalized size of antiderivative = 5.63

$$\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(3/2))/(d*f*Sqrt[c + d*Tan[e + f*x]]) + ((-2*(-a +
 I*b)^(3/2)*(B + I*(A - C))*d*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f
*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((-c + I*d)^(3/2)*f) - (
2*(a + I*b)^(3/2)*(B - I*(A - C))*d*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[
e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/((c + I*d)^(3/2)*f)
+ (2*(I*a + b)*(A - I*B - C)*d*Sqrt[a + b*Tan[e + f*x]])/((c - I*d)*f*Sqrt
[c + d*Tan[e + f*x]]) - (2*(I*a - b)*(A + I*B - C)*d*Sqrt[a + b*Tan[e + f*
x]])/((c + I*d)*f*Sqrt[c + d*Tan[e + f*x]]) - (6*c*C*(b*c - a*d)*(b/((b^2*
c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(3/2)*((b^2*c)/(b*c - a*d) - (a*b*d
)/(b*c - a*d))^2*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a
+ b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)
)))*(-((b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b
*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b
*c - a*d) - (a*b*d)/(b*c - a*d)))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*S
qrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d)
 - (a*b*d)/(b*c - a*d)])]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[
(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)]*Sqrt[1 + (b*d*(a + b*Tan[e + f*
x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))])))/(b*d^2*
f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]*Sqrt[1 + (b*d*(a + b*T
an[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))...
```

3.154.3 Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.204$, Rules used = {3042, 4128, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2\int \frac{\sqrt{a+b\tan(e+fx)}\left(b\left(3Cc^2-2Bdc+(2A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+3bd)+(3bc-ad)(cC-Bd)\right)}{2\sqrt{c+d\tan(e+fx)}}dx}{d\left(c^2+d^2\right)} - \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(b\left(3Cc^2-2Bdc+(2A+C)d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+3bd)+(3bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{d\left(c^2+d^2\right)} - \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(b\left(3Cc^2-2Bdc+(2A+C)d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+3bd)+(3bc-ad)(cC-Bd)\right)}{\sqrt{c+d\tan(e+fx)}}dx}{d\left(c^2+d^2\right)} - \frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 4130}$$

$$\int -\frac{-2\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-2a(Ad(ac+3bd)+(3bc-ad)(cC-Bd))d+b(3bcC-3adC-2bBd)\left(c^2+d^2\right)\tan^2(e+fx)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$$d$$

$$d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 27

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\int\frac{-2\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-2a(Ad(ac}{\sqrt{a}}$$

$$d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\int\frac{-2\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-2a(Ad(ac}{\sqrt{a}}$$

$$d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4138

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\int\frac{-2\left((Bc-(A-C)d)a^2+2b(Ac-Cc+Bd)a-b^2(Bc-(A-C)d)\right)\tan(e+fx)d^2-2a(Ad(ac}{\sqrt{a+b\tan(e}}$$

$$d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 2348

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+$$

$$\frac{b\left(d^2(2A+C)-2Bcd+3c^2C\right)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\int\left(\frac{b(3bcC-3adC-2bBd)\left(c^2+d^2\right)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{2Ab^2d^3-2a^2Ad^3+4abBd^3+2a^2Cd^3-2b^2Cd^3}{}\right.$$

$\downarrow$ 2009

3.154. $\displaystyle\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{df\left(c^2+d^2\right)\sqrt{c+d\tan(e+fx)}}+\frac{\frac{b(d^2(2A+C)-2Bcd+3c^2C)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}{df}-\frac{\frac{2d^2(a-ib)^{3/2}(-d+ic)(A-iB-C)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}}-\frac{2d^2(a+ib)^3}{}}{}}{d\left(c^2+d^2\right)}$$

input
```
Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*S
qrt[c + d*Tan[e + f*x]]) + (-1/2*((2*(a - I*b)^(3/2)*(A - I*B - C)*(I*c -
d)*d^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqr
t[c + d*Tan[e + f*x]])])/Sqrt[c - I*d] - (2*(a + I*b)^(3/2)*(I*A - B - I*C
)*(c - I*d)*d^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a +
 I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I*d] + (2*Sqrt[b]*(3*b*c*C - 2*
b*B*d - 3*a*C*d)*(c^2 + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(S
qrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[d])/(d*f) + (b*(3*c^2*C - 2*B*c*d
+ (2*A + C)*d^2)*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f))
/(d*(c^2 + d^2))
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.154.4 Maple [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(fx+e))^{\frac{3}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(c+d\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.154.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.154.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan\left(\right.\right.}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(c + d*tan(e + f*x))**(3/2), x)
```

3.154.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.154.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^{3/2}\,(C\tan(e+}{(c+d\tan\,(e+}$$

input
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2), x)
```

3.155 $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 49, antiderivative size = 299

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a-ib}(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{3/2}f} - \frac{\sqrt{a+ib}(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}f} + \frac{2\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{3/2}f} - \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{d(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
a+I*b)^(1/2)/(c+I*d)^(3/2)/f+2*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^(1/2)/b^
(1/2)/(c+d*tan(f*x+e))^(1/2))*b^(1/2)/d^(3/2)/f-2*(A*d^2-B*c*d+C*c^2)*(a+b
*tan(f*x+e))^(1/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{-a+ib}(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+}}\right.}{(-c+id)^{3/2}}}{}$$

input
```
Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(c + d*Tan[e + f*x])^(3/2),x]
```

output
```
((Sqrt[-a + I*b]*(I*A + B - I*C)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + (I
*Sqrt[a + I*b]*(A + I*B - C)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x
]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(3/2) + ((B + I*(
A - C))*Sqrt[a + b*Tan[e + f*x]])/((c - I*d)*Sqrt[c + d*Tan[e + f*x]]) + (
((-I)*A + B + I*C)*Sqrt[a + b*Tan[e + f*x]])/((c + I*d)*Sqrt[c + d*Tan[e +
 f*x]]) + (2*C*(-(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]) + Sqrt[b*c - a*d]*ArcS
inh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/Sqrt[b*c - a*d]]*Sqrt[(b*(c + d*Tan
[e + f*x]))/(b*c - a*d)]))/(d^(3/2)*Sqrt[c + d*Tan[e + f*x]]))/f
```

3.155.3 Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4128, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan(e+fx)^2)}{(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 27

$$\frac{\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{bC(c^2+d^2)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 4138

$$\frac{\int \frac{bC(c^2+d^2)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}(\tan^2(e+fx)+1)}d\tan(e+fx)}{df(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}$$

$\big\downarrow$ 2348

$$-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{\int \left(\frac{bC(c^2+d^2)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{aAd^2-bBd^2-aCd^2-Abcd-aBcd+bcCd+i(Abd^2+aBd^2-bCd^2+aAcd-bBcd-acCd)}{2(i-\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{-aAd^2+bB}{}\right.}{df(c^2+d^2)}$$

$\big\downarrow$ 2009

$$-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{d\sqrt{a-ib}(c+id)(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c-id}}+\frac{d\sqrt{a+ib}(d+ic)(A+iB-C)\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{c+id}}+\frac{2\sqrt{b}C(c^2-}{}}{df(c^2+d^2)}$$

input
```
Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
+ d*Tan[e + f*x])^(3/2),x]
```

3.155. $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

output
```
(-((Sqrt[a - I*b]*(I*A + B - I*C)*(c + I*d)*d*ArcTanh[(Sqrt[c - I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c - I
*d]) + (Sqrt[a + I*b]*(A + I*B - C)*d*(I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c +
 I*d] + (2*Sqrt[b]*C*(c^2 + d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]]
)/(Sqrt[b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[d])/(d*(c^2 + d^2)*f) - (2*(c^
2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(d*(c^2 + d^2)*f*Sqrt[c + d
*Tan[e + f*x]])
```

3.155.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

3.155. $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.155.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan\left(fx+e\right)}\,\left(A+B\tan\left(fx+e\right)+C\tan\left(fx+e\right)^{2}\right)}{\left(c+d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69574 vs. $2(237) = 474$.

Time = 246.04 (sec) , antiderivative size = 139175, normalized size of antiderivative = 465.47

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.155.6 Sympy [F]

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+b\tan{(e+fx)}}(A+B\tan{(e+}}{(c+d\tan{(e+f}}$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(c + d*tan(e + f*x))**(3/2), x)
```

3.155.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.155.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a+b\tan\left(e+f\,x\right)}\,\left(C\tan(e+f\,x\right)}{\left(c+d\tan\left(e+f\right.\right.}$$

input
```
int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(3/2), x)
```

3.156 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx$

3.156.1 Optimal result

Integrand size = 49, antiderivative size = 251

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{(B+i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}(c-id)^{3/2}f} + \frac{(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}(c+id)^{3/2}f} + \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{(bc-ad)(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(c-I*d)^(3/2)/f/(a-I*b)^(1/2)+(I*A-B-I*C)*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
c+I*d)^(3/2)/f/(a+I*b)^(1/2)+2*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(1/2)/
(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 3.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{(bc-ad)\left(\frac{(iA+B-iC)(c+id)\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{(A+iB-C)(ic+d)\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right)+\frac{2(c^2C}{}}{(-bc+ad)(c^2+d^2)f}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(3/2)),x]
```

output
```
-(((b*c - a*d)*(((I*A + B - I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a
+ b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a +
I*b]*Sqrt[-c + I*d]) + ((A + I*B - C)*(I*c + d)*ArcTanh[(Sqrt[c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a
+ I*b]*Sqrt[c + I*d])) + (2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x
]])/Sqrt[c + d*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f))
```

3.156.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{4132}$$
$$\frac{2\int \frac{(bc-ad)(Ac-Cc+Bd)+(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(bc-ad)(Ac-Cc+Bd)+(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(bc-ad)(Ac-Cc+Bd)+(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 4099$$

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+$$
$$\frac{\frac{1}{2}(c-id)(A+iB-C)(bc-ad)\int \frac{1-i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c+id)(A-iB-C)(bc-ad)\int \frac{i\,\mathrm{ta}}{\sqrt{a+b\tan(e+}}}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+$$
$$\frac{\frac{1}{2}(c-id)(A+iB-C)(bc-ad)\int \frac{1-i\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx+\frac{1}{2}(c+id)(A-iB-C)(bc-ad)\int \frac{i\,\mathrm{ta}}{\sqrt{a+b\tan(e+}}}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \; 4098$$

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+$$
$$\frac{\frac{(c+id)(A-iB-C)(bc-ad)\int \frac{1}{(1-i\tan(e+fx))\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}d\tan(e+fx)}{2f}+\frac{(c-id)(A+iB-C)(bc-ad)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{a+b\tan(e}}}{2f}}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \; 104$$

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+$$
$$\frac{\frac{(c-id)(A+iB-C)(bc-ad)\int \frac{1}{-ia+b+\frac{(ic-d)(a+b\tan(e+fx))}{c+d\tan(e+fx)}}d\frac{\sqrt{a+b\tan(e+fx)}}{\sqrt{c+d\tan(e+fx)}}}{f}+\frac{(c+id)(A-iB-C)(bc-ad)\int \frac{1}{ia+b-\frac{(ic+d)(a+b\tan(e+fx))}{c+d\tan(e+fx)}}d\frac{\sqrt{a+b\,\mathrm{ta}}}{\sqrt{c+d\,\mathrm{ta}}}}{f}}{(c^2+d^2)(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}+$$
$$\frac{\frac{i(c-id)(A+iB-C)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a+ib}\sqrt{c+id}}-\frac{i(c+id)(A-iB-C)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{f\sqrt{a-ib}\sqrt{c-id}}}{(c^2+d^2)(bc-ad)}$$

3.156. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(3/2)),x]
```

output
```
(((-I)*(A - I*B - C)*(c + I*d)*(b*c - a*d)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a +
 b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b
]*Sqrt[c - I*d]*f) + (I*(A + I*B - C)*(c - I*d)*(b*c - a*d)*ArcTanh[(Sqrt[
c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]]
)])/(Sqrt[a + I*b]*Sqrt[c + I*d]*f))/((b*c - a*d)*(c^2 + d^2)) + (2*(c^2*C
 - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sqr
t[c + d*Tan[e + f*x]])
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.156.4 Maple [F(-1)]

Timed out.

$$\int \frac{A + B\tan(fx+e) + C\tan(fx+e)^2}{\sqrt{a+b\tan(fx+e)}\,(c+d\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.156.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.156.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{\sqrt{a+b\tan{(e+fx)}}\,(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x)
)*(c + d*tan(e + f*x))**(3/2)), x)
```

3.156.7 Maxima [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx=\int \frac{C\tan{(fx+e)}^2+B\tan{(fx+e)}+A}{\sqrt{b\tan{(fx+e)}+a}(d\tan{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)/(sqrt(b*tan(f*x + e) + a
)*(d*tan(f*x + e) + c)^(3/2)), x)
```

3.156.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
 + d*tan(e + f*x))^(3/2)),x)
```

output
```
\text{Hanged}
```

3.157 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

3.157.1 Optimal result

Integrand size = 49, antiderivative size = 383

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = -\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{3/2}f} - \frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}(c+id)^{3/2}f} - \frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} - \frac{2d(b^2c(cC-Bd)-abB(c^2+d^2)+a^2(2c^2C-Bcd+Cd^2)+A(a^2d^2+b^2(c^2+2d^2)))\sqrt{a+b\tan(e+fx)}}{(a^2+b^2)(bc-ad)^2(c^2+d^2)f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
a+I*b)^(3/2)/(c+I*d)^(3/2)/f-2*(A*b^2-a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/
(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)-2*d*(b^2*c*(-B*d+C*c)-a*b*B*
(c^2+d^2)+a^2*(-B*c*d+2*C*c^2+C*d^2)+A*(a^2*d^2+b^2*(c^2+2*d^2)))*(a+b*tan
(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 6.87 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.26

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$
$$-\frac{2\left(\frac{(bc-ad)^2\left(\frac{(a+ib)(iA+B-iC)(c+id)\operatorname{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}+\frac{(ia+b)(A+iB-C)(c-id)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}\sqrt{c+id}}\right)}{2(-bc+ad)(c^2+d^2)f}-2\ldots\right)}{(a^2+b^2)(bc-a\ldots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3
/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*(((b*c - a*d)^2*(((a + I*b)*(I*A + B
 - I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a + I*b]*Sqrt[-c + I*d]) + ((
I*a + b)*(A + I*B - C)*(c - I*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c +
 I*d])))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(-(c*(-(c*(A*b^2 - a*(b*B -
 a*C))*d) + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(2*A*b^2*d - a*A*
(b*c - a*d) - (b*B - a*C)*(b*c + a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/((-(b
*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/((a^2 + b^2)*(b*c - a
*d))
```

3.157.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.245$, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}dx$$

$\Big\downarrow$ 4132

$$-\frac{2\int \frac{(A+C)da^2-b(Ac-Cc+Bd)a+2\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-b^2(Bc-2Ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{(A+C)da^2-b(Ac-Cc+Bd)a+2\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-b^2(Bc-2Ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{(A+C)da^2-b(Ac-Cc+Bd)a+2\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-b^2(Bc-2Ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4132

$$-\frac{\frac{2\int -\frac{(bBc-b(A-C)d+a(Ac-Cc+Bd))(bc-ad)^2+(aBc+bCc-bBd+aCd-A(bc+ad))\tan(e+fx)(bc-ad)^2}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2\right.\right.}{f(c^2+d^2)}}{\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\int \frac{(bBc-b(A-C)d+a(Ac-Cc+Bd))(bc-ad)^2}{\sqrt{a+b\tan}}}{(}}{\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 3042

3.157. $\displaystyle\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\int\frac{(bBc-b(A-C)d+a(Ac-Cc+Bd))(bc-ad)^2}{\sqrt{a+b\tan}\ldots}}{(\ldots}}{\left(a^2+b^2\right)(bc-ad)}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$

$\Big\downarrow$ 4099

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{1}{2}(a-ib)(c-id)(A+iB-C)(bc-ad)^2\int\frac{\ldots}{\sqrt{a+\ldots}}\ldots}{\ldots}}{\left(a^2+b^2\right)(bc-ad)}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{1}{2}(a-ib)(c-id)(A+iB-C)(bc-ad)^2\int\frac{\ldots}{\sqrt{a+\ldots}}\ldots}{\ldots}}{\left(a^2+b^2\right)(bc-ad)}$$

$\Big\downarrow$ 4098

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{(a+ib)(c+id)(A-iB-C)(bc-ad)^2\int\frac{\ldots}{(1-i\tan(e\ldots}\ldots}{\ldots}}{\ldots}}{\left(a^2+b^2\right)(bc\ldots}$$

$\Big\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{(a-ib)(c-id)(A+iB-C)(bc-ad)^2\int\frac{\ldots}{-ia+b+\frac{(i\ldots}{\ldots}}\ldots}{f}\ldots}{\ldots}}{\left(a^2+b^2\right)(bc-ad)}$$

$\Big\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+2c^2C+Cd^2\right)-abB\left(c^2+d^2\right)+Ab^2\left(c^2+2d^2\right)+b^2c(cC-Bd)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{\frac{i(a-ib)(c-id)(A+iB-C)(bc-ad)^2\text{arctanh}\left(\ldots\right)}{f\sqrt{a+ib}\sqrt{c+id}\ldots}\ldots}{\ldots}}{\left(a^2+b^2\right)(bc-ad)}$$

3.157. $\displaystyle\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
 + d*Tan[e + f*x])^(3/2)),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (-((((-I)*(a + I*b)*(A - I*B - C)*(c +
I*d)*(b*c - a*d)^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[
a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) + (I*
(a - I*b)*(A + I*B - C)*(c - I*d)*(b*c - a*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a
+ I*b]*Sqrt[c + I*d]*f))/((b*c - a*d)*(c^2 + d^2))) + (2*d*(a^2*A*d^2 + b^
2*c*(c*C - B*d) - a*b*B*(c^2 + d^2) + A*b^2*(c^2 + 2*d^2) + a^2*(2*c^2*C -
 B*c*d + C*d^2))*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sqrt
[c + d*Tan[e + f*x]]))/((a^2 + b^2)*(b*c - a*d))
```

3.157.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.157.4 Maple [F(-1)]

Timed out.

$$\int \frac{A + B\tan(fx+e) + C\tan(fx+e)^2}{(a + b\tan(fx+e))^{\frac{3}{2}}(c + d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.157.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.157.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{(a+b\tan{(e+fx)})^{\frac{3}{2}}\,(c+d\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(
3/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

3.157.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.157.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}\,dx=\text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
 + d*tan(e + f*x))^(3/2)),x)
```

output
```
\text{Hanged}
```

3.158 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

3.158.1 Optimal result

Integrand size = 49, antiderivative size = 598

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{(iA+B-iC)\text{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{5/2}(c-id)^{3/2}f}$$
$$-\frac{(B-i(A-C))\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{5/2}(c+id)^{3/2}f}$$
$$-\frac{2(Ab^2-a(bB-aC))}{3(a^2+b^2)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$
$$-\frac{2(7a^3bBd-4a^4Cd+b^4(3Bc-4Ad)+ab^3(6Ac-6cC+Bd)-a^2b^2(3Bc+2(5A-C)d))}{3(a^2+b^2)^2(bc-ad)^2f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}$$
$$-\frac{2d(8a^3bBd(c^2+d^2)+2ab^3(3Ac-3cC+Bd)(c^2+d^2)-a^4d(8c^2C-3Bcd+(3A+5C)d^2)-a^2b^2(3Bc^3}{3(a^2+b^2)^2(bc-ad)^3}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(a-I*b)^(5/2)/(c-I*d)^(3/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
a+I*b)^(5/2)/(c+I*d)^(3/2)/f-2/3*(7*a^3*b*B*d-4*a^4*C*d+b^4*(-4*A*d+3*B*c)
+a*b^3*(6*A*c+B*d-6*C*c)-a^2*b^2*(3*B*c+2*(5*A-C)*d))/(a^2+b^2)^2/(-a*d+b*
c)^2/f/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e))^(1/2)-2/3*d*(8*a^3*b*B*d*(c
^2+d^2)+2*a*b^3*(3*A*c+B*d-3*C*c)*(c^2+d^2)-a^4*d*(8*c^2*C-3*B*c*d+(3*A+5*
C)*d^2)-a^2*b^2*(11*A*c^2*d+17*A*d^3+3*B*c^3-3*B*c*d^2+5*C*c^2*d-C*d^3)-b^
4*(d*(5*A*c^2+8*A*d^2+3*C*c^2)-3*B*(c^3+2*c*d^2)))*(a+b*tan(f*x+e))^(1/2)/
(a^2+b^2)^2/(-a*d+b*c)^3/(c^2+d^2)/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*b^2-a*(
B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(c+d*tan(f*x+e))^(1/2)/(a+b*tan(f*x+e))^(
3/2)
```

3.158.2 Mathematica [A] (verified)

Time = 7.17 (sec) , antiderivative size = 902, normalized size of antiderivative = 1.51

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{2(Ab^2-a(bB-aC))}{3\left(a^2+b^2\right)(bc-ad)f(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$-\frac{2\left(-\frac{2\left(-a\left(-2a\left(Ab^2-a(bB-aC)\right)d+\frac{3}{2}b(Ab-aB-bC)(bc-ad)\right)+\frac{1}{2}b^2\left(4Ab^2d-3aA(bc-ad)-(bB-aC)(3bc+ad)\right)\right)}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{2\left(-\frac{3(bc-ad)^3\left(\frac{(a+ib)^2 \cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5
/2)*(c + d*Tan[e + f*x])^(3/2)),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f
*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) - (2*((-2*(-(a*(-2*a*(A*b^2 - a*(b*B
- a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2)) + (b^2*(4*A*b^2*d - 3*
a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/((a^2 + b^2)*(b*c - a*d)
*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]) - (2*((-3*(b*c - a*d
)^3*(((a + I*b)^2*(I*A + B - I*C)*(c + I*d)*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a +
 I*b]*Sqrt[-c + I*d]) + ((a - I*b)^2*(A + I*B - C)*(I*c + d)*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]
])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2
*(d^2*((-1/2*(b*c) - (a*d)/2)*(-2*a*(A*b^2 - a*(b*B - a*C))*d + (3*b*(A*b
- a*B - b*C)*(b*c - a*d))/2) + ((b^2*d - (a*(b*c - a*d))/2)*(4*A*b^2*d - 3
*a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2) - c*((d*(b*c - a*d)*(-2*
b*(A*b^2 - a*(b*B - a*C))*d - (3*a*(A*b - a*B - b*C)*(b*c - a*d))/2 + (b*(
4*A*b^2*d - 3*a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))/2 - c*d*(-
(a*(-2*a*(A*b^2 - a*(b*B - a*C))*d + (3*b*(A*b - a*B - b*C)*(b*c - a*d))/2
)) + (b^2*(4*A*b^2*d - 3*a*A*(b*c - a*d) - (b*B - a*C)*(3*b*c + a*d)))/2))
)*Sqrt[a + b*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
 + f*x]])))/((a^2 + b^2)*(b*c - a*d))))/(3*(a^2 + b^2)*(b*c - a*d))
```

3.158.3 Rubi [A] (verified)

Time = 4.39 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{2\int \frac{4Adb^2+4\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-3aA(bc-ad)-(bB-aC)(3bc+ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{4Adb^2+4\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-3aA(bc-ad)-(bB-aC)(3bc+ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{4Adb^2+4\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-3aA(bc-ad)-(bB-aC)(3bc+ad)+3(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{3/2}}dx}{3\left(a^2+b^2\right)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{2\int \frac{3\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)^2-2d\left(-4C\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\int \frac{3\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)^2-2d\left(-4Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{\int \frac{3\left(Ba^2-2b(A-C)a-b^2B\right)\tan(e+fx)(bc-ad)^2-2d\left(-4Cd\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$$\Big\downarrow \; 4132$$

3.158. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{2\int\frac{3\left((bc-ad)^3\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac\right.\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 27

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{3\int\frac{(bc-ad)^3\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac-\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{3\int\frac{(bc-ad)^3\left((Ac-Cc+Bd)a^2+2b(Bc-(A-C)d)a-b^2(Ac-\right.}{}}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}$$

$\downarrow$ 4099

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right)+8a^3\right.}{}}{}}{}$$

$\downarrow$ 3042

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right)+8a^3\right.}{}}{}}{}$$

$\downarrow$ 4098

3.158. $\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right)+8a^3}{}}{}}{}$$

$\big\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right)+8a^3}{}}{}}{}$$

$\big\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{3f\left(a^2+b^2\right)(bc-ad)(a+b\tan(e+fx))^{3/2}\sqrt{c+d\tan(e+fx)}}-$$

$$\frac{\frac{2\left(-4a^4Cd+7a^3bBd-a^2b^2(10Ad+3Bc-2Cd)+ab^3(6Ac+Bd-6cC)+b^4(3Bc-4Ad)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}-\frac{-\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^4(-d)\left(d^2(3A+5C)-3Bcd+8c^2C\right)+8a^3}{}}{}}{}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(5/2)*(c
 + d*Tan[e + f*x])^(3/2)),x]
```

3.158. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/(3*(a^2 + b^2)*(b*c - a*d)*f*(a + b*Tan[e + f
*x])^(3/2)*Sqrt[c + d*Tan[e + f*x]]) - ((2*(7*a^3*b*B*d - 4*a^4*C*d + b^4*
(3*B*c - 4*A*d) + a*b^3*(6*A*c - 6*c*C + B*d) - a^2*b^2*(3*B*c + 10*A*d -
2*C*d)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Ta
n[e + f*x]]) - ((3*(((-I)*(a + I*b)^2*(A - I*B - C)*(c + I*d)*(b*c - a*d)^
3*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) + (I*(a - I*b)^2*(A +
I*B - C)*(c - I*d)*(b*c - a*d)^3*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c +
 I*d]*f)))/((b*c - a*d)*(c^2 + d^2)) - (2*d*(8*a^3*b*B*d*(c^2 + d^2) + 2*a
*b^3*(3*A*c - 3*c*C + B*d)*(c^2 + d^2) - a^4*d*(8*c^2*C - 3*B*c*d + (3*A +
 5*C)*d^2) - a^2*b^2*(3*B*c^3 + 11*A*c^2*d + 5*c^2*C*d - 3*B*c*d^2 + 17*A*
d^3 - C*d^3) - b^4*(d*(5*A*c^2 + 3*c^2*C + 8*A*d^2) - 3*B*(c^3 + 2*c*d^2))
)*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e +
f*x]]))/((a^2 + b^2)*(b*c - a*d)))/(3*(a^2 + b^2)*(b*c - a*d))
```

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.158. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx$

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.158.4 Maple [F(-1)]

Timed out.

$$\int \frac{A+B\tan(fx+e)+C\tan(fx+e)^2}{(a+b\tan(fx+e))^{\frac{5}{2}}(c+d\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(f*x+e)
)^(3/2),x)
```

3.158.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.158.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = \int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{\frac{5}{2}}(c+d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(5/2)/(c+d*ta
n(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(
5/2)*(c + d*tan(e + f*x))**(3/2)), x)
```

3.158.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{5/2}(c+d\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.158.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{5/2}(c + d\tan(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(5/2)/(c+d*tan(
f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{5/2}(c + d\tan(e + fx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(5/2)*(c
 + d*tan(e + f*x))^(3/2)),x)
```

output
```
\text{Hanged}
```

3.159 $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.159.1 Optimal result

Integrand size = 49, antiderivative size = 549

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{(a-ib)^{5/2}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}$$

$$-\frac{(a+ib)^{5/2}(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$

$$-\frac{b^{3/2}(5bcC-2bBd-5aCd)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{7/2}f}$$

$$-\frac{2(c^2C-Bcd+Ad^2)\,(a+b\tan(e+fx))^{5/2}}{3d\,(c^2+d^2)\,f(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{2(b(5c^4C-2Bc^3d-c^2(A-11C)d^2-8Bcd^3+5Ad^4)+3ad^2(2c(A-C)d-B(c^2-d^2)))\,(a+b\tan(e+}{3d^2\,(c^2+d^2)^2\,f\sqrt{c+d\tan(e+fx)}}$$

$$+\frac{b(b(5c^4C-2Bc^3d+10c^2Cd^2-6Bcd^3+(4A+C)d^4)+2ad^2(2c(A-C)d-B(c^2-d^2)))\,\sqrt{a+b\tan(e+}{d^3\,(c^2+d^2)^2\,f}$$

output
```
-(a-I*b)^(5/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f-(a+I*b)^(5/2)*(B-I*(A-
C))*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(c+I*d)^(5/2)/f-b^(3/2)*(-2*B*b*d-5*C*a*d+5*C*b*c)*arctanh(d^
(1/2)*(a+b*tan(f*x+e))^(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(7/2)/f+b*(
b*(5*c^4*C-2*B*c^3*d+10*C*c^2*d^2-6*B*c*d^3+(4*A+C)*d^4)+2*a*d^2*(2*c*(A-C
)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)*(c+d*tan(f*x+e))^(1/2)/d^3/(c^2+d
^2)^2/f-2/3*(b*(5*c^4*C-2*B*c^3*d-c^2*(A-11*C)*d^2-8*B*c*d^3+5*A*d^4)+3*a*
d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(3/2)/d^2/(c^2+d^2)^2/f/(c
+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(5/2)/d/(c^2
+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.159.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2650 vs. $2(549) = 1098$.

Time = 9.79 (sec) , antiderivative size = 2650, normalized size of antiderivative = 4.83

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(c + d*Tan[e + f*x])^(5/2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(5/2))/(d*f*(c + d*Tan[e + f*x])^(3/2)) + ((2*(I*a
 + b)*(A - I*B - C)*d*(a + b*Tan[e + f*x])^(3/2))/(3*(c - I*d)*f*(c + d*Ta
n[e + f*x])^(3/2)) - (2*(I*a - b)*(A + I*B - C)*d*(a + b*Tan[e + f*x])^(3/
2))/(3*(c + I*d)*f*(c + d*Tan[e + f*x])^(3/2)) + (2*(a - I*b)^2*(I*A + B -
 I*C)*d*((Sqrt[-a + I*b]*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])
/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + Sqrt[a + b
*Tan[e + f*x]]/((c - I*d)*Sqrt[c + d*Tan[e + f*x]])))/((c - I*d)*f) + (2*(
a + I*b)^2*(I*A - B - I*C)*d*((Sqrt[a + I*b]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(
3/2) - Sqrt[a + b*Tan[e + f*x]]/((c + I*d)*Sqrt[c + d*Tan[e + f*x]])))/((c
 + I*d)*f) + (10*c*C*(b*c - a*d)*(b/((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c -
a*d)))^(5/2)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^3*Sqrt[(b*(c + d*
Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*
((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2*((b^2*d^2*(a + b*Tan[e + f
*x])^2)/(3*(b*c - a*d)^2*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))^2*(-1
 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/
(b*c - a*d))))^2) - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c
- a*d) - (a*b*d)/(b*c - a*d))*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c - a*d
)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))) - (Sqrt[b]*Sqrt[d]*ArcSin
h[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2...
```

3.159.3 Rubi [A] (verified)

Time = 4.99 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-2Bdc+(2A+3C)d^2\right)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+5bd)+(5bc-3ad)(cC-}{2(c+d\tan(e+fx))^{3/2}}}{3d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-2Bdc+(2A+3C)d^2\right)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+5bd)+(5bc-3ad)(cC-Bc}{(c+d\tan(e+fx))^{3/2}}}{3d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(e+fx))^{3/2}\left(b\left(5Cc^2-2Bdc+(2A+3C)d^2\right)\tan(e+fx)^2+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+5bd)+(5bc-3ad)(cC-Bc}{(c+d\tan(e+fx))^{3/2}}}{3d\left(c^2+d^2\right)}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4128

$$2\int \frac{\sqrt{a+b\tan(e+fx)}\left(3((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+3bd)(Ad(3ac+5bd)+(5bc-3ad)(cC-B\right.}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(3((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+3bd)(Ad(3ac+5bd)+(5bc-3ad)(cC-Bd\right.}{}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

3.159. $$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$$

$$\int \frac{\sqrt{a+b\tan(e+fx)}(3((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+3bd)(Ad(3ac+5bd)+(5bc-3ad)(cC-Bd}{}$$

$$\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4130

$$\int \frac{3(-c(5Cc^4-2Bdc^3+10Cd^2c^2-6Bd^3c+(4A+C)d^4)b^3-(5bcC-5adC-2bBd)(c^2+d^2)^2\tan^2(e+fx)b^2+ad(5Cc^4-2(3A-8C)d^2c^2-12Bd^3c+(6A-C)d^4)b^2+6a^2}{}$$

$$\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$3\int \frac{-c(5Cc^4-2Bdc^3+10Cd^2c^2-6Bd^3c+(4A+C)d^4)b^3-(5bcC-5adC-2bBd)(c^2+d^2)^2\tan^2(e+fx)b^2+ad(5Cc^4-2(3A-8C)d^2c^2-12Bd^3c+(6A-C)d^4)b^2+6a^2}{}$$

$$\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$3\int \frac{-c(5Cc^4-2Bdc^3+10Cd^2c^2-6Bd^3c+(4A+C)d^4)b^3-(5bcC-5adC-2bBd)(c^2+d^2)^2\tan(e+fx)^2b^2+ad(5Cc^4-2(3A-8C)d^2c^2-12Bd^3c+(6A-C)d^4)b^2+6a^2}{}$$

$$\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4138

$$3\int \frac{-c(5Cc^4-2Bdc^3+10Cd^2c^2-6Bd^3c+(4A+C)d^4)b^3-(5bcC-5adC-2bBd)(c^2+d^2)^2\tan^2(e+fx)b^2+ad(5Cc^4-2(3A-8C)d^2c^2-12Bd^3c+(6A-C)d^4)b^2+6a^2}{}$$

$$\frac{2(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{5/2}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 2348

3.159. $\displaystyle\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{\frac{3b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(2a\left(2c(A-C)d-B\left(c^2-d^2\right)\right)d^2+b\left(5Cc^4-2Bdc^3+10Cd^2c^2-6Bd^3c+(4A+C)d^4\right)\right)}{df}+\frac{3\int\left(-\frac{b^2(5bcC-5adC-2bBd)\left(c^2+d^2\right)^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\right.}{}}{}$$

$$\frac{2\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^{5/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{5/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2(a+b\tan(e+fx))^{3/2}\left(3ad^2\left(2cd(A-C)-B(c^2-d^2)\right)+b\left(-c^2d^2(A-11C)+5Ad^4-2Bc^3d-8Bcd^3+5c^4C\right)\right)}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{3b\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\right.}{}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(5/2))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) + ((-2*(b*(5*c^4*C - 2*B*c^3*d - c^2*(A - 11*
C)*d^2 - 8*B*c*d^3 + 5*A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*(
a + b*Tan[e + f*x])^(3/2))/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]) + ((
3*((-2*(a - I*b)^(5/2)*(I*A + B - I*C)*(c + I*d)^2*d^3*ArcTanh[(Sqrt[c - I
*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c + d*Tan[e + f*x]])])/S
qrt[c - I*d] - (2*(a + I*b)^(5/2)*(B - I*(A - C))*(c - I*d)^2*d^3*ArcTanh[
(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e +
 f*x]])])/Sqrt[c + I*d] - (2*b^(3/2)*(5*b*c*C - 2*b*B*d - 5*a*C*d)*(c^2 +
d^2)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[
e + f*x]])])/Sqrt[d]))/(2*d*f) + (3*b*(b*(5*c^4*C - 2*B*c^3*d + 10*c^2*C*d
^2 - 6*B*c*d^3 + (4*A + C)*d^4) + 2*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))
*Sqrt[a + b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]])/(d*f))/(d*(c^2 + d^2))
)/(3*d*(c^2 + d^2))
```

3.159. $\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.159. $\int \frac{(a+b\tan(e+fx))^{5/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.159.4 Maple [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(fx+e))^{\frac{5}{2}}\left(A+B\tan(fx+e)+C\tan(fx+e)^{2}\right)}{(c+d\tan(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

output
```
int((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

3.159.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^{2}(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.159.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+fx))^{\frac{5}{2}}\left(A+B\tan(\right.}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(c + d*tan(e + f*x))**(5/2), x)
```

3.159.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.159.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{5/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^{5/2}\,\left(C\tan(e+\right.}{(c+d\tan\left(e+\right.}$$

input
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(5/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2), x)
```

3.160 $\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.160.1 Optimal result

Integrand size = 49, antiderivative size = 407

$$\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{(a-ib)^{3/2}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f} - \frac{(a+ib)^{3/2}(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f} + \frac{2b^{3/2}C\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b\tan(e+fx)}}{\sqrt{b}\sqrt{c+d\tan(e+fx)}}\right)}{d^{5/2}f} - \frac{2(c^2C-Bcd+Ad^2)(a+b\tan(e+fx))^{3/2}}{3d(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} - \frac{2(b(c^4C-c^2(A-3C)d^2-2Bcd^3+Ad^4)+ad^2(2c(A-C)d-B(c^2-d^2)))\sqrt{a+b\tan(e+fx)}}{d^2(c^2+d^2)^2f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(a-I*b)^(3/2)*(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a
-I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f-(a+I*b)^(3/2)*(B-I*(A-
C))*arctanh((c+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*
x+e))^(1/2))/(c+I*d)^(5/2)/f+2*b^(3/2)*C*arctanh(d^(1/2)*(a+b*tan(f*x+e))^
(1/2)/b^(1/2)/(c+d*tan(f*x+e))^(1/2))/d^(5/2)/f-2*(b*(c^4*C-c^2*(A-3*C)*d^
2-2*B*c*d^3+A*d^4)+a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)
/d^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a+b*tan
(f*x+e))^(3/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.160.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1135 vs. 2(407) = 814.

Time = 7.17 (sec) , antiderivative size = 1135, normalized size of antiderivative = 2.79

$$\int \frac{(a+b\tan(e+fx))^{3/2}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx=\frac{(B+i(A-C))(a+b\tan(e+fx))^{3/}}{3(c-id)f(c+d\tan(e+fx))^{3/2}}$$
$$-\frac{(iA-B-iC)(a+b\tan(e+fx))^{3/2}}{3(c+id)f(c+d\tan(e+fx))^{3/2}}$$
$$+\frac{(ia+b)(A-iB-C)\left(\frac{\sqrt{-a+ib}\,\text{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(-c+id)^{3/2}}+\frac{\sqrt{a+b\tan(e+fx)}}{(c-id)\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)f}$$
$$+\frac{(ia-b)(A+iB-C)\left(\frac{\sqrt{a+ib}\,\text{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{3/2}}-\frac{\sqrt{a+b\tan(e+fx)}}{(c+id)\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)f}$$
$$-\frac{2C(bc-ad)\left(\frac{b}{\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}}\right)^{5/2}\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)^3\sqrt{\frac{b(c+d\tan(e+fx))}{bc-ad}}\left(-1-\frac{bd(a+b\tan(e+fx))}{(bc-ad)\left(\frac{b^2c}{bc-ad}-\frac{abd}{bc-ad}\right)}\right)^2\left(\frac{}{3(bc-ad)^2\left(\right.}\right.}{b^2d^3f\sqrt{a+b\tan(e+f}}$$

input
```
Integrate[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]
^2))/(c + d*Tan[e + f*x])^(5/2),x]
```

output
```
((B + I*(A - C))*(a + b*Tan[e + f*x])^(3/2))/(3*(c - I*d)*f*(c + d*Tan[e +
 f*x])^(3/2)) - ((I*A - B - I*C)*(a + b*Tan[e + f*x])^(3/2))/(3*(c + I*d)*
f*(c + d*Tan[e + f*x])^(3/2)) + ((I*a + b)*(A - I*B - C)*((Sqrt[-a + I*b]*
ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(-c + I*d)^(3/2) + Sqrt[a + b*Tan[e + f*x]]/((c - I*d)
*Sqrt[c + d*Tan[e + f*x]])))/((c - I*d)*f) + ((I*a - b)*(A + I*B - C)*((Sq
rt[a + I*b]*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b
]*Sqrt[c + d*Tan[e + f*x]])])/(c + I*d)^(3/2) - Sqrt[a + b*Tan[e + f*x]]/(
(c + I*d)*Sqrt[c + d*Tan[e + f*x]])))/((c + I*d)*f) - (2*C*(b*c - a*d)*(b/
((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)))^(5/2)*((b^2*c)/(b*c - a*d) -
(a*b*d)/(b*c - a*d))^3*Sqrt[(b*(c + d*Tan[e + f*x]))/(b*c - a*d)]*(-1 - (b
*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c
- a*d))))^2*((b^2*d^2*(a + b*Tan[e + f*x])^2)/(3*(b*c - a*d)^2*((b^2*c)/(b
*c - a*d) - (a*b*d)/(b*c - a*d))^2*(-1 - (b*d*(a + b*Tan[e + f*x]))/((b*c
- a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))))^2) - (b*d*(a + b*Tan[
e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d))*(-1 -
(b*d*(a + b*Tan[e + f*x]))/((b*c - a*d)*((b^2*c)/(b*c - a*d) - (a*b*d)/(b*
c - a*d)))))) - (Sqrt[b]*Sqrt[d]*ArcSinh[(Sqrt[b]*Sqrt[d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) - (a*b*d)/(b*c - a*d)])
]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b*c - a*d]*Sqrt[(b^2*c)/(b*c - a*d) -...
```

3.160.3 Rubi [A] (verified)

Time = 2.85 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.204$, Rules used = {3042, 4128, 27, 3042, 4128, 27, 3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{3\sqrt{a+b\tan(e+fx)}\left(bC\left(c^2+d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{2(c+d\tan(e+fx))^{3/2}}dx}{3d\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(bC\left(c^2+d^2\right)\tan^2(e+fx)+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sqrt{a+b\tan(e+fx)}\left(bC\left(c^2+d^2\right)\tan(e+fx)^2+d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(ac+bd)+(bc-ad)(cC-Bd)\right)}{(c+d\tan(e+fx))^{3/2}}dx}{d\left(c^2+d^2\right)}-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{\frac{2\int \frac{((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+bd)(Ad(ac+bd)+(bc-ad)(cC-Bd))d+b^2C\left(c^2+d^2\right)^2\tan^2}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}}{d}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+bd)(Ad(ac+bd)+(bc-ad)(cC-Bd))d+b^2C\left(c^2+d^2\right)^2\tan^2(}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{d(c^2+d^2)}}{d\,(}$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

3.160. $\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$\int \frac{((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+bd)(Ad(ac+bd)+(bc-ad)(cC-Bd))d+b^2C\left(c^2+d^2\right)^2\tan(e\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}} \Big/ d(c^2+d^2) \;\Big/\; d\,(\cdots$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4138$$

$$\int \frac{((ac+bd)((A-C)(bc-ad)+B(ac+bd))-(bc-ad)(bBc-b(A-C)d-a(Ac-Cc+Bd)))\tan(e+fx)d^2+(ac+bd)(Ad(ac+bd)+(bc-ad)(cC-Bd))d+b^2C\left(c^2+d^2\right)^2\tan^2(\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)} \Big/ df(c^2+d^2) \;\cdots$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 2348$$

$$\int\left(\frac{b^2C\left(c^2+d^2\right)^2}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}+\frac{2aAbd^4+a^2Bd^4-b^2Bd^4-2abCd^4-2Ab^2cd^3+2a^2Acd^3-4abBcd^3-2a^2cCd^3+2b^2cCd^3-2aAbc^2d^2-a^2Bc^2d^2+b^2Bc^2d^2+\cdots}{2(i-\tan(\cdots}\right.\;\cdots$$

$$\frac{2\left(Cc^2-Bdc+Ad^2\right)(a+b\tan(e+fx))^{3/2}}{3d\left(c^2+d^2\right)f(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{3/2}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{-\frac{2\sqrt{a+b\tan(e+fx)}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(A-3C)+Ad^4-2Bcd^3+c^4C\right)\right)}{df(c^2+d^2)\sqrt{c+d\tan(e+fx)}}+\frac{-\frac{d^2(a-ib)^{3/2}(c+id)^2(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\cdots}{\sqrt{a-ib}\cdots}\right.\cdots}{\sqrt{c-id}}\cdots}{d\left(c^2+d^2\cdots\right.}$$

input
```
Int[((a + b*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(
c + d*Tan[e + f*x])^(5/2),x]
```

3.160. $\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

output
```
(-2*(c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(3/2))/(3*d*(c^2 + d^2)*f
*(c + d*Tan[e + f*x])^(3/2)) + ((-(((a - I*b)^(3/2)*(I*A + B - I*C)*(c + I
*d)^2*d^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*
Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c - I*d]) - ((a + I*b)^(3/2)*(B - I*(A -
C))*(c - I*d)^2*d^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt
[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I*d] + (2*b^(3/2)*C*(c^2 +
d^2)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[b]*Sqrt[c + d*Tan[
e + f*x]])])/Sqrt[d])/(d*(c^2 + d^2)*f) - (2*(b*(c^4*C - c^2*(A - 3*C)*d^2
 - 2*B*c*d^3 + A*d^4) + a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a + b*
Tan[e + f*x]])/(d*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/(d*(c^2 + d^2))
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.160.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

output
```
int((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

3.160.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.160.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+fx))^{\frac{3}{2}}\left(A+B\tan(\right.}{(c+d\tan(e+}$$

input
```
integrate((a+b*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)*
*2)/(c + d*tan(e + f*x))**(5/2), x)
```

3.160.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.160.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^{3/2}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^{3/2}\,\left(C\tan(e+\cdots\right.}{(c+d\tan\left(e+\cdots\right.}$$

input
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(3/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2), x)
```

3.161 $\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.161.1 Optimal result

Integrand size = 49, antiderivative size = 373

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{a-ib}(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c-id)^{5/2}f}$$
$$-\frac{\sqrt{a+ib}(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(c+id)^{5/2}f}$$
$$-\frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3d(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}$$
$$+\frac{2(b(c^4C+2Bc^3d-c^2(5A-7C)d^2-4Bcd^3+Ad^4)+3ad^2(2c(A-C)d-B(c^2-d^2)))\sqrt{a+b\tan(e+fx)}}{3d(bc-ad)(c^2+d^2)^2f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))*(a-I*b)^(1/2)/(c-I*d)^(5/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))*(
a+I*b)^(1/2)/(c+I*d)^(5/2)/f+2/3*(b*(c^4*C+2*B*c^3*d-c^2*(5*A-7*C)*d^2-4*B
*c*d^3+A*d^4)+3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)/d/
(-a*d+b*c)/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)-2/3*(A*d^2-B*c*d+C*c^2)*(a
+b*tan(f*x+e))^(1/2)/d/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.161.2 Mathematica [A] (verified)

Time = 7.11 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{C\sqrt{a+b\tan(e+fx)}}{df(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{-\frac{2\left(\frac{1}{2}d^2(-bcC-a(2A-3C)d)-c\left(-((Ab+aB-bC)d^2)-\frac{1}{2}c(-bcC-2bBd+aCd)\right)\right)\sqrt{a+b\tan(e+fx)}}{3(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(-\frac{3d(bc-ad)^2\left(\frac{\sqrt{-a+ib}(iA+B-iC)(c+\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2
))/(c + d*Tan[e + f*x])^(5/2),x]
```

output
```
-((C*Sqrt[a + b*Tan[e + f*x]])/(d*f*(c + d*Tan[e + f*x])^(3/2))) - ((-2*((
d^2*(-(b*c*C) - a*(2*A - 3*C)*d))/2 - c*(-((A*b + a*B - b*C)*d^2) - (c*(-(
b*c*C) - 2*b*B*d + a*C*d))/2))*Sqrt[a + b*Tan[e + f*x]])/(3*(-(b*c) + a*d)
*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((-3*d*(b*c - a*d)^2*((Sqr
t[-a + I*b]*(I*A + B - I*C)*(c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a + b
*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[-c + I*d]
 + (Sqrt[a + I*b]*(B - I*(A - C))*(c - I*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[
a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/Sqrt[c + I
*d]))/(2*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2*(-1/2*(d^2*(b*c - a*d)*(3*a*d*
(A*c - c*C + B*d) + b*(c^2*C - B*c*d + A*d^2))) - c*((-3*d^2*(b*c - a*d)*(
A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d))/2 + (b*c*(b*c - a*d)*(c^2*
C + 2*B*c*d - (2*A - 3*C)*d^2))/2))*Sqrt[a + b*Tan[e + f*x]])/((-(b*c) + a
*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]])))/(3*(-(b*c) + a*d)*(c^2 + d^2
)))/d
```

3.161.3 Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.245$, Rules used = {3042, 4128, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4128}$$

$$\frac{2\int \frac{b(Cc^2+2Bdc-(2A-3C)d^2)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+bd)+(bc-3ad)(cC-Bd)}{2\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(Cc^2+2Bdc-(2A-3C)d^2)\tan^2(e+fx)+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+bd)+(bc-3ad)(cC-Bd)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Cc^2+2Bdc-(2A-3C)d^2)\tan(e+fx)^2+3d((A-C)(bc-ad)+B(ac+bd))\tan(e+fx)+Ad(3ac+bd)+(bc-3ad)(cC-Bd)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3d(c^2+d^2)}-\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3df(c^2+d^2)(c+d\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4132}$$

$$\frac{2\int -\frac{3\left(d(bc-ad)\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)-b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)+d(bc-ad)\left(2aAcd-2acCd-Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)+b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}}{(c^2+d^2)(bc-ad)}$$

$$3d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}} - \frac{3\int \frac{d(bc-ad)\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right.\right.\right.}{}}{}$$

$$3d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}} - \frac{3\int \frac{d(bc-ad)\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right.\right.\right.}{}}{}$$

$$3d\left(c^2+d^2\right)$$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4099

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}} - \frac{3\left(-\frac{1}{2}d(a+ib)(c-id)^2(A+iB-C)(bc-ad\right.}{}$$

$$3d\left(c^2+d^2\right)$$

$\downarrow$ 3042

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}} - \frac{3\left(-\frac{1}{2}d(a+ib)(c-id)^2(A+iB-C)(bc-ad\right.}{}$$

$$3d\left(c^2+d^2\right)$$

$\downarrow$ 4098

3.161. $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{d(a+ib)(c-id)^2(A+iB-C)(bc-ad)\int\frac{}{(i}}{}\right.}{}}{3d\,(c}$$

$\downarrow$ 104

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{d(a+ib)(c-id)^2(A+iB-C)(bc-ad)\int\frac{}{-}}{}\right.}{}}{3d\left(c^2+d^2\right)}$$

$\downarrow$ 221

$$-\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3df\left(c^2+d^2\right)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)+b\left(-c^2d^2(5A-7C)+Ad^4+2Bc^3d-4Bcd^3+c^4C\right)\right)}{f\left(c^2+d^2\right)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(\frac{id\sqrt{a-ib}(c+id)^2(A-iB-C)(bc-ad)\text{arct}}{f\sqrt{c-id}}\right.}{}}{3d\left(c^2+d^2\right)}$$

input
```
Int[(Sqrt[a + b*Tan[e + f*x]]*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c
+ d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*d*(c^2 + d^2)*f*(
c + d*Tan[e + f*x])^(3/2)) + ((-3*((I*Sqrt[a - I*b]*(A - I*B - C)*(c + I*d
)^2*d*(b*c - a*d)*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a
 - I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c - I*d]*f) - (I*Sqrt[a + I*b]*(
A + I*B - C)*(c - I*d)^2*d*(b*c - a*d)*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*T
an[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[c + I*d]*f)
))/((b*c - a*d)*(c^2 + d^2)) + (2*(b*(c^4*C + 2*B*c^3*d - c^2*(5*A - 7*C)*
d^2 - 4*B*c*d^3 + A*d^4) + 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2)))*Sqrt[a
 + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/
(3*d*(c^2 + d^2))
```

3.161. $\int \frac{\sqrt{a+b\tan(e+fx)}\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^{5/2}}\,dx$

3.161.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

3.161. $\int \frac{\sqrt{a+b\tan(e+fx)}\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.161.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(fx+e)}\,(A+B\tan(fx+e)+C\tan(fx+e)^2)}{(c+d\tan(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

output
```
int((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

3.161.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.161.6 Sympy [F]

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+$$

$$(c+d\tan(e+f$$

input
```
integrate((a+b*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(a + b*tan(e + f*x))*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)
/(c + d*tan(e + f*x))**(5/2), x)
```

3.161.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((2*b*d+2*a*c)^2>0)', see `assum
e?` for mo
```

3.161.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(e+fx)}(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a+b\tan(e+fx)}\,(C\tan(e+fx)}{(c+d\tan(e+f}$$

input
```
int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2),x)
```

output
```
int(((a + b*tan(e + f*x))^(1/2)*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(
c + d*tan(e + f*x))^(5/2), x)
```

3.162 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$

3.162.1 Optimal result

Integrand size = 49, antiderivative size = 379

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{(B+i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a-ib}(c-id)^{5/2}f} + \frac{(iA-B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{a+ib}(c+id)^{5/2}f} + \frac{2(c^2C-Bcd+Ad^2)\sqrt{a+b\tan(e+fx)}}{3(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}} + \frac{2(b(2c^4C-5Bc^3d+4c^2(2A-C)d^2+Bcd^3+2Ad^4)-3ad^2(2c(A-C)d-B(c^2-d^2)))\sqrt{a+b\tan(e+}{3(bc-ad)^2(c^2+d^2)^2f\sqrt{c+d\tan(e+fx)}}$$

output
```
-(B+I*(A-C))*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(c-I*d)^(5/2)/f/(a-I*b)^(1/2)+(I*A-B-I*C)*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
c+I*d)^(5/2)/f/(a+I*b)^(1/2)+2/3*(b*(2*c^4*C-5*B*c^3*d+4*c^2*(2*A-C)*d^2+B
*c*d^3+2*A*d^4)-3*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2)))*(a+b*tan(f*x+e))^(1/2)/
(-a*d+b*c)^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))^(1/2)+2/3*(A*d^2-B*c*d+C*c^2)*
(a+b*tan(f*x+e))^(1/2)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.162.2 Mathematica [A] (verified)

Time = 6.00 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.06

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx = \frac{3(bc-ad)^2\left(\frac{(iA+B-iC)(c+id)^2\operatorname{arctanh}\left(\frac{\sqrt{-c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{-a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{\sqrt{-a+ib}\sqrt{-c+id}}\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]
]*(c + d*Tan[e + f*x])^(5/2)),x]
```

output
```
(3*(b*c - a*d)^2*(((I*A + B - I*C)*(c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqr
t[a + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-
a + I*b]*Sqrt[-c + I*d]) + (I*(A + I*B - C)*(c - I*d)^2*ArcTanh[(Sqrt[c +
I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/
(Sqrt[a + I*b]*Sqrt[c + I*d])) + (2*(b*c - a*d)*(c^2 + d^2)*(c^2*C - B*c*d
 + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(c + d*Tan[e + f*x])^(3/2) + (2*(b*(2*
c^4*C - 5*B*c^3*d + 4*c^2*(2*A - C)*d^2 + B*c*d^3 + 2*A*d^4) + 3*a*d^2*(2*
c*(-A + C)*d + B*(c^2 - d^2)))*Sqrt[a + b*Tan[e + f*x]])/Sqrt[c + d*Tan[e
+ f*x]])/(3*(b*c - a*d)^2*(c^2 + d^2)^2*f)
```

3.162.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.245$, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{2\int \frac{2Abd^2+2b(Cc^2-Bdc+Ad^2)\tan^2(e+fx)+3Ac(bc-ad)-(bc-3ad)(cC-Bd)+3(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\int \frac{2Abd^2+2b(Cc^2-Bdc+Ad^2)\tan^2(e+fx)+3Ac(bc-ad)-(bc-3ad)(cC-Bd)+3(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{2Abd^2+2b(Cc^2-Bdc+Ad^2)\tan(e+fx)^2+3Ac(bc-ad)-(bc-3ad)(cC-Bd)+3(bc-ad)(Bc-(A-C)d)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}dx}{3(c^2+d^2)(bc-ad)}+\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4132

$$\frac{\frac{2\int -\frac{3\left(\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)(bc-ad)^2+\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2\right)}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}dx}{(c^2+d^2)(bc-ad)}+\frac{2\sqrt{a+b\tan(e+fx)}(b(4c^2d^2(2A-C)+2Ad^4-5Bc}{f(c^2+d^2)(bc-ad)}}{3(c^2+d^2)(bc-ad)}$$
$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}(b(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C)-3ad^2(2cd(A-C)-B(c^2-d^2)))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\int \frac{\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)(bc-ad)^2}{\sqrt{a+b\tan(e+f}}}{(c^2+d^2}}{3(c^2+d^2)(bc-ad)}$$
$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}(b(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C)-3ad^2(2cd(A-C)-B(c^2-d^2)))}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\int \frac{\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)(bc-ad)^2}{\sqrt{a+b\tan(e+f}}}{(c^2+d^2}}{3(c^2+d^2)(bc-ad)}$$
$$\frac{2(Ad^2-Bcd+c^2C)\sqrt{a+b\tan(e+fx)}}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}$$

$\big\downarrow$ 4099

3.162. $\displaystyle\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{1}{2}(c-id)^2(A+iB-C)(bc-ad)^2\int\frac{}{\sqrt{a+}}\right.}{}}{3\left(c^2+d^2\right)(bc-ad)}$$

$\Big\downarrow$ 3042

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{1}{2}(c-id)^2(A+iB-C)(bc-ad)^2\int\frac{}{\sqrt{a+}}\right.}{}}{3\left(c^2+d^2\right)(bc-ad)}$$

$\Big\downarrow$ 4098

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{(c+id)^2(A-iB-C)(bc-ad)^2\int\frac{}{(1-i\tan(}}{}\right.}{}}{3\left(c^2+d^2\right)(bc}$$

$\Big\downarrow$ 104

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(-\frac{(c-id)^2(A+iB-C)(bc-ad)^2\int\frac{}{-ia+b+}}{}\right.}{}}{3\left(c^2+d^2\right)(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{2\left(Ad^2-Bcd+c^2C\right)\sqrt{a+b\tan(e+fx)}}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+$$

$$\frac{\frac{2\sqrt{a+b\tan(e+fx)}\left(b\left(4c^2d^2(2A-C)+2Ad^4-5Bc^3d+Bcd^3+2c^4C\right)-3ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)\right)}{f(c^2+d^2)(bc-ad)\sqrt{c+d\tan(e+fx)}}-\frac{3\left(\frac{i(c+id)^2(A-iB-C)(bc-ad)^2\text{arctanh}\left(\right.}{f\sqrt{a-ib}\sqrt{c-id}}\right.}{}}{3\left(c^2+d^2\right)(bc-ad)}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c +
 d*Tan[e + f*x])^(5/2)),x]
```

3.162. $\int\frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$

output
```
(2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])^(3/2)) + ((-3*((I*(A - I*B - C)*(c + I*d)^2*(
b*c - a*d)^2*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*
b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) - (I*(A + I
*B - C)*(c - I*d)^2*(b*c - a*d)^2*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e
+ f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c
+ I*d]*f)))/((b*c - a*d)*(c^2 + d^2)) + (2*(b*(2*c^4*C - 5*B*c^3*d + 4*c^2
*(2*A - C)*d^2 + B*c*d^3 + 2*A*d^4) - 3*a*d^2*(2*c*(A - C)*d - B*(c^2 - d^
2)))*Sqrt[a + b*Tan[e + f*x]])/((b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e
 + f*x]]))/(3*(b*c - a*d)*(c^2 + d^2))
```

3.162.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

3.162. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.162.4 Maple [F(-1)]

Timed out.

$$\int \frac{A + B\tan(fx + e) + C\tan(fx + e)^2}{\sqrt{a + b\tan(fx + e)}\,(c + d\tan(fx + e))^{\frac{5}{2}}}dx$$

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

3.162.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.162.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{A+B\tan{(e+fx)}+C\tan^2{(e+fx)}}{\sqrt{a+b\tan{(e+fx)}}\,(c+d\tan{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(1/2)/(c+d*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/(sqrt(a + b*tan(e + f*x)
)*(c + d*tan(e + f*x))**(5/2)), x)
```

3.162.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.162.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(1/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(1/2)*(c
 + d*tan(e + f*x))^(5/2)),x)
```

output
```
\text{Hanged}
```

3.163 $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 49, antiderivative size = 651

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{(iA+B-iC)\operatorname{arctanh}\left(\frac{\sqrt{c-id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a-ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a-ib)^{3/2}(c-id)^{5/2}f}$$
$$-\frac{(B-i(A-C))\operatorname{arctanh}\left(\frac{\sqrt{c+id}\sqrt{a+b\tan(e+fx)}}{\sqrt{a+ib}\sqrt{c+d\tan(e+fx)}}\right)}{(a+ib)^{3/2}(c+id)^{5/2}f}$$
$$-\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$
$$-\frac{2d(b^2c(cC-Bd)-3abB(c^2+d^2)+a^2(4c^2C-Bcd+3Cd^2)+A(a^2d^2+b^2(3c^2+4d^2)))\sqrt{a+b\tan(e+}}{3(a^2+b^2)(bc-ad)^2(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}$$
$$-\frac{2d(b^3c(5c^3C-8Bc^2d-cCd^2-2Bd^3)+a^2b(8c^4C-8Bc^3d+5c^2Cd^2-2Bcd^3+3Cd^4)+3a^3d^2(2cCd+}{3(a^2+b}$$

output
```
-(I*A+B-I*C)*arctanh((c-I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a-I*b)^(1/2)/(c
+d*tan(f*x+e))^(1/2))/(a-I*b)^(3/2)/(c-I*d)^(5/2)/f-(B-I*(A-C))*arctanh((c
+I*d)^(1/2)*(a+b*tan(f*x+e))^(1/2)/(a+I*b)^(1/2)/(c+d*tan(f*x+e))^(1/2))/(
a+I*b)^(3/2)/(c+I*d)^(5/2)/f-2/3*d*(b^3*c*(-8*B*c^2*d-2*B*d^3+5*C*c^3-C*c*
d^2)+a^2*b*(-8*B*c^3*d-2*B*c*d^3+8*C*c^4+5*C*c^2*d^2+3*C*d^4)+3*a^3*d^2*(2
*C*c*d+B*(c^2-d^2))+3*a*b^2*(2*c*C*d^3-B*(c^4+c^2*d^2+2*d^4))-A*(6*a^3*c*d
^3+6*a*b^2*c*d^3-a^2*b*d^2*(11*c^2+5*d^2)-b^3*(3*c^4+17*c^2*d^2+8*d^4)))*(
a+b*tan(f*x+e))^(1/2)/(a^2+b^2)/(-a*d+b*c)^3/(c^2+d^2)^2/f/(c+d*tan(f*x+e)
)^(1/2)-2*(A*b^2-a*(B*b-C*a))/(a^2+b^2)/(-a*d+b*c)/f/(a+b*tan(f*x+e))^(1/2
)/(c+d*tan(f*x+e))^(3/2)-2/3*d*(b^2*c*(-B*d+C*c)-3*a*b*B*(c^2+d^2)+a^2*(-B
*c*d+4*C*c^2+3*C*d^2)+A*(a^2*d^2+b^2*(3*c^2+4*d^2)))*(a+b*tan(f*x+e))^(1/2
)/(a^2+b^2)/(-a*d+b*c)^2/(c^2+d^2)/f/(c+d*tan(f*x+e))^(3/2)
```

3.163.2 Mathematica [A] (verified)

Time = 7.21 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.39

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))}{(a^2+b^2)(bc-ad)f\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$$-\frac{2\left(-\frac{2\left(-c\left(-2c(Ab^2-a(bB-aC))d+\frac{1}{2}(Ab-aB-bC)d(bc-ad)\right)+\frac{1}{2}d^2\left(4Ab^2d-aA(bc-ad)-(bB-aC)(bc+3ad)\right)\right)\sqrt{a+b\tan(e+fx)}}{3(-bc+ad)(c^2+d^2)f(c+d\tan(e+fx))^{3/2}}-\frac{2\left(\frac{3(bc\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3
/2)*(c + d*Tan[e + f*x])^(5/2)),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]]*(c + d*Tan[e + f*x])^(3/2)) - (2*((-2*(-(c*(-2*c*(A*b^2 - a*(b*B -
a*C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)) + (d^2*(4*A*b^2*d - a*A*(b
*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)*Sqrt[a + b*Tan[e + f*x]])/(3*(-
(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^(3/2)) - (2*((3*(b*c - a*d
)^3*(((a + I*b)*(I*A + B - I*C)*(c + I*d)^2*ArcTanh[(Sqrt[-c + I*d]*Sqrt[a
 + b*Tan[e + f*x]])/(Sqrt[-a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[-a +
 I*b]*Sqrt[-c + I*d]) + ((I*a + b)*(A + I*B - C)*(c - I*d)^2*ArcTanh[(Sqrt
[c + I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]
])])/(Sqrt[a + I*b]*Sqrt[c + I*d])))/(4*(-(b*c) + a*d)*(c^2 + d^2)*f) - (2
*(d^2*(((b*c)/2 - (3*a*d)/2)*(-2*c*(A*b^2 - a*(b*B - a*C))*d + ((A*b - a*B
 - b*C)*d*(b*c - a*d))/2) + ((b*d^2 - (3*c*(-(b*c) + a*d))/2)*(4*A*b^2*d -
 a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2) - c*((3*d*(-(b*c) + a*d)
*(-2*(A*b^2 - a*(b*B - a*C))*d^2 - (c*(A*b - a*B - b*C)*(b*c - a*d))/2 + (
d*(4*A*b^2*d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2))/2 - b*c*(
-(c*(-2*c*(A*b^2 - a*(b*B - a*C))*d + ((A*b - a*B - b*C)*d*(b*c - a*d))/2)
) + (d^2*(4*A*b^2*d - a*A*(b*c - a*d) - (b*B - a*C)*(b*c + 3*a*d)))/2)))*S
qrt[a + b*Tan[e + f*x]])/((-(b*c) + a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e +
f*x]])))/(3*(-(b*c) + a*d)*(c^2 + d^2))))/((a^2 + b^2)*(b*c - a*d))
```

3.163.3 Rubi [A] (verified)

Time = 4.41 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)+C\tan(e+fx)^2}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4132}$$

$$-\frac{2\int \frac{4Adb^2+4(Ab^2-a(bB-aC))d\tan^2(e+fx)-aA(bc-ad)-(bB-aC)(bc+3ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{2\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}dx}{(a^2+b^2)(bc-ad)}-\frac{2(Ab^2-a(bB-aC))}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\int \frac{4Adb^2+4\left(Ab^2-a(bB-aC)\right)d\tan^2(e+fx)-aA(bc-ad)-(bB-aC)(bc+3ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}dx}{(a^2+b^2)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\int \frac{4Adb^2+4\left(Ab^2-a(bB-aC)\right)d\tan(e+fx)^2-aA(bc-ad)-(bB-aC)(bc+3ad)+(Ab-Cb-aB)(bc-ad)\tan(e+fx)}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{5/2}}dx}{(a^2+b^2)(bc-ad)}-\frac{2\left(Ab^2-a(bB-aC)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4132

$$-\frac{2\int \frac{-3(aBc+bCc-bBd+aCd-A(bc+ad))\tan(e+fx)(bc-ad)^2+2bd\left(Ad^2a^2+\left(4Cc^2-Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(3c^2+4d^2\right)\right)\tan^2(e+fx \cdots}{2\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx \cdots} \quad 3(c^2+d^2)(bc-ad) \cdots}{\cdots}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\int \frac{-3(aBc+bCc-bBd+aCd-A(bc+ad))\tan(e+fx)(bc-ad)^2+2bd\left(Ad^2a^2+\left(4Cc^2-Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(3c^2+4d^2\right)\right)\tan^2(e+fx)\cdots}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3\cdots}} \quad 3(c^2+d^2)(bc-ad) \cdots}{\cdots}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\int \frac{-3(aBc+bCc-bBd+aCd-A(bc+ad))\tan(e+fx)(bc-ad)^2+2bd\left(Ad^2a^2+\left(4Cc^2-Bdc+3Cd^2\right)a^2-3bB\left(c^2+d^2\right)a+b^2c(cC-Bd)+Ab^2\left(3c^2+4d^2\right)\right)\tan(e+fx)^2\cdots}{\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3\cdots}} \quad 3(c^2+d^2)(bc-ad) \cdots}{\cdots}$$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f(a^2+b^2)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4132

3.163. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{\frac{2\int \frac{3\left(\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)(bc-ad)^3+\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)-b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)\cdots}{2\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\cdots}{\left(c^2+d^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{3\int \frac{\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)(bc-ad)^3+\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)-b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)(b\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\cdots}{\left(c^2+d^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{3\int \frac{\left(a\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+b\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\right)(bc-ad)^3+\left(2aAcd-2acCd+Ab\left(c^2-d^2\right)-aB\left(c^2-d^2\right)-b\left(Cc^2-2Bdc-Cd^2\right)\right)\tan(e+fx)(b\cdots}{\sqrt{a+b\tan(e+fx)}\sqrt{c+d\tan(e+fx)}}\cdots}{\left(c^2+d^2\right)(bc-ad)}\cdots}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}$$

$\downarrow$ 4099

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+4c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+4d^2\right)+b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(3a^3d^2\left(B\left(c^2-d^2\right)+2c\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+4c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+4d^2\right)+b^2c(cC-Bd)\right)}{3f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(3a^3d^2\left(B\left(c^2-d^2\right)+2c\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 4098

3.163. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+4c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+4d^2\right)+b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(3a^3d^2\left(B\left(c^2-d^2\right)+2c\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 104

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+4c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+4d^2\right)+b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(3a^3d^2\left(B\left(c^2-d^2\right)+2c\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 221

$$-\frac{2\left(Ab^2-a(bB-aC)\right)}{f\left(a^2+b^2\right)(bc-ad)\sqrt{a+b\tan(e+fx)}(c+d\tan(e+fx))^{3/2}}-\frac{\frac{2d\sqrt{a+b\tan(e+fx)}\left(a^2Ad^2+a^2\left(-Bcd+4c^2C+3Cd^2\right)-3abB\left(c^2+d^2\right)+Ab^2\left(3c^2+4d^2\right)+b^2c(cC-Bd)\right)}{3f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^{3/2}}+\frac{2d\sqrt{a+b\tan(e+fx)}\left(3a^3d^2\left(B\left(c^2-d^2\right)+2c\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Int[(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2)/((a + b*Tan[e + f*x])^(3/2)*(c
 + d*Tan[e + f*x])^(5/2)),x]
```

3.163. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

output
```
(-2*(A*b^2 - a*(b*B - a*C)))/((a^2 + b^2)*(b*c - a*d)*f*Sqrt[a + b*Tan[e +
 f*x]]*(c + d*Tan[e + f*x])^(3/2)) - ((2*d*(a^2*A*d^2 + b^2*c*(c*C - B*d)
- 3*a*b*B*(c^2 + d^2) + A*b^2*(3*c^2 + 4*d^2) + a^2*(4*c^2*C - B*c*d + 3*C
*d^2))*Sqrt[a + b*Tan[e + f*x]])/(3*(b*c - a*d)*(c^2 + d^2)*f*(c + d*Tan[e
 + f*x])^(3/2)) + ((3*((I*(a + I*b)*(A - I*B - C)*(c + I*d)^2*(b*c - a*d)^
3*ArcTanh[(Sqrt[c - I*d]*Sqrt[a + b*Tan[e + f*x]])/(Sqrt[a - I*b]*Sqrt[c +
 d*Tan[e + f*x]])])/(Sqrt[a - I*b]*Sqrt[c - I*d]*f) - (I*(a - I*b)*(A + I*
B - C)*(c - I*d)^2*(b*c - a*d)^3*ArcTanh[(Sqrt[c + I*d]*Sqrt[a + b*Tan[e +
 f*x]])/(Sqrt[a + I*b]*Sqrt[c + d*Tan[e + f*x]])])/(Sqrt[a + I*b]*Sqrt[c +
 I*d]*f)))/((b*c - a*d)*(c^2 + d^2)) + (2*d*(b^3*c*(5*c^3*C - 8*B*c^2*d -
c*C*d^2 - 2*B*d^3) + a^2*b*(8*c^4*C - 8*B*c^3*d + 5*c^2*C*d^2 - 2*B*c*d^3
+ 3*C*d^4) + 3*a^3*d^2*(2*c*C*d + B*(c^2 - d^2)) + 3*a*b^2*(2*c*C*d^3 - B*
(c^4 + c^2*d^2 + 2*d^4)) - A*(6*a^3*c*d^3 + 6*a*b^2*c*d^3 - a^2*b*d^2*(11*
c^2 + 5*d^2) - b^3*(3*c^4 + 17*c^2*d^2 + 8*d^4)))*Sqrt[a + b*Tan[e + f*x]]
)/((b*c - a*d)*(c^2 + d^2)*f*Sqrt[c + d*Tan[e + f*x]]))/(3*(b*c - a*d)*(c^
2 + d^2)))/((a^2 + b^2)*(b*c - a*d))
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.163. $\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx$

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

3.163.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

output
```
int((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(f*x+e)
)^(5/2),x)
```

3.163.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.163.6 Sympy [F]

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx=\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{\frac{3}{2}}\,(c+d\tan(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(a+b*tan(f*x+e))**(3/2)/(c+d*ta
n(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*tan(e + f*x) + C*tan(e + f*x)**2)/((a + b*tan(e + f*x))**(
3/2)*(c + d*tan(e + f*x))**(5/2)), x)
```

3.163.7 Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(e+fx)+C\tan^2(e+fx)}{(a+b\tan(e+fx))^{3/2}(c+d\tan(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.163.8 Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}(c + d\tan(e + fx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(a+b*tan(f*x+e))^(3/2)/(c+d*tan(
f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(e + fx) + C\tan^2(e + fx)}{(a + b\tan(e + fx))^{3/2}(c + d\tan(e + fx))^{5/2}}\, dx = \text{Hanged}$$

input
```
int((A + B*tan(e + f*x) + C*tan(e + f*x)^2)/((a + b*tan(e + f*x))^(3/2)*(c
 + d*tan(e + f*x))^(5/2)),x)
```

output
```
\text{Hanged}
```

3.164 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n (A+B\tan(e$

3.164.1 Optimal result

Integrand size = 45, antiderivative size = 376

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx =$$

$$-\frac{(B+i(A-C))\,\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}(}{2(a-ib)f(1+m)}$$

$$-\frac{(A+iB-C)\,\text{AppellF1}\left(1+m,-n,1,2+m,-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}(c}{2(ia-b)f(1+m)}$$

$$+\frac{C\,\text{Hypergeometric2F1}\left(1+m,-n,2+m,-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx)}{bf(1+m)}$$

output
```
-1/2*(B+I*(A-C))*AppellF1(1+m,1,-n,2+m,(a+b*tan(f*x+e))/(a-I*b),-d*(a+b*ta
n(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^n/(a-I*b)/f/
(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*c))^n)-1/2*(A+I*B-C)*AppellF1(1+m,1,-n,
2+m,(a+b*tan(f*x+e))/(a+I*b),-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+
e))^(1+m)*(c+d*tan(f*x+e))^n/(I*a-b)/f/(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*
c))^n)+C*hypergeom([-n, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*ta
n(f*x+e))^(1+m)*(c+d*tan(f*x+e))^n/b/f/(1+m)/((b*(c+d*tan(f*x+e))/(-a*d+b*
c))^n)
```

3.164.2 Mathematica [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2), x]
```

3.164.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3042, 4138, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)(c+d\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)(c+d\tan(e+fx))^n dx$$

$$\Big\downarrow \text{4138}$$

$$\frac{\int \frac{(a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(C\tan^2(e+fx)+B\tan(e+fx)+A\right)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2348}$$

$$\frac{\int \left(C(c+d\tan(e+fx))^n(a+b\tan(e+fx))^m+\frac{(i(A-C)-B)(c+d\tan(e+fx))^n(a+b\tan(e+fx))^m}{2(i-\tan(e+fx))}+\frac{(B+i(A-C))(c+d\tan(e+\ldots}{2(\tan(e+\ldots}\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(B+i(A-C))(a+b\tan(e+fx))^{m+1}(c+d\tan(e+fx))^n\left(\frac{b(c+d\tan(e+fx))}{bc-ad}\right)^{-n}\operatorname{AppellF1}\left(m+1,-n,1,m+2,-\frac{d(a+b\tan(e+fx))}{bc-ad},\frac{a+b\tan(e+fx)}{a-ib}\right)}{2(m+1)(a-ib)}-$$

input
```
Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

output
```
(-1/2*((B + I*(A - C))*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a + b*Tan[e + f
*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(
1 + m)*(c + d*Tan[e + f*x])^n)/((a - I*b)*(1 + m)*((b*(c + d*Tan[e + f*x])
)/(b*c - a*d))^n) - ((A + I*B - C)*AppellF1[1 + m, -n, 1, 2 + m, -((d*(a +
 b*Tan[e + f*x]))/(b*c - a*d)), (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan
[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^n)/(2*(I*a - b)*(1 + m)*((b*(c + d
*Tan[e + f*x]))/(b*c - a*d))^n) + (C*Hypergeometric2F1[1 + m, -n, 2 + m, -
((d*(a + b*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m)*(c +
d*Tan[e + f*x])^n)/(b*(1 + m)*((b*(c + d*Tan[e + f*x]))/(b*c - a*d))^n))/f
```

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

3.164.4 Maple [F]

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e))^n \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right) dx$$

input
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

output
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

3.164.5 Fricas [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m(d\tan(fx+e)+c)^n dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(d
*tan(f*x + e) + c)^n, x)
```

3.164.6 Sympy [F(-2)]

Exception generated.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**n*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.164.7 Maxima [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m(d\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(
d*tan(f*x + e) + c)^n, x)
```

3.164.8 Giac [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m(d\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^n*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m*(
d*tan(f*x + e) + c)^n, x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^n \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^n\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

input
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^n*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2), x)
```

3.165 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\,(A+B\tan(e$

3.165.1 Optimal result

Integrand size = 45, antiderivative size = 560

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$= \frac{(bc(2+m)\left(b^2d(Bc+(A-C)d)(3+m)(4+m)-2(bc-ad)(3aCd-b(3cC+Bd(4+m)))\right)+d(b^3(2c(\cdots}{\cdots}$$

$$+\frac{(A-iB-C)(c-id)^3\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}$$

$$-\frac{(A+iB-C)(c+id)^3\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia-b)f(1+m)}$$

$$+\frac{d(b^2d(Bc+(A-C)d)(3+m)(4+m)-2(bc-ad)(3aCd-b(3cC+Bd(4+m))))\tan(e+fx)(a+b\cdots}{b^3f(2+m)(3+m)(4+m)}$$

$$-\frac{(3aCd-b(3cC+Bd(4+m)))(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^2}{b^2f(3+m)(4+m)}$$

$$+\frac{C(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^3}{bf(4+m)}$$

3.165.
$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

output
```
(b*c*(2+m)*(b^2*d*(B*c+(A-C)*d)*(3+m)*(4+m)-2*(-a*d+b*c)*(3*C*a*d-b*(3*C*c
+B*d*(4+m))))+d*(b^3*(2*c*(A-C)*d+B*(c^2-d^2))*(2+m)*(3+m)*(4+m)-a*(b^2*d*
(B*c+(A-C)*d)*(3+m)*(4+m)-2*(-a*d+b*c)*(3*C*a*d-b*(3*C*c+B*d*(4+m))))))*(a
+b*tan(f*x+e))^(1+m)/b^4/f/(1+m)/(2+m)/(3+m)/(4+m)+1/2*(A-I*B-C)*(c-I*d)^3
*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)
/(I*a+b)/f/(1+m)-1/2*(A+I*B-C)*(c+I*d)^3*hypergeom([1, 1+m],[2+m],(a+b*tan
(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/f/(1+m)+d*(b^2*d*(B*c+(A-
C)*d)*(3+m)*(4+m)-2*(-a*d+b*c)*(3*C*a*d-b*(3*C*c+B*d*(4+m))))*tan(f*x+e)*(
a+b*tan(f*x+e))^(1+m)/b^3/f/(2+m)/(3+m)/(4+m)-(3*C*a*d-b*(3*C*c+B*d*(4+m))
)*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^2/b^2/f/(3+m)/(4+m)+C*(a+b*tan(f
*x+e))^(1+m)*(c+d*tan(f*x+e))^3/b/f/(4+m)
```

3.165.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1390 vs. 2(560) = 1120.

Time = 6.54 (sec) , antiderivative size = 1390, normalized size of antiderivative = 2.48

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{C(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^3}{bf(4+m)}$$
$$+\frac{\frac{(3bcC-3aCd+bBd(4+m))(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^2}{bf(3+m)}+\frac{d\left(b^2d(Bc+(A-C)d)(3+m)(4+m)+2(bc-ad)(3bcC-3aCd+bBd(4+m))\right)\tan(\ldots}{bf(2+m)}}{\ldots}$$

input
```
Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^3)/(b*f*(4 + m)) + ((
(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Ta
n[e + f*x])^2)/(b*f*(3 + m)) + ((d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m
) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e + f*x]*(a + b
*Tan[e + f*x])^(1 + m))/(b*f*(2 + m)) - (((-(b*c*(2 + m)*(b^2*d*(B*c + (A
- C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)
))) + d*(-(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) +
a*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*
C*d + b*B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/
2)*(a*d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C
- 3*a*C*d + b*B*d*(4 + m))) + b*c*(2 + m)*(b^2*d*(B*c + (A - C)*d)*(3 + m)
*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) - b*c*(2 + m
)*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))) + b*c*(3 +
 m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m)))) - d*(-(b^3*(2*c*(A - C)*d +
 B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m)) + a*(b^2*d*(B*c + (A - C)*d)*(3 +
 m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))) - I*b*(2
 + m)*(b^2*c*(2*c*(A - C)*d + B*(c^2 - d^2))*(3 + m)*(4 + m) - d*(b^2*d*(B
*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d
*(4 + m))) + d*(-((2*a*d + b*c*(1 + m))*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)
)) + b*c*(3 + m)*(A*b*c*(4 + m) - C*(3*a*d + b*c*(1 + m))))))*Hypergeom...
```

3.165.3 Rubi [A] (verified)

Time = 3.41 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.356$, Rules used = {3042, 4130, 3042, 4130, 25, 3042, 4120, 25, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^3(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+d\tan(e+fx))^3(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \; 4130$$

$$\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left((3bcC-3adC+bBd(m+4))\tan^2(e+fx)+b(Bc+(A-C)d)(m-\right.}{b(m+4)}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 3042

$$\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left((3bcC-3adC+bBd(m+4))\tan(e+fx)^2+b(Bc+(A-C)d)(m-\right.}{b(m+4)}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 4130

$$\frac{\frac{\int -(a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)(m+3)(m+4)\tan(e+fx)b^2\right)-c(m+3)(Abc(m+4)-C(3ad+bc(m+1)))b-(d(}{b(m+3)}}{}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 25

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)(m+3\right.\right.}{}}{}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 3042

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)(m+3\right.\right.}{}}{}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 4120

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{-\frac{\int -(a+b\tan(e+fx))^m\left(-\left(\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)(m+2)\left(m^2+7m\right.\right.\right.}{}}{}}{}$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 25

3.165.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)} - \int(a+b\tan(e+fx))^m\left(-\left(\left(\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)(m+2)\left(m^2+7m+12\right.\right.\right.\right.$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 3042

$$\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)} - \int(a+b\tan(e+fx))^m\left(-\left(\left(\left((A-C)d\left(3c^2-d^2\right)+B\left(c^3-3cd^2\right)\right)(m+2)\left(m^2+7m+12\right.\right.\right.\right.$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 4113

$$\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)} - \int(a+b\tan(e+fx))^m\left(-\left(\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)(m+2)(m+3\right.\right.$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 3042

$$\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)} - \int(a+b\tan(e+fx))^m\left(-\left(\left(Ac^3-Cc^3-3Bdc^2-3Ad^2c+3Cd^2c+Bd^3\right)(m+2)(m+3\right.\right.$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}$$

$\downarrow$ 4022

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}+$$

$$\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)} - -\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(\right.}{bf(m+2)}$$

$\downarrow$ 3042

3.165.

$$\int(a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}+$$

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{-\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(\right.}{bf(m+2)}}{}}{}$$

$\downarrow$ 4020

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}+$$

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{-\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(\right.}{bf(m+2)}}{}}{}$$

$\downarrow$ 25

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}+$$

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{-\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(\right.}{bf(m+2)}}{}}{}$$

$\downarrow$ 78

$$\frac{C(c+d\tan(e+fx))^3(a+b\tan(e+fx))^{m+1}}{bf(m+4)}+$$

$$\frac{\frac{(c+d\tan(e+fx))^2(-3aCd+bBd(m+4)+3bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+3)}-\frac{-\frac{d\tan(e+fx)(a+b\tan(e+fx))^{m+1}\left(2(bc-ad)(-3aCd+bBd(m+4)+3bcC)+b^2d(\right.}{bf(m+2)}}{}}{}$$

input
```
Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^3*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

3.165.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

output
```
(C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^3)/(b*f*(4 + m)) + ((
(3*b*c*C - 3*a*C*d + b*B*d*(4 + m))*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Ta
n[e + f*x])^2)/(b*f*(3 + m)) - (-((d*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 +
 m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 + m)))*Tan[e + f*x]*(a +
 b*Tan[e + f*x])^(1 + m))/(b*f*(2 + m))) + (-(((b*c*(2 + m)*(b^2*d*(B*c +
(A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C*d + b*B*d*(4 +
 m))) + d*(b^3*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m)*(4 + m) - a
*(b^2*d*(B*c + (A - C)*d)*(3 + m)*(4 + m) + 2*(b*c - a*d)*(3*b*c*C - 3*a*C
*d + b*B*d*(4 + m)))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m))) + ((I/
2)*b^3*(A - I*B - C)*(c - I*d)^3*(2 + m)*(3 + m)*(4 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1
+ m))/((a - I*b)*f*(1 + m)) - ((I/2)*b^3*(A + I*B - C)*(c + I*d)^3*(2 + m)
*(3 + m)*(4 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(
a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)))/(b*(2 + m))
)/(b*(3 + m)))/(b*(4 + m))
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

3.165.
$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.165.

$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

3.165.4 Maple [F]

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e))^3 \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right) dx$$

input
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

output
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

3.165.5 Fricas [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \int (C\tan(fx+e)^2 + B\tan(fx+e) + A)(d\tan(fx+e)+c)^3(b\tan(fx+e)+a)^m dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
integral((C*d^3*tan(f*x + e)^5 + (3*C*c*d^2 + B*d^3)*tan(f*x + e)^4 + A*c^
3 + (3*C*c^2*d + 3*B*c*d^2 + A*d^3)*tan(f*x + e)^3 + (C*c^3 + 3*B*c^2*d +
3*A*c*d^2)*tan(f*x + e)^2 + (B*c^3 + 3*A*c^2*d)*tan(f*x + e))*(b*tan(f*x +
 e) + a)^m, x)
```

3.165.6 Sympy [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \int (a+b\tan(e+fx))^m (c+d\tan(e+fx))^3 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**3*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**3*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

3.165.7 Maxima [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
Timed out
```

3.165.8 Giac [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)^3\left(b\tan\left(fx+e\right)+a\right)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^3*(
b*tan(f*x + e) + a)^m, x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^3\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))^3\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

input
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^3*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2), x)
```

3.166 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2 \left(A+B\tan(e\right.$

3.166.1 Optimal result

Integrand size = 45, antiderivative size = 363

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2 \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= \frac{(2a^2Cd^2-abd(2cC+Bd)(3+m)+b^2(2+m)\left(2c^2C+2Bcd(3+m)+(A-C)d^2(3+m)\right))\,(a+b\tan(}{b^3f(1+m)(2+m)(3+m)}$$
$$+\frac{(A-iB-C)(c-id)^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}$$
$$+\frac{(iA-B-iC)(c+id)^2\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)f(1+m)}$$
$$-\frac{d(2aCd-b(2cC+Bd(3+m)))\tan(e+fx)(a+b\tan(e+fx))^{1+m}}{b^2f(2+m)(3+m)}$$
$$+\frac{C(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^2}{bf(3+m)}$$

output
```
(2*a^2*C*d^2-a*b*d*(B*d+2*C*c)*(3+m)+b^2*(2+m)*(2*c^2*C+2*B*c*d*(3+m)+(A-C
)*d^2*(3+m)))*(a+b*tan(f*x+e))^(1+m)/b^3/f/(1+m)/(2+m)/(3+m)+1/2*(A-I*B-C)
*(c-I*d)^2*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x
+e))^(1+m)/(I*a+b)/f/(1+m)+1/2*(I*A-B-I*C)*(c+I*d)^2*hypergeom([1, 1+m],[2
+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/f/(1+m)-d*(2*
C*a*d-b*(2*C*c+B*d*(3+m)))*tan(f*x+e)*(a+b*tan(f*x+e))^(1+m)/b^2/f/(2+m)/(
3+m)+C*(a+b*tan(f*x+e))^(1+m)*(c+d*tan(f*x+e))^2/b/f/(3+m)
```

3.166.2 Mathematica [A] (verified)

Time = 6.37 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.39

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{C(a+b\tan(e+fx))^{1+m}(c+d\tan(e+fx))^2}{bf(3+m)}$$
$$+\frac{\frac{d(2bcC-2aCd+bBd(3+m))\tan(e+fx)(a+b\tan(e+fx))^{1+m}}{bf(2+m)}-\frac{\left(-bc(2+m)(2bcC-2aCd+bBd(3+m))-d\left(b^2(Bc+(A-C)d)(2+m)(3+m)-a(2bcC-\right.\right.}{bf(1+m)}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x
] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m)) + ((
d*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1
 + m))/(b*f*(2 + m)) - (((-(b*c*(2 + m)*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m)
)) - d*(b^2*(B*c + (A - C)*d)*(2 + m)*(3 + m) - a*(2*b*c*C - 2*a*C*d + b*B
*d*(3 + m))))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/2)*(-(b^2*
(A*c^2 - c^2*C - 2*B*c*d - A*d^2 + C*d^2)*(2 + m)*(3 + m)) - I*b^2*(2*c*(A
 - C)*d + B*(c^2 - d^2))*(2 + m)*(3 + m))*Hypergeometric2F1[1, 1 + m, 2 +
m, ((-I)*a - I*b*Tan[e + f*x])/((-I)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))
/((a + I*b)*f*(1 + m)) - ((I/2)*(-(b^2*(A*c^2 - c^2*C - 2*B*c*d - A*d^2 +
C*d^2)*(2 + m)*(3 + m)) + I*b^2*(2*c*(A - C)*d + B*(c^2 - d^2))*(2 + m)*(3
 + m))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a + I*b*Tan[e + f*x])/((-I)
*a - b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)))/(b*(2 + m))
)/(b*(3 + m))
```

3.166.3 Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4130, 3042, 4120, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))^2(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

3.166. $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

$$\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))^2(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\downarrow \text{4130}$$

$$\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left((2bcC-2adC+bBd(m+3))\tan^2(e+fx)+b(Bc+(A-C)d)(m+\right.}{b(m+3)}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\downarrow \text{3042}$$

$$\frac{\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left((2bcC-2adC+bBd(m+3))\tan(e+fx)^2+b(Bc+(A-C)d)(m+\right.}{b(m+3)}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\downarrow \text{4120}$$

$$\frac{\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{\int (a+b\tan(e+fx))^m\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)(m+2)(m+3)\tan(e+fx)b^2\right)\right.}{}}{}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{\int (a+b\tan(e+fx))^m\left(-\left(\left(2c(A-C)d+B\left(c^2-d^2\right)\right)(m+2)(m+3)\tan(e+fx)b^2\right)\right.}{}}{}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\downarrow \text{4113}$$

$$\frac{\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{\int (a+b\tan(e+fx))^m\left(-\left(\left(Ac^2-Cc^2-2Bdc-Ad^2+Cd^2\right)(m+2)(m+3)b^2\right)-(2c(\right.}{}}{b(m}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\downarrow \text{3042}$$

3.166.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - \int(a+b\tan(e+fx))^m\left(-\left(\left(Ac^2-Cc^2-2Bdc-Ad^2+Cd^2\right)(m+2)(m+3)b^2\right)-(2c\right.$$

$$b(m$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)}$$

$$\Big\downarrow \text{ 4022}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} +$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - -\frac{1}{2}b^2(m+2)(m+3)(c+id)^2(A+iB-C)\int(1-i\tan(e+fx))(a+b\tan(e+fx))^m d$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} +$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - -\frac{1}{2}b^2(m+2)(m+3)(c+id)^2(A+iB-C)\int(1-i\tan(e+fx))(a+b\tan(e+fx))^m d$$

$$\Big\downarrow \text{ 4020}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} +$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - -\frac{ib^2(m+2)(m+3)(c-id)^2(A-iB-C)\int -\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f} + ib^2(m+$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} +$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - \frac{ib^2(m+2)(m+3)(c-id)^2(A-iB-C)\int \frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f} - ib^2(m+2)(m$$

$$b($$

$$\Big\downarrow \text{ 78}$$

$$\frac{C(c+d\tan(e+fx))^2(a+b\tan(e+fx))^{m+1}}{bf(m+3)} +$$

$$\frac{d\tan(e+fx)(-2aCd+bBd(m+3)+2bcC)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} - -\frac{(a+b\tan(e+fx))^{m+1}\left(2a^2Cd^2-abd(m+3)(Bd+2cC)+b^2(m+2)\left(d^2(m+3)(A-C)+2B\right.\right.}{bf(m+1)}$$

3.166.

$$\int(a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^2*(A + B*Tan[e + f*x] + C*
Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(1 + m)*(c + d*Tan[e + f*x])^2)/(b*f*(3 + m)) + ((
d*(2*b*c*C - 2*a*C*d + b*B*d*(3 + m))*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1
 + m))/(b*f*(2 + m)) - (-(((2*a^2*C*d^2 - a*b*d*(2*c*C + B*d)*(3 + m) + b^
2*(2 + m)*(2*c^2*C + 2*B*c*d*(3 + m) + (A - C)*d^2*(3 + m)))*(a + b*Tan[e
+ f*x])^(1 + m))/(b*f*(1 + m))) + ((I/2)*b^2*(A - I*B - C)*(c - I*d)^2*(2
+ m)*(3 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a -
I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)) - ((I/2)*b^2*(A
+ I*B - C)*(c + I*d)^2*(2 + m)*(3 + m)*Hypergeometric2F1[1, 1 + m, 2 + m,
(a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f
*(1 + m)))/(b*(2 + m)))/(b*(3 + m))
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

3.166.4 Maple [F]

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e))^2 \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right) dx$$

input
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

output
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),
x)
```

3.166.5 Fricas [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (C\tan(fx+e)^2+B\tan(fx+e)+A)(d\tan(fx+e)+c)^2(b\tan(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="fricas")
```

output
```
integral((C*d^2*tan(f*x + e)^4 + (2*C*c*d + B*d^2)*tan(f*x + e)^3 + A*c^2
+ (C*c^2 + 2*B*c*d + A*d^2)*tan(f*x + e)^2 + (B*c^2 + 2*A*c*d)*tan(f*x + e
))*(b*tan(f*x + e) + a)^m, x)
```

3.166.6 Sympy [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan(e+fx))^m\,(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))**2*(A+B*tan(f*x+e)+C*tan(f*
x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))**2*(A + B*tan(e + f*
x) + C*tan(e + f*x)**2), x)
```

3.166.7 Maxima [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (C\tan(fx+e)^2+B\tan(fx+e)+A)(d\tan(fx+e)+c)^2(b\tan(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^2*(
b*tan(f*x + e) + a)^m, x)
```

3.166.8 Giac [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (C\tan(fx+e)^2+B\tan(fx+e)+A)(d\tan(fx+e)+c)^2(b\tan(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+
e)^2),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)^2*(
b*tan(f*x + e) + a)^m, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))^2\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int (a+b\tan(e+fx))^m\,(c+d\tan(e+fx))^2\,\left(C\tan(e+fx)^2+B\tan(e+fx)+A\right)\,dx$$

input
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))^2*(A + B*tan(e + f*x) + C*
tan(e + f*x)^2), x)
```

3.167 $\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\,(A+B\tan(e$ -

3.167.1 Optimal result

Integrand size = 43, antiderivative size = 247

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$= -\frac{(aCd-b(cC+Bd)(2+m))(a+b\tan(e+fx))^{1+m}}{b^2f(1+m)(2+m)} + \frac{(A-iB-C)(c-id)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)} - \frac{(A+iB-C)(c+id)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia-b)f(1+m)} + \frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{1+m}}{bf(2+m)}$$

output
```
-(C*a*d-b*(B*d+C*c)*(2+m))*(a+b*tan(f*x+e))^(1+m)/b^2/f/(1+m)/(2+m)+1/2*(A
-I*B-C)*(c-I*d)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*ta
n(f*x+e))^(1+m)/(I*a+b)/f/(1+m)-1/2*(A+I*B-C)*(c+I*d)*hypergeom([1, 1+m],[
2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/f/(1+m)+C*d*
tan(f*x+e)*(a+b*tan(f*x+e))^(1+m)/b/f/(2+m)
```

3.167.2 Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.82

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\frac{(a+b\tan(e+fx))^{1+m}\left(\frac{-2aCd+2b(cC+Bd)(2+m)}{b(1+m)}-\frac{ib(A-iB-C)(c-id)(2+m)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)}{(a-ib)(1+m)}\right.}{2bf(2+m)}$$

input
```
Integrate[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x]
+ C*Tan[e + f*x]^2),x]
```

output
```
((a + b*Tan[e + f*x])^(1 + m)*((-2*a*C*d + 2*b*(c*C + B*d)*(2 + m))/(b*(1
+ m)) - (I*b*(A - I*B - C)*(c - I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2
 + m, (a + b*Tan[e + f*x])/(a - I*b)])/((a - I*b)*(1 + m)) + (I*b*(A + I*B
 - C)*(c + I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e +
f*x])/(a + I*b)])/((a + I*b)*(1 + m)) + 2*C*d*Tan[e + f*x]))/(2*b*f*(2 + m
))
```

3.167.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4120, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+d\tan(e+fx))(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\tan(e+fx))(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)dx$$

$$\Big\downarrow \text{4120}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-$$
$$\frac{\int (a+b\tan(e+fx))^m\left((aCd-b(cC+Bd)(m+2))\tan^2(e+fx)-b(Bc+(A-C)d)(m+2)\tan(e+fx)+aC\right.}{b(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{\int (a+b\tan(e+fx))^m \left((aCd-b(cC+Bd)(m+2))\tan(e+fx)^2-b(Bc+(A-C)d)(m+2)\tan(e+fx)+aC\right.}{b(m+2)}$$

$$\Big\downarrow \text{4113}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{\int (a+b\tan(e+fx))^m(-b(Ac-Cc-Bd)(m+2)-b(Bc+(A-C)d)\tan(e+fx)(m+2))dx+\frac{(aCd-b(m+2)(Bd}{}}{b(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{\int (a+b\tan(e+fx))^m(-b(Ac-Cc-Bd)(m+2)-b(Bc+(A-C)d)\tan(e+fx)(m+2))dx+\frac{(aCd-b(m+2)(Bd}{}}{b(m+2)}$$

$$\Big\downarrow \text{4022}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{-\frac{1}{2}b(m+2)(c+id)(A+iB-C)\int(1-i\tan(e+fx))(a+b\tan(e+fx))^m dx-\frac{1}{2}b(m+2)(c-id)(A-iB-C)}{b(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{-\frac{1}{2}b(m+2)(c+id)(A+iB-C)\int(1-i\tan(e+fx))(a+b\tan(e+fx))^m dx-\frac{1}{2}b(m+2)(c-id)(A-iB-C)}{b(m+2)}$$

$$\Big\downarrow \text{4020}$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)} -$$
$$\frac{-\frac{ib(m+2)(c-id)(A-iB-C)\int -\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}+\frac{ib(m+2)(c+id)(A+iB-C)\int -\frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}+\frac{(aCd}{}}{b(m+2)}$$

$$\Big\downarrow \text{25}$$

3.167.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{\frac{ib(m+2)(c-id)(A-iB-C)\int\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}-\frac{ib(m+2)(c+id)(A+iB-C)\int\frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}+\frac{(aCd-b(m}{}}{b(m+2)}$$

$$\downarrow 78$$

$$\frac{Cd\tan(e+fx)(a+b\tan(e+fx))^{m+1}}{bf(m+2)}-\frac{\frac{ib(m+2)(c-id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(a-ib)}-\frac{ib(m+2)(c+id)(A+iB-C)(a+b\tan(e+f}{}}{b(m+2)}$$

input
```
Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])*(A + B*Tan[e + f*x] + C*Ta
n[e + f*x]^2),x]
```

output
```
(C*d*Tan[e + f*x]*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(2 + m)) - (((a*C*d -
 b*(c*C + B*d)*(2 + m))*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) + ((I/
2)*b*(A - I*B - C)*(c - I*d)*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (a
 + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*f*(
1 + m)) - ((I/2)*b*(A + I*B - C)*(c + I*d)*(2 + m)*Hypergeometric2F1[1, 1
+ m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^(1 + m))/
((a + I*b)*f*(1 + m)))/(b*(2 + m))
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.167.
$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

3.167.4 Maple [F]

$$\int (a+b\tan(fx+e))^m (c+d\tan(fx+e)) \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right) dx$$

input
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

3.167.5 Fricas [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)\left(b\tan\left(fx+e\right)+a\right)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="fricas")
```

output
```
integral((C*d*tan(f*x + e)^3 + (C*c + B*d)*tan(f*x + e)^2 + A*c + (B*c + A
*d)*tan(f*x + e))*(b*tan(f*x + e) + a)^m, x)
```

3.167.6 Sympy [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(a+b\tan\left(e+fx\right)\right)^m\left(c+d\tan\left(e+fx\right)\right)\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e
)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x)
+ C*tan(e + f*x)**2), x)
```

3.167.7 Maxima [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx))\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx$$
$$=\int \left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(d\tan\left(fx+e\right)+c\right)\left(b\tan\left(fx+e\right)+a\right)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)*(b*
tan(f*x + e) + a)^m, x)
```

3.167.8 Giac [F]

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \int (C\tan{(fx+e)}^2 + B\tan{(fx+e)} + A)(d\tan{(fx+e)} + c)(b\tan{(fx+e)} + a)^m dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)
^2),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(d*tan(f*x + e) + c)*(b*
tan(f*x + e) + a)^m, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m(c+d\tan(e+fx)) \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$
$$= \int (a+b\tan(e+f\,x))^m\,(c+d\tan(e+f\,x))\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)\,dx$$

input
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^m*(c + d*tan(e + f*x))*(A + B*tan(e + f*x) + C*ta
n(e + f*x)^2), x)
```

3.168 $\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e$

3.168.1 Optimal result

Integrand size = 33, antiderivative size = 178

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{C(a+b\tan(e+fx))^{1+m}}{bf(1+m)}+\frac{(A-iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)f(1+m)}+\frac{(iA-B-iC)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)f(1+m)}$$

output
```
C*(a+b*tan(f*x+e))^(1+m)/b/f/(1+m)+1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],
(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a+b)/f/(1+m)+1/2*(I*A-
B-I*C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))
^(1+m)/(a+I*b)/f/(1+m)
```

3.168.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.76

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)\,dx=\frac{\left(\frac{2C}{b}-\frac{i(A-iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{i(A+iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right)}{2f(1+m)}$$

input
```
Integrate[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x
]
```

output
```
(((2*C)/b - (I*(A - I*B - C)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan
[e + f*x])/(a - I*b)])/(a - I*b) + (I*(A + I*B - C)*Hypergeometric2F1[1, 1
 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[e + f*
x])^(1 + m))/(2*f*(1 + m))
```

3.168.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right) dx$$

$$\Big\downarrow \text{ 4113}$$

$$\int (a+b\tan(e+fx))^m(A-C+B\tan(e+fx))dx+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\tan(e+fx))^m(A-C+B\tan(e+fx))dx+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$$\Big\downarrow \text{ 4022}$$

$$\frac{1}{2}(A+iB-C)\int (1-i\tan(e+fx))(a+b\tan(e+fx))^m dx+\frac{1}{2}(A-iB-C)\int (i\tan(e+fx)+1)(a+b\tan(e+fx))^m dx+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(A+iB-C)\int(1-i\tan(e+fx))(a+b\tan(e+fx))^m dx+\frac{1}{2}(A-iB-C)\int(i\tan(e+fx)+1)(a+b\tan(e+fx))^m dx+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$\Big\downarrow$ 4020

$$\frac{i(A-iB-C)\int-\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}-\frac{i(A+iB-C)\int-\frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$\Big\downarrow$ 25

$$-\frac{i(A-iB-C)\int\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}+\frac{i(A+iB-C)\int\frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

$\Big\downarrow$ 78

$$-\frac{i(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a-ib}\right)}{2f(m+1)(a-ib)}+\frac{i(A+iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}+\frac{C(a+b\tan(e+fx))^{m+1}}{bf(m+1)}$$

input
```
Int[(a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2),x]
```

output
```
(C*(a + b*Tan[e + f*x])^(1 + m))/(b*f*(1 + m)) - ((I/2)*(A - I*B - C)*Hype
rgeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[
e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)) + ((I/2)*(A + I*B - C)*Hypergeome
tric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*
x])^(1 + m))/((a + I*b)*f*(1 + m))
```

3.168. $\int(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)dx$

3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.168.4 Maple [F]

$$\int (a+b\tan (fx+e))^m \left(A+B\tan (fx+e)+C\tan (fx+e)^2\right) dx$$

input
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

output
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x)
```

3.168.5 Fricas [F]

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \int \left(C\tan (fx+e)^2+B\tan (fx+e)+A\right)(b\tan (fx+e)+a)^m dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"fricas")
```

output
```
integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m, x
)
```

3.168.6 Sympy [F]

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \int (a+b\tan (e+fx))^m \left(A+B\tan (e+fx)+C\tan^2 (e+fx)\right) dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2),x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2),
 x)
```

3.168.7 Maxima [F]

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m,
x)
```

3.168.8 Giac [F]

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \int (C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
"giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m,
x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan^2(e+fx)\right) dx = \int (a+b\tan(e+f\,x))^m \left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right) dx$$

input
```
int((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2),x)
```

output
```
int((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2), x)
```

3.169 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$

3.169.1 Optimal result

Integrand size = 45, antiderivative size = 258

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx = -\frac{(iA+B-iC)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a-ib)(c-id)f(1+m)} - \frac{(A+iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia-b)(c+id)f(1+m)} + \frac{(c^2C-Bcd+Ad^2)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\,(c^2+d^2)\,f(1+m)}$$

output
```
-1/2*(I*A+B-I*C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*t
an(f*x+e))^(1+m)/(a-I*b)/(c-I*d)/f/(1+m)-1/2*(A+I*B-C)*hypergeom([1, 1+m],
[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(I*a-b)/(c+I*d)/f/(
1+m)+(A*d^2-B*c*d+C*c^2)*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*
d+b*c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(1+m)
```

3.169.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.79

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$= \frac{\left(\frac{(A-iB-C)(-ic+d)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)}{a-ib}+\frac{(A+iB-C)(ic+d)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)}{a+ib}\right.}{2\left(c^2+d^2\right)f(1+m)}$$

input
```
Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x]),x]
```

output
```
((((A - I*B - C)*((-I)*c + d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Ta
n[e + f*x])/(a - I*b)])/(a - I*b) + ((A + I*B - C)*(I*c + d)*Hypergeometri
c2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)])/(a + I*b) + (2*(c^2
*C - B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*Tan[e + f
*x]))/(-(b*c) + a*d)])/(b*c - a*d))*(a + b*Tan[e + f*x])^(1 + m))/(2*(c^2
+ d^2)*f*(1 + m))
```

3.169.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 4136, 3042, 4022, 3042, 4020, 25, 78, 4117, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{c+d\tan(e+fx)}dx$$
$$\Big\downarrow \text{4136}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan^2(e+fx)+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\int (a+b\tan(e+fx))^m(Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx))dx}{c^2+d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\tan(e+fx))^m(Ac-Cc+Bd+(Bc-(A-C)d)\tan(e+fx))dx}{c^2+d^2}+\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}$$

$$\Big\downarrow \text{4022}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\frac{1}{2}(c-id)(A+iB-C)\int (1-i\tan(e+fx))(a+b\tan(e+fx))^mdx+\frac{1}{2}(c+id)(A-iB-C)\int (i\tan(e+fx)+1}{c^2+d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\frac{1}{2}(c-id)(A+iB-C)\int (1-i\tan(e+fx))(a+b\tan(e+fx))^mdx+\frac{1}{2}(c+id)(A-iB-C)\int (i\tan(e+fx)+1}{c^2+d^2}$$

$$\Big\downarrow \text{4020}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\frac{i(c+id)(A-iB-C)\int -\frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}-\frac{i(c-id)(A+iB-C)\int -\frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}}{c^2+d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\frac{i(c-id)(A+iB-C)\int \frac{(a+b\tan(e+fx))^m}{i\tan(e+fx)+1}d(-i\tan(e+fx))}{2f}-\frac{i(c+id)(A-iB-C)\int \frac{(a+b\tan(e+fx))^m}{1-i\tan(e+fx)}d(i\tan(e+fx))}{2f}}{c^2+d^2}$$

$$\Big\downarrow \text{78}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2+1\right)}{c+d\tan(e+fx)}dx}{c^2+d^2}+\frac{\frac{i(c-id)(A+iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}-\frac{i(c+id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hyperge}}{2f(m+1)(a}}{c^2+d^2}$$

$$\Big\downarrow \text{4117}$$

3.169. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}dx$

$$\frac{\left(Ad^2-Bcd+c^2C\right)\int \frac{(a+b\tan(e+fx))^m}{c+d\tan(e+fx)}d\tan(e+fx)}{f\left(c^2+d^2\right)}+\frac{\frac{i(c-id)(A+iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}-\frac{i(c+id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hyperge}}{2f(m+1)(a}}{c^2+d^2}$$

$\downarrow$ 78

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{d(a+b\tan(e+fx))}{bc-ad}\right)}{f(m+1)\left(c^2+d^2\right)(bc-ad)}+\frac{\frac{i(c-id)(A+iB-C)(a+b\tan(e+fx))^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{a+b\tan(e+fx)}{a+ib}\right)}{2f(m+1)(a+ib)}-\frac{i(c+id)(A-iB-C)(a+b\tan(e+fx))^{m+1}\text{Hyperge}}{2f(m+1)(a}}{c^2+d^2}$$

input
```
Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x]),x]
```

output
```
((c^2*C - B*c*d + A*d^2)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*Ta
n[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2
 + d^2)*f*(1 + m)) + (((-1/2*I)*(A - I*B - C)*(c + I*d)*Hypergeometric2F1[
1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan[e + f*x])^(1 +
 m))/((a - I*b)*f*(1 + m)) + ((I/2)*(A + I*B - C)*(c - I*d)*Hypergeometric
2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a + I*b)]*(a + b*Tan[e + f*x])^
(1 + m))/((a + I*b)*f*(1 + m)))/(c^2 + d^2)
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.169. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{c+d\tan(e+fx)}\,dx$

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.169.4 Maple [F]

$$\int \frac{(a+b\tan(fx+e))^m \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{c+d\tan(fx+e)}\,dx$$

input
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

output
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e)),x)
```

3.169.5 Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$=\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{d\tan\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="fricas")
```

output
```
integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
*tan(f*x + e) + c), x)
```

3.169.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$=\int \frac{\left(a+b\tan\left(e+fx\right)\right)^m\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)}{c+d\tan\left(e+fx\right)}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e)),x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x)), x)
```

3.169.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx$$
$$=\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{d\tan\left(fx+e\right)+c}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(
d*tan(f*x + e) + c), x)
```

3.169.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx = \int \frac{(C\tan(fx+e)^2+B\tan(fx+e)+A)(b\tan(fx+e)+a)^m}{d\tan(fx+e)+c}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e)),x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(
d*tan(f*x + e) + c), x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{c+d\tan(e+fx)}\,dx = \int \frac{(a+b\tan(e+f\,x))^m\,(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A)}{c+d\tan(e+f\,x)}\,dx$$

input
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x)),x)
```

output
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x)), x)
```

3.170 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.170.1 Optimal result

Integrand size = 45, antiderivative size = 403

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$= \frac{(A-iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^2f(1+m)}$$
$$+\frac{(iA-B-iC)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)(c+id)^2f(1+m)}$$
$$-\frac{(ad^2(2c(A-C)d-B(c^2-d^2))-b(Ad^2(c^2(2-m)-d^2m)-Bcd(c^2(1-m)-d^2(1+m))-c^2C(c^2n}{(bc-ad)^2\left(c^2+d^2\right.}$$
$$+\frac{\left(c^2C-Bcd+Ad^2\right)(a+b\tan(e+fx))^{1+m}}{(bc-ad)\left(c^2+d^2\right)f(c+d\tan(e+fx))}$$

output
```
1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(
f*x+e))^(1+m)/(I*a+b)/(c-I*d)^2/f/(1+m)+1/2*(I*A-B-I*C)*hypergeom([1, 1+m]
,[2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/(c+I*d)^2/
f/(1+m)-(a*d^2*(2*c*(A-C)*d-B*(c^2-d^2))-b*(A*d^2*(c^2*(2-m)-d^2*m)-B*c*d*
(c^2*(1-m)-d^2*(1+m))-c^2*C*(c^2*m+d^2*(2+m))))*hypergeom([1, 1+m],[2+m],-
d*(a+b*tan(f*x+e))/(-a*d+b*c))*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^2/(c^2+d^
2)^2/f/(1+m)+(A*d^2-B*c*d+C*c^2)*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^
2)/f/(c+d*tan(f*x+e))
```

3.170.2 Mathematica [A] (verified)

Time = 6.24 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$
$$= -\frac{(Ad^2-c(-cC+Bd))\,(a+b\tan(e+fx))^{1+m}}{(-bc+ad)\,(c^2+d^2)\,f(c+d\tan(e+fx))}$$
$$-\frac{-\frac{(-cd(bc-ad)(Bc-(A-C)d)-bc^2(c^2C-Bcd+Ad^2)m+d^2((cC-Bd)(ad-bc(1+m))-A(acd-b(c^2-d^2m))))\ \text{Hypergeometric2F1}\left(1,1+m,2\right.}{(-bc+ad)(c^2+d^2)f(1+m)}}{}$$

input
```
Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^2,x]
```

output
```
-(((A*d^2 - c*(-(c*C) + B*d))*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) + a*d
)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (-(((-(c*d*(b*c - a*d)*(B*c - (A
- C)*d)) - b*c^2*(c^2*C - B*c*d + A*d^2)*m + d^2*((c*C - B*d)*(a*d - b*c*(
1 + m)) - A*(a*c*d - b*(c^2 - d^2*m))))*Hypergeometric2F1[1, 1 + m, 2 + m,
 (d*(a + b*Tan[e + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 + m))/((
-(b*c) + a*d)*(c^2 + d^2)*f*(1 + m))) + (((I/2)*(-((b*c - a*d)*(c^2*C - 2*
B*c*d - C*d^2 - A*(c^2 - d^2))) - I*(b*c - a*d)*(2*c*(A - C)*d - B*(c^2 -
d^2)))*Hypergeometric2F1[1, 1 + m, 2 + m, ((-I)*a - I*b*Tan[e + f*x])/((-I
)*a + b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)) - ((I/2)*(-(
(b*c - a*d)*(c^2*C - 2*B*c*d - C*d^2 - A*(c^2 - d^2))) + I*(b*c - a*d)*(2*
c*(A - C)*d - B*(c^2 - d^2)))*Hypergeometric2F1[1, 1 + m, 2 + m, -((I*a +
I*b*Tan[e + f*x])/((-I)*a - b))]*(a + b*Tan[e + f*x])^(1 + m))/((a - I*b)*
f*(1 + m)))/(c^2 + d^2))/((-(b*c) + a*d)*(c^2 + d^2))
```

3.170.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4132, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 78, 4117, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(e+fx))^m \left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^2}dx$$

$$\Big\downarrow \ 4132$$

$$\frac{\int \frac{(a+b\tan(e+fx))^m\left(-b\left(Cc^2-Bdc+Ad^2\right)m\tan^2(e+fx)+(bc-ad)(Bc-(A-C)d)\tan(e+fx)+(cC-Bd)(ad-bc(m+1))-A\left(acd-b\left(c^2-d^2m\right)\right)\right)}{c+d\tan(e+fx)}d}{\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{(a+b\tan(e+fx))^m\left(-b\left(Cc^2-Bdc+Ad^2\right)m\tan(e+fx)^2+(bc-ad)(Bc-(A-C)d)\tan(e+fx)+(cC-Bd)(ad-bc(m+1))-A\left(acd-b\left(c^2-d^2m\right)\right)\right)}{c+d\tan(e+fx)}d}{\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \ 4136$$

$$\frac{\frac{\int -(a+b\tan(e+fx))^m\left((bc-ad)\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+(bc-ad)\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right.}{}}{\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{\int(a+b\tan(e+fx))^m\left((bc-ad)\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+(bc-ad)\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right.}{}}{\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{\int(a+b\tan(e+fx))^m\left((bc-ad)\left(Cc^2-2Bdc-Cd^2-A\left(c^2-d^2\right)\right)+(bc-ad)\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right.}{}}{\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}$$

$$\Big\downarrow \ 4022$$

3.170. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}dx$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2\right.}{c+d\tan(e+fx)}}{c^2+d^2}}{\left(c^2+\right.}$$

$\downarrow$ 3042

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2\right.}{c+d\tan(e+fx)}}{c^2+d^2}}{\left(c^2+\right.}$$

$\downarrow$ 4020

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2\right.}{c+d\tan(e+fx)}}{c^2+d^2}}{\left(c^2+d^2\right)(bc-}$$

$\downarrow$ 25

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2\right.}{c+d\tan(e+fx)}}{c^2+d^2}}{\left(c^2+d^2\right)(bc-}$$

$\downarrow$ 78

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m\left(\tan(e+fx)^2\right.}{c+d\tan(e+fx)}}{c^2+d^2}}{}$$

$\downarrow$ 4117

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$
$$-\frac{\frac{\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\int\frac{(a+b\tan(e+fx))^m}{c+d\tan(e+fx)}d\tan(e+fx}{f\left(c^2+d^2\right)}}{}$$

$\downarrow$ 78

3.170. $\int\frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))}+$$

$$-\frac{\frac{(a+b\tan(e+fx))^{m+1}\left(ad^2\left(2cd(A-C)-B\left(c^2-d^2\right)\right)-b\left(Ac^2d^2(2-m)-Ad^4m-B\left(c^3d(1-m)-cd^3(m+1)\right)+c^4(-C)m-c^2Cd^2(m+2)\right)\right)\text{Hyperge}}{f(m+1)(c^2+d^2)(bc-ad)}}{}$$

input
```
Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^2,x]
```

output
```
((c^2*C - B*c*d + A*d^2)*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(c^2 +
 d^2)*f*(c + d*Tan[e + f*x])) + (-(((a*d^2*(2*c*(A - C)*d - B*(c^2 - d^2))
 - b*(A*c^2*d^2*(2 - m) - c^4*C*m - A*d^4*m - c^2*C*d^2*(2 + m) - B*(c^3*d
*(1 - m) - c*d^3*(1 + m))))*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b
*Tan[e + f*x]))/(b*c - a*d))]*(a + b*Tan[e + f*x])^(1 + m))/((b*c - a*d)*(
c^2 + d^2)*f*(1 + m))) - (((I/2)*(A - I*B - C)*(c + I*d)^2*(b*c - a*d)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a - I*b)]*(a + b*Tan
[e + f*x])^(1 + m))/((a - I*b)*f*(1 + m)) - ((I/2)*(A + I*B - C)*(c - I*d)
^2*(b*c - a*d)*Hypergeometric2F1[1, 1 + m, 2 + m, (a + b*Tan[e + f*x])/(a
+ I*b)]*(a + b*Tan[e + f*x])^(1 + m))/((a + I*b)*f*(1 + m)))/(c^2 + d^2))/
((b*c - a*d)*(c^2 + d^2))
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.170. $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2} dx$

3.170.4 Maple [F]

$$\int \frac{(a+b\tan(fx+e))^m \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(c+d\tan(fx+e))^2}\,dx$$

input
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,
x)
```

output
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^2,
x)
```

3.170.5 Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^m (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \int \frac{\left(C\tan(fx+e)^2+B\tan(fx+e)+A\right)(b\tan(fx+e)+a)^m}{(d\tan(fx+e)+c)^2}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="fricas")
```

output
```
integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
^2*tan(f*x + e)^2 + 2*c*d*tan(f*x + e) + c^2), x)
```

3.170.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(e+fx))^m (A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.170.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{\left(d\tan\left(fx+e\right)+c\right)^2}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="maxima")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(
d*tan(f*x + e) + c)^2, x)
```

3.170.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{\left(d\tan\left(fx+e\right)+c\right)^2}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^2,x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(
d*tan(f*x + e) + c)^2, x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^2}\,dx = \int \frac{(a+b\tan(e+f\,x))^m\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\left(c+d\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^2,x)
```

output
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^2, x)
```

3.170. $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^2}\,dx$

3.171 $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

3.171.1 Optimal result

Integrand size = 45, antiderivative size = 702

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$$

$$= \frac{(A-iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a-ib}\right)(a+b\tan(e+fx))^{1+m}}{2(ia+b)(c-id)^3f(1+m)}$$

$$+\frac{(A+iB-C)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{a+b\tan(e+fx)}{a+ib}\right)(a+b\tan(e+fx))^{1+m}}{2(a+ib)(ic-d)^3f(1+m)}$$

$$+\frac{(2a^2d^3((A-C)d(3c^2-d^2)-B(c^3-3cd^2))-2abd^2(B(6c^2d^2-c^4(2-m)-d^4m)+2c(A-C)d(c^2(3}{}$$

$$+\frac{(c^2C-Bcd+Ad^2)(a+b\tan(e+fx))^{1+m}}{2(bc-ad)(c^2+d^2)f(c+d\tan(e+fx))^2}$$

$$-\frac{(2ad^2(2c(A-C)d-B(c^2-d^2))-b(c^4C(1-m)+Ad^4(1-m)-Bc^3d(3-m)+Bcd^3(1+m)+c^2d}{2(bc-ad)^2(c^2+d^2)^2f(c+d\tan(e+fx))}$$

output
```
1/2*(A-I*B-C)*hypergeom([1, 1+m],[2+m],(a+b*tan(f*x+e))/(a-I*b))*(a+b*tan(
f*x+e))^(1+m)/(I*a+b)/(c-I*d)^3/f/(1+m)+1/2*(A+I*B-C)*hypergeom([1, 1+m],[
2+m],(a+b*tan(f*x+e))/(a+I*b))*(a+b*tan(f*x+e))^(1+m)/(a+I*b)/(I*c-d)^3/f/
(1+m)+1/2*(2*a^2*d^3*((A-C)*d*(3*c^2-d^2)-B*(c^3-3*c*d^2))-2*a*b*d^2*(B*(6
*c^2*d^2-c^4*(2-m)-d^4*m)+2*c*(A-C)*d*(c^2*(3-m)-d^2*(1+m)))-b^2*(A*d^2*(d
^4*(1-m)*m+2*c^2*d^2*(-m^2+3*m+1)-c^4*(m^2-5*m+6))+B*c*d*(d^4*m*(1+m)-2*c^
2*d^2*(-m^2+m+3)+c^4*(m^2-3*m+2))+c^2*C*(c^4*(1-m)*m+2*c^2*d^2*(-m^2-m+3)-
d^4*(m^2+3*m+2))))*hypergeom([1, 1+m],[2+m],-d*(a+b*tan(f*x+e))/(-a*d+b*c)
)*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)^3/(c^2+d^2)^3/f/(1+m)+1/2*(A*d^2-B*c*d
+C*c^2)*(a+b*tan(f*x+e))^(1+m)/(-a*d+b*c)/(c^2+d^2)/f/(c+d*tan(f*x+e))^2-1
/2*(2*a*d^2*(2*c*(A-C)*d-B*(c^2-d^2))-b*(c^4*C*(1-m)+A*d^4*(1-m)-B*c^3*d*(
3-m)+B*c*d^3*(1+m)+c^2*d^2*(A*(5-m)-C*(3+m))))*(a+b*tan(f*x+e))^(1+m)/(-a*
d+b*c)^2/(c^2+d^2)^2/f/(c+d*tan(f*x+e))
```

3.171.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2238 vs. 2(702) = 1404.

Time = 6.33 (sec) , antiderivative size = 2238, normalized size of antiderivative = 3.19

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$$
= Result too large to show

input
```
Integrate[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))
/(c + d*Tan[e + f*x])^3,x]
```

output
```
-1/2*((A*d^2 - c*(-(c*C) + B*d))*(a + b*Tan[e + f*x])^(1 + m))/((-(b*c) +
a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x])^2) - (-(((-(c*(2*d*(b*c - a*d)*(B*
c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c
- a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))))*(a + b*Tan[e
 + f*x])^(1 + m))/((-(b*c) + a*d)*(c^2 + d^2)*f*(c + d*Tan[e + f*x]))) - (
-(((-(c*d*(-(b*c) + a*d)*(-2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C
- B*c*d + A*d^2)*(1 - m) + d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C -
 B*d)*(2*a*d - b*c*(1 + m))))) - b*c^2*m*(-(c*(2*d*(b*c - a*d)*(B*c - (A -
 C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))) + d^2*(A*(2*c*(b*c - a*d) +
 b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))) + d^2*((2*d*(b*c - a
*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(1 - m))*(-(a*d) + b*c
*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(b*c - a*d) + b*d^2*(1
 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m)))))*Hypergeometric2F1[1, 1 + m,
2 + m, (d*(a + b*Tan[e + f*x]))/(-(b*c) + a*d)]*(a + b*Tan[e + f*x])^(1 +
m))/((-(b*c) + a*d)*(c^2 + d^2)*f*(1 + m))) + (((I/2)*(d*(-(b*c) + a*d)*(-
2*c*(b*c - a*d)*(B*c - (A - C)*d) - b*d*(c^2*C - B*c*d + A*d^2)*(1 - m) +
d*(A*(2*c*(b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))
)) + c*((2*d*(b*c - a*d)*(B*c - (A - C)*d) - b*c*(c^2*C - B*c*d + A*d^2)*(
1 - m))*(-(a*d) + b*c*(1 + m)) + (-(c*(-(b*c) + a*d)) - b*d^2*m)*(A*(2*c*(
b*c - a*d) + b*d^2*(1 - m)) + (c*C - B*d)*(2*a*d - b*c*(1 + m))) + b*m*...
```

3.171.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan(e+fx)^2\right)}{(c+d\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4132}$$

$$\frac{\int \frac{(a+b\tan(e+fx))^m\left(b\left(Cc^2-Bdc+Ad^2\right)(1-m)\tan^2(e+fx)+2(bc-ad)(Bc-(A-C)d)\tan(e+fx)+A\left(b(1-m)d^2+2c(bc-ad)\right)+(cC-Bd)(2ad-b}{(c+d\tan(e+fx))^2}}{2\left(c^2+d^2\right)(bc-ad)}$$
$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a+b\tan(e+fx))^m\left(b\left(Cc^2-Bdc+Ad^2\right)(1-m)\tan(e+fx)^2+2(bc-ad)(Bc-(A-C)d)\tan(e+fx)+A\left(b(1-m)d^2+2c(bc-ad)\right)+(cC-Bd)(2ad-b\right.}{(c+d\tan(e+fx))^2}}{2\left(c^2+d^2\right)(bc-ad)}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 4132

$$\int -\frac{(a+b\tan(e+fx))^m\left(2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-bm\left(2ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)-b\left(C(1-m)c^4-Bd(3-m)c^3+d^2(A(5-m)-C(m+3))c\right.\right.\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\int \frac{(a+b\tan(e+fx))^m\left(2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-bm\left(2ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)-b\left(C(1-m)c^4-Bd(3-m)c^3+d^2(A(5-m)-C(m+3))\right.\right.\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 3042

$$-\int \frac{(a+b\tan(e+fx))^m\left(2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)\tan(e+fx)(bc-ad)^2-bm\left(2ad^2\left(2c(A-C)d-B\left(c^2-d^2\right)\right)-b\left(C(1-m)c^4-Bd(3-m)c^3+d^2(A(5-m)-C(m+3))\right.\right.\right.}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 4136

$$-\frac{\frac{\int -2(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\right.\right.}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 27

3.171. $\displaystyle\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

$$-\frac{-\frac{(2a^2d^3(d(A-C)(3c^2-d^2)-B(c^3-3cd^2))-2abd^2(2cd(A-C)(c^2(3-m)-d^2(m+1))+B(-(c^4(2-m))+6c^2d^2-d^4m))-b^2(Ad^2(-(c^4(m^2-5m+6))+2c^2d \cdots}{\cdots}}{\cdots}$$

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m((bc-ad)^2(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3)-(bc-ad)^2((A-C)d(3c^2-d^2)-B(c^3-3cd^2))\tan(e+fx))dx}{c^2+d^2}-\frac{(2a^2d^3(d(A-C) \cdots}{\cdots}}{\cdots}$$

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{(2a^2d^3(d(A-C)(3c^2-d^2)-B(c^3-3cd^2))-2abd^2(2cd(A-C)(c^2(3-m)-d^2(m+1))+B(-(c^4(2-m))+6c^2d^2-d^4m))-b^2(Ad^2(-(c^4(m^2-5m+6))+2c^2d \cdots}{\cdots}}{\cdots}$$

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m((bc-ad)^2(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3)-(bc-ad)^2((A-C)d(3c^2-d^2)-B(c^3-3cd^2))\tan(e+fx))dx}{c^2+d^2}-\frac{(2a^2d^3(d(A-C) \cdots}{\cdots}}{\cdots}$$

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{(2a^2d^3(d(A-C)(3c^2-d^2)-B(c^3-3cd^2))-2abd^2(2cd(A-C)(c^2(3-m)-d^2(m+1))+B(-(c^4(2-m))+6c^2d^2-d^4m))-b^2(Ad^2(-(c^4(m^2-5m+6))+2c^2d \cdots}{\cdots}}{\cdots}$$

$$\frac{(Ad^2-Bcd+c^2C)(a+b\tan(e+fx))^{m+1}}{2f(c^2+d^2)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

3.171. $\int \frac{(a+b\tan(e+fx))^m(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

3.171. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

3.171. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$$\Big\downarrow \ 25$$

3.171. $\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx$

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\right.\right.}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2\int -(a+b\tan(e+fx))^m\left((bc-ad)^2\left(Ac^3-Cc^3+3Bdc^2-3Ad^2c+3Cd^2c-Bd^3\right)-(bc-ad)^2\left((A-C)d\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)\tan(e+fx)\right)dx}{c^2+d^2}-\frac{\left(2a^2d^3\left(d(A-C)\right.\right.}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\left(2a^2d^3\left(d(A-C)\left(3c^2-d^2\right)-B\left(c^3-3cd^2\right)\right)-2abd^2\left(2cd(A-C)\left(c^2(3-m)-d^2(m+1)\right)+B\left(-\left(c^4(2-m)\right)+6c^2d^2-d^4m\right)\right)-b^2\left(Ad^2\left(-\left(c^4\left(m^2-5m+6\right)\right)+2c^2d\right.\right.}{}}{}$$

$$\frac{\left(Ad^2-Bcd+c^2C\right)(a+b\tan(e+fx))^{m+1}}{2f\left(c^2+d^2\right)(bc-ad)(c+d\tan(e+fx))^2}$$

input
```
Int[((a + b*Tan[e + f*x])^m*(A + B*Tan[e + f*x] + C*Tan[e + f*x]^2))/(c +
d*Tan[e + f*x])^3,x]
```

output
```
$Aborted
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

3.171.4 Maple [F]

$$\int \frac{(a+b\tan(fx+e))^m \left(A+B\tan(fx+e)+C\tan(fx+e)^2\right)}{(c+d\tan(fx+e))^3}dx$$

input
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,
x)
```

output
```
int((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+e))^3,
x)
```

3.171.5 Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx = \int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{\left(d\tan\left(fx+e\right)+c\right)^3}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="fricas")
```

output
```
integral((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(d
^3*tan(f*x + e)^3 + 3*c*d^2*tan(f*x + e)^2 + 3*c^2*d*tan(f*x + e) + c^3),
x)
```

3.171.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx = \int \frac{\left(a+b\tan\left(e+fx\right)\right)^m\left(A+B\tan\left(e+fx\right)+C\tan^2\left(e+fx\right)\right)}{\left(c+d\tan\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**m*(A+B*tan(f*x+e)+C*tan(f*x+e)**2)/(c+d*tan(f*
x+e))**3,x)
```

output
```
Integral((a + b*tan(e + f*x))**m*(A + B*tan(e + f*x) + C*tan(e + f*x)**2)/
(c + d*tan(e + f*x))**3, x)
```

3.171.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^m\left(A+B\tan(e+fx)+C\tan^2(e+fx)\right)}{(c+d\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.171.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$=\int \frac{\left(C\tan\left(fx+e\right)^2+B\tan\left(fx+e\right)+A\right)\left(b\tan\left(fx+e\right)+a\right)^m}{\left(d\tan\left(fx+e\right)+c\right)^3}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^m*(A+B*tan(f*x+e)+C*tan(f*x+e)^2)/(c+d*tan(f*x+
e))^3,x, algorithm="giac")
```

output
```
integrate((C*tan(f*x + e)^2 + B*tan(f*x + e) + A)*(b*tan(f*x + e) + a)^m/(
d*tan(f*x + e) + c)^3, x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^m\,(A+B\tan(e+fx)+C\tan^2(e+fx))}{(c+d\tan(e+fx))^3}\,dx$$
$$=\int \frac{(a+b\tan(e+f\,x))^m\,\left(C\tan(e+f\,x)^2+B\tan(e+f\,x)+A\right)}{\left(c+d\tan\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^3,x)
```

output
```
int(((a + b*tan(e + f*x))^m*(A + B*tan(e + f*x) + C*tan(e + f*x)^2))/(c +
d*tan(e + f*x))^3, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
        else:
            return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```